Examination for Economics 201B (1st half) (with Answers)

Instructions

- Do ***not*** open this exam until told to do so by an authority figure. Do, however, take this time (i) to count that your copy of the exam has 9 pages; and (ii) to read all the instructions on this cover sheet.
- ***Right now print your name legibly on your blue books.***
- You may *not* use any of the following: a computer, a PDA, notes, or books. A calculator is permissible.
- This is an *individual* exercise. You are *not* to communicate with anyone other than the individuals administering the exam during the exam.
- As a courtesy to your fellow students, please make sure that all cell phones, pagers, and wrist-watch alarms are *turned off.*
- Please answer three (3) of the following six (6) questions. Each question counts for 60 points. Each sub-part of a question is worth an equal fraction of the 60 points (*i.e.,* if a question has n parts, each part is worth $60/n$).
- You must answer at least one question from Part A (*i.e.,* at least one of questions 1–3). You must answer at least one question from Part B (*i.e.,* at least one of questions 4–6).
- Do *not* answer more than three questions; if you do, your grade will be based on your three worst answers.
- Note you are *not* to answer three questions from one part only. If you do, your grade will be based on your two worst answers.
- Please make sure it is clear which questions you are answering and which answer goes with which question.
- Where you are asked to come up with either a numerical or algebraic answer, please put a box around your answer $\boxed{\text{like this.}}$
- If you use graphs or other illustrations in your answer, please make sure to label all components clearly *including, when present, the axes.*
- Unless instructed otherwise, you are expected to show your work or justify your answers. Do, however, try to be brief.
- Please write as legibly as possible. If your answer cannot be read, it will receive a mark of zero.

Part A

Answer at least one question from this part. Answer no more than two questions from this part.

Question 1: Consider a profit-maximizing monopolist. For convenience, assume its marginal cost (MC) is zero. It faces market demand, $D(p)$, that satisfies the equation $B \times D(p) = A - p$, where p is price and A and B are positive constants.

(a) Under simple monopoly pricing, what is the profit-maximizing quantity for the firm to sell?

Answer: Inverse demand is $P(x) = A - Bx$. Hence, $MR(x) = A - 2Bx$. Equating MR and MC yields $\boxed{x^* = \frac{A}{2B}}$.

(b) Does the quantity you found in part (a) maximize social welfare? How do you know?

Answer: $\boxed{\text{No, it doesn't.}}$ The social welfare-maximizing quantity solves $P(x) = MC$; hence $x^{**} = A/B > x^*$.

(c) Suppose, now, that the monopolist knows it faces N consumers all of whom have the same preferences. The preference of any one of these consumers is captured by the utility function

$$u(x, y) = \int_0^x (a - bt)dt + y \,,$$

where x is consumption of the good sold by the monopolist, y is income, and a and b are positive constants. If these preferences are consistent with the *market* demand given above, solve for a and b as functions of A, B, and N.

Answer: Observe that the inverse demand of each individual is $p(x) = a - bx$. Hence, each individual's demand is $d(p) = (a - p)/b$. The sum of individual demand is aggregate demand;[1] hence,

$$N\frac{a-p}{b} = \frac{A-p}{B} \,. \tag{1}$$

Observe this holds for *all* p, so we can differentiate both sides; that is, we see $-N/b = -1/B$. So $\boxed{b = BN}$. Substituting back into (1), we find $\boxed{a = A}$.

(d) Maintaining the assumptions of part (c), what is a profit-maximizing pricing scheme? Is there a second way to implement that scheme (and, if so, what is it)?

[1] Remember, you *never* aggregate *inverse* demand.

Answer: One option is to use a two-part tariff. The per-unit charge, p, equals marginal cost, here 0. The entry fee, f, equals individual consumer surplus when $p = MC$; that is,

$$f = \int_0^{a/b} (a - bt)dt = at - \frac{1}{2}bt^2 \bigg|_0^{a/b} = \frac{a^2}{2b}\,.$$

A second option is to put a/b units in a package and sell the package at a price of $\frac{a^2}{2b}$.

Question 2: A profit-maximizing monopolist has a constant marginal cost of $c \geq 0$ (*i.e.*, $MC = c$). It sells in two geographically distinct markets. Aggregate demand in market #1 is $D_1(p) = A_1 - p$ and aggregate demand in market #2 is $D_2(p) = A_2 - p$. Assume $A_1 > c$ and $A_2 > c$.

