Understanding Firm Value and Corporate Governance*

Benjamin E. Hermalin†

Abstract

An impressive volume of careful empirical studies finds evidence that the strength of firms' corporate governance tends to be positively correlated with their financial performance; that is, firms that score higher on some measure of governance tend to outperform those which score worse. These findings are a puzzle insofar as we expect those who decide how a firm is organized, including its corporate governance, to do so in a manner that maximizes firm value subject to the relevant constraints. If the governance we observe is constrained optimal, then why, in equilibrium, should any correlation—positive or negative—exist between it and firm performance? This paper offers an answer. In doing so, the paper also makes predictions about the correlation between firm size and strength of governance, provides new explanations for the correlation between firm size and executive compensation, and provides insights into why empirical estimates of managerial incentives are often deemed too low.

Keywords: corporate governance, executive compensation, firm heterogeneity, trends in governance.

*The author thanks seminar participants at the Universities of Arizona, California (Berkeley), Chicago, and Missouri, Ohio State University, and the 13th Finsia Conference, Melbourne, for valuable feedback on earlier versions. As usual, they are exonerated with respect to the paper's residual shortcomings. The financial support of the Thomas and Alison Schneider Distinguished Professorship in Finance is gratefully acknowledged.

†University of California • Walter A. Haas School of Business • S545 Student Services Building #1900 • Berkeley, CA 94720–1900. email: hermalin@haas.berkeley.edu.

Draft 8/3/2010—Version 12

Contents

1 Introduction

Suppose one had a sample of solutions to the program of maximizing payoffs $1-(x-k)^2$, where x is the choice variable and k a parameter that varies across observations. Suppose, too, the sample contained the realized payoffs. Even though the choice of x is critical to maximizing payoff, no correlation would exist between the observed xs (which would equal the various ks) and the realized payoffs, as the latter would all be 1. Since at least Demsetz and Lehn (1985), it has been known that a similar problem could well plague studies of corporate governance.[1] If firms' governance structures represent the constrained optimal solution to the firms' agency problems, then—as Figure 1 heuristically illustrates—there is no obvious reason one should expect to observe any particular correlation between a measure of governance (*e.g.,* board size, index of governance, etc.) and a measure of firm performance (*e.g.,* ROA, Tobin's Q, etc.). Yet, many carefully executed empirical studies find evidence of correlation; moreover, this correlation tends to be positive, measures that arguably indicate stronger governance (*e.g.,* smaller boards, higher governance indices, etc.) go hand-in-hand with better firm performance and *vice versa*.[2] Why, in a world of constrained value-maximizing governance structures, do we observe these positive correlations? This is the question this paper seeks, in part, to answer.

The answer starts with the following: A concern, dating at least as far back as Adam Smith, is that managers will abuse, misuse, or even misappropriate the resources of the firm.[3] Corporate governance is the security that owners put in place to protect their interests against such agency problems. Rationality then dictates that those firms with the strongest governance will be those in which the owners perceive the greatest benefit of such protection.

But why does a greater benefit from governance correlate positively with good firm performance (*e.g.,* greater profits)? For instance, in the simple optimization program above, entities with greater ks had greater xs, but no correlation existed between the xs and payoffs. Indeed, couldn't stronger governance be of greater importance in a struggling firm than a successful one? The answer is arguably no for the following reason: Consider two firms, A and B, with B having the greater *marginal* return to resources. The marginal cost to B's own-

---

[1] Other articles that have made this point include Hermalin and Weisbach (1998), Himmelberg et al. (1999), Palia (2001), Hermalin and Wallace (2001), and Coles et al. (2007).

[2] See, *e.g.,* Bhagat and Jefferis (2002), Becht et al. (2003), and Adams et al. (2010), among others, for surveys covering this empirical literature. Examples of articles finding a negative correlation between board size and performance (*i.e.,* that small boards are associated with better performance) include Yermack (1996) and Eisenberg et al. (1998) (but see, also, Coles et al., 2008 for partially contradictory evidence). Examples of articles finding a positive correlation between indices of good governance and performance include Gompers et al. (2003) and Bebchuk et al. (2009).

[3] "The directors of companies, however, being the managers rather of other people's money than of their own, it cannot well be expected, that they should watch over it with the same anxious vigilance . . . negligence and profusion, therefore, must always prevail, more or less, in the management of the affairs of such a company." — Smith (1776).

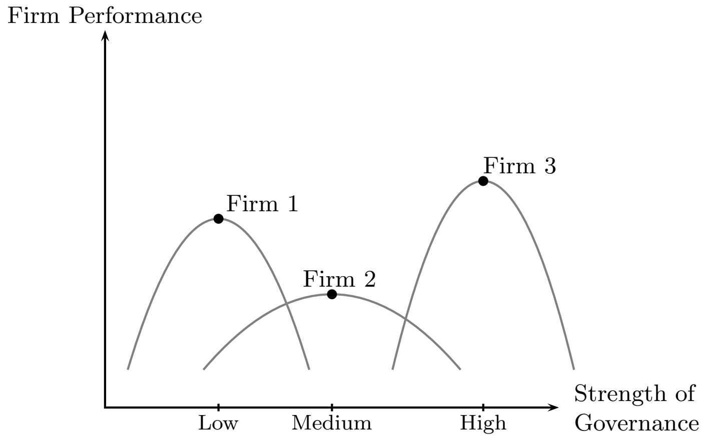

Figure 1: A Heuristic Figure: Given firms choose governance optimally, should governance and performance be correlated? Parabolas illustrate the indicated firms' optimization problems.

ers of an abused, misused, or misappropriated dollar of resources is, thus, greater than it is for A's. Consequently, the marginal return to B's owners of investing in greater security—that is, stronger corporate governance—is greater than it is for A's. In equilibrium, firm B will have stronger governance than firm A. Furthermore, under mild conditions, greater marginal returns imply greater total returns, so firm B will be more profitable than firm A in equilibrium. Profits and strength of governance will, therefore, be positively correlated. The majority of the paper is given to exploring the predictions this insight generates and its implications for interpreting existing empirical findings.

One paper that can be seen as offering a similar insight is the contemporaneous work of Coles et al. (2007). Although their paper is primarily empirical, it does contain a brief model of optimal managerial ownership in which the firm's productivity, among other factors, influences the amount of managerial stock ownership. They do not, however, analyze the general relation between profit potential and governance; nor do they identify the key marginal effects that are at work.

In what follows, Section 2 presents the model. The key assumption is that firms vary in their marginal returns from resources utilized for the owners' benefit (*i.e.,* total resources less those lost due to a managerial agency problem). It is shown that firms with higher marginal returns—*higher-type firms*—will have stronger governance in equilibrium than those with lower marginal returns—*lower-type firms*. Because, under a mild condition, higher-type firms will have greater profits than lower-type firms, there will be a positive correlation between profits and the level of governance.

In Section 2, a firm's total resources are fixed exogenously. In many contexts, it is better to think of them as endogenously determined, with the necessary funds coming from the capital markets. Section 3 extends the basic model to allow the firm's owners to also determine how much capital is to be raised. Higher-type firms will raise more capital, have stronger governance, and generate greater profits than will lower-type firms. To the extent the amount of capital raised or profits are indicators of firm size or correlated with other measures of size, the analysis in this section predicts a positive correlation should exist between firm size and the strength of governance. It is also shown that the level of governance could well be independent of a firm's capital structure.

A particularly important form of governance is incentive compensation. Section 4 explores the implications of the model for compensation. It is shown there that higher-type firms pay more in expectation than lower-type firms; that is, executives are paid more not only as a function of how their firms *actually* do, but also as a function of how they are *expected* to do. Given that profits are correlated with standard measures of firm size, this insight offers a novel explanation for the positive correlation between firm size and executive pay commonly found in the data.

Much of Section 4 concerns the implications for regressions of executive compensation on performance. It is shown there are reasons to believe that the coefficient on performance (profits, ROA, change in stock price, etc.) in such regressions will be biased downward, so that it understates the true strength of the incentives executives have. This finding suggests the ongoing debate over whether real-life executives are given sufficiently strong incentives could be relying on misleading evidence.

Section 5, addresses the degree to which the analysis of the earlier sections continues to hold if governance is a multi-dimensional variable. That is, in that section, the fact the governance can vary simultaneously across firms on many dimensions, such as board structure, incentive compensation, shareholder activism, and so forth, is explicitly considered. It is readily shown that firms with better profit potentials will spend more on governance than firms with weaker profit potentials. As will be shown, this does not, however, mean that higher-type firms have stronger governance on all dimensions. Under certain, arguably strong, conditions, however, it will be true that higher-type firms have stronger governance on all dimensions.

Section 6 contains a brief discussion of how the analysis in this paper could shed light on trends in corporate governance over the past twenty to thirty years. Section 7 is a brief conclusion, which focuses on the implications of the analysis for future empirical work in the area.

Proofs not given in the text can be found in Appendix A.

2 The Basic Model

Consider a firm's manager, who must make a decision about the allocation of total resources, $Y \in \mathbb{R}_+$. There are two purposes to which resources can be put. One is a productive use that raises the shareholders' payoffs. The other is what might be called "private use," which is shorthand for various "agency" actions such as allocating resources to pet projects not in the owners' interest, using funds for empire building, acquiring perks, or misusing assets for private benefit. For the version of the model in which Y is exogenous (*i.e.,* is not a function of capital raised), Y could also be interpreted as the total available managerial time, managerial effort, or managerial attention (in this sense, the model encompasses the standard principal-agent model). Let $S \in \mathbb{R}_+$ denote the amount allocated to private use (think "S" for self). The amount allocated to purposes desired by the shareholders is, thus, $Y - S$.

Assume the manager's utility is given by the function defined as

$$(Y, S, g) \mapsto S + v(Y - S, g)\,,$$

where $g \in \mathbb{R}_+$ is a measure of the strength of corporate governance. The variable g could represent the percentage of independent directors on the board or on key board committees, a measure of the directors' diligence, a measure of the effectiveness of the monitoring and auditing systems in place, some measure of the strength of the incentives given the manager, or perhaps some index of governance strength.[4] One interpretation, in particular, is worth considering: given the many dimensions of governance, think of g as the firm's total expenditure on governance. Provided the owners set the dimensions of governance optimally, spending more on governance must correspond to stronger governance. Section 5 explores this interpretation in greater depth.

The function $v : \mathbb{R}_+^2 \to \mathbb{R}$ represents the benefit the manager derives from behaving in a manner desired by the firm's owners (*i.e.,* not diverting funds or assets for private use). This benefit is, in part, a function of the level of governance.[5] This reflects the idea that governance structures operate to reward the manager for good behavior.

The function $v(\cdot, \cdot)$, like all functions in this paper, is assumed to be twice continuously differentiable in each of its arguments. Throughout, the convention f_n is used to denote the derivative with respect to the nth argument of function f and f_{nm} to denote the second derivative with respect to the nth and mth arguments.

[4] Some readers have suggested that v should vary with firm type: "Independence from firm type means that a manager who delivers \$9 million of profit will receive the same incentive pay (or threat of being fired) regardless of whether the firm had the potential to deliver \$10 million or \$100 million." This claim is false because it overlooks the fact that g is *endogenous.* As will be shown, the manager of the firm with the greater potential will operate under stricter governance in equilibrium, so his reward for delivering \$9 million would be far less than if he were managing the firm with lower potential.

The conclusions of the paper would not change if the manager's utility were $b(S) + v(Y - S, g)$, where $b(\cdot)$ is an increasing and, at least weakly, concave function. Provided, as in this and the next section, there is no risk, then the model is robust to any positive monotonic transformation of $S + v(Y - S, g)$.

