Problem Set #3
ECONOMICS 220A

This problem set is due at the beginning of class on November 9, 2005.

1. Consider an upstream monopolist with constant marginal cost, c, who sells to two downstream retailers. The retailers are located at the opposite ends of town. The consumers are uniformly distributed across the town from one end to another. Normalize the length of the town to one. A consumer's utility if he or she buys the good is \( u = b - td - p \), where \( b > c \) is a constant common to all consumers, \( t > 0 \) is a constant common to all consumers, \( d \) is the distance traveled by the consumer, and \( p \) is the price paid. Assume the downstream retailers' marginal cost is the wholesale price, \( w \), they pay the manufacturer (the upstream firm). In your analysis, assume the parameters are such that the market is spanned in equilibrium.

   (a) In equilibrium, what is the wholesale price set by the upstream firm? What is the upstream firm's profit?
   (b) How does the upstream firm's profit vary with \( t \)? Briefly interpret.

2. Consider an upstream monopolist with constant marginal cost, c, who sells to two downstream retailers. Assume the retailers are Cournot competitors. Let end-user inverse demand be \( P = a + s_1 + s_2 - b(x_1 + x_2) \), where \( a > c \) and \( b \) are fixed parameters, \( s_i \) is the sales effort of downstream firm \( i \) and \( x_i \) is the number of units that downstream firm \( i \) orders from the upstream monopolist. Assume the timing is (i) upstream monopolist sets wholesale price, \( w \); (ii) downstream firms decide how much sales effort to make at a cost of \( s_i^2 / 2 \); once made this is a sunk cost; (iii) downstream firms decide how many units to order from the upstream firm; and (iv) the units ordered are sold to the end users at the price \( P \) determined by the expression above. At each stage, the firms know what happened at previous stages, but the downstream firms play simultaneously within a stage. Assume the downstream retailers’ marginal cost is the wholesale price, \( w \), they pay the manufacturer (the upstream firm).

   (a) Solve for the equilibrium values of \( w, s_1, s_2, x_1, \) and \( x_2 \).
   (b) Suppose that the upstream firm buys one of the downstream firms and vertically forecloses the other. What are the equilibrium values of \( x \) and \( s \)?
(c) Suppose that the upstream firm is made to divest itself of its downstream division, so that there is now one upstream firm and one downstream firm. The downstream firm is, now, totally independent of the upstream firm. What are the equilibrium values of $w$, $x$, and $s$?

(d) From a social-welfare perspective, rank the three outcomes above.

(e) From the perspective of the upstream firm’s profit, rank the three outcomes above.