Exam

Economics 201B — First Half

INSTRUCTIONS. Answer all *four* (4) questions. Each question counts equally. Please remember to write your name *clearly* on all material you hand in. Neither the GSI nor I are trained in reading cuneiform, linear B, or hieroglyphics, so please write neatly.

QUESTIONS

1. Consider a monopoly seller of a given beverage. Despite its monopoly status, however, the firm is still relatively small. Consequently, it is not economically feasible for it to use custom-sized bottles. Instead, it is limited to buying standard-sized bottles. Assume there are just two kinds of standard-sized bottles, one-liter bottles and the two-liter bottles. Assume that, when buying bottles from the bottle maker, a one-liter bottle and two-liter bottle cost the beverage maker the same. Assume the marginal cost of the beverage is $c \geq 0$ on a per-liter basis.

   (a) Suppose that all consumers have identical preferences given by the utility function $u(x, y) = ax - \frac{1}{2}bx^2 + y$, where x is *consumption* of the beverage (measured in liters), y is consumption of the numeraire good, and a and b are positive constants satisfying $\frac{3}{2}b > a > b$. What is the profit-maximizing price for the beverage maker to charge for a two-liter bottle?

   (b) Same assumptions as part (a). Would this firm do better to sell one-liter bottles or two-liter bottles?

2. Recall from *Star Trek* that Vulcans and Romulans, although from different planets, are physically indistinguishable. Both engage in space travel and assume both their spaceships require dilithium crystals. Specifically, Vulcan ships require one crystal, while Romulan ships require two. Consider Diane, the owner of Diane's Dilithium Emporium, who has a monopoly on dilithium crystals in her part of the galaxy. Although she cannot tell Romulans from Vulcans, she does know that $\phi \times 100\%$ of her customers are Vulcans (the rest are Romulans). She also knows their utilities are quasi-linear of the form $u_\theta(x) + y$, where x is number of crystals purchased, y is the numeraire good, and θ is type (*i.e.,* Vulcan or Romulan). She further knows

$$u_V(x) = \begin{cases} 0\,, & x < 1 \\ v\,, & x \geq 1 \end{cases} \quad \text{and } u_R(x) = \begin{cases} 0\,, & x < 2 \\ 2r\,, & x \geq 2 \end{cases}\,,$$

   where V denotes Vulcan, R denotes Romulan, and v and r are positive constants. For convenience, imagine the marginal cost of a dilithium crystal is zero. Finally, assume $2r > v > r$.

(a) Suppose Diane engages in simply monopoly pricing. What unit price would she set as a function of r, v, and ϕ?

(b) Suppose Diane can sell crystals individually and in two-packs (*i.e.,* two crystals in one box). What price should she optimally charge for a single crystal? For a two-pack? How does her expected profit per customer compare to part (a)? What is this an example of?

3. Consider a society with I citizens and a benevolent social planner. The I citizens have preferences $v_i(x, \theta_i) + y_i$, where i indexes the citizen, x is the level of some public good, θ_i is citizen i's type, and y_i is citizen's i's allocation of a transferable good (money). Although the social planner is benevolent, she likes some citizens more than others. Hence, she seeks to maximize the following function

$$\sum_{i=1}^{I} \lambda_i v_i(x, \theta_i),$$

where the λ_i vary across citizens but are all positive. Assume that there exists a finite $x^*(\boldsymbol{\theta})$ that maximizes that sum for all $\boldsymbol{\theta} \in \Theta \equiv \Theta_1 \times \cdots \times \Theta_I$, where Θ_i is the possible type space for citizen i. Assume that the citizens' types are realized independently (*i.e.,* knowledge of his own type does not affect a citizen's ability to predict the types of the other citizens). Find a Bayesian mechanism that implements $x^*(\cdot)$ and is balanced budget.

4. Consider the following hidden-action principal-agent model. A principal makes a take-it-or-leave-it offer to an agent. The agent's utility function is $w - d(a)$, where $d(\cdot)$ is a non-negative, strictly increasing, and strictly convex function, w is the agent's pay, and $a \geq 0$ is the agent's action. The probability of the good state occurring conditional on the agent's action is qa, $0 < q < 1$. The probability of the other state (the bad state) is, thus, $1 - qa$. Let R_s denote the principal's revenue in state s, $s \in \{b, g\}$, denoting bad state and good state respectively. Assume $R_g = G > 0 = R_b$. The principal's net payoff is $R_s - w_s$ in state s. Assume $G - d(\frac{1}{q}) < 0$, but that $qaG - d(a) > 0$ for some $a \in (0, \frac{1}{q})$. Normalize the agent's reservation utility to zero.

(a) Suppose the principal offers the agent a sharing contract (*i.e.,* the agent's compensation is ϕR_s, $\phi \in (0, 1)$). What sharing contract maximizes the principal's expected payoff?

(b) A sharing contract is not the optimal contract form. What is the optimal contract? How optimal is it relative to the first-best solution?

(c) Relate your answers to parts (a) and (b) to a monopoly pricing problem. Please write *no* more than a paragraph.

(d) What *non*-standard assumption of this hidden-action principal-agent model allowed you to find such an optimal contract in part (b)? Why would the standard assumption prevent obtaining such an optimal result? [Note: A verbal answer is not only sufficient, but desired.]