Problem Set #2

Economics 201b

1. Recall, from page 22 of "Lecture Notes for Economics," that *packaging* is equivalent to a two-part tariff when consumers are homogeneous. Using the notation from §3.2 of that reading,[1] this problem asks you to explore the design of optimal packaging with *heterogeneous* consumers. In what follows, assume (i) a constant marginal cost of c; (ii) that $x(p, \theta) = \theta - p$; and (iii) that $\Psi(\cdot)$ is the uniform distribution on $[\theta_0, \theta_1]$, where $\theta_1 \geq 2\theta_0$ and $\theta_1 > \theta_0 \geq 0$.

   (a) Suppose the firm in question sells a package with $\hat{x}$ units, $\hat{x} < 2(\theta_1 - c)$, in it for $\hat{s}$ dollars (note $\hat{s}$ is the price for the *entire* package and is *not* a per-unit price). What is the profit-maximizing value of $\hat{s}$? [*Hint:* derive the demand curve for packages of size $\hat{x}$.]

   (b) Using your answer from (a), write the firm's expected profits as a function of $\hat{x}$. What size package maximizes expected profits?

   (c) Suppose that instead of packaging, the firm engaged in linear pricing. How would its expected profits compare to its expected profits in (b)?

   (d) Prove that, with heterogeneous consumers and constant marginal cost, packaging yields strictly less profits than the optimal two-part tariff.

2. Redo the welfare analysis of third-degree price discrimination in §4.1 of "Lecture Notes for Economics" (pages 28–29), except now assume that the cost of $x_1 + x_2$ is $C(x_1 + x_2)$, where $C(\cdot)$ is a strictly increasing, differentiable, and strictly convex function. In particular, establish that a necessary condition for welfare to increase with third-degree price discrimination is that *total* output increase and that a sufficient condition for welfare to increase is that *each* type consumes more under 3rd-degree price discrimination.

3. A theater has K seats and faces two types of customers who can be identified by the theater (*e.g.,* the types are students and non-students). The two types have inverse demand $p_1(x) = a_1 - bx$ and $p_2(x) = a_2 - bx$, respectively, where, note, the slopes are the same. Assume $a_2 > a_1$. For convenience, assume a constant marginal cost of 0. Prove that, if

$$\frac{a_2 - a_1}{2b} < K < \frac{a_2 + a_1}{2b},$$

[1] The symbol §stands for "section."

then the seat constraint is binding, but the theater still engages in third-degree price discrimination. If $K = 100$, $a_2 = 200$, $a_1 = 80$, and $b = 1$, what is the welfare loss from 3rd-degree price discrimination?

4. Consider second-degree price discrimination via quantity discounts (see, *e.g.,* §5.2 of "Lecture Notes for Economics"). Assume the firm has constant marginal cost 1. Assume there are two types $\theta \in \{1, 2\}$ and each type has a valuation function $v(x, \theta) = 5\theta x - \frac{1}{2}x^2$. Assume the two types occur with equal probability in the population. What is the optimal second-degree price discrimination scheme?