[5] Of course, because a cost is just a negative benefit, this formulation also incorporates specifications in which the governance structure punishes the manager for behaving at odds with the shareholders' preferences.

Consistent with the view that better performance is better rewarded, assume

$$v_1(\cdot, g) > 0 \; \forall g > 0\,. \tag{1}$$

In what follows, it vastly simplifies the analysis if, for any g, there is a unique value of S that maximizes the manager's utility. This property will hold if the manager's benefit function exhibits diminishing marginal returns with respect to good behavior; that is, if

$$v_{11}(\cdot, g) < 0 \; \forall g\,. \tag{2}$$

It is a common feature of most two-good allocation problems that the marginal cost of allocating more to one good in terms of forgone consumption of the other is increasing, a property that (2) captures.

By stronger governance, one means an increase in the governance parameter that results in a reduction in agency behavior; that is, leads the manager to choose a smaller S (assuming he is not at a corner solution). Formally, assume:[6]

Assumption 1. *Let g and g' be two different levels of governance, $g > g'$ and let S and S', respectively, be the levels of private use chosen by the manager in response to those governance levels. Then $S \leq S'$. Moreover, if S or S' is an interior solution to the manager's choice problem, then $S < S'$.*

Allowing for corner solutions in which the manager allocates all resources to private use complicates the analysis without adding much insight. The following assumption rules out such allocations except, possibly, when there is no governance.

$$v_1(0, g) > 1 \;\; \forall g > 0\,. \tag{3}$$

Again, primarily to keep the analysis straightforward, it is convenient to rule out there being a level of governance so strict that the manager never diverts resources to private use no matter how great the firm's resources are. Formally, assume

$$\lim_{Y \to \infty} v_1(Y, g) < 1 \;\; \forall g\,. \tag{4}$$

A consequence of the above assumptions is

Lemma 1. *For all governance levels, $g \in \mathbb{R}_+$, there exists an amount $Y(g)$, such that, in equilibrium, the manager diverts a positive amount if and only if total resources exceed $Y(g)$ (i.e., iff $Y > Y(g)$). The equilibrium amount of diversion is $S = \max\{Y - Y(g), 0\}$. Moreover, $Y(g)$ is strictly increasing and differentiable in g.*

To avoid dealing with corner solutions in the level of governance, assume $v_1(0, 0) = 1$; this implies $Y(0) = 0$—in the absence of governance, the manager

[6] A sufficient condition for this assumption to be true is that $v_{12}(\cdot, \cdot) > 0$; see Lemma 2 *infra*.

will divert all available funds to his private use.[7] Because the function $Y(\cdot)$ is monotone, it is invertible. Let $G(\cdot)$ denote its inverse.

Much of the analysis in this paper relies on the following well-known revealed-preference result, which is worth stating once, at a general level, for the sake of completeness and to avoid unnecessary repetition.

Lemma 2. *Let $f(\cdot,\cdot):\mathbb{R}^2 \to \mathbb{R}$ be a twice-differentiable function. Suppose that $f_{12}(\cdot,\cdot) > 0$. Let $\hat{x}$ maximize $f(x,z)$ and let $\hat{x}'$ maximize $f(x,z')$, where $z > z'$. Then $\hat{x} \geq \hat{x}'$. Moreover, if $\hat{x}'$ is an interior maximum, then $\hat{x} > \hat{x}'$.*

The owners of the firm are assumed to have a payoff given by

$$B(Y - S, \tau) - C(g)\,,$$

where $\tau \in \mathcal{T} \subset \mathbb{R}$ is an index of the firm's type. The amount $C(g)$ is the cost of implementing governance level g; it is, for instance, the cost of establishing and maintaining auditing and monitoring procedures, the cost of incentive pay, etc. It could also include the cost owners incur overcoming managerial resistance to more oversight. Because it is a cost function, $C(\cdot)$ is increasing. To avoid, however, corner solutions at zero governance, assume $C'(0) = 0$.[8] Otherwise marginal cost of governance is assumed to be bounded away from zero: Formally, there exist $\xi \in (0,\infty)$ and $\underline{g} \in (0,\infty)$ such that $C'(g) \geq \xi$ for all $g \geq \underline{g}$.

The amount $B(Y-S,\tau)$ is the benefit a type-τ firm's owners realize when the net resources utilized are $Y - S$. Consistent with the discussion above, assume the more net resources utilized, the more the owners' benefit (*i.e.*, $B_1(\cdot,\tau) > 0$). Firm type is defined as a condition about the firm's *marginal* benefit schedule: The marginal benefit of more net resources is greater for higher-index types than for lower-index types. Formally,

$$B_{12}(\cdot,\cdot) > 0\,. \tag{5}$$

As one of many possible examples, suppose $B(y,\tau) = \tau\psi(y)$, where $\psi(\cdot)$ is an increasing function that relates the net amount invested to expected production and τ is the average price margin. Alternatively, $\psi(\cdot)$ could be the probability of a successful R&D innovation and τ the profit from such an innovation. A different formulation is to imagine firm demand is $\tau - p$, where p is price, and its constant marginal cost of production is $m(y)$, $m'(y) < 0$ (greater resource allocation yields lower marginal cost). It is readily seen that the resulting $B(y,\tau) = \big(\tau - m(y)\big)^2/4$ is consistent with (5).

The timing of the model is that the owners choose the level of governance, g, then the manager chooses how much to divert, S. Assume for the moment that Y is fixed exogenously. From Lemma 1, net resources will be $Y(g)$ if $Y(g) < Y$

[7] An example of a v function satisfying all the assumptions given so far is $v(Y - S, g) = 2g\sqrt{Y - S}$.

[8] This assumption is not critical. There are other assumptions that could be made to avoid corner solutions at $g = 0$. Moreover, corner solutions only mean that some of the strict comparative static results below become weak comparative static results.

(the manager diverts $Y - Y(g)$); or Y if $Y(g) \geq Y$. Given that raising g is costly and $Y(\cdot)$ is strictly increasing, the owners will never choose a level of g such that $Y(g) > Y$. The owners' problem is, thus,

$$\max_{g \in \mathbb{R}_+} B\bigl(Y(g), \tau\bigr) - C(g) \,. \tag{6}$$

The owners' problem has at least one solution:

Lemma 3. *For any firm type, τ, the owners' problem of choosing a governance level to maximize profit has at least one finite and positive solution.*

Let $g(\tau)$ denote the solution adopted by a type-τ firm. If a solution exists to the equation $Y(g) = Y$—call it $\overline{g}$—then the reasoning above implies $g(\tau) \leq \overline{g}$.

It can now be established that there will heterogeneity in the level of governance across firms. Specifically,

Proposition 1. *Higher-type firms adopt at least as great a level of governance as lower-type firms (i.e., if $\tau > \tau'$, then $g(\tau) \geq g(\tau')$). Moreover, if either there is no maximum level of governance (i.e., the equation $Y(g) = Y$ has no solution) or if a lower-type firm has not adopted that maximum (i.e., $g(\tau') < \overline{g}$), then a higher-type firm will have a strictly greater level of governance (i.e., $g(\tau) > g(\tau')$).*

Proof: Given Lemma 2, the proposition follows if the cross-partial derivative of

$$B\bigl(Y(g), \tau\bigr) - C(g)$$

with respect to τ and g is positive everywhere. That cross-partial derivative is $B_{12}\bigl(Y(g), \tau\bigr)Y'(g)$, which is positive by Lemma 1 and expression (5). ∎

Assume that, as seems plausible, if no resources are put to productive use, then benefits are zero, regardless of type. Because the derivative of a constant is zero, this implies $B_2(0, \tau) \equiv 0$. Given (5), it follows from the fundamental theorem of calculus that

$$B_2(y, \tau) = B_2(0, \tau) + \int_0^y B_{12}(z, \tau)dz > 0 \tag{7}$$

for $y > 0$. In other words, total benefit, not just the marginal benefit of resources, is increasing in type. A consequence of this is the following relationship between firm profits and governance:

Proposition 2. *Under the assumptions of the basic model and assuming a common level of resources, a firm that will be more profitable in equilibrium than another has at least as high a level of governance as the other firm.*

Proof: Equilibrium profits are

$$\pi(\tau) \equiv B\Bigl(Y\bigl(g(\tau)\bigr), \tau\Bigr) - C\bigl(g(\tau)\bigr) \,.$$

By the envelope theorem,

$$\pi'(\tau) = B_2\Big(Y\big(g(\tau)\big), \tau\Big) > 0\,.$$

So $\tau > \tau'$ implies $\pi(\tau) > \pi(\tau')$. From Proposition 1, $\tau > \tau'$ implies $g(\tau) \geq g(\tau')$. ■

In other words, profitability potential (type) causes stronger governance and it means higher profits in equilibrium. Consequently, profits and governance end up positively correlated.

Suppose that τ were invariant across firms and, instead, that firms differed in terms of the gross resources, Y, available to them. Suppose further that for all firms, the equation $Y = Y(g)$ has a solution (*i.e.,* $G(Y)$ is defined for all firms).[9] Each firm's owners would solve

$$\max_{g \in [0, G(Y)]} B\big(Y(g)\big) - C(g)\,, \tag{8}$$

where τ has been suppressed because it is assumed constant across firms. Let $\hat{g}(Y)$ denote the solution to (8). Because $G(\cdot)$ is an increasing function, $\hat{g}(\cdot)$ is non-decreasing. Moreover, a higher-Y firm has more options than a lower-Y firm (*i.e.,* $[0, G(Y')] \subset [0, G(Y)]$ if $Y' < Y$), which means its profits are weakly greater as well. Hence,

Proposition 3. *Assume all firms are the same type, but they differ as to the gross resources, Y, available to them. Then a firm with more resources will have at least as great a level of governance as a firm with fewer resources. It will also have at least as much profits as a firm with fewer resources. That is, under these assumptions, gross resources and the level of governance are non-negatively correlated and profits and the level of governance are non-negatively correlated. The correlation between profits and the level of governance will, therefore, be non-negative.*

3 Endogenous Investment

To this point, gross resources, Y, were fixed exogenously. Now consider a model in which resources must be funded from the capital market. Let I denote funds raised from the market. The resources potentially available for productive investment are $Y = I - C(g)$. Of these, S will be diverted, leaving $N = I - C(g) - S$ available to be actually utilized. Normalize the model so the marginal opportunity cost of funds is 1.

Denote financial returns by r. Assume $r \sim F(\cdot|N, \tau)$. Assume the expectation

$$B(N, \tau) \equiv \int_{-\infty}^{\infty} r dF(r|N, \tau)$$

[9] A sufficient condition for $Y = Y(g)$ to be solvable for all Y is that for all $y \in \mathbb{R}_+$ $\liminf_{g\to\infty} v_1(y, g) > 1$. The previously given example of $v(y, g) = 2g\sqrt{y}$ satisfies this condition.

exists. Maintain the previously made assumptions about $B(\cdot,\cdot)$.[10] To these, add the assumption that, for all $\tau \in \mathcal{T}$,

$$B_1(0,\tau) > 1 > B_1(n,\tau) \tag{9}$$

for $n > \bar{n}$, where $\bar{n}$ is a finite constant. The first inequality in (9) implies it is profitable to invest in firms; the second inequality rules out infinite investment as being optimal. The second inequality implies that there is no loss of generality if the set of possible investment levels is treated as bounded above; this will insure interior maxima for the optimization programs below.