(a) Suppose that the cost of transporting the good in question is sufficiently great that arbitrage between the two markets is impossible. What are the profit-maximizing prices charged in the two markets?

Answer: Observe that inverse demand in market n is $P_n(x_n) = A_n - x_n$. Hence, $MR_n(x_n) = A_n - 2x_n$. Equating MR and MC, we have $x_n^* = (A_n - c)/2$. Observe

$$P_n(x_n^*) = A_n - \frac{A_n - c}{2} = \frac{A_n + c}{2}\,.$$

Hence, $\boxed{p_1^* = \frac{1}{2}(A_1 + c) \text{ and } p_2^* = \frac{1}{2}(A_2 + c)}$.

(b) Suppose a law is passed that prohibits the monopolist from charging different prices in the two markets. Prove that if

$$A_2 \leq \frac{A_1 + 2c}{3}\,, \tag{2}$$

then this law reduces total welfare.

Answer: First, note that, because $A_1 > c$,

$$A_2 \leq \frac{A_1 + 2c}{3} < A_1\,.$$

Recall §4.1 of "Lecture Notes for Economics." There we proved that if one market's consumption went down, while the other's remained the same, then the change reduced welfare. Observe that aggregate demand is

$$D(p) = \begin{cases} A_1 - p\,, & \text{if } p \geq A_2 \\ A_1 + A_2 - 2p\,, & \text{if } p < A_2 \end{cases}.$$

Aggregate *inverse* demand is, therefore,

$$P(x) = \begin{cases} A_1 - x\,, & \text{if } x \leq A_1 - A_2 \\ \frac{1}{2}(A_1 + A_2) - x/2\,, & \text{if } x > A_1 - A_2 \end{cases}.$$

Marginal revenue is, therefore,

$$MR(x) = \begin{cases} A_1 - 2x\,, & \text{if } x \leq A_1 - A_2 \\ \frac{1}{2}(A_1 + A_2) - x\,, & \text{if } x > A_1 - A_2 \end{cases}\,.$$

Observe, for $x > A_1 - A_2$, that

$$MR(x) < \frac{1}{2}(A_1 + A_2) - (A_1 - A_2) = \frac{3A_2 - A_1}{2} \leq c\,.$$

Therefore $x^* \leq A_1 - A_2$, but this means nothing is sold to market-2 people and, moreover, that the price will be $\frac{1}{2}(A_1 + c)$, which means the same amount is sold market-1 people as under 3rd-degree price discrimination. It follows, thus, that welfare has been reduced.

(c) Suppose, contrary to what was assumed in part (b), that the monopolist can charge different prices in the two markets. Assume, however, now that there has been a change in transportation technology or a reduction in cross-border tariffs such that it is now feasible for others to transport a unit of the good from one market to the other at a cost of $t > 0$. Assume, further, that

$$\frac{A_1 - A_2}{2} > t\,.$$

Assume that

$$\frac{A_1 + c}{2} < A_2\,.$$

As functions of A_1, A_2, c, and t, what are the profit-maximizing prices to charge in the two markets?

Answer: Observe that $A_1 > A_2$. Does the change matter? From part (a), the difference in the prices the monopolist would ideally like to charge is

$$\frac{A_1 - A_2}{2}\,.$$

Hence, it would be profitable to arbitrage this difference by transporting units from market #2 to market #1. Hence, a no-arbitrage constraint,

$$A_1 - x_1 - (A_2 - x_2) = (A_1 - A_2) - x_1 + x_2 \leq t\,,$$

is binding. Because it is binding we can use it to substitute out x_1 in the monopolist's profit-maximization program

$$\begin{aligned} &\max_{\{(x_1,x_2)|(A_1-A_2)-x_1+x_2=t\}} x_1(A_1 - x_1 - c) + x_2(A_2 - x_2 - c) \\ &\equiv \max_{x_2}(A_1 - A_2 - t + x_2)\big(A_1 - (A_1 - A_2 - t + x_2) - c\big) + x_2(A_2 - x_2 - c) \\ &\equiv \max_{x_2}(A_1 - A_2 - t + x_2)\big(A_2 + t - x_2 - c\big) + x_2(A_2 - x_2 - c)\,. \end{aligned}$$

