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# INFORMATION AND THE HOLD-UP PROBLEM

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## ABSTRACT

We examine situations in which a party must make a sunk investment prior to contracting with a second party to purchase an essential complementary input. We study how the resulting hold-up problem is affected by the seller's information about the investing party's likely returns from its investment. Our principal focus is on the effects of the investment's being observable by the non-investing party. We establish conditions under which the seller's ability to observe the buyer's investment harms the seller, benefits the buyer, and reduces equilibrium investment and total surplus. We also note conditions under which investment and welfare rise when investment is observable.

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# 1 Introduction

The hold-up problem is a central issue in economic analysis.<sup>1</sup> It arises when one party makes a sunk, relationship-specific investment and then engages in bargaining with an economic trading partner. That partner may be able to appropriate some of the gains from the sunk investment, thus distorting investment incentives, either toward too little investment or toward investments that are less subject to appropriation. Examples include a buyer who requires the seller's facility to market the buyer's products (*e.g.*, a coal mine reliant on the local railroad or a web-based application provider reliant on an Internet service provider), a buyer who must invest in complementary assets to be used in conjunction with the seller's product (*e.g.*, a firm undertaking marketing expenditures or investment in specialized facilities in order to distribute a manufacturer's product), investment in R&D or specialized production assets early on in a procurement process, and private investment subject to later government regulation (*e.g.*, construction of a regulated oil or gas pipeline).

In the present paper, we analyze the effects of the information structure on the hold-up problem when pre-investment contracting is infeasible.<sup>2</sup> Our principal focus is on the effects of the investment's being observable by the non-investing party. The situation we have in mind is the following. There is an initial stage in which a buyer invests in complementary assets that are necessary to generate value from a seller's product and which have no value in alternative uses. After the results of the buyer's investment have been realized, the seller makes the buyer a take-it-or-leave-it offer.<sup>3</sup> In deciding

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<sup>1</sup>For classic analyses of the hold-up problem, see Klein (1988) and Williamson (1975, 1976). More recent work is discussed below.

<sup>2</sup>Intellectual property licensing represents an interesting case in which pre-investment contracting is particularly difficult because neither the intellectual property owner nor the party producing an infringing product may be aware of the infringement until after the producer has sunk its investment and begun operations. One difference from our formal model below is that with positive probability the producer does not have to obtain a license. Incorporating this feature requires a minor and obvious modification of our model.

<sup>3</sup>As should become evident, our analysis applies equally well to settings in which the seller makes an investment that lowers its costs and the buyer then makes a take-it-or-leave-it offer. In each case, the party making the investment increases its own value of exchanging the input. For an analysis of situations in which investment by one party raises the other party's value of exchange, see Che and Hausch (1999).

the price to offer, the seller may have information about (*i.e.*, receive a signal of) the buyer's realized value for the seller's product. At one extreme, the signal could be perfect and reveal the buyer's realized value. Then, absent any *ex ante* pricing commitments to do otherwise, the seller will set a price that fully extracts the buyer's surplus. Anticipating such pricing, the buyer expects to earn zero profits gross of its investment expenses regardless of its level of investment. Hence, a rational buyer makes no investment. In other words, as is well known, perfect information leads to complete hold up and destroys buyer investment incentives.

It is readily shown that both the buyer's profits and investment incentives can be positive when the seller is perfectly ignorant of the buyer's realized value. Given that perfect information drives both to zero, one might suspect that improving the seller's information lowers the buyer's profits and investment incentives, even when the improved information is itself imperfect. As we will demonstrate, however, there are important circumstances in which neither comparative static obtains. It is perhaps not surprising that "anything can happen" absent sufficient structure. Suppose one restricts attention to settings in which investment improves the distribution of the buyer's returns in the sense of first-order stochastic dominance and a higher value of the seller's signal leads to an improvement in the conditional distribution of the buyer's returns in the sense of first-order stochastic dominance. With this structure, it seems intuitively clear that the seller's price is increasing in the signal value and that, in comparison with an uninformative signal, an informative signal lowers the equilibrium levels of investment, buyer profits, and joint profits.<sup>4</sup> As we will show, however, all of these claims are false.

Our analysis proceeds as follows. After describing the model and characterizing a baseline case in which the seller is perfectly uninformed about the buyer's investment level and the realized value of trading, we examine settings in which the seller can observe—and condition its price on—the buyer's investment level. We demonstrate that, when the seller cannot commit to a price schedule prior to the buyer's sinking its investment, the observability of investment may, in general, raise or lower the buyer's equilibrium investment level and the seller's price may be increasing or decreasing in the investment level. We derive conditions under which the seller's price is increasing in investment and the additional information reduces equilibrium buyer in-

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<sup>4</sup>We base this statement about intuitive clarity on our experience in presenting this material to numerous seminar audiences.

vestment, in accord with the common intuition that additional information allows the seller to appropriate more of the returns to investment and thus reduces the buyer's investment incentives. Even in this case, however, we obtain the surprising—but quite general—result that the additional information results in the *buyer's* equilibrium profits rising *vis-à-vis* the situation in which the seller cannot observe investment. We also derive conditions under which the observability of investment reduces the seller's profits. In other words, we show that, even when the additional information gives the seller a greater ability to extract rents from the buyer at the margin, the additional information reduces the seller's ability to extract rents overall.

We also show that, because there are two opposing forces at work, the net effect of investment-based pricing on total surplus is ambiguous even when such pricing lowers buyer investment further below the efficient level. First, investment-based pricing induces the buyer to invest less, which tends to lower welfare. But, second, the seller lowers its price in response to lower investment, which increases the social benefits associated with a given level of investment because the seller is less likely to inefficiently price the buyer out of the market (*i.e.*, to cause the buyer to shut down). We demonstrate that a necessary condition for investment-based pricing to increase welfare is that it raise the equilibrium probability of trade.

