Problem Set #6*

Economics 201B

1. A social planner faces N citizens (agents). Each agent has a quasi-linear utility over the amount of a public good, $a \in \mathbb{R}_+$, and a transferable good (*e.g.,* money). Specifically, each agent n's utility is $v_n(a|\tau_n) + y$, where y_n is the amount of the transferable good and $\tau_n \in \mathcal{T}_n$ is the agent's type. Assume the social planner's objective is to choose the a that maximizes $\sum_{n=1}^N v_n(a|\tau_n)$ (*i.e.,* the program given in expression (13.13) of "Lecture Notes for Economics"). For each of the following, construct a *dominant-strategy* mechanism to implement that a (*i.e.,* $a^*(\cdot)$). For each mechanism indicate whether it will be balanced budget or not.

   (a) $v_n(a|\tau_n) = \lambda_n \times \left(\tau_n a - a^2/2\right)$, where λ_n is a *known* constant, $N > 2$, and $\mathcal{T}_n = [1, 2]$.

   (b) $v_n(a|\tau_n) = \tau_n \ln(a) - a$, $N \geq 2$, and $\mathcal{T}_n = [1, 2]$.

   (c) $v_n(a|\tau_n) = \tau_n - a$ if $a \geq 1$ and $v_n(a|\tau_n) = -a$ if $a < 1$, $\mathcal{T}_n = [0, 2]$, and $N \geq 2$.

2. Repeat the above, except design the optimal, balanced-budget Bayesian mechanism. You should assume that the types are distributed independently.

3. Consider the following social-planning situation. A benevolent social planner must allocate a single item to one of I agents (indexed by i) in the society. Each agent has a valuation, $\nu \in [0, \bar{\nu}]$, for the item. Each agent has utility $\delta\nu + y$, where y is the numeraire good ("money") and $\delta \in \{0, 1\}$ is an indicator variable denoting whether the agent has been allocated the item ($\delta = 1$) or not allocated it ($\delta = 0$). Let the ν_i be distributed independently and identically according to the common differentiable distribution function F. Let Δ_i be the vector that has a 1 in the ith place and a 0 in all other places. Let $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_I)$. The benevolent social planner wishes to maximize, over i, $\Delta_i \cdot \boldsymbol{\nu}$; that is, allocate the item to the agent who values it most. Observe that the probability that two agents share the highest value is zero and, hence, ties can be ignored. The social planner's objective is to find a mechanism, transfers $\left(p_1(\boldsymbol{\nu}), \ldots, p_I(\boldsymbol{\nu})\right)$, such that $\delta_i = 1$ if and only if $\nu_i > \nu_j$ for all $j \neq i$ (note, an agent's utility is $\delta_i\nu_i - p_i$). Limit attention to dominant strategy mechanisms.

   (a) Use a revealed preference argument to show that if $\delta_i(\nu_i, \boldsymbol{\nu}_{-i}) = \delta_i(\nu_i', \boldsymbol{\nu}_{-i})$ for $\nu_i \neq \nu_i'$, then $p_i(\nu_i, \boldsymbol{\nu}_{-i}) = p_i(\nu_i', \boldsymbol{\nu}_{-i})$.

*Recall there is no Problem Set #5.

(b) Show that incentive compatibility requires that $p_i(\nu_i, \boldsymbol{\nu}_{-i})$ have a common value for all ν_i such that $\delta_i(\nu_i, \boldsymbol{\nu}_{-i}) = 1$; similarly, show that $p_i(\nu_i, \boldsymbol{\nu}_{-i})$ must have a common value for all ν_i such that $\delta_i(\nu_i, \boldsymbol{\nu}_{-i}) = 0$. Call these two common values $P_1(\boldsymbol{\nu}_{-i})$ and $P_0(\boldsymbol{\nu}_{-i})$.

(c) Define ν^{**} to be the *second* greatest value of ν among the agents. Show that if the mechanism successfully implements the desired allocation rule (*i.e.,* the one that allocates the good to the agent with the greatest value of ν, then $P_1(\boldsymbol{\nu}_{-i}) - P_0(\boldsymbol{\nu}_{-i}) = \nu^{**}$ for all i.

(d) Find a balanced dominant-strategy mechanism that implements the social planner's objective.