Answers to Problem Set #1

Economics 201b

1. **Footnote 3:** An affine inverse demand is $P(x) = a - bx$, where a and b are positive constants. Recall that $MR(x) = P(x) + xP'(x) = a - bx + x \cdot (-b) = a - 2bx$. Clearly the slope is twice as great as that of inverse demand.

   Footnote 4: Setting $MR = MC$ yields

   $$a - 2bx = c$$

   (recall the previous question/footnote). Solving, $x_M^* = \frac{(a-c)}{2b}$. Hence

   $$P(x_M^*) = a - b \times \frac{a-c}{2b} = \frac{a+c}{2}\,.$$

   Footnote 5: Profit is $x_M^* P(x_M^*) - cx_M^*$. Using the last problem/footnote, profit is

   $$\frac{a-c}{2b} \times \left(\frac{a+c}{2} - c\right) = \frac{1}{b} \times \left(\frac{a-c}{2}\right)^2\,.$$

   Footnote 6: Consumer surplus is the integral under the inverse demand from 0 to x_M^* units. This integral is

   $$\begin{aligned} \int_0^{x_M^*} \left(a - bt - \frac{a+c}{2}\right) dt &= \left.\frac{a-c}{2}t - \frac{b}{2}t^2\right|_0^{x_M^*} \\ &= \frac{a-c}{2} \times \frac{a-c}{2b} - \frac{b}{2}\left(\frac{a-c}{2b}\right)^2 \\ &= \frac{(a-c)^2}{8b}\,. \end{aligned}$$

   Question at end of example: Deadweight loss is the integral of the area between inverse demand and marginal cost from the monopoly output to the welfare-maximizing output. The welfare-maximizing output, x_W^*, equates inverse demand and marginal cost:

   $$a - bx = c\,,$$

so $x_W^* = (a-c)/b$. Therefore,

$$\begin{aligned}
\text{DWL} &= \int_{x_M^*}^{x_W^*} (a - bt - c)dt \\
&= (a-c)t - \frac{b}{2}t^2 \Big|_{x_M^*}^{x_W^*} \\
&= (a-c)\left(\frac{a-c}{b} - \frac{a-c}{2b}\right) - \frac{1}{2}\left(\frac{(a-c)^2}{b} - \frac{(a-c)^2}{4b}\right) \\
&= \frac{(a-c)^2}{2b} - (a-c)^2\frac{3}{8b} \\
&= \frac{(a-c)^2}{8b}\,.
\end{aligned}$$

2. The elasticity of demand is

$$\frac{dX}{dp} \times \frac{p}{X} = -2p^{-3}\frac{p}{p^-2} = -2\,.$$

Recalling the Lerner markup rule,

$$\frac{p - MC}{p} = -\frac{1}{\varepsilon} = \frac{1}{2}\,;$$

hence $p = 2MC = 2$.

If $X(p) = p^{-1/2}$, then $\varepsilon = -1/2$. From the Lerner markup rule, this would imply that $p - MC = 2p$, which has no solution. Because demand is everywhere *in*elastic, the firm always increases profit by raising price; there is, thus, no profit-maximizing price.

3. Note that $s = c - r$. Note, too, that expected surplus is

$$\mathcal{S} = \int_{c-r}^{\infty} \left(\int_r^{\infty} (b_F + b_Y - c)g_F(b_F)db_F\right) g_Y(b_Y)db_Y\,.$$

(a) Observe, here, $G_F(b) = G_Y(b) = b$ and $g_F(b) = g_Y(b) = 1$. So surplus is

$$\mathcal{S} = \int_{c-r}^{1} \left(\int_r^{1} (b_F + b_Y - c)db_F\right) db_Y\,.$$

So

$$\begin{aligned}
\frac{d\mathcal{S}}{dr} &= -\int_{c-r}^{1} (r + b_Y - c)\,db_Y + \int_r^1 (b_F + (c-r) - c)db_F \\
&= c - \frac{c^2}{2} - 2r + cr \\
&= -(c-2)\frac{c}{2} + (c-2)r\,.
\end{aligned}$$

Hence, setting $d\mathcal{S}/dr = 0$ yields $r = c/2$. Observe that, because $c < 2$, the second-order condition is met.

(b) Because $c < f+Y$, we know that trade (communications) are surplus enhancing at least some of the time. Hence, it can't be surplus maximizing to set $r > f$—this would eliminate all communications. Regulators require $r+s = c$, so $r \leq c$. This establishes that $r \leq \min\{c, f\}$. Recall $s = c - r$, so surplus is

$$\mathcal{S} = \int_{c-r}^{\infty} (b + f - c) g_Y(b) db \, .$$

Observe $\mathcal{S}$ is increasing in r, so we want r to be as large as possible; that is, $r = \min\{c, f\}$.

(c) Ideally, we should have communication when $b_Y + b_F \geq c$ and no communication when $b_Y + b_F < c$. In part (b), we achieve this optimum. This is not surprising because in part (b) there is only one problem—getting you to call when it's appropriate—and we have one instrument, r. But in part (a) we have two problems—getting you to call when it's appropriate and getting your friend to "answer" when appropriate—yet we still have just one instrument, r (remember s is fixed as $c-r$). Not surprisingly the optimum in (a) doesn't match the ideal. This is a further illustration of the two-instruments principle.

4. The firm's profit is

$$x_d P(x_d) + p_0 x_o - c \cdot (x_d + x_o)^2 \, ,$$

where d denotes domestic market and o denotes overseas market. First-order conditions are

$$\begin{aligned} P(x_d) + x_d P'(x_d) &= 2(x_d + x_o)c \equiv MC \text{ and} \\ p_0 &= 2(x_d + x_o)c \equiv MC \, . \end{aligned}$$

The second equation shows that we can think of p_0 as the cost of the marginal unit sold domestically because it could be sold overseas for p_0; that is, if the firm suddenly was given an additional unit, the opportunity *cost* of selling it domestically would be p_0. Note, however, that we can't think of p_0 as being the marginal cost for *all* domestic units.

5. First, we need to determine aggregate demand. Inverting individual inverse demand to get individual demand yields $x(p) = 20 - 2p$. Aggregate demand is, thus, 100 times individual demand or $2000 - 200p$. The optimal two-part tariff sets the unit price to equate inverse aggregate demand and marginal cost and the unit fee as one-hundredth of the aggregate consumer surplus. Aggregate inverse demand is $P(x) = 10 - x/200$. Equating to marginal cost, yields $10 - x/200 = x/200$ or $x^* = 1000$. Unit price is, thus, 5. Aggregate consumer surplus is $\int_0^{1000} (10 - t/200 - 5) dt = 2500$. So the entry fee is 25.