Lastly, we briefly examine markets in which the seller conditions its price on a general, noisy signal of the returns realized from the buyer's investment. We derive conditions under which the seller's price is an increasing function of the signal's value and the buyer's equilibrium investment is less than the second-best level. However, we also observe that the investment and welfare effects of increased seller information are generally ambiguous even under strong regularity conditions.

Before presenting our analysis, it is useful to put it in context. Economists have devoted considerable attention to the hold-up problem under various assumptions concerning the information structure and contracting institutions.<sup>5</sup> Like us, Rogerson (1992) and Hermalin and Katz (1993) consider situations in which the buyer's value of trade remains his private information. Unlike us, they assume that contracting prior to the buyer's investment is feasible, and they establish conditions under which the first-best outcome is attainable.

Tirole (1986), Gul (2001), and Lau (2008) examine situations in which

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<sup>5</sup>For a recent survey, see Schmitz (2001).

contracting prior to investment is infeasible. *Inter alia*, these authors demonstrate how the observability of investment affects the equilibrium outcome. Specifically, Tirole focuses on the change in equilibrium investment when observability implies the parties can contract on the level of investment.<sup>6</sup> In contrast, we assume observability does not imply contractibility. Gul shows that the hold-up problem is solved when the buyer’s investment is unobservable, all of the offers are made by the seller, and the time between offers is small. Lau (2008) looks at an intermediate case in which—at the time that the buyer invests—it is uncertain whether the seller will observe the buyer’s investment. She shows that welfare can be greater than at either of the extremes of no information (less holdup but less efficient trade) and perfect information (complete holdup but efficient trade) because intermediate information “balances” the conflicting tensions. Both Gul and Lau assume that the buyer’s value of the seller’s product is a deterministic function of investment.<sup>7</sup> In a departure from these authors, we allow for the more realistic case of stochastic returns to investment. In this setting, even when the non-investing party observes the investing party’s level of investment and the non-investing party has all the bargaining power, the non-investing party is typically unable to appropriate the investing party’s surplus fully.

Like us, Skrzypacz (2005) allows for investment with noisy returns. However, Skrzypacz focuses on the limiting case of a bargaining process in which the degree of *ex post* inefficiency goes to zero. In contrast, we limit ourselves to letting the non-investing party make a single, take-it-or-leave-it offer. Our simpler bargaining process gives rise to the possibility of *ex post* inefficiency, which we believe is an important feature of many settings of interest.

## 2 The Model

We examine a setting in which there is a single buyer that requires the output of an upstream monopoly seller to generate value by selling a downstream product. For example, the monopoly seller might control a bottleneck facility through which the buyer reaches its market. Alternatively, the buyer might

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<sup>6</sup>Tirole briefly considers the case of observable, but non-contractible investment as well (see his Proposition 3); his analysis is consistent with our Proposition 6 below.

<sup>7</sup>We briefly examine the deterministic-investment case below and show that another important difference is that we, unlike Gul and Lau, assume that there is no value of exchange absent buyer investment.

be a distributor of a monopoly manufacturer's product. Or the buyer might need to license the seller's intellectual property. We assume the buyer is a monopoly provider of its downstream product. This assumption avoids complications that arise when there are multiple buyers that are downstream competitors and, consequently, have interdependent demands.

The timing of the baseline game is as follows:

- The buyer chooses and sinks its investment,  $I$ , in its product. The buyer's investment yields a conditional distribution of product-market quasi-rents (*i.e.*, buyer profits gross of the investment cost and any payments to the seller). As a shorthand, we refer to these quasi-rents as the buyer's return,  $r \in \mathbb{R}_+$ . We assume  $r$  is the buyer's private information.
- The seller observes a signal,  $s$ , which may contain information about the buyer's benefit of trade,  $r$ . The seller then makes a take-it-or-leave-it offer to sell one unit of its output at price  $p(s)$ .
- After observing the realized values of  $r$  and  $p(s)$ , the buyer chooses whether to shut down or continue. If the buyer shuts down, it loses its investment,  $I$ , earns no returns, and makes no payment to the seller. If the buyer continues operation, it earns profits of  $r - p(s) - I$  and the seller receives payment  $p(s)$ . For simplicity, we assume the seller incurs no marginal costs to produce output.<sup>8</sup>

Formally, returns have the conditional distribution  $F(r, s|I)$  with the corresponding density function  $f(r, s|I)$ .<sup>9</sup> We assume  $F(r, s|I)$  is at least twice differentiable in  $I$  for all  $I \in (0, \infty)$ ,  $r$ , and  $s$ . Let  $F_r(r|I)$  denote the corresponding marginal distribution and  $f_r(r|I)$  its corresponding density. We assume the set of  $r$  for which the latter is positive (*i.e.*, the support) is an interval with a greatest lower bound of 0. Let

$$h(r|I) = \frac{f_r(r|I)}{1 - F_r(r|I)}$$

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<sup>8</sup>Positive marginal costs would have no effects on the qualitative results as long as the seller incurred those costs only after the buyer placed a firm order for the good.

<sup>9</sup>For much of our analysis, the assumption that  $F(\cdot, \cdot|I)$  is a continuous distribution is unnecessary. Generally,  $f(r, s|I)$  could be interpreted as a probability mass on the point  $(r, s)$ .

denote the corresponding hazard rate.

We assume that investment is essential (*i.e.*, the buyer earns no returns if it makes no investment) and that the problem is nontrivial insofar as neither zero nor infinite investment maximizes total surplus:

**Assumption 1**  $F_r(0|0) = 1$ , and there exist  $I > 0$  such that

$$\int_0^\infty (1 - F_r(r|I)) dr > I \quad (1)$$

and a finite  $\bar{I} > 0$  such that

$$\int_0^\infty (1 - F_r(r|I)) dr < I \quad (2)$$

for all  $I > \bar{I}$ .<sup>10</sup>

Our solution concept is perfect Bayesian equilibrium and, as usual, we solve the game working backwards.

In what follows, the relationship between investment and the distribution of returns plays a critical role. We consider three assumptions:

- *Productive Investment*: An increase in  $I$  raises the expected value of  $r$ .
- *FOSD Improvement*: An increase in  $I$  improves the distribution of  $r$  in the sense of first-order stochastic dominance.<sup>11</sup>
- *Monotone Hazard*: For any  $I > 0$  and  $r$  in the support of  $f_r(\cdot|I)$ , the hazard rate  $h(r|I)$  is decreasing in  $I$ .

The Monotone Hazard Condition can be interpreted as a statement about the price elasticity of demand for the seller's product,  $\epsilon$ . Demand for the seller's product is  $D(p|I) \equiv 1 - F_r(p|I)$ , the conditional survival function. By definition

$$\epsilon(p|I) \equiv -\frac{d \log(D(p|I))}{d \log p} = \frac{p f_r(p|I)}{1 - F_r(p|I)} = p h(p|I). \quad (3)$$

<sup>10</sup>The derivation of (1) will become clear below.

<sup>11</sup>Throughout, when we refer to first-order stochastic dominance, we mean it in the following strict sense: If  $I > I'$ , then  $F_r(r|I) < F_r(r|I')$  for any  $r$  such that  $F_r(r|I) < 1$ . Because  $F_r(r|\cdot)$  is differentiable, it is equivalent to express the first-order stochastic dominance ordering as  $\partial F_r(r|I)/\partial I < 0$  for any  $r$  and  $I$  such that  $F_r(r|I) < 1$ .

The Monotone Hazard Condition thus implies that the buyer's demand becomes less price elastic as the buyer's investment increases, holding price constant.

It is useful to consider a few examples that satisfy the Monotone Hazard Condition and Assumption 1:

$$D(r|I) = \exp\left(-\frac{\alpha r}{\sqrt{I}}\right), \quad (4)$$

$$D(r|I) = 1 - \frac{\alpha r}{I^2} \text{ for } r \in \left[0, \frac{I^2}{\alpha}\right]; \quad (5)$$

and

$$D(r|I) = \frac{I}{I+1} - \alpha r \text{ for } r \in \left(0, \frac{I}{\alpha(I+1)}\right], \quad (6)$$

where  $\alpha$  is a positive constant. Investment rotates demand clockwise about the point  $(1, 0)$  in the second example and generates a parallel outward shift in the third.

It is also useful to observe that the three relationships between investment and returns are nested. As is well known, the FOSD Improvement Condition implies the Productive Investment Condition. Lemma A.1 in the Appendix establishes that the Monotone Hazard Condition implies the FOSD Improvement Condition (the converse, however, does not hold).

We consider the Productive Investment Condition to be a weak requirement. In contrast, we believe there are important settings in which FOSD Improvement is too strong a condition. Specifically, we have in mind settings in which low-cost investment projects give rise to moderate returns with near certainty but expensive, breakthrough projects have significant probabilities of yielding very low and very high returns. Hence, in what follows, we always assume that the Productive Investment Condition is satisfied but consider situations in which the FOSD Improvement and Monotone Hazard Conditions are not.

In addition to examining the effects of the information structure on equilibrium investment and profits, we examine the effects on equilibrium welfare. We take expected total surplus as our welfare measure. We assume that the buyer's customers (if any) derive zero consumer surplus from consumption of the buyer's output. We do this for expositional convenience and because it is



well known that a supplier (here, “the buyer”) tends to underinvest when an increase in investment generates consumer benefits that the supplier is unable to appropriate. Our interest is in the new phenomena that arise directly as a consequence of the upstream seller’s pricing conditional on its signal.

Given the assumption that  $r$  captures the full social benefits derived from the production and consumption of the seller’s output, expected total surplus is

$$W(p(\cdot), I) \equiv \int_0^\infty \int_{p(s)}^\infty r f(r, s|I) dr ds - I. \quad (7)$$

As long as  $p(\cdot)$  is Lebesgue integrable,  $W(p(\cdot), I)$  is continuous in  $I$ . By (2), there is no loss of generality in assuming that  $I$  is chosen from the compact interval  $[0, \bar{I}]$ . Hence, there exists at least one welfare-maximizing investment level given  $p(\cdot)$ .

It is apparent from (7) that the welfare-maximizing (first-best) price schedule entails marginal-cost pricing:  $p(s) \equiv 0$ . The resulting welfare is

$$W(0, I) = \int_0^\infty r f_r(r|I) dr - I = \int_0^\infty D(r|I) dr - I, \quad (8)$$

where the second equality follows from integration by parts. Observe that  $W(0, I)$  is the buyer’s expected surplus less the cost of the buyer’s investment when the seller prices at marginal cost. Assumption 1 implies there exists an  $I > 0$  such that  $W(0, I) > 0$ ; that is, the welfare-maximizing investment level is positive.

In the remainder of the paper, we consider three cases:

1. The seller’s signal is completely uninformative about  $I$  and the realized value of  $r$ . We refer to this as the “uninformed-seller” case.
2. The seller’s signal is perfectly informative about  $I$ , but provides no information about the realized value of  $r$  beyond that contained in knowledge of the value of  $I$ . We refer to this as the “observable-investment” case.
3.  $s$  is an arbitrary noisy, but informative signal of  $r$ . We refer to this as the “noisy-signal-of-returns” case.

The difference between cases 2 and 3 is that, in the former, we can make use of our investment conditions to put additional structure on the relationship between the signal and the buyer’s returns. Circumstances exist in which

either case is the more appropriate model of an informed seller. At one pole, suppose the input is essential to some business activity that is conducted by an organizational unit that makes public reports of its financial performance at the unit level. In this case, the financial reports could be interpreted as a noisy signal of returns. At the other pole, suppose that a firm undertakes many different activities, only one of which requires the input in question, and the firm does not report financial performance broken down by activity. In this case, investment in specialized plant and equipment may be more readily observable and our observable-investment model is more relevant.