The first-order condition is

$$A_2 + t - x_2 - c - A_1 + A_2 + t - x_2 + A_2 - x_2 - c - x_2 = 0$$
$$= 3A_2 + 2(t - c) - A_1 - 4x_2 \,.$$

Hence, $x_2 = \frac{1}{4}\big(3A_2 - A_1 + 2(t-c)\big)$ and $x_1 = \frac{1}{4}\big(3A_1 - A_2 - 2(t+c)\big)$. Because assumption (2) doesn't hold,[2] we can be sure that $x_2 > 0$. All this translates into $\boxed{p_2 = \frac{1}{4}\big(A_2 + A_1 - 2(t-c)\big) \text{ and } p_1 = \frac{1}{4}\big(A_1 + A_2 + 2(t+c)\big)}$.

Question 3: A profit-maximizing monopolist sells in two markets, market #1 and market #2. Assume that market $\#i$ has N_i consumers ($i = 1, 2$). Assume that within each market $\#i$ the consumers are homogeneous. Specifically, assume the utility of a representative consumer from market $\#i$ is $u_i(x, y) = \int_0^x (a_i - t)dt + y$, where x is the amount of the good in question that the consumer receives and y is income. Assume $a_2 > a_1$. For convenience, assume the monopolist's marginal cost is zero (*i.e.*, $MC = 0$).

(a) Suppose the markets are geographically separate and it would be impossible for a consumer in market $\#i$ to buy a package that is sold in market $\#j$, $i \neq j$, and *vice versa*. What is the profit-maximizing package size in each market?

Answer: Each customer has an inverse demand of $a_i - x$. Given constant marginal cost (here, 0), the optimal package size assuming no arbitrage solves $a_i - x = MC$. Hence, the profit-maximizing package sizes are $\boxed{a_1 \text{ in market \#1 and } a_2 \text{ in market \#2}}$.

(b) Suppose, now, that the markets are not geographically separate and that it is, now, possible for a consumer in market (segment) $\#i$ to buy a package that is intended for market (segment) $\#j$, $i \neq j$, and *vice versa*. Now what is the profit-maximizing package size for each market segment?

Answer: From §5.3 of "Lecture Notes for Economics," we know that there is no distortion at the top. Hence, $\boxed{x_2 = a_2}$. The first-order condition for x_1 is

$$a_1 - x_1 - \frac{N_2}{N_1}\big((a_2 - x_1) - (a_1 - x_1)\big) = 0\,.$$

Hence, $\boxed{x_1 = \max\left\{\frac{N_1+N_2}{N_1}a_1 - \frac{N_2}{N_1}a_2,\, 0\right\}}$.

[2] Observe that, because $A_1 > c$, we have

$$A_2 > \frac{A_1 + c}{2} > \frac{A_1 + 2c}{3}\,;$$

that is, (2) doesn't hold.

Part B

Answer at least one question from this part. Answer no more than two questions from this part.

Question 4: Consider the following hidden-information agency model. A utility regulator (the principal) wishes to set the number of kilowatts, k, that a utility (the agent) will produce and the payment, s, that it will receive. The regulator's utility function is $v(k) - s$, where $v(\cdot)$ is increasing and concave. Assume $v(0) = 0$. The utility's profit is $s - c(k, \theta)$, where $\theta \in [0, 1]$ is a measure of the utility's efficiency. Only the utility knows its efficiency. Assume that θ is distributed uniformly on $[0, 1]$ (*i.e.,* $F(\theta) = \theta$) and that $c(k, \theta) = h(\theta)\gamma(k)$, where $h(\cdot)$ is positive, strictly decreasing, and convex and $\gamma(\cdot)$ is strictly increasing, strictly convex, and $\gamma(0) = 0$. Finally, assume that $v'(0) - \partial c(0, \theta)/\partial k > 0$ for all $\theta > 0$.

(a) Does this problem satisfy the Spence-Mirrlees condition? Prove that it does or doesn't.

Answer: It satisfies the Spence-Mirrlees condition if

$$\frac{\partial^2\bigl(-c(k,\theta)\bigr)}{\partial\theta\partial k} > 0\,.$$

The cross-partial derivative is $-h'(\theta)\gamma'(k)$, which is positive because $h'(\cdot) < 0$ and $\gamma'(\cdot) > 0$.

(b) What is the optimal mechanism from the regulator's perspective? Be sure to show that the mechanism satisfies the IC and IR conditions. For this problem you may take the utility's reservation profit level as being 0.

Answer: The virtual surplus function is

$$\Sigma(k,\theta) = v(k) - c(k,\theta) - \frac{1-\theta}{1}\bigl(-h'(\theta)\bigr)\gamma(k)\,.$$

Observe that condition 1 on page 96 of "Lecture Notes for Economics" is met (*i.e.,* $\partial u/\partial\theta = -h'(\theta)\gamma(k) > 0$). Condition 2 holds because

$$\frac{\partial^2\Sigma(k,\theta)}{\partial k^2} = v''(k) - h(\theta)\gamma''(k) + (1-\theta)h'(\theta)\gamma''(k) < 0\,.$$

Condition 3 holds because

$$\frac{\partial^2\Sigma(k,\theta)}{\partial\theta\partial k} = -h'(\theta)\gamma'(k) + (1-\theta)h''(\theta)\gamma'(k) > 0\,. \tag{3}$$

Hence the optimal $k^*(\theta)$ is defined by

$$\frac{\partial\Sigma\bigl(k^*(\theta),\theta\bigr)}{\partial k} = 0$$

or

$$v'\big(k^*(\theta)\big) - h(\theta)\gamma'\big(k^*(\theta)\big) + (1-\theta)h'(\theta)\gamma'\big(k^*(\theta)\big) = 0\,.$$

Given (3), $k^*(\cdot)$ is monotonic. Hence, IC is guaranteed by setting

$$s(\theta) = c\big(k^*(\theta),\theta\big) - \int_0^\theta h'(t)\gamma\big(k^*(t)\big)dt$$

(this is Theorem 1, p. 92). It is readily seen that this payment schedule satisfies IR and, moreover, is the cheapest schedule to do so.

Question 5: A society consists of N consumers and one utility company. A benevolent social planner is trying to figure out the welfare-maximizing generation capacity, G, for the utility company to have. Each consumer has a utility function of the form $v(G,\tau)+y$, where $\tau \in \mathcal{T}$ is his type and y is money. Further assume

$$v(G,\tau) = \frac{G}{N} - \tau\frac{G^2}{2}$$

(interpret this as the consumer enjoys G/N units of electricity, but suffers $\tau G^2/2$ disutility from the pollution that the generation of G kilowatts of electricity entails). The utility company has a profit function $s+\theta G^2/2$, where $\theta \in \Theta \subset \mathbb{R}_-$ is its type. Note $\theta \leq 0$. Assume Θ is a closed interval. Observe that the social planner wants to maximize

$$\theta\frac{G^2}{2} + \sum_{n=1}^{N}\left(\frac{G}{N} - \tau_n\frac{G^2}{2}\right)\,. \tag{4}$$

(a) What is a dominant-strategy mechanism that solves the social planner's problem?

Answer: Given that pollution is not liked, we know $\tau_n \geq 0$ for all n. Observe (i) that (4) is 0 if $G=0$; (ii) its derivative at $G=0$ is $1>0$; (iii) (4) is concave (its second derivative is $\theta - \sum_{n=1}^N \tau_n < 0$—recall $\theta < 0$); and (iv) $\lim_{G\to\infty}(4) = -\infty$. Hence, a unique interior maximum exists and, moreover, yields strictly positive social welfare.

The solution to the social planner's problem is found by solving the first-order condition

$$\theta G + \sum_{n=1}^{N}\left(\frac{1}{N} - \tau_n G\right) = 0\,.$$

Hence,

$$G^*(\theta,\boldsymbol{\tau}) = \frac{1}{-\theta + \sum_{n=1}^N \tau_n}\,.$$

From Groves-Clarke, we know that a dominant-strategy mechanism that implements that $G^*(\cdot)$ is found by setting the transfer to each party equal to the sum of the utilities of all *other* players plus, possibly, a positive constant:

$$\begin{aligned} P_U(\theta,\boldsymbol{\tau}) &= \sum_{n=1}^{N}\left(\frac{G^*(\theta,\boldsymbol{\tau})}{N} - \tau_n\frac{G^*(\theta,\boldsymbol{\tau})^2}{2}\right) + h_U(\boldsymbol{\tau}) \qquad (5)\\ &= \frac{1}{-\theta+\sum_{n=1}^{N}\tau_n} - \frac{\sum_{n=1}^{N}\tau_n}{2(-\theta+\sum_{n=1}^{N}\tau_n)^2} + h_U(\boldsymbol{\tau})\\ &= \frac{-2\theta+\sum_{n=1}^{N}\tau_n}{2(-\theta+\sum_{n=1}^{N}\tau_n)^2} + h_U(\boldsymbol{\tau}) \end{aligned}$$

and

$$\begin{aligned} P_n(\theta,\boldsymbol{\tau}) &= \theta\frac{G^*(\theta,\boldsymbol{\tau})^2}{2} + \sum_{j\neq n}\left(\frac{G^*(\theta,\boldsymbol{\tau})}{N} - \tau_j\frac{G^*(\theta,\boldsymbol{\tau})^2}{2}\right) + h_n(\theta,\boldsymbol{\tau}_{-n})\\ &= \frac{N-1}{N(-\theta+\sum_{n=1}^{N}\tau_n)} + \frac{\theta-\sum_{j\neq n}\tau_j}{2(-\theta+\sum_{n=1}^{N}\tau_n)^2} + h_n(\theta,\boldsymbol{\tau}_{-n})\,, \end{aligned}$$

where the subscript U denotes the utility company and the subscript n denotes the nth consumer.

(b) Suppose that the utility can never receive a negative transfer. Can the mechanism you found in part (a) be modified so that this property holds? If so, show how this modification is constructed. If not, prove that it would be impossible.

Answer: We know that (4) is positive at $G^*(\theta,\boldsymbol{\tau})$ and we know $\theta G^*(\theta,\boldsymbol{\tau})/2 < 0$; hence, the summation in (5) must be positive. The utility company, therefore, receives a non-negative transfer provided $h_U(\boldsymbol{\tau}) \geq 0$ for all $\boldsymbol{\tau}$.

Question 6: Consider a standard two-action, two-outcome hidden-action principal-agent problem. Assume the principal's utility is $x - s$, where x is the realization of the signal and s is her payment to the agent. Assume the agent's utility is $\ln(s) - c(a)$, where a is the action he takes. Assume his reservation utility is zero. Assume the principal offers an incentive contract on a TIOLI basis. Assume $a \in \{0,1\}$ and $x \in \{x_L, x_H\}$. Assume $\Pr\{x = x_H|a\} = a/2$. Assume $c(a) = a\ln(2)$.

(a) What is the optimal contract for the principal to offer if she wishes to implement $a = 1$?

Answer: We know that both IR and IC are binding. Hence, when there are two actions and two possible outcomes, the contract is fully determined by the constraints:

$$\frac{1}{2}\ln(s_H) + \frac{1}{2}\ln(s_L) - \ln(2) \geq \ln(s_L) \qquad \text{(IC)}$$

and

$$\frac{1}{2}\ln(s_H) + \frac{1}{2}\ln(s_L) - \ln(2) \geq 0\,. \tag{IR}$$

Subtracting (IR) from (IC) yields $\ln(s_L) = 0$. Hence, $\boxed{s_L = 1}$. Substituting that into (IR) yields

$$\frac{1}{2}\ln(s_H) = \ln(2) \Rightarrow \ln(s_H) = 2\ln(2) \Rightarrow \ln(s_H) = \ln(2^2) = \ln(4)\,.$$

Hence, $\boxed{s_H = 4}$.

(b) Suppose, now, that there are three possible actions. Specifically, $a \in \{0, 1/2, 1\}$. Assume $\Pr\{x = x_H|1/2\} = p < 1/2$ and $c(1/2) = \ln(2)/2$. Maintain the other assumptions. Derive a condition that p must satisfy for $a = 1/2$ to be implementable.

Answer: It is clear that a convex combination (mixed strategy) exists over $a = 0$ and $a = 1$ that induces the same density over outcomes as $a = 1/2$. Hence, for $a = 1/2$ to be implementable, it must be that any such convex combination costs more than $c(1/2)$. Observe that such a convex combination is a σ that solves

$$\sigma\frac{1}{2} + (1-\sigma)0 = p\,.$$

Hence, $\sigma = 2p$. We require that

$$c(1/2) = \ln(2)/2 \leq 2p\ln(2) + (1-2p)0\,.$$

Hence, $\boxed{1/4 \leq p}$.