Answers to Problem Set #2

Economics 201B

1. Observe that $\Delta W = \Delta v_1 + \Delta v_2 - \Delta C$, where, with the exception of ΔC, the notation is the same as §5.1 of "A Primer on Pricing." Define $\Delta C = C(x_1^* + x_2^*) - C(x_1^U + x_2^U)$. Because $C(\cdot)$ is strictly convex, it lies everywhere above any line tangent to it; that is,

$$C(x_1^* + x_2^*) > C(x_1^U + x_2^U) + C'(x_1^U + x_2^U)(\Delta x_1 + \Delta x_2) \text{ and}$$
$$C(x_1^U + x_2^U) > C(x_1^* + x_2^*) - C'(x_1^* + x_2^*)(\Delta x_1 + \Delta x_2)\,.$$

Hence,

$$-C'(x_1^* + x_2^*)(\Delta x_1 + \Delta x_2) < -\Delta C < -C'(x_1^U + x_2^U)(\Delta x_1 + \Delta x_2)\,. \quad (1)$$

Define $MC^U = C'(x_1^U + x_2^U)$ and $MC^* = C'(x_1^* + x_2^*)$. Recalling expression (36) of "A Primer on Pricing," we can conclude that

$$(p^U - MC^U)(\Delta x_1 + \Delta x_2) > \Delta W > (p_1^* - MC^*)\Delta x_1 + (p_2^* - MC^*)\Delta x_2\,. \quad (2)$$

Observe that expression (2) has the same interpretation as expression (37) of "A Primer on Pricing."

2. Recall that for linear inverse demand of the form $p = A - Bx$, with constant marginal cost, c, the profit-maximizing quantity under linear pricing is $\frac{A-c}{2B}$ (see, *e.g.,* footnote 6 of "A Primer on Pricing"). Hence, under an *un*constrained third-degree price discrimination scheme (*i.e.,* linear pricing to the different populations or segments), total sales are $\frac{a_2+a_1}{2b}$ (recall, here, $c = 0$). Given the condition

$$\frac{a_2 - a_1}{2b} < K < \frac{a_2 + a_1}{2b}\,, \quad (3)$$

this proves the seat constraint is binding. Given the constraint is binding, let x_2 be the seats sold to population 2 and $K - x_2$ be, therefore, the seats sold to population 1. The optimal x_2 is an interior solution (*i.e.,* between 0 and K) if equating the marginal revenues has an interior solution:

$$a_1 - 2b(K - x_2) = a_2 - 2bx_2\,.$$

Obviously $x_2 > 0$ because $a_2 > a_1$. So the question is whether $x_2 < K$. Solving the last equation for x_2:

$$x_2 = \frac{1}{2}K + \frac{1}{2}\left(\frac{a_2 - a_1}{2b}\right)\,, \quad (4)$$

which is less than K from the first inequality of (3).

If $K = 100$, $a_2 = 200$, $a_1 = 80$, and $b = 1$, then (4) yields $x_2 = 80$; that is, 20 seats are sold to population-1 consumers. Note that $p_1(0) = 80 < p_2(100) = 100$; this means that it is inefficient to sell *any* seats to population-1 consumers. The welfare loss from doing so is, thus, the forgone benefit of selling the last 20 seats to population-2 consumers less the benefit of selling them to population-1 consumers:

$$\begin{aligned}\int_{80}^{100} p_2(t)dt - \int_0^{20} p_1(t)dt &= \int_0^{20} \left(\left[200-(t+80)\right] - \left[80-t\right]\right) dt \\ &= \int_0^{20} 40dt \\ &= 800\,.\end{aligned}$$

3. Observe that $\partial v(x_2, 2)/\partial x = 10 - x$. Equate to marginal cost (recall, no distortion at the top), so $x_2^* = 9$. Now consider, x_1. $I(x) = v(x, 2) - v(x, 1) = 5x$. The left-hand side of expression (60) from "A Primer on Pricing" is, thus,

$$5 - x_1 - 5 - 1 = -x_1 - 1\,.$$

This cannot equal zero for a positive x_1; hence, we can conclude that population-1 is excluded from buying. In other words, the seller sells only to population 2. The solution is, therefore, $x_1^* = T_1 = 0$ and $x_2^* = 9$ and $T_2 = v(9, 2) = 49.5$.