### 3 An Uninformed Seller

We begin by characterizing the equilibrium outcome when the seller's signal is perfectly uninformative and, thus, the seller bases pricing on its inference of the equilibrium value of investment and the corresponding distribution of returns. We focus on pure-strategy equilibria.

First, consider the seller's best response to the buyer's choice of investment level. If the buyer invests  $I$  and the seller charges price  $p$ , then the seller's profits are

$$\pi^S(p, I) \equiv pD(p|I). \quad (9)$$

If  $I = 0$ , then expression (9) is identically zero (recall  $F_r(0|0) = 1$ ) and any  $p$  is a best response. Because both a zero price and an infinite price yield zero profits, any maximizer of (9) is an element of the interior of the support of  $F(\cdot|I)$  when  $I > 0$ .<sup>12</sup> To ensure that the seller has a unique best response to any  $I > 0$ , we make the following assumption, which is satisfied by examples (4)–(6) above:

**Assumption 2** *The buyer's demand is log concave in price over the interval for which demand is positive.*<sup>13</sup>

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<sup>12</sup>That an infinite price yields zero profits is immediate if the support of  $r$  is bounded above. For an unbounded support, the log concavity of demand (Assumption 2 *infra*) implies that, if revenue is ever decreasing in price, then it must tend to zero as price goes to infinity. If, instead, revenue were everywhere non-decreasing, then that fact and the log concavity of demand (the survival function) would imply a hazard rate of zero everywhere, which is impossible (*i.e.*, revenue cannot be everywhere non-decreasing).

<sup>13</sup>One can also state a sufficient condition for demand to be log concave in price in terms of the density function. By a theorem of Prékopa (1971) (Theorem 13.20 of Pečarić et al.,

Under this assumption, the seller's objective function, (9), is log concave in price and there exists a unique best response to any  $I > 0$ , which we denote by  $p^*(I)$ .

Conditional on the buyer's choice of  $I$ , the seller faces a standard monopoly pricing problem with a marginal cost of zero. Given the log concavity of the seller's optimization problem, the solution is given by the well-known Lerner markup rule:  $1 = 1/\epsilon$ . Under the Monotone Hazard Condition,  $\epsilon$  is decreasing in  $I$ . Under Assumption 2,  $\epsilon$  is increasing in  $p$ . Hence, the Lerner condition can be maintained only if an increase in  $I$  is offset by an appropriate increase in  $p$ . Therefore,

**Proposition 1** *If the Monotone Hazard Condition is satisfied, then the seller's profit-maximizing price is increasing in the buyer's investment level whenever the investment level is positive.*

It is worth noting that the weaker conditions, Productive Investment and FOSD Improvement, do not impose enough structure on the way the demand curves shift with an increase in investment to imply that the seller's best response is increasing in the buyer's investment. For instance, the FOSD Improvement Condition does not rule out situations in which the price elasticity of demand is increasing in  $I$  (e.g., although demand rises with investment, most of the increase comes at low values of  $r$ ), which would cause the seller to lower price in response to an increase in investment.

Next, consider the buyer's best response to the seller's price. When the price rises, the returns to investment are realized in a smaller set of states (i.e., when  $r > p$ ) and the buyer earns less in those states (i.e.,  $r - p$ ). These effects tend to reduce the buyer's investment incentives. However, there can also be an effect running in the opposite direction. Suppose that a higher level of investment corresponds to a riskier project: it has a greater chance of performing very well, but also a greater chance of performing very poorly. The seller's price acts as a hurdle, where only those returns that clear the hurdle are realized. Setting a higher hurdle encourages the buyer to adopt a riskier project because the returns from a safer project are unlikely to clear the hurdle. Thus, there can be a range of values over which the buyer's investment rises with the seller's price.

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1992), log-concavity of the density  $f_r(\cdot|I)$  implies log-concavity of the survival function  $D(\cdot|I)$ .

This risk effect arises only when the demand curves corresponding to different investment levels cross. Hence, although it is not strong enough to guarantee that the seller's profit-maximizing price is increasing in the buyer's investment level, the FOSD Improvement Condition is strong enough to insure that the buyer's best-response investment level falls as the seller's price rises.<sup>14</sup>

**Proposition 2** *Suppose that the FOSD Improvement Condition is satisfied, and let  $p_1$  and  $p_2$  be any two prices such that  $p_2 > p_1$ . Then any best-response investment level for  $p_2$  is less than any best-response investment level for  $p_1$  unless both investment levels are zero.*

Another property of interest is how the buyer's choice of investment level compares with a welfare-maximizing one. Let  $I^w(p)$  denote a socially optimal level of investment given price,  $p$ . We refer to  $I^w(p)$  as the second-best investment level conditional on  $p$  and  $I^w(0)$  as the first-best investment level. When the FOSD Improvement Condition is satisfied, the buyer invests too little from a welfare perspective:

**Proposition 3** *Suppose that the FOSD Improvement Condition is satisfied. Then, given any price,  $p \in (0, \infty)$ , any best-response investment level for the buyer is less than any second-best amount unless both are zero.*

The underlying intuition is clear. When the FOSD Improvement Condition holds, an increase in  $I$ , holding price fixed, raises the probability of trade and, thus, the seller's profits. The buyer, unlike a social planner, does not take the increase in the seller's profits into account in choosing its investment level. Observe that, when the FOSD Improvement Condition does not hold, an increase in  $I$  could lower the probability of trade, thus harming the seller and creating an incentive wedge in the other direction.<sup>15</sup>

Now, consider equilibrium. There always exist degenerate equilibria in which the buyer believes the seller will charge such a high price that the buyer's best response is to invest nothing. As discussed above, if the seller

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<sup>14</sup>Proofs not given in the text may be found in the Appendix.

<sup>15</sup>It is for this reason that Schmitz's (2008) conclusion that giving the buyer greater bargaining power increases investment is not fully general. Absent the FOSD Improvement Condition, which he implicitly assumes, a gain in bargaining power could lower the buyer's incentives to invest.

believes  $I = 0$ , then the seller is indifferent as to the price it quotes and, so, it is a weak best response for it to charge a high price.<sup>16</sup>

In some circumstances, these degenerate equilibria are the only equilibria. In particular, there is no equilibrium with positive investment when the seller is able to appropriate a sufficiently high percentage of the quasi-rents that hold up renders buyer investment unprofitable even though it is socially desirable. This is the case, for example, when the realization of the buyer's return is a deterministic function of investment:

**Proposition 4** *Suppose that the seller's signal is perfectly uninformative and that investment  $I$  yields  $r(I)$  with certainty, where  $r(0) = 0$  and  $r(\cdot)$  is an increasing continuous function. Then, in equilibrium, the buyer's expected investment level is zero.*

Intuitively, if the buyer's expected investment level is positive, then the seller can ensure itself positive profits and the infimum of the prices that the seller charges with positive probability is greater than zero. It follows that the infimum of non-zero investment levels played by the buyer with positive probability is also greater than zero given that  $r(0) = 0$ . The seller, however, will never charge less than the infimum of  $r(I)$  over the set of investment levels chosen with positive probability. Hence, the buyer would suffer losses. Therefore, the buyer's equilibrium investment must be zero.<sup>17</sup>

When the returns to investment are stochastic, there are circumstances in which the seller's ability to extract the buyer's quasi-rents is sufficiently limited that there exist equilibria in which the buyer invests positive amounts. For instance, it can be shown that pure-strategy equilibria exist for each of our three examples, expressions (4)–(6) above.<sup>18</sup>

<sup>16</sup>The zero-investment equilibrium can survive trembles if there exists an investment level,  $I_0$ , such that  $\pi^B(p^*(I_0), I) - I < 0$  for all  $I > 0$ .

<sup>17</sup>Gul (2001, Proposition 1) and Lau (2008, Section 3) also examine settings in which returns are a deterministic function of investment. They both find equilibria with positive probabilities of investment. Critically, they both assume that  $r(0) > 0$ .

<sup>18</sup>For example (5), we require  $\alpha \leq 1/112$  if a pure-strategy positive-investment equilibrium is to exist. The equilibrium values are

$$I = \frac{1}{2} \left( \alpha + \sqrt{2\alpha + \alpha^2} \right) \quad \text{and} \quad p = \frac{1 + \alpha + \sqrt{2\alpha + \alpha^2}}{4}.$$

For example (6), we require  $\alpha \leq 1/32$ . A closed-form solution for the equilibrium values does not exist. For (4), the equilibrium is  $p = 1/(\alpha^2 e)$  and  $I = 1/(\alpha e)^2$ , where  $e$  is the base

We close this section with a result that will be useful in our later analysis. One question of interest is how the seller's profits vary with the investment level. One can readily construct examples in which the expected value of  $r$  is increasing in  $I$ , but the seller's profits fall because the share of surplus that the seller can appropriate falls. The share effect cannot dominate when increased investment leads to an increase in returns in the sense of first-order stochastic dominance (*i.e.*, when greater investment leads to everywhere greater demand). Formally, the seller's profits are  $\pi^S(p, I) = p(1 - F_r(p|I))$ , and first-order stochastic dominance implies that  $F_r(p|I)$  falls as  $I$  rises. We have established:

**Lemma 1** *If the FOSD Improvement Condition is satisfied, then the seller's profit,  $\pi^S(p, I)$ , is increasing in  $I$  for any  $p \in (0, \infty)$ .*

## 4 Observable Investment

With the uninformed-seller case as a benchmark, we now examine the equilibrium outcome when the seller can observe the buyer's investment level and condition its price on it.<sup>19</sup>

If the buyer invests  $I$ , then perfection requires that the seller charge price  $p^*(I)$ . The buyer chooses  $I$  to maximize

$$\begin{aligned} \pi^B(p^*(I), I) - I &= \int_0^\infty \max\{0, r - p^*(I)\} f(r|I) dr - I \\ &= \int_{p^*(I)}^\infty (1 - F_r(r|I)) dr - I. \end{aligned}$$

Because a log-transformation of the seller's optimization problem is concave in  $p$  for all  $I$  and continuous in both  $p$  and  $I$ , it follows that that  $p^*(I)$  is

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of the natural logarithm. Additional examples in which pure-strategy positive-investment equilibria exist can be constructed from (4) by replacing  $\sqrt{I}$  with other increasing concave functions of  $I$ . A Mathematica program for calculating these equilibria is available from the authors upon request.

<sup>19</sup>For any pure-strategy equilibrium, it is readily shown that—as long as the support of the signal is independent of the value of  $I$ —allowing the seller to observe a noisy measure of  $I$ , but not  $I$  itself, is equivalent to the uninformed-seller case previously analyzed. Observe, too, that one consequence of being a noisy measure of  $I$  is that the signal has no value in predicting the value of  $r$  conditional on knowing  $I$ .

continuous in  $I$ . Given the assumed properties of  $F_r(r|I)$ , it follows that  $\pi^B(p^*(I), I) - I$  is continuous in  $I$  and thus achieves a maximum over the compact interval  $[0, \bar{I}]$ . Therefore, at least one perfect equilibrium exists.

Recall that, when the seller is perfectly uninformed, there exist degenerate equilibria with  $I = 0$ . When the seller can observe the buyer's investment level, there is a Nash equilibrium with  $I = 0$ . But this outcome cannot be a subgame perfect equilibrium if there exists any  $I$  such that  $\pi^B(p^*(I), I) > I$ .

As when buyer investment is unobservable, if the FOSD Improvement Condition is satisfied, then the buyer invests less than the socially optimal amount given the seller's pricing strategy because the buyer ignores the benefits conferred on the seller:

**Proposition 5** *If the seller can observe the buyer's investment level and the FOSD Improvement Condition is satisfied, then, in any equilibrium, the buyer's investment level is less than the second-best amount unless both are zero.*

It is worth comparing the equilibrium investment levels with observable and unobservable investment given that public policy makers often are concerned with effects on investment.<sup>20</sup>

**Proposition 6** *If the Monotone Hazard Condition is satisfied, then the buyer's equilibrium investment level when the seller can observe investment is lower than the equilibrium investment level when the seller's signal is perfectly uninformative unless both investment levels are zero.*<sup>21</sup>

**Proof:** Let an “ $o$ ” or “ $u$ ” superscript denote the equilibrium value of a variable when the seller can base price on  $I$  or not, respectively. By revealed preference,

$$\pi^B(p^*(I^o), I^o) - I^o \geq \pi^B(p^*(I^u), I^u) - I^u \geq \pi^B(p^*(I^u), I^o) - I^o. \quad (10)$$

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<sup>20</sup>Outside of our formal model, one might expect buyer investment to generate positive externalities, either real (*e.g.*, technological spillovers) or pecuniary (*e.g.*, consumer surplus enjoyed by the buyer's customers), both of which could cause policy makers to care about the level of investment.

<sup>21</sup>Tirole (1986) derives a similar result. He assumes that, in the unobservable-investment case, the buyer does better, *ceteris paribus*, the lower the seller believes the buyer's investment to have been. In our analysis, the buyer does better the lower the seller's price and by our Proposition 1 the price is lower the less the seller believes the buyer to have invested.

Suppose  $I^o > I^u$ . Then, by Proposition 1,  $p^*(I^o) > p^*(I^u)$ . But then

$$\pi^B(p^*(I^u), I^o) > \pi^B(p^*(I^o), I^o),$$

which contradicts (10). Hence  $I^o \leq I^u$ .

To establish  $I^o \neq I^u$  when  $I^u > 0$ , observe that such an  $I^u$  would satisfy the first-order condition

$$\int_{p^*(I^u)}^{\infty} \frac{-\partial F_r(r|I^u)}{\partial I} dr - 1 = 0. \quad (11)$$

In contrast,  $I^o$  satisfies the first-order condition

$$\int_{p^*(I^o)}^{\infty} \frac{-\partial F_r(r|I^o)}{\partial I} dr - p^{*'}(I^o) \left( 1 - F_r(p^*(I^o)|I^o) \right) - 1 = 0.^{22} \quad (12)$$

If  $I^o = I^u$ , then  $p^*(I^o) = p^*(I^u)$ . Making those substitutions into (12) and using (11) implies  $p^{*'}(I^o) = 0$ , which contradicts Proposition 1. Hence  $I^o < I^u$ . ■

Not surprisingly, given our earlier discussion, Proposition 6 depends critically on our assumption about the distribution of returns given investment. Absent such an assumption, examples can be constructed in which investment-based pricing increases the equilibrium investment level. For instance, one can readily construct an example in which the buyer invests more than the first-best amount when the buyer's investment is observable and less than the first-best amount when the seller's signal is perfectly uninformative, and in which equilibrium welfare is positive when investment is observable and zero when it is not.<sup>23</sup>

This example builds on a broader and well-known phenomenon with respect to investment and the hold-up problem: in some situations, the buyer's investment choice affects the *share* of the returns that the buyer retains, and the buyer's incentives are biased toward investments that increase the buyer's share of the total.<sup>24</sup> The intuition underlying the example is as follows. If

<sup>22</sup>The derivative  $p^{*'}(I^o)$  exists by the implicit function theorem.

<sup>23</sup>An earlier version of this paper contained this example, which is available from the authors upon request.

<sup>24</sup>Actions to affect the buyer's share include: randomization of the choice of  $I$  (see Gul, 2001); investment in projects with noisy returns (see, *e.g.*, Skrzypacz, 2005); adoption of flexible technologies, which improve the buyer's bargaining disagreement point; and second-sourcing (see, *e.g.*, Farrell and Gallini, 1988).



the buyer chooses to invest the efficient amount, then it earns a deterministic level of returns, which the seller can fully appropriate if it knows that amount has been invested. When its investment level is observable, the buyer can credibly “show” the seller that the buyer has chosen a higher investment level that leads to noisy and, thus, less-than-fully appropriable returns. This effect does not arise when investment is unobservable and, in this example, the buyer consequently chooses a lower level of investment.

We next consider the effects of investment-based pricing on social welfare. Such pricing can lower welfare by inefficiently reducing equilibrium investment. However, it can be shown by example that investment-based pricing can also raise welfare in some circumstances where such pricing lowers equilibrium investment (*i.e.*, even when the assumptions of Proposition 6 hold).<sup>25</sup> Intuitively, price also falls by a sufficient amount that the probability of trade is higher at the low-investment/low-price outcome than at the high-investment/high-price outcome. In other words, there is an *ex post* efficiency improvement associated with the outcome under investment-based pricing.

In general, the sign of the welfare effects of investment-based pricing is ambiguous and depends on specific market characteristics. The following result characterizes one set of markets in which the sign is unambiguous:

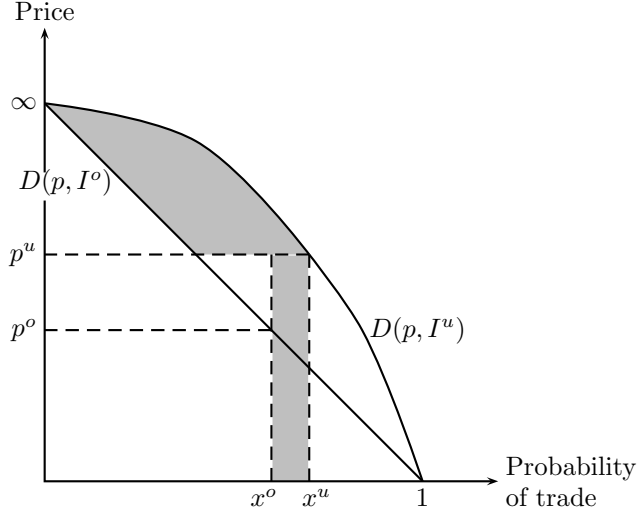
**Proposition 7** *Suppose that the Monotone Hazard Condition is satisfied. If the equilibrium probability of trade is lower when the seller can observe the buyer’s investment than when its signal is perfectly uninformative, then the improvement in the seller’s information lowers equilibrium welfare.*<sup>26</sup>

**Proof:** Let  $x^o$  denote the equilibrium probability of trade when the seller can observe the buyer’s investment level, and let  $x^u$  denote the corresponding probability when it cannot. By the previous proposition,  $I^o < I^u$ . Proposition 1 then implies  $p^o \equiv p^*(I^o) < p^*(I^u) \equiv p^u$ . Figure 1 illustrates the change in total surplus gross of the buyer’s investment cost. The relative positions of the demand curves follow because the Monotone Hazard Condition implies the FOSD Improvement Condition. Observe that the change in *total* surplus

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<sup>25</sup>An earlier version of this paper contained this example, which is available from the authors upon request.

<sup>26</sup>This result is suggestive of the well-known result that third-degree price discrimination lowers welfare if it lowers equilibrium output. The mechanisms at work are, however, different.



**Figure 1:** The welfare effects of investment-based pricing.

gross of investment costs exceeds the two shaded regions in Figure 1. The area of these regions are

$$\pi^B(p^u, I^u) - \pi^B(p^u, I^o) + p^u(x^u - x^o). \quad (13)$$

The result follows if (13) exceeds the incremental cost of investment,  $I^u - I^o$ . That, in turn, follows if

$$(\pi^B(p^u, I^u) - I^u) - (\pi^B(p^u, I^o) - I^o) + (x^u - x^o)p^u > 0.$$

By revealed preference, the difference in the first two terms is positive. And the third term is positive by hypothesis. Therefore, total surplus must be higher when the seller cannot observe the buyer's investment than when it can. ■

**Corollary 1** *Suppose that the Monotone Hazard Condition is satisfied and the price elasticity of demand at any probability of trade is increasing in*

*investment (i.e., demand curves get “flatter” as investment increases). Then equilibrium welfare is lower when the seller can observe  $I$  than when its signal is perfectly uninformative.*

**Proof:** Given Proposition 7, it is sufficient to show that  $x^u > x^o$ . By Proposition 6,  $I^u > I^o$  and, thus, the result follows if we can show that the seller’s marginal revenue as a function of  $x$  is increasing in  $I$ . Letting  $P(x, I)$  denote the inverse demand curve and  $\epsilon(x, I)$  the price elasticity of demand at quantity  $x$  given  $I$ , marginal revenue is

$$P(x, I) + x \frac{\partial P(x, I)}{\partial x} = P(x, I) \left( 1 - \frac{1}{\epsilon(x, I)} \right). \quad (14)$$

By assumption,  $\epsilon(x, I)$  and  $P(x, I)$  are increasing in  $I$ . The result follows. ■

Above, we provided examples, expressions (4)–(6), that satisfy the Monotone Hazard Condition. In all three examples, the price elasticity of demand is non-decreasing with investment, holding the probability of trade constant.<sup>27</sup> Consequently, for these examples, equilibrium welfare is lower when investment is observable than when it is not.

Lastly, we examine the distributional effects of the observability of the buyer’s investment level. A simple revealed preference argument demonstrates that the improvement in the seller’s information raises the buyer’s equilibrium profits under very general conditions:

$$\pi^B(p^*(I^o), I^o) - I^o \geq \pi^B(p^*(I^u), I^u) - I^u = \pi^B(p^u, I^u) - I^u.$$

**Proposition 8** *The buyer’s equilibrium expected profits are weakly greater when the seller can observe the buyer’s investment than when the seller’s signal is perfectly uninformative.*

One might have expected increased information to provide the seller with a greater ability to extract rents from the buyer. However, under a pure-strategy equilibrium, the seller already predicts the buyer’s investment level with certainty. The only effect of the seller’s being able to observe  $I$  is that it allows the buyer to behave as a Stackelberg leader.

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<sup>27</sup>Calculations are available from the authors upon request.

This result has a simple but powerful implication: if the seller prefers to engage in investment-based pricing, it is socially optimal for the seller to do so. Hence, banning a seller from using such information could be welfare improving only in those circumstances in which the seller cannot commit to ignoring the information. This result also highlights the difference between marginal and total profit effects on investment incentives. Even though the buyer earns higher profits when the seller is better informed, we have seen that the buyer may also invest less.<sup>28</sup>

Although the buyer gains from the improvement in the seller's information, the seller loses, at least under the Monotone Hazard Condition:

**Proposition 9** *If the Monotone Hazard Condition is satisfied, then the seller's equilibrium expected profits are lower when the seller can observe the buyer's investment than when the seller's signal is perfectly uninformative.*

**Proof:** By Proposition 6,  $I^u > I^o$ . Using the fact that the Monotone Hazard Condition implies the FOSD Improvement Condition, the result follows from application of Lemma 1. ■

Under the conditions of Proposition 9, the seller would wish to commit *ex ante* not to price on the basis of investment. Observe, however, that an *ex ante* contractual agreement with the buyer would be insufficient if renegotiation were possible. A contractual agreement could prevent the seller from unilaterally raising the price. But suppose the buyer invested  $I'$ , where  $I' < I^u$ , and proposed to the seller that it lower its price from  $p^u$  to  $p^*(I')$ . It would be in both the buyer and seller's interests to renegotiate the contract in this way rather than maintain  $p = p^u$ . Anticipating renegotiation, the buyer would solve the program

$$\max_I \begin{cases} \int_{p^*(I)}^{\infty} (1 - F_r(r|I)) dr - I, & \text{if } I \leq I^u \\ \int_{p^u}^{\infty} (1 - F_r(r|I)) dr - I, & \text{if } I > I^u \end{cases}.$$

When the Monotone Hazard Condition is satisfied, the solution to this program is  $I^o < I^u$ . In other words, it is not necessarily enough for the seller

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<sup>28</sup>In a different context, Inderst and Wey (2006) also find that a change that raises the investor's overall level of returns can lower investment incentives. Specifically, they find that a decrease in buyer power, which raises the seller's profits, can decrease the seller's incentives to invest in reducing its costs.

to commit not to price opportunistically; it could also be necessary for the seller to commit not to negotiate discounts off its posted price.

In some settings, the seller may be able to establish a reputation for neither making use of the available information nor engaging in hold up. Lafontaine and Shaw (1999) examined a large panel of franchise contracts (specifically, royalty rates and franchise fees) over 13 years. The authors found large differences across franchise systems but that the franchise fee and royalty rate are generally the same for all franchisees joining a given system at a given time (*i.e.*, there is no customization of contracts to idiosyncratic conditions). Moreover, they found that renewals occur at the then-current terms for new franchisees. Although there are other explanations for this behavior, we observe that it is consistent with the implications of the present analysis.

The welfare results above compare the polar cases of unobservable and observable investment. An interesting question is whether the welfare effects of intermediate cases lie between those of the two poles. We explore the answer to this question by making use of the information structure of Lau (2008) to generate a parameterized family of intermediate cases:

**Proposition 10** *Suppose that the Monotone Hazard Condition is satisfied and that with probability  $\zeta$  the buyer's investment level is revealed to the seller after the buyer has chosen its investment level but before the seller has set its price. Then the equilibrium investment level and seller's profits are weakly decreasing in  $\zeta$ , while the buyer's profits are weakly increasing in  $\zeta$ .*

## 5 A Noisy Signal of Returns

When investment is observable, it serves as a signal of  $r$ . The fact that the signal corresponds to investment allowed us to put considerable structure on the relationship between the signal and  $r$ . We now allow  $s$  to be an arbitrary but informative signal of  $r$ . Conditional on the signal  $s$  and the seller's anticipated value of  $I$ ,  $I_0$ , the seller seeks to maximize

$$p(1 - G(p|s, I_0)), \quad (15)$$

where  $G(r|s, I)$  is the distribution of  $r$  conditional on  $s$  and  $I$ .<sup>29</sup>

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<sup>29</sup>Note that, in any pure-strategy equilibrium,  $s$  does not serve as a signal of  $I$ . Specifically, observation of the value of  $s$  never leads to the seller to revise its beliefs along

Given a price schedule  $p^*(s, I_0)$ , the buyer's investment problem is

$$\begin{aligned} \max_I \int_0^\infty \int_{p^*(s, I_0)}^\infty (r - p^*(s, I_0)) f(r, s|I) dr ds - I \\ = \max_I \int_0^\infty \left( \int_{p^*(s, I_0)}^\infty (1 - G(r|s, I)) dr \right) f_s(s|I) ds - I, \end{aligned} \quad (16)$$

where  $f_s(\cdot|I)$  denotes the density function of the signal conditional on investment.

As we saw in our examination of  $s \equiv I$ , it is necessary to put structure on the problem in order to obtain definitive results. To that end, we assume that  $s$  is a noisy signal of  $r$  such that  $s = 0$  with probability 1 if  $I = 0$  and an increase in  $I$  leads to an improvement in the marginal distribution of  $s$  in the sense of first-order stochastic dominance. We also extend the Monotone Hazard Condition to include  $s$ :

**Assumption 3** *For any  $I > 0$ , the hazard rate associated with the distribution of the buyer's returns conditional on its investment and the signal is decreasing in both the signal and the level of investment. In addition, conditional demand,  $1 - G(\cdot|s, I)$ , is log concave when positive.*

**Proposition 11** *Suppose that an increase in investment leads to an improvement in the distribution of  $s$  in the sense of first-order stochastic dominance and that Assumption 3 is satisfied. Then:*

- (i) *the seller's profit-maximizing price increases with both the signal and the anticipated value of investment;*
- (ii) *an increase in investment by the buyer raises the seller's profits;*
- (iii) *given the equilibrium price schedule chosen by the seller, the buyer's equilibrium investment level is less than the second-best amount unless the latter is zero; and*
- (iv) *depending on the parameter values, pricing based on a noisy-but-informative signal of returns either raises or lowers the equilibrium level of the buyer's investment and profits relative to pricing based on a perfectly uninformative signal.*

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the equilibrium path, and even off of the equilibrium path the seller revises its beliefs only if the distribution of  $s$  conditional on  $I$  has a shifting support and the value of  $s$  is inconsistent with the equilibrium value of  $I$ .

We prove parts (i) through (iii), which extend the results of the observable-investment case (*i.e.*, Proposition 1, Lemma 1, and Proposition 5) in the Appendix. We prove part (iv), which differs from the observable-investment case (*i.e.*, Propositions 6 and 8) by example. The fact that, with a general signal, the buyer could be worse off than when the seller's signal is perfectly uninformative is easily shown (*e.g.*, when  $s$  is a near-perfect signal of  $r$  and, thus, allows near-perfect rent extraction by the seller). Proving that the investment effects are ambiguous requires a considerably more complex example, which is available from the authors upon request.

The reason for the lack of definitive results despite strong assumptions about the distributions (*e.g.*, Assumption 3) is that there are several effects at work. Signal-based pricing typically affects both the level and slope of the seller's price as a function of  $s$ . When  $s$  conveys no information about  $r$ , this function is flat. When the Monotone Hazard Condition is extended to encompass  $s$ , the seller tends to charge the buyer more when the buyer's revenues are high than when they are low.<sup>30</sup> Although it might seem that the upward slope would tend to discourage investment, we have constructed an example (again, available upon request) in which, for a given level of investment, the equilibrium price when the seller cannot observe the signal is always at least as high and sometimes strictly higher than the price charged when the seller can observe the signal; yet, the buyer's investment incentives are higher when the seller cannot observe the signal. The effects of the increase in the seller's information depend on whether the price rises faster or slower than the buyer's returns conditional on  $s$ . Relatedly, it can