Initially, assume that the owners self finance. Because every dollar provided over $Y(g)+C(g)$ will be diverted by the manager, the owners will never provide funding in excess of $Y(g)+C(g)$. The owners problem can, thus, be stated as

$$\max_{Y} \int_{-\infty}^{\infty} r\, dF(r|Y,\tau) - C\big(G(Y)\big) - Y\,, \tag{10}$$

where, recall, $G(\cdot)$ is the inverse function of $Y(\cdot)$.[11]

Proposition 4. *Under the above assumptions, there will be a strictly positive correlation between the amount the owners invest in a firm and its level of governance. Furthermore, if there is no return without investment (i.e., $B(0,\tau)$ is zero for all τ), then there will be a strictly positive correlation between firm profit and level of governance.*

Proof: Let $y^*(\tau)$ denote a solution to (10). By the assumptions above, $0 < y^*(\tau) < \infty$ for all τ; that is, it is an interior solution. The first part of the proposition follows immediately from Lemma 2 provided the cross-partial derivative of

$$B(Y,\tau) - C\big(G(Y)\big) - Y$$

with respect to Y and τ is positive everywhere. That it is follows from assumption (5).

The "furthermore" part follows from the envelope theorem, which establishes that equilibrium profits are increasing in type, and from the first part of the proposition, which established that investment is increasing in type. ■

One imagines that firm size is positively correlated with the amount invested in it. Indeed, the amount invested—firm capitalization—could be a definition of size. Hence,

Corollary 1. *If investment in a firm is positively correlated with its size, then firm size and level of governance are positively correlated.*

---

[10] Those assumptions would hold, for instance, if $r = \tau\sqrt{N} + \eta$, where η is a random variable drawn independently of N and τ.

[11] For the case when $Y = Y(g)$ may not have a solution for all g, there is the implicit constraint on (10) that $Y \leq \lim_{g\to\infty} Y(g)$. This is not a binding constraint as the owners never wish to invest more than $Y(g) + C(g)$.

What if the firm owners must raise capital? Consider two timing possibilities: first, the owners can wait to set g until they have received outside capital; second, they must set it and expend the money to do so prior to seeking capital. In the latter case, it follows that $g \leq C^{-1}(I_0)$, where I_0 is the owners' available capital. A firm's type is taken to be common knowledge; in particular, it is known to would-be investors.

Consider the first possibility. Because every dollar of capital over $Y(g)+C(g)$ will be diverted by the manager, total capital invested will be $Y + C\big(G(Y)\big)$. Let $I \in [0, I_0]$ be the amount of capital the owners self finance and $Y + C\big(G(Y)\big) - I$, therefore, be the amount they must raise from outside investors. Let $Z(\cdot)$ denote the financial contract; that is, the owners repay the outside investors $Z(r)$ when the firm's gross profit is r.[12] Observe, this encompasses standard forms of financing such as debt, equity, a combination of debt and equity, or more exotic securities. Given the owners are the residual claimants, they get the cash left in the firm at the end, r, less what the outside investors are due. Hence, their problem is

$$\max_{\{Y,I,Z(\cdot)\}} \int_{-\infty}^{\infty} \big(r - Z(r)\big) dF(r|Y,\tau) - I \tag{11}$$

subject to

$$\int_{-\infty}^{\infty} Z(r) dF(r|Y,\tau) = Y + C\big(G(Y)\big) - I\,, \tag{12}$$

where (12) is the condition for investors to be willing to provide the required capital. Note $Z(\cdot)$ is *endogenous*. Using (12) to substitute out I in (11) and, then, simplifying yields (10); hence, the total amount invested is unaffected by the need to raise funds and the financial structure of the firm (*i.e.,* the $Z(\cdot)$ function) is indeterminate. This establishes

Proposition 5. *If a firm's owners are not obligated to fund the level of governance before raising capital from the market, then the level of governance will be the same as if the owners could self finance. Moreover, there is not necessarily any correlation between the firm's capital structure and its level of governance.*

This result is, in essence, a simple version of Modigliani and Miller (1958). Like Modigliani and Miller, Proposition 5 can be criticized insofar as capital-structure indeterminancy may fail to hold in a richer model. Nevertheless, it indicates that governance need not be a driver of capital structure nor even correlated with it.

A further potential criticism is there is a significant literature that argues that the capital structure is itself part of the governance structure; that is, because g has entered the model in reduced form, the analysis could be overlooking the possibility that g is a function of the capital structure. For instance, it has been argued that debt can be used to force managers not to divert funds (see,

[12] An alternative, but ultimately equivalent, accounting would be to define profit as $r - C\big(G(Y)\big)$. What is relevant for the owners is the cash left in the firm less what they must repay outside investors.

e.g., Grossman and Hart, 1982, Jensen, 1986, Hart and Moore, 1998). While this literature offers many insights, it remains true that there are a number of other governance instruments, such as incentive schemes, board oversight, and outside auditing, that have nothing to do with the capital structure. Moreover, because of access to these other governance mechanisms, one wonders to what extent firms would utilize capital structure for this purpose. After all, there could be competing motives (*e.g.,* the tax advantage of debt) affecting capital structure; and it could be difficult or costly to adjust capital structure with sufficient precision to deal with governance issues.

A related concern is that agency issues can lead managers to distort a firm's capital structure (see, *e.g.,* Jensen and Meckling, 1976, Harris and Raviv, 1988, Stulz, 1988; for an empirical examination see Berger et al., 1997). In particular, weak governance could mean more agency problems, which could mean a preference for one form of financing over another; that is, a correlation exists between governance and capital structure. To an extent, this issue can be captured within the framework of the previous section: Interpret S as funds raised via one form of financing and $Y - S = Y(g)$ the funds raised via a second. In light of Lemma 1, stronger governance would be positively correlated with the use of the second form of financing. On the other hand, one might question why the manager is not simply barred contractually from changing the capital structure.

What if the owners must fix and pay for the level of governance before seeking outside capital? Because the owners can subsequently acquire capital from the market at the same rate that their own investments in the market would earn, there is no loss in generality in assuming that, beyond the funding of the governance level, the owners invest none of their own money in the firm. The owners' problem is, therefore,

$$\max_{\{g,Z(\cdot)\}} \int_{-\infty}^{\infty} \big(r - Z(r)\big)\, dF\big(r|Y(g),\tau\big) - C(g) \tag{13}$$

subject to

$$C(g) \leq I_0 \text{ and} \tag{14}$$

$$\int_{-\infty}^{\infty} Z(r)\, dF\big(r|Y(g),\tau\big) = Y(g)\,, \tag{15}$$

where, recall, I_0 equals the owners' available funds. Using (15) to substitute out $\int Z dF$ in (13), the owners' problem becomes

$$\max_{g} B\big(Y(g),\tau\big) - C(g) - Y(g) \tag{16}$$

subject to (14). Expression (16) is equivalent to (10); hence, if $I_0 > C\big(G\big(y^*(\tau)\big)\big)$, then the solution is the same as in Proposition 4. If, instead, the constraint binds, then the equilibrium level of governance is less than the unconstrained optimum. The basic conclusion of Proposition 5 continues, however, to hold.

Corollary 2. *Suppose a firm's owners are obligated to set and pay for the level of governance before raising capital from the market. Then the level of governance they choose is independent of the firm's capital structure.*

4 Managerial Compensation

In this section, the focus is on the use of managerial compensation as a governance mechanism. To that end a model of compensation consistent with the model developed above needs to be constructed. Assume, as is customary in agency models, the manager's utility is additively separable in action (*i.e.,* choice of S) and income, w, and exhibits risk aversion with respect to the latter. Specifically, assume his utility is $S+V(w)$, where $V(\cdot)$ is increasing and concave. Suppose that there are two possible outcomes, success and failure, upon which the manager's compensation can be based. Let w_s and w_f be compensation for success and failure, respectively. Let the probability of success be $P(Y-S)$, where $P'(\cdot)>0$ and $P''(\cdot)<0$. Assume $P(0)=0$. Define $g=V(w_s)-V(w_f)$. Observe g is a measure of the strength of the incentives.

$$v(Y-S,g)=P(Y-S)g+V(w_f)\,.$$

Observe $v_{11}=P''(Y-S)g<0$ and $v_{12}=P'(Y-S)>0$, as required.[13]

Here, $C(g)$, the cost of governance, must be determined as part of the analysis. To keep the analysis straightforward, assume Y is sufficiently big that the constraint $S\leq Y$ never binds in equilibrium. Given g, the manager will choose S to solve

$$-P'(Y-S)g+1=0$$

if a solution exists for $S\in[0,Y)$; and he chooses $S=Y$ otherwise. Observe if $g=0$, the manager will choose $S=Y$. In that case, because $P(0)=0$, it is irrelevant what w_s is because the manager will be paid w_f with certainty.

The first-order condition for the manager's choice problem can rewritten as

$$P'(Y-S)=P'\big(\hat{N}(g)\big)=\frac{1}{g}\,;$$

note the implicit definition of $\hat{N}(\cdot)$. Because $P(\cdot)$ is concave, $\hat{N}(\cdot)$ is an increasing function.

To close the model, assume that the manager has, as an alternative to working for the firm in question, an opportunity that would yield him utility $\underline{U}$. Normalize this reservation utility to zero (*i.e.,* $\underline{U}=0$). Conditional on $V(w_s)-V(w_f)=g$, the firm will set w_s and w_f as low as possible, which means the manager's participation constraint,

$$P\big(\hat{N}(g)\big)\underbrace{\big(V(w_s)-V(w_f)\big)}_{g}+V(w_f)+Y-\hat{N}(g)\geq 0\,,$$

[13] Via Lemma 2, it is readily shown that $v_{12}>0$ implies Assumption 1.

is binding. It follows that

$$w_f(g) = V^{-1}\left(\hat{N}(g) - Y - P\big(\hat{N}(g)\big)g\right) \text{ and}$$
$$w_s(g) = V^{-1}\left(g + \hat{N}(g) - Y - P\big(\hat{N}(g)\big)g\right).$$

The expected cost of providing g in incentives is, therefore,

$$C(g) = P\big(\hat{N}(g)\big)\big(w_s(g) - w_f(g)\big) + w_f(g).$$

Lemma 4. *The function $C(\cdot)$ is increasing.*

Finally, suppose that owners get τ if the manager is successful and 0 if he fails.[14] The benefit function is, thus, $B(N,\tau) = \tau P(N)$. Let $c(N)$ denote the minimum cost to the owners of inducing the manager to divert only $S = Y - N$. Note $c(N)$ is the manager's expected compensation, which by Lemma 4 is an increasing function of N. Because $\hat{N}(g)$ is increasing in g it is invertible, so a higher N also means the manager has more high-powered incentives. Given that $B_{12}(\cdot,\cdot) > 0$ and $B_2(\cdot,\cdot) > 0$ when $B(N,\tau) = \tau P(N)$, it follows that N, and thus expected compensation and the power of the manager's incentives will (i) be non-decreasing in τ by Lemma 2; and (ii) that therefore there will be a non-negative correlation between firm profits and managers' expected compensation and between profits and strength of incentives. By Lemma 2 "non-decreasing" and "non-negative" can be replaced by "increasing" and "positive" if the owners' problem always admits an interior solution (for instance, if $c(\cdot)$ is convex and $c'(0) < \tau P'(0)$ for almost every $\tau \in \mathcal{T}$). To summarize:

Proposition 6. *Under the agency model presented here in which the owners earn profit τ if the manager is successful and 0 if he fails, an increase in the relative value of success (i.e., τ) means, in the new equilibrium, that*

(i) resources allocated to productive use, N, are weakly greater than before;

(ii) the manager's expected compensation, $c(N)$, is weakly greater than before; and

(iii) the power of the manager's incentives, g, is weakly greater than before.

If the owners' problem has an interior solution for all $\tau \in \mathcal{T}$, then "weakly" can be replaced by "strictly." Furthermore, the following correlations will hold:

(iv) Firm profits and managerial compensation will be non-negatively correlated; and

(v) Firm profits and the power of managerial incentives will be non-negatively correlated.

[14]The amount τ could either be profit or the gain in the value of the firm (*i.e.,* if τ is the present value of the returns generated by success).

If the owners' problem has an interior solution for all $\tau \in \mathcal{T}$, then "non-negatively" can be replaced by "positively."

Proposition 6 holds two important implications for empirical analysis. One concerns the cross-sectional relation between compensation and performance, the other the cross-sectional relation between compensation and firm size. With respect to the first, the proposition predicts that there will be a positive correlation between managerial compensation and the financial performance of the firm. Firms that are likely to be more profitable (*e.g.,* higher τ firms) will have both a higher level of g and a higher probability of paying it. To understand the possible implications for empirical analysis, consider the regression

$$\text{Compensation}_i = \delta_0 + \delta_1 \text{Firm Performance}_i + \eta_i \,, \tag{17}$$

where i indexes firms, the δs are coefficients to be estimated, η_i is an error term, and an explicit list of other controls is omitted for the sake of simplicity.[15] The analysis above indicates that regression estimate of δ_1 would be positive. How should such a finding be interpreted? Observe that the manager of a more profitable firm has greater expected compensation than the manager of a less profitable firm solely because he was employed by a firm that anticipated being more profitable. In other words, his expected compensation is due to the inherent profitability of the firm that employes him.

Another issue this analysis points out is that, because different type firms will have different values of δ_0 and δ_1, heterogeneity across firms could make (17) a questionable specification. To appreciate this point, for each firm, its δ_0 is $w_f(g)$ and its δ_1 is $\big(w_s(g) - w_f(g)\big)/\tau$. Because g varies with τ (Proposition 6(iii)), the coefficients are varying with τ and hence either or both are not constant across firms as specification (17) assumes.

This last discussion leads to a final comment about (17). There has been a lengthy debate about whether executive compensation is sufficiently sensitive to firm performance (see, *e.g.,* Jensen and Murphy, 1990, Haubrich, 1994, Hall and

[15] This specification is somewhat agnostic as to what measure of firm performance is used. Given the simple model here, firm performance could simply be profit. Alternatively (see discussion in footnote 14 *supra*), it could be the increase in firm value (change in stock price). Note the precise measure is not crucial to the point being made. For instance, suppose firm performance were profit. One might object to this specification, arguing it would be better to regress compensation (or the change in compensation) on changes in stock price; that is,

$$\text{Compensation}_i = \delta_0 + \alpha_1 \Delta \text{Stock Price}_i + \eta_i \,.$$

That objection is readily answered: Typically, the change in stock price is a function of how realized earnings (profit) deviates from expected earnings. Taking the change to be proportional to the deviation from expectation, this regression is equivalent to

$$\text{Compensation}_i = \delta_0 + \delta_1 (\text{Profit}_i - \overline{\text{Profit}_i}) + \eta_i \,,$$

where $\overline{\text{Profit}}$ is expected profit. That term can be, for instance, proxied by lagged profit and thus go into the other, non-listed controls; hence, (17) attains. If the left-hand side variable is change in compensation, then past compensation can be made a control, again yielding (17).

For purpose of this discussion Compensation_i and $\text{Firm Performance}_i$ are in levels. As will become clear, a specification in logs would not change the conclusions of this analysis.

Coefficient	Estimate ($V(w) = w$ case)	Estimate ($V(w) = -1/w$ case)
δ_0	-191.6	.0327
	(-7329.)	(399.6)
δ_1	.5855	1.997×10^{-3}
	(194.9)	(247.1)
R^2	.6550	.7532

Table 1: Estimation of equation (17) using data generated as described in the text. Dependent variable is realized compensation. Independent variable is realized shareholder gain (gross of compensation), its coefficient is δ_1. The coefficient δ_0 is the intercept. Numbers in parentheses are t-statistics.

Liebman, 1998, Hermalin and Wallace, 2001). A rough summary of the debate is that it asks whether δ_1 is big enough. But δ_1 could be the wrong measure on which to focus: The strength of the manager's incentives are reflected by g, not δ_1. Moreover, the estimated δ_1 will likely be biased.[16]

To illustrate the problems with specification (17), the following simulations were done using the specification $P(N) = \frac{N}{N+1}$ and $V(w) = w$ or $V(w) = -1/w$.[17] Data were created for 20,000 firms as follows. For each firm, its τ was a random draw from the Pareto distribution $\tau \sim 1-(6/\tau)^3 : [6, \infty)$. Gabaix and Landier (2008) provide evidence for why a Pareto distribution reflects reality. Then, for each firm, the optimal contract was calculated, as was the N its manager would choose in equilibrium. Whether that firm was successful or not was determined by whether a uniformly distributed random variable on the unit interval was less than $P(N)$—successful—or was above $P(N)$—failure. Once the data were constructed, equation (17) was estimated. The results are shown in Table 1.[18]

The estimated δ_1, while estimated with great accuracy, is a poor measure of actual incentives in these samples. For any given firm, the true coefficients solve

$$w_f = \delta_0 + \delta_1 \times 0 \quad \text{and} \quad w_s = \delta_0 + \delta_1 \tau \, . \tag{18}$$

Hence a firm's true δ_1 is given by

$$\delta_1 = \frac{w_s - w_f}{\tau} \, .$$

For the case in which $V(w) = w$, calculations reveal that the true $\delta_1 \equiv 1$ regardless of τ. Hence, in this case, the estimated value, .5855, understates

[16]Hermalin and Wallace also present reasons, different than those discussed here, for why the δ_1 estimated from (17) using cross-sectional data could be a biased-downward measure of incentive strength.

[17]In both cases, Y was set large enough to ensure the constraint $N \leq Y$ didn't bind and, for the case, $V(w) = -1/w$, so that w_f and w_s would never be negative.

[18]The Mathematica program used to generate the data is available from the author upon request.

the actual strength of incentives for *every* firm in the sample. Indeed, it does so by almost a factor of two! Nor is this a consequence of assuming no risk aversion: For the case, in which $V(w) = -1/w$, the estimated δ_1 equals the δ_1 of a firm with $\tau \approx 59.23$. Because $\delta_1(\tau)$ is a *decreasing* function of τ (the difference $w_s - w_f$ grows slower than τ given the assumed functional forms), this means all firms with $\tau < 59.23$ will have greater coefficients. Given the assumed distribution, the probability that $\tau < 59.23$ is .999; that is, nearly every firm has a greater δ_1 than the estimated one.

The two-outcome model is a useful model because, given (18), if one knew the true δ_0 and δ_1 for each firm, one can easily back out the true underlying contract. On the other hand, two outcomes may not be especially realistic. Fortunately, the number of outcomes is not what is driving these results. In Appendix B, I show that the results from estimating (17) are similarly misleading even when there is a continuum of outcomes. In particular, the estimated coefficient δ_1 remains biased downward (see Table 2 and connected discussion *infra*).

Why are cross-sectional regressions prone to yielding estimates that understate the strength of incentives? Figure 2 illustrates the problem. High-type firms will have strong incentives and, hence, in equilibrium their managers' expectation is that they will receive a large payoff due to these incentives (although, if unlucky, their compensation could be low). Consequently, these firms do not need to offer particularly high levels of "base pay" (*i.e.,* w_f will tend to be low) in order to satisfy their managers' participation constraint. Low-type firms, in contrast, will have weak incentives and, hence, in equilibrium their managers' expectation is that the bulk of their pay is base pay (*i.e.,* w_f). To satisfy their managers' participation constraints, base pay will have to be sufficiently high. Because low-type firms' managers will tend to perform more poorly, while high-type managers will tend to perform better, the data will largely consist of observation pairs of (i) poor performance and high base pay; and (ii) good performance and high incentive pay. Since, in a loose sense, all pay is "high," the estimated regression line will tend to be "flat," understating the true incentives of most or all firms in the sample.

A second cross-sectional relation implied by Proposition 6 is the following. Suppose firm size is positively correlated with firm profits.[19] Given firms that will be more profitable (higher τ firms) pay more than their counterparts (Proposition 6), the following result obtains.

Corollary 3. *If firm size is positively correlated with firm profits, then there will be a positive correlation between firm size and managerial compensation under the assumptions of Proposition 6.*

That there is a positive correlation between firm size and executive compensation is a well-documented phenomenon (see, among others, Baker et al., 1988, Rose and Shepard, 1997, and Frydman and Saks; Gabaix and Landier,

[19] In empirical analyses, firm size is often measured as market value (see, *e.g.,* Frydman and Saks, in press), which, as the present discounted value of profits, should be correlated with profits.

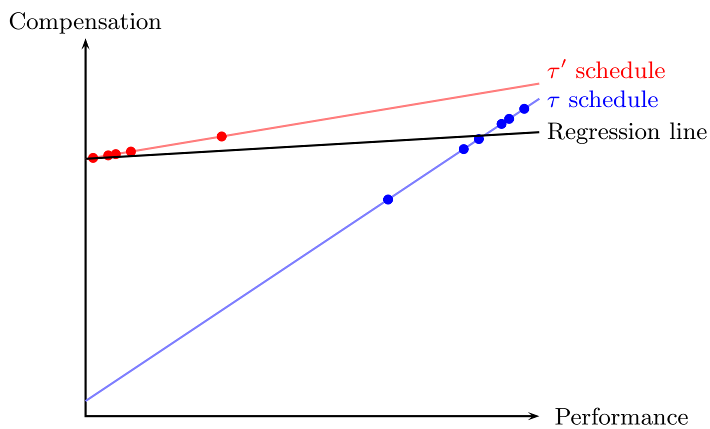

Figure 2: Heuristic illustration of downward bias in estimates of incentive strength. Two firm types, $\tau > \tau'$. The schedules are the contracts offered by the two types of firms. Small circles indicate observations; those on the τ schedule are observations from τ-type firms and those on the τ' schedule are from τ'-type firms. The estimated regression line is flatter than either schedule.

2008, provide a brief survey of the literature). Corollary 3 offers a potential explanation for this phenomenon.[20]

The amount of firm resources, Y, is an alternative measure of size. The following proposition provides comparative statics when Y varies, but τ is fixed (to improve readability, τ is, thus, suppressed in the following).

Proposition 7. *Assume firms don't vary in type, but have different levels of resources, Y. A manager of firm with a higher value of Y is paid at least as much in expectation as a manager of a firm with a lower value of Y; moreover, there is an interval of Y, starting at 0, such that the manager's expected compensation is strictly increasing with Y.*

Proof: A firm's profit can be written as

$$\hat{\pi} = B\big(\hat{N}(g)\big) - C(g)\,. \tag{19}$$

Because $\hat{N}(\cdot)$ is strictly increasing, it is invertible and, hence, (19) can be rewritten as

$$\hat{\pi} = B(N) - c(N)\,, \tag{20}$$

where $c(N) = C\big(\hat{N}^{-1}(N)\big)$. The firm's objective is to maximize (20) with respect to N subject to the boundary constraint $N \leq Y$. For Y small, the constraint binds, so relaxing it means a larger N, which in turn means a larger g, which in turn means higher expected compensation, $C(g)$. ■

Corollary 4. *Under the assumptions of Proposition 7, there is a positive correlation between firm size, measured as available resources (assets), and managerial compensation.*

5 Governance as a Multi-Dimensional Problem

There are multiple dimensions to governance. There is board structure, compensation, shareholder activism, and so forth. Heretofore, however, governance has been treated as a scalar, g. In this section, the model is extended to allow governance to be a vector, $\mathbf{g} \in \mathbb{R}^n_+$, $n > 1$.

Return to the model of Section 2 and let the manager's utility be

$$u = S + v(Y - S, \mathbf{g})\,.$$

Assume that $v(\cdot,\cdot)$ continues to satisfy conditions (1)–(4), with $\mathbf{g}$ substituted for g. In lieu of Assumption 1, assume

$$v_{1j}(\cdot,\cdot) > 0 \text{ for all } 2 \leq j \leq n+1\,; \tag{21}$$

[20] As such, Corollary 3 complements other explanations that have been offered in the literature. Two such explanations are models based on the distribution of managerial talent (Terviö, 2008, and Gabaix and Landier); and the correlation between firm size, hierarchical depth, and pay at the top of the hierarchy (Calvo and Wellisz, 1979). It is worth noting that both these explanations rely, in part, on heterogeneity in managerial ability whereas Corollary 3 does not.

that is, an increase in any dimension of governance lowers the marginal benefit of diverting resources.

Lemma 1′. *For all governance levels, $\mathbf{g} \in \mathbb{R}^n_+$, there exists an amount $Y(\mathbf{g})$, such that, in equilibrium, the manager diverts a positive amount if and only if total resources exceed $Y(\mathbf{g})$ (i.e., iff $Y > Y(\mathbf{g})$). The equilibrium amount of diversion is $S = \max\{Y - Y(\mathbf{g}), 0\}$. Moreover, $Y(\cdot)$ is strictly increasing and differentiable in each argument (i.e., $\partial Y/\partial g_j$ exists and is positive).*

As before, to save having to deal with corner solutions, assume $v_1(0, \mathbf{0}) = 1$, so $Y(\mathbf{0}) = 0$.

Similar to Section 2, the analysis in this section does not depend on $Y(\mathbf{g}) = Y$ having a solution in $\mathbf{g}$ for all Y. Assuming it does, however, simplifies the analysis greatly. Hence, assume

$$\forall y \; \exists \mathbf{g} = (g_1, \ldots, g_n) \text{ such that } g_j < \infty \; \forall j \text{ and } v_1(y, \mathbf{g}) \geq 1\,. \tag{22}$$

Let the owners' profits be

$$B(Y - S, \tau) - C(\mathbf{g})\,,$$

where the previous assumptions hold and $C(\cdot)$ is strictly increasing in each of its arguments. Because increasing $\mathbf{g}$ along any dimension is costly, the owners will never choose a $\mathbf{g}$ such that $Y(\mathbf{g}) > Y$. Define

$$\mathcal{G}(Y) = \left\{\mathbf{g} \middle| Y(\mathbf{g}) \leq Y\right\}.$$

In light of the assumption that $v_1(0, \mathbf{0}) = 1$, condition (22), and the continuity of $Y(\cdot)$ as established by Lemma 1′, $\mathcal{G}(Y)$ is compact. The owners problem is, thus,

$$\max_{\mathbf{g} \in \mathcal{G}(Y)} B\big(Y(\mathbf{g}), \tau\big) - C(\mathbf{g})\,.$$

Because $\mathcal{G}(Y)$ is compact and all functions are continuous, at least one solution must exist. Let $\mathbf{g}(\tau)$ be the solution adopted by a type-τ firm.

The main comparative static result is the following:

Proposition 8. *Higher-type firms spend at least as much on governance as do lower-type firms (i.e., if $\tau > \tau'$, then $C\big(\mathbf{g}(\tau)\big) \geq C\big(\mathbf{g}(\tau')\big)$). Moreover, if a lower-type firm has not blocked all resource diversion (i.e., $Y\big(\mathbf{g}(\tau')\big) < Y$), then the higher-type firm spends strictly more (i.e., $C\big(\mathbf{g}(\tau)\big) > C\big(\mathbf{g}(\tau')\big)$).*

Recall the interpretation set forth in Section 2 that g be thought of as the expenditure on governance; that is, $g = C(\mathbf{g})$. In light of Proposition 8, one is free to view the owners as solving a two-step process: first, for each y solve the problem

$$\min_{\mathbf{g}} C(\mathbf{g}) \text{ subject to } Y(\mathbf{g}) = y\,.$$

Let $\mathbf{g}(y)$ denote the solution. Then associate to each y a $g \equiv C(\mathbf{g}(y))$; this yields a one-to-one strictly monotonic mapping. This mapping can be inverted to yield a function mapping g into y. By construction, that function is equivalent to the $Y(g)$ function used in Section 2. Observe, in this case, the cost of g is just g.[21]

A related question is whether higher-type firms employ stronger governance on all dimensions than lower-type firms; that is, does $\tau > \tau'$ imply $\mathbf{g}(\tau) \geq \mathbf{g}(\tau')$, where the order over vectors is the usually piecewise ordering? It is readily shown this cannot be true generally. For instance, suppose $n = 2$, $v(Y - S, \mathbf{g}) = v\big(Y - S, \max\{g_1, g_2\}\big)$, and

$$C(\mathbf{g}) = g_1 + \frac{1}{2}g_2 + \frac{3}{2}\left(g_2 - \min\left\{g_2, \frac{2}{3}\right\}\right),$$

then the optimal $\mathbf{g}$ to achieve effective governance level $g = \max\{g_1, g_2\}$ is $(0, g)$ for $g \leq 1$ and $(g, 0)$ for $g > 1$. Hence, if $g(\tau') < 1 < g(\tau)$, then $\mathbf{g}(\tau')$ and $\mathbf{g}(\tau)$ cannot be compared (*i.e.,* neither $\mathbf{g}(\tau) \geq \mathbf{g}(\tau')$ nor $\mathbf{g}(\tau') \geq \mathbf{g}(\tau)$ are true).

In the preceding example, the two dimensions of governance are perfect substitutes. If the dimensions of governance are complements, then the desired implication, $\tau > \tau' \Rightarrow \mathbf{g}(\tau) \geq \mathbf{g}(\tau')$, follows from Topkis's Monotonicity Theorem (Milgrom and Roberts, 1990, p. 1262):

Proposition 9. *Suppose that the manager's* marginal *benefit from behaving in a manner desired by the owners, $v_1(y, \mathbf{g})$, exhibits complementarities in governance; specifically, assume it is supermodular in $\mathbf{g}$. Suppose it also exhibits increasing differences; that is, if $y > y'$ and $\mathbf{g} \geq \mathbf{g}'$, then*

$$v_1(y, \mathbf{g}) - v_1(y', \mathbf{g}) > v_1(y, \mathbf{g}') - v_1(y', \mathbf{g}').$$

Finally suppose that the marginal cost of governance in one dimension is non-increasing in any other dimension (i.e., $\partial^2 C(\mathbf{g})/(\partial g_i \partial g_j) \leq 0$, $i \neq j$).[22] *Then the governance of a higher-type firm on any given dimension is no less than that of a lower-type firm on that dimension; that is, $\tau > \tau'$ implies $\mathbf{g}(\tau) \geq \mathbf{g}(\tau')$.*

Observe that if (i) $v(y, \mathbf{g})$ has the form

$$v(y, \mathbf{g}) = \gamma(\mathbf{g})\upsilon(y) \tag{23}$$

plus, possibly, additional terms with zero cross-partial derivatives in g_i and y for all i; if (ii) $\gamma(\cdot)$ is increasing in its arguments, with positive cross-partial derivatives; and if (iii) $\upsilon(\cdot)$ is increasing, then the conditions on $v_1(y, \mathbf{g})$ set forth in Proposition 9 all hold. Observe further that (23) has the following

[21] To rule out corner solutions at no governance (*i.e.,* $g = 0$) it was assumed in Section 2 that $C'(0) = 0$. Under the interpretation here, $C'(g) \equiv 1$. Hence, an alternative solution is needed to rule out solutions; one such assumption would be $B_1(0, \tau) > 1$ for all τ.

[22] Observe this condition would hold if $C(\mathbf{g})$ were additive across the dimensions; that is, if

$$C(\mathbf{g}) = \sum_{i=1}^{n} c_i(g_i),.$$

where $c_i(\cdot)$ is strictly increasing for all i.

interpretation: given a choice of $\mathbf{g}$, the firm has an "effective" level of governance $\gamma(\mathbf{g})$.[23] The owners' problem can, thus, be seen as choosing, for each effective level of governance γ, the cost-minimizing vector $\mathbf{g}(\gamma)$. The cost of such an effective level is $C\big(\mathbf{g}(\gamma)\big) \equiv \hat{C}(\gamma)$. Viewing γ as the equivalent of g in Sections 2 and 3, the analysis in those earlier sections can be seen as short-hand for a more elaborate model in which the owners set governance on many dimensions in a cost-minimizing way to achieve an effective level of governance (the g in those sections).

Whether different dimensions of governance are complements or substitutes is an empirical question. This question does not seem to have attracted much attention. A partial exception is Hermalin and Wallace (2001), which studies, *inter alia*, whether firms base incentive compensation on the same measures or different measures. They find evidence that if a firm heavily weights one measure, it will tend not to weight another; whereas if a firm heavily weights the other, it will tend not to weight the one. With respect to compensation, these findings support a view that dimensions of governance are substitutes. On the other hand, they neglect many other dimensions, so the overall issue of complements versus substitutes must be seen as open.

6 Trends in Governance

There have been numerous trends in corporate governance over the past twenty to thirty years (see, *e.g.,* Becht et al. and Holmstrom and Kaplan, 2001, for surveys). As noted above, the proportion of outside directors on boards has steadily increased in the United States and other countries (Borokhovich et al.; Dahya et al.; and Huson et al.). There has been growing use of stock-based incentives for directors over the period 1989 to 1997 (Huson et al.). Kaplan and Minton (2008) find evidence that CEO turnover rates in the period 1998–2005 are significantly greater than in the period 1992–1998; and the rate in that period is greater than found in studies for the pre-1992 period. They interpret this as evidence of better monitoring by boards of directors. Consistent with this interpretation is the finding of Huson et al. that firings, as a percentage of all CEO successions, were trending upward in the period 1971 to 1994. These trends can all be interpreted as evidence that governance has been getting stronger over the past twenty to thirty years.

At the same time, there is evidence that firms' profit potential and resources have been increasing during this period. As Gabaix and Landier note, there has been a six-fold increase in the market value of the top 500 US firms between 1980 and 2003. From 1973 to 2003, there has been a three-fold increase in patents granted in the US; and, since the late 1980s, evidence of increased productivity in R&D (Hall, 2004). Technological progress has been remarkable in this period, especially with respect to information technology and telecom-

[23] The effective level of governance is increasing in each dimension of governance and the *marginal* effective level in any one dimension is increasing in any other dimension (*e.g.,* $\gamma(\cdot)$ exhibits complementarities).

munications. Since the mid-1990s, there has been a growth in productivity that has not resulted in a significant increase in wages (DeLong, 2003).

The analysis in Sections 2–4 offers a way of tying these two trends together. As the potential profitability and resources of firms increased, the value of improved governance also rose. Consequently, governance got stronger (on average—the model does not predict any kind of convergence across firms).

This is not to say the process was necessarily smooth. As noted by many authors, one might expect management to resist improved governance. This resistance could have led to more aggressive forms of change, such as the takeovers, leveraged buyouts, and proxy fights that characterized the 1980s. It could also have motivated shareholders to seek change through legislation or changes in the listing requirements of exchanges. But over time, as suggested by Holmstrom and Kaplan, a new equilibrium with stronger governance has emerged.

Tying the change in governance to changes in the resources and potential of firms also serves to explain why changes in governance occurred when they did. After all, commentators have been complaining about the state of governance for a long time (consider, *e.g.,* Berle and Means, 1932), so presumably something had to occur to motivate action. Until the point that the payoff from improved governance made imposing it worthwhile, investors were not willing to walk the talk.

The model set forth above offers a broader explanation for change than that set forth by Terviö or Gabaix and Landier, which are concerned with executive compensation only; moreover, their models suggest a relatively smooth process in which growth in firm size increases executive compensation.[24] Like the model here, Hermalin (2005) offers an explanation for improvements, broadly, in governance, but his explanation is based on the rise of institutional investors.[25] His explanation can be seen as complementary to the one set forth here insofar as greater in holdings by institutional investors can be seen as a rise in τ—as a larger proportion of profits accrue to these investors, their incentive to push for stronger governance increases.

7 Conclusions

This paper has sought to explain why a pattern should exist between strength of governance and firm performance. Given that governance should be constrained optimal, there is not necessarily an obvious reason why the two should be correlated at all, to say nothing of positively correlated. This paper offers, as an explanation for the demonstrated positive correlation, the idea that firms in which the marginal value of resources are greatest will have face the greatest opportunity cost of misuse of resources and, thus, have the greatest desire for

[24] Another explanation primarily focused on executive compensation is the model of Murphy and Zábojník (2003), which argues that changes in the CEO labor market, specifically a greater emphasis on general versus firm-specific knowledge, explains the rise in CEO compensation.

[25] Huson et al. report that the percentage of US equity held by institutional investors has increased from 20% in 1971 to 45% in 1994. Gompers and Metrick (2001) report a similar doubling from 1980 to 1996.

strong governance. Because, under mild conditions, a greater marginal value to resources implies greater overall profitability, a positive correlation will exist between firm performance and the strength of corporate governance.

This insight holds important implications for empirical work in corporate governance and, to an extent, for the study of organizations generally. The endogenous characteristics of an organization are, presumably, chosen to facilitate the organization's objectives. If organizations are behaving optimally, then variation in how well they do are not necessarily explained by variation in their characteristics. However, as shown here, the variation in their characteristics could well be tied to variation in the potentials they have to do well.

What might these insights hold for empirical work? One course suggested by this paper is to consider exogenous firm attributes that plausibly predict profitability and see whether they predict patterns in governance. A second course is, for some aspects of governance such as compensation, to make greater use of panel data and employ random-coefficient or similar models to estimate firm-specific coefficients. For instance, in the pay-for-performance regression (17), estimate the coefficients δ_0 and δ_1 on a firm-by-firm basis.[26] A third course is to examine the consequences of regulated changes that are binding on some firms (*e.g.,* those resulting from the Sarbanes-Oxley Act). If firms were optimizing prior to the regulated change, then those firms for which the regulations are binding should suffer poorer performance subsequent to the regulations than firms for which the regulations were not binding (*i.e.,* than firms that were meeting the regulations prior to their enactment).

Beyond empirical work, future research may wish to model the dynamics of governance change. Although some sense of how the model might be extended to a dynamic setting was given in Section 6, this is far from a complete analysis. Moreover, in a dynamic setting, many models presume managers can entrench themselves or otherwise gain influence over how they are governed (the survey by Becht et al. discusses such models). In particular, as the bargaining theory of governance (set forth, *e.g.,* by Hermalin and Weisbach) suggests, a consequence of past good performance is managers' bargaining for looser governance. In terms of the model presented above, this suggests that the cost of governance (*i.e.,* $C(g)$) could rise over time if the same management team remains in office. Particularly if the *marginal* cost increases, then governance strength will decline and, consequently, so will profits.

Other factors that influence the play of a dynamic game would be adjustment costs related to governance. In short, numerous issues will arise when this analysis is extended to a dynamic framework. Nonetheless, the basic messages of the paper concerning causality and the importance of firm heterogeneity are unlikely to be overturned by such an extension.

[26] Hermalin and Wallace essentially employ this approach in their study of pay for performance; they find evidence for a much stronger pay-for-performance relationship using this approach than suggested by a cross-sectional analysis of the same data.

Appendix A: Technical Notes and Proofs

Proof of Lemma 1: Fix a g. Given (3), (4), and the continuity and monotonicity of $v_1(\cdot, g)$, there exists a $Y(g) \in (0, \infty)$ such that

$$v_1\big(Y(g), g\big) = 1\,. \tag{24}$$

The manager's problem is

$$\max_{S \geq 0} S + v(Y - S, g)\,.$$

$$\text{The derivative} = 1 - v_1(Y - S, g) \begin{cases} < 0\,, & \text{if } Y - S < Y(g) \\ > 0\,, & \text{if } Y - S > Y(g) \end{cases}\,,$$

where the inequalities follow from (24) because $v(\cdot, g)$ is strictly concave for all g. It follows the manager does best to set $S = 0$ if $Y < Y(g)$ and $S > 0$ if $Y > Y(g)$. In the latter case, it is readily seen the manager's optimal $S = Y - Y(g)$. The moreover part follows from Assumption 1: Consider $g > g'$. Let $Y \geq \max\big\{Y(g), Y(g')\big\}$. The amount $S = Y - Y(g)$ is an interior maximum for the manager given governance level g and resource level Y and similarly for $S' = Y - Y(g')$. By Assumption 1, $S < S'$, implying $Y(g) > Y(g')$. That $Y(\cdot)$ is differentiable follows from (24) given the implicit function theorem. ■

Proof of Lemma 2: By the definition of an optimum (revealed preference):

$$f(\hat{x}, z) \geq f(\hat{x}', z) \text{ and} \tag{25}$$

$$f(\hat{x}', z') \geq f(\hat{x}, z')\,. \tag{26}$$

Expressions (25) and (26) imply

$$\begin{aligned} 0 &\leq \big(f(\hat{x}, z) - f(\hat{x}', z)\big) - \big(f(\hat{x}, z') - f(\hat{x}', z')\big) \\ &= \int_{\hat{x}'}^{\hat{x}} \big(f_1(x, z) - f_1(x, z')\big) dx = \int_{\hat{x}'}^{\hat{x}} \left(\int_{z'}^{z} f_{12}(x, \zeta) d\zeta \right) dx\,, \end{aligned}$$

where the integrals follow from the fundamental theorem of calculus. The inner integral in the rightmost term is positive because $f_{12}(\cdot, \cdot) > 0$ and the direction of integration is left to right. It follows that the direction of integration in the outer integral must be weakly left to right; that is, $\hat{x}' \leq \hat{x}$. To establish the moreover part, because $f_1(\cdot, \zeta)$ is a differentiable function for all ζ, if $\hat{x}'$ is an interior maximum, then it must satisfy the first-order condition

$$0 = f_1(\hat{x}', z')\,.$$

Because $f_{12}(\cdot, \cdot) > 0$ implies $f_1(\hat{x}', z) > f_1(\hat{x}', z')$, it follows that $\hat{x}'$ does not satisfy the necessary first-order condition for maximizing $f_1(x, z)$. Therefore $\hat{x}' \neq \hat{x}$; so, by the first half of the lemma, $\hat{x}' < \hat{x}$. ■

Proof of Lemma 3: Define $\overline{g}$ as the solution to $Y(g) = Y$ should such a solution exist; that is, $\overline{g} = G(Y)$. There are two cases to consider, one in which $\overline{g}$ exists and one in which it does not.

Suppose $\overline{g}$ exists. As noted in the text, the owners will never choose a level of governance greater than $\overline{g}$ (recall $Y(\cdot)$ is increasing). Hence, in this case, the owners' problem is equivalent to

$$\max_{g\in[0,\overline{g}]} B\bigl(Y(g),\tau\bigr) - C(g)\,.$$

Because $[0,\overline{g}]$ is compact and all functions continuous, at least one solution must exist.

Suppose $\overline{g}$ doesn't exist. Given $Y(0) = 0$ and $Y'(\cdot) > 0$, this means $Y(g) < Y$ for all $g \in \mathbb{R}_+$. From the fundamental theorem of calculus,

$$Y > \int_0^g Y'(z)dz \tag{27}$$

for all g. It follows that $Y'(g) \to 0$ as $g \to \infty$. Define $\beta = \max_{x\in[0,Y]} B_1(x,\tau)$. Because this is again optimization of a continuous function over a compact space, such a maximum exists. Consider $g > \underline{g}$. It follows that the derivative of (6) satisfies

$$B_1\bigl(Y(g),\tau\bigr)Y'(g) - C'(g) \le \beta Y'(g) - \xi\,. \tag{28}$$

If the rightmost term in (28) is negative for all $g > \underline{g}$, define $\tilde{g} = \underline{g}$. If the rightmost term in (28) is not negative for all $g > \underline{g}$, consider only $g > g_0$, where $g_0 < \infty$ is such that $Y'(g) < \xi/\beta$ for all $g > g_0$ (such a g_0 must exist because $Y'(g) \to 0$ as $g \to \infty$). Then, for all $g > g_0$, (28) implies

$$B_1\bigl(Y(g),\tau\bigr)Y'(g) - C'(g) < 0\,.$$

For this case, define $\tilde{g} = g_0$. Because the derivative of the owners' profit function is negative in governance level for all $g > \tilde{g}$, the owners would never wish to choose such a g and, hence, there is no loss in defining their problem as

$$\max_{g\in[0,\tilde{g}]} B\bigl(Y(g),\tau\bigr) - C(g)\,.$$

Because $[0,\tilde{g}]$ is compact and all functions continuous, at least one solution must exist. ■

Proof of Lemma 4: Given that $\hat{N}(\cdot)$ is increasing, it is sufficient to show that the standard agency problem of implementing N at minimum cost yields a cost function $c(N)$ that is increasing in N. The standard problem is

$$\min_{\{v_s,v_f\}} P(N)V^{-1}(v_s) + \bigl(1-P(N)\bigr)V^{-1}(v_f) \tag{P}$$

subject to

$$P'(N)(v_s - v_f) - 1 = 0 \text{ and} \tag{IC}$$

$$P(N)v_s + \bigl(1-P(N)\bigr)v_f + Y - N \ge 0\,. \tag{IR}$$

The solution is readily shown to be

$$v_f = -\frac{P(N)}{P'(N)} + N - Y \quad\text{and}\quad v_s = \frac{1-P(N)}{P'(N)} + N - Y\,.$$

It follows that

$$c(N) = P(N)V^{-1}\left(\frac{1-P(N)}{P'(N)} + N - Y\right) + \bigl(1-P(N)\bigr)V^{-1}\left(-\frac{P(N)}{P'(N)} + N - Y\right).$$

Observe

$$P(N)\frac{1-P(N)}{P'(N)} + \bigl(1-P(N)\bigr)\frac{-P(N)}{P'(N)} \equiv 0\,. \tag{29}$$

The result will be shown to be a consequence of the following:

Lemma A.1. *Consider the following differentiable functions from $\mathbb{R}$ to $\mathbb{R}$: q, a, b, and f, where f is increasing and convex, q is increasing, b is decreasing, and $a(x) > b(x)$ for all x. Then*

$$q(x)a(x) + \bigl(1-q(x)\bigr)b(x) \equiv k \tag{30}$$

for all x, k a constant, implies

$$q(x)f\bigl(a(x)\bigr) + \bigl(1-q(x)\bigr)f\bigl(b(x)\bigr) \tag{31}$$

is increasing in x.

Proof of Lemma A.1: Differentiating (30) with respect to x implies

$$q'(x)\bigl(a(x)-b(x)\bigr) + q(x)a'(x) + \bigl(1-q(x)\bigr)b'(x) \equiv 0 \tag{32}$$

for all x. The derivative of (31) with respect to x is

$$q'(x)\Bigl(f\bigl(a(x)\bigr) - f\bigl(b(x)\bigr)\Bigr) + q(x)f'\bigl(a(x)\bigr)a'(x) + \bigl(1-q(x)\bigr)f'\bigl(b(x)\bigr)b'(x)\,.$$

Because $f'\bigl(a(x)\bigr) > 0$, its derivative has the same sign as

$$q'(x)\frac{f\bigl(a(x)\bigr) - f\bigl(b(x)\bigr)}{f'\bigl(a(x)\bigr)} + q(x)a'(x) + \bigl(1-q(x)\bigr)\frac{f'\bigl(b(x)\bigr)}{f'\bigl(a(x)\bigr)}b'(x)\,. \tag{33}$$

Because $f(\cdot)$ is convex, $a(x) > b(x)$, and $b'(x) < 0$, expression (33) is greater than

$$q'(x)\frac{f\bigl(a(x)\bigr) - f\bigl(b(x)\bigr)}{f'\bigl(a(x)\bigr)} + q(x)a'(x) + \bigl(1-q(x)\bigr)b'(x)\,. \tag{34}$$

Because $f(\cdot)$ is convex, it exceeds a first-order Taylor approximation of it:

$$f\bigl(b(x)\bigr) \geq f\bigl(a(x)\bigr) + f'\bigl(a(x)\bigr)\bigl(b(x)-a(x)\bigr)\,.$$

Hence,

$$\frac{f\bigl(a(x)\bigr) - f\bigl(b(x)\bigr)}{f'\bigl(a(x)\bigr)} \geq a(x) - b(x)\,.$$

Consequently, (34) is no less than the left-hand side of (32) and, thus, no less than 0. By transitivity, (33) is greater than zero. ■

Continuation of Proof of Lemma 4: With respect to Lemma A.1, q corresponds to P, a to $\big(1-P(N)\big)/P'(N)$, b to $-P(N)/P'(N)$, and f to the function defined by

$$z \mapsto V^{-1}(z+n-y)\,.$$

Note

$$\frac{d}{dN}\left(-\frac{P(N)}{P'(N)}\right) = -1 + \frac{P(N)P''(N)}{P'(N)^2} < 0$$

(recall $P''(\cdot) < 0$). Consider $N > N'$. Invoking Lemma A.1 yields the chain

$$\begin{aligned}
&c(N') \\
&= P(N')V^{-1}\left(\frac{1-P(N')}{P'(N')} + N' - Y\right) + \big(1-P(N')\big)V^{-1}\left(-\frac{P(N')}{P'(N')} + N' - Y\right) \\
&< P(N)V^{-1}\left(\frac{1-P(N)}{P'(N)} + N' - Y\right) + \big(1-P(N)\big)V^{-1}\left(-\frac{P(N)}{P'(N)} + N' - Y\right) \\
&< P(N)V^{-1}\left(\frac{1-P(N)}{P'(N)} + N - Y\right) + \big(1-P(N)\big)V^{-1}\left(-\frac{P(N)}{P'(N)} + N - Y\right) \\
&= c(N)\,, \qquad (35)
\end{aligned}$$

where the first inequality is a consequence of Lemma A.1 and the second because $V^{-1}(\cdot)$ is an increasing function. ■

Proof of Lemma 1′: The proof up to the "moreover" part mimics that of Lemma 1 and is omitted for the sake of brevity. The moreover part follows because raising any element of $\mathbf{g}$ increases the left-hand side of (24) by (21), hence, by concavity, $Y(\mathbf{g})$ must increase to restore equality. That $Y(\cdot)$ is differentiable follows from the implicit function theorem. ■

Proof of Proposition 8: Let $\tau > \tau'$. To reduce notational clutter, let $\mathbf{g} = \mathbf{g}(\tau)$ and $\mathbf{g}' = \mathbf{g}(\tau')$. To prove the first part of the proposition it is sufficient to show that $Y(\mathbf{g}) \geq Y(\mathbf{g}')$; because, if $Y(\mathbf{g}) \geq Y(\mathbf{g}')$ but $C(\mathbf{g}) < C(\mathbf{g}')$, then the τ'-type firm cannot be optimizing—it could weakly increase its benefit and strictly lower its costs by switching to $\mathbf{g}$. By revealed preference:

$$B\big(Y(\mathbf{g}),\tau\big) - C(\mathbf{g}) \geq B\big(Y(\mathbf{g}'),\tau\big) - C(\mathbf{g}') \text{ and} \qquad (36)$$

$$B\big(Y(\mathbf{g}'),\tau'\big) - C(\mathbf{g}') \geq B\big(Y(\mathbf{g}),\tau'\big) - C(\mathbf{g})\,. \qquad (37)$$

Expressions (36) and (37) can be combined to yield

$$B\big(Y(\mathbf{g}),\tau\big) - B\big(Y(\mathbf{g}'),\tau\big) \geq B\big(Y(\mathbf{g}),\tau'\big) - B\big(Y(\mathbf{g}'),\tau'\big)\,.$$

Twice applying the fundamental theorem of calculus, this last expression can be rewritten as

$$\int_{\tau'}^{\tau}\int_{Y(\mathbf{g}')}^{Y(\mathbf{g})} B_{12}(y,t)\,dy\,dt \geq 0\,. \qquad (38)$$

Because $B_{12}(\cdot,\cdot) > 0$ and $\tau > \tau'$, (38) can be non-negative only if the direction of integration for the inner integral is left to right; that is, only if $Y(\mathbf{g}) \geq Y(\mathbf{g}')$. As noted, this implies $C(\mathbf{g}) \geq C(\mathbf{g}')$, as was to be shown.

Turning to the moreover part, the goal is to show $Y(\mathbf{g}) > Y(\mathbf{g}')$. Suppose, instead, that $Y(\mathbf{g}) = Y(\mathbf{g}')$. One of the types would, therefore, have to be playing non-optimally if $C(\mathbf{g}) \neq C(\mathbf{g}')$; hence, this supposition implies $C(\mathbf{g}) = C(\mathbf{g}')$. The τ'-type firm is at an interior solution, so there must be at least one g_j' such that

$$B_1\big(Y(\mathbf{g}'),\tau'\big)Y_j(\mathbf{g}') - C_j(\mathbf{g}') = 0\,.$$

Because $B_{12}(\cdot,\cdot) > 0$, this implies

$$B_1\big(Y(\mathbf{g}'),\tau\big)Y_j(\mathbf{g}') - C_j(\mathbf{g}') > 0\,.$$

It follows there exists a governance vector $\tilde{\mathbf{g}}$ that has slightly more on the jth dimension such that

$$B\big(Y(\tilde{\mathbf{g}}),\tau\big) - C(\tilde{\mathbf{g}}) > B\big(Y(\mathbf{g}'),\tau\big) - C(\mathbf{g}') = B\big(Y(\mathbf{g}),\tau\big) - C(\mathbf{g})\,,$$

where the equality follows because $Y(\mathbf{g}) = Y(\mathbf{g}')$ and $C(\mathbf{g}) = C(\mathbf{g}')$. But then, $\mathbf{g}$ was not optimal for the τ-type firm, a contradiction. By contradiction, $Y(\mathbf{g}) \neq Y(\mathbf{g}')$, which, given the first part of the proposition, entails $Y(\mathbf{g}) > Y(\mathbf{g}')$. It must then be that $C(\mathbf{g}) > C(\mathbf{g}')$ because otherwise the τ'-type firm is not behaving optimally. ■

Proof of Proposition 9: In light of Proposition 8, define

$$\tilde{C}(y) = \min_{\mathbf{g}\in\mathcal{G}(Y)} C(\mathbf{g}) \;\text{ subject to }\; y = Y(\mathbf{g})\,. \tag{39}$$

The cost, $C(\mathbf{g})$, is increasing in each dimension and so is $Y(\mathbf{g})$ (the latter follows from Lemma 1′). Consequently, $\tilde{C}(\cdot)$ is an increasing function. The owners' problem can be reëxpressed as

$$\max_{y\leq Y} B(y,\tau) - \tilde{C}(y)\,. \tag{40}$$

Let $y(\tau)$ be the solution to (40) selected by a type-τ firm. Utilizing Lemma 2, it is readily shown that $\tau > \tau'$ implies $y(\tau) \geq y(\tau')$. The proposition follows if it can be shown that $y(\tau) \geq y(\tau')$ implies $\mathbf{g}(\tau) \geq \mathbf{g}(\tau')$.

To that end, observe that minimization program in (39) is equivalent to the program

$$\max_{\mathbf{g}\in\mathcal{G}(Y)} -C(\mathbf{g}) \quad \text{subject to} \tag{41}$$

$$v_1(y,\mathbf{g}) = 1 \tag{42}$$

Let λ be the Lagrange multiplier on (42). The program given by (41) is, thus, equivalent to

$$\max_{\mathbf{g},\lambda} -C(\mathbf{g}) + \lambda\big(v_1(y,\mathbf{g}) - 1\big)\,. \tag{43}$$

This expression is supermodular in $(\mathbf{g}, \lambda)$ in light of Topkis's characterization theorem (Milgrom and Roberts, 1990, p. 1261) because $v_{1i}(y, \mathbf{g}) > 0$ by (21) and $v_{1ij}(y, \mathbf{g}) \geq 0$ and $-C_{ij}(\mathbf{g}) \geq 0$ by the assumptions of the proposition.[27] By the increasing-differences assumption of the proposition, (43) exhibits increasing differences in y and g_i for any i. Let $\mathbf{g}(y)$ denote the solution to (43). It follows, therefore, from Topkis's monotonicity theorem (Milgrom and Roberts, p. 1262) that $y > y'$ implies the $\mathbf{g}(y) \geq \mathbf{g}(y')$. ■

Appendix B: Issues in the Estimation of the Pay-for-Performance Relation

Consider the following agency model. The manager's utility is

$$\frac{-1}{\exp\bigl(\underbrace{Y - N}_{S} + \delta_0 + \delta_1 \pi\bigr)},$$

where his compensation contract is $\delta_0 + \delta_1 \pi$, π being realized profits gross of compensation. Assume that[28]

$$\pi \sim \mathcal{N}\bigl(\tau \log(N), \sigma^2\bigr),$$

In what follows, assume Y is sufficiently large that the constraint $N \leq Y$ never binds.

Using the formula for the moment-generating function of a normal random variable, the manager's expected utility is

$$\frac{-1}{\exp\bigl(Y - N + \delta_0 + \delta_1 \tau \log(N) - \frac{1}{2}\delta_1^2 \sigma^2\bigr)}.$$

A monotonic transformation is

$$Y - N + \delta_0 + \delta_1 \tau \log(N) - \frac{1}{2}\delta_1^2 \sigma^2. \tag{44}$$

Given δ_0 and δ_1, the manager maximizes (44) with respect to N. This yields

$$N = \delta_1 \tau.$$

[27] v_{1ij} denotes the third partial derivative of v with respect to y, g_i, and g_j.

[28] Note the assumed compensation contract is not second-best optimal under these assumptions (see Mirrlees, 1974). A reformulation of this simple model along the lines of Holmstrom and Milgrom (1987)—in particular, assuming the manager decides how much to divert on a continuous basis with the resulting net funds controlling the drift of a Brownian motion—would, however, yield an optimal compensation contract of this form. One can thus view the model here as a simple approximation of that more complex model. In any case, given the issue is the econometric consequences of heterogeneity, the optimality of the contract is not essential to the analysis.

Hence, if the owners want to induce a particular N, they must set δ_1 to satisfy

$$\delta_1 = \frac{N}{\tau}\,. \tag{45}$$

Assume, as an alternative to working for the firm, the manager could get a job that paid a flat wage of $\underline{w}$. Hence, the owners must set δ_0 to satisfy

$$Y - N + \delta_0 + \underbrace{\frac{N}{\tau}}_{\delta_1} \tau \log(N) - \frac{1}{2} \underbrace{\frac{N^2}{\tau^2}}_{\delta_1^2} \sigma^2 \geq \underline{w}\,.$$

Given the owners' profit is decreasing in δ_0, the constraint will bind and, thus,

$$\delta_0 = \underline{w} - Y + N - N\log(N) + \frac{1}{2} N^2 \frac{\sigma^2}{\tau^2}\,. \tag{46}$$

The owners seek to choose N to maximize their expected profits. Using (45) and (46), their expected profits can be written as

$$\tau \log(N) - \underline{w} + Y - N - \frac{1}{2} N^2 \frac{\sigma^2}{\tau^2}\,.$$

The first-order condition is

$$\frac{\tau}{N} - 1 - N\frac{\sigma^2}{\tau^2} = 0\,.$$

Solving for the root that satisfies the second-order condition yields

$$N = \frac{\tau^{3/2}\sqrt{4\sigma^2 + \tau} - \tau^2}{2\sigma^2}\,. \tag{47}$$

Plugging (47) into (45) and (46) yields expressions for δ_1 and δ_0, respectively, in terms of the model's primitives (τ, σ^2, Y, and $\underline{w}$).

Data were created for 10,000 firms as follows. For each firm, its τ was a random draw from the uniform distribution $\tau \sim \mathcal{U} : [10, 20]$. To insure that $N \leq Y$ was never binding, $Y = 400$. The reservation wage, $\underline{w}$, was set to zero. Then, for each firm, the optimal contract was calculated, as was the N its manager would choose in equilibrium. Profits gross of compensation, π, were then generated for that firm as a draw from $\mathcal{N}(\tau \log(N), 1)$ and the resulting compensation for the manager calculated. Once the data were constructed, equation (17) was estimated with π as the independent variable. The results are shown in Table 2.[29]

The true δ_1s for firms with $\tau \in [10, 20]$ ranges from $[.916, .954]$; hence, the estimate δ_1 is less than the true δ_1 for 100% of the firms. The reason why (17) does so poorly is that while the true δ_1s are fairly constant across τ, the intercepts—the true δ_0s—vary considerably. Because low τ firms have greater δ_0s and tend to have lower πs, while high τ firms have lower δ_0s and tend to have higher πs, the estimated slope between pay and profits is flattened. Consequently, the estimated δ_1 is biased downward.

[29]The Mathematica program used to generate the data is available from the author upon request.

Coefficient	Estimate
δ_0	-396.46 (-15321.)
δ_1	.27568 (440.69)
R^2	.95104

Table 2: Estimation of equation (17) using data generated as described in this appendix. Dependent variable is realized pay. Independent variable is profit (gross of compensation), its coefficient is δ_1. The coefficient δ_0 is the intercept. Numbers in parentheses are t-statistics.

References

Adams, Renée B., Benjamin E. Hermalin, and Michael S. Weisbach, "The Role of Boards of Directors in Corporate Governance: A Conceptual Framework and Survey," *Journal of Economic Literature*, 2010, *48*, 59–108.

Baker, George P., Michael C. Jensen, and Kevin J. Murphy, "Compensation and Incentives: Practice vs. Theory," *Journal of Finance*, 1988, *43* (3), 593–616.

Bebchuk, Lucian Arye, Alma Cohen, and Allen Ferrell, "What Matters in Corporate Governance?," *Review of Financial Studies*, 2009, *22* (2), 783–827.

Becht, Marco, Patrick Bolton, and Ailsa Röell, "Corporate Governance and Control," in George Constantinides, Milton Harris, and René Stulz, eds., *The Handbook of the Economics of Finance*, Amsterdam: North-Holland, 2003.

Berger, Philip G., Eli Ofek, and David L. Yermack, "Managerial Entrenchment and Capital Structure Decisions," *Journal of Finance*, September 1997, *52* (4), 1411–1438.

Berle, Adolph A. and Gardiner C. Means, *The Modern Corporation and Private Property*, New York: MacMillan, 1932.

Bhagat, Sanjai and Richard H. Jefferis, *The Econometrics of Corporate Governance Studies*, Cambridge, MA: MIT Press, 2002.

Borokhovich, Kenneth A., Robert P. Parrino, and Teresa Trapani, "Outside Directors and CEO Selection," *Journal of Financial and Quantitative Analysis*, September 1996, *31* (3), 337–355.

Calvo, Guillermo A. and Stanislaw Wellisz, "Hierarchy, Ability, and Income Distribution," *Journal of Political Economy*, October 1979, *87* (5, Part 1), 991–1010.

Coles, Jeffrey L., Michael L. Lemmon, and J. Felix Meschke, "Structural Models and Endogeneity in Corporate Finance: The Link Between Managerial Ownership and Corporate Finance," 2007. Working Paper, Arizona State University.

_ , Naveen D. Daniel, and Lalitha Naveen, "Boards: Does One Size Fit All?," *Journal of Financial Economics*, February 2008, *87* (2), 329–356.

Dahya, Jay, John J. McConnell, and Nickolaos G. Travlos, "The Cadbury Committee, Corporate Performance, and Top Management Turnover," *Journal of Finance*, 2002, *57* (1), 461–483.

DeLong, J. Bradford, "Productivity Growth in the 2000s," in Mark Gertler and Kenneth Rogoff, eds., *NBER Macroeconomics Annual 2002*, Cambridge, MA: MIT Press, 2003.

Demsetz, Harold and Kenneth Lehn, "The Structure of Corporate Ownership: Causes and Consequences," *Journal of Political Economy*, December 1985, *93* (6), 1155–1177.

Eisenberg, Theodore, Stefan Sundgren, and Martin T. Wells, "Larger Board Size and Decreasing Firm Value in Small Firms," *Journal of Financial Economics*, April 1998, *48* (1), 35–54.

Frydman, Carola and Raven E. Saks, "Executive Compensation: A New View from a Long-Term Perspective, 1936–2005," *Review of Financial Studies*, in press.

Gabaix, Xavier and Augustin Landier, "Why Has CEO Pay Increased So Much?," *Quarterly Journal of Economics*, 2008, *123* (1), 49–100.

Gompers, Paul A. and Andrew Metrick, "Institutional Investors and Equity Prices," *Quarterly Journal of Economics*, February 2001, *116* (1), 229–259.

_ , _ , and Joy Ishii, "Corporate Governance and Equity Prices," *Quarterly Journal of Economics*, February 2003, *118* (1), 107–155.

Grossman, Sanford J. and Oliver D. Hart, "Corporate Financial Structure and Managerial Incentives," in John J. McCall, ed., *The Economics of Information and Uncertainty*, University of Chicago, 1982.

Hall, Brian J. and Jeffrey B. Liebman, "Are CEOs Really Paid Like Bureaucrats?," *Quarterly Journal of Economics*, August 1998, *CXIII* (3), 653–691.

Hall, Bronwyn H., "Exploring the Patent Explosion," *Journal of Technology Transfer*, December 2004, *30* (1–2), 35–48.

Harris, Milton and Artur Raviv, "Corporate Control Contests and Capital Structure," *Journal of Financial Economics*, 1988, *20*, 55–86.

Hart, Oliver D. and John H. Moore, "Default and Renegotiation: A Dynamic Model of Debt," *Quarterly Journal of Economics*, February 1998, *113* (1), 1–41.

Haubrich, Joseph G., "Risk Aversion, Performance Pay, and the Principal-Agent Problem," *Journal of Political Economy*, April 1994, *102* (2), 258–276.

Hermalin, Benjamin E., "Trends in Corporate Governance," *Journal of Finance*, 2005, *60* (5), 2351–2384.

_ **and Michael S. Weisbach**, "Endogenously Chosen Boards of Directors and Their Monitoring of the CEO," *American Economic Review*, March 1998, *88* (1), 96–118.

_ **and Nancy E. Wallace**, "Firm Performance and Executive Compensation in the Savings and Loan Industry," *Journal of Financial Economics*, 2001, *61*, 139–170.

Himmelberg, Charles P., R. Glenn Hubbard, and Darius Palia, "Understanding the Determinants of Managerial Ownership and the Link between Ownership and Performance," *Journal of Financial Economics*, September 1999, *53* (3), 353–384.

Holmstrom, Bengt and Paul Milgrom, "Aggregation and Linearity in the Provision of Intertemporal Incentives," *Econometrica*, March 1987, *55* (2), 303–328.

_ **and Steven N. Kaplan**, "Corporate Governance and Merger Activity in the United States: Making Sense of the 1980s and 1990s," *Journal of Economic Perspectives*, Spring 2001, *15* (2), 121–144.

Huson, Mark R., Robert Parrino, and Laura T. Starks, "Internal Monitoring Mechanisms and CEO Turnover: A Long-Term Perspective," *Journal of Finance*, December 2001, *55* (6), 2265–2297.

Jensen, Michael C., "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers," *American Economic Review*, May 1986, *76* (2), 323–329. Papers and Proceedings of the Ninety-Eighth Annual Meeting of the American Economic Association.

_ **and Kevin J. Murphy**, "Performance Pay and Top-Management Incentives," *Journal of Political Economy*, 1990, *98*, 225–264.

_ **and William H. Meckling**, "Theory of the Firm: Managerial Behavior, Agency Costs, and Capital Structure," *Journal of Financial Economics*, 1976, *3*, 305–360.

Kaplan, Steven N. and Bernadette A. Minton, "How has CEO Turnover Changed?," September 2008. Unpublished working paper, University of Chicago GSB.

Milgrom, Paul and John Roberts, "Rationalizability, Learning, and Equilibrium in Games with Strategic Complementarities," *Econometrica*, 1990, *58*, 1255–1277.

Mirrlees, James, "Notes on Welfare Economics, Information, and Uncertainty," in Michael S. Balch, Daniel L. McFadden, and Shih-Yen Wu, eds., *Essays on Economic Behavior Under Uncertainty*, Amsterdam: North-Holland, 1974.

Modigliani, Franco and Merton H. Miller, "The Cost of Capital, Corporate Finance, and the Theory of Corporation Finance," *American Economic Review*, 1958, *48*, 261–297.

Murphy, Kevin J. and Ján Zábojník, "Managerial Capital and the Market for CEOs," December 2003. Marshall School of Business Working Paper, University of Southern California.

Palia, Darius, "The Endogeneity of Managerial Compensation in Firm Value: A Solution," *Review of Financial Studies*, Autumn 2001, *14* (3), 735–764.

Rose, Nancy L. and Andrea Shepard, "Firm Diversification and CEO Compensation: Managerial Ability or Executive Entrenchment?," *RAND Journal of Economics*, Autumn 1997, *28* (3), 489–514.

Smith, Adam, *An Inquiry into the Nature and Causes of the Wealth of Nations*, London: W. Strahan and T. Cadeli, in the Strand, 1776.

Stulz, René M., "Managerial Control of Voting Rights : Financing Policies and the Market for Corporate Control," *Journal of Financial Economics*, January-March 1988, *20*, 25–54.

Terviö, Marko, "The Difference That CEOs Make: An Assignment Model Approach," *American Economic Review*, 2008, *98* (3), 642–668.

Yermack, David, "Higher Valuation of Companies with a Small Board of Directors," *Journal of Financial Economics*, 1996, *40* (2), 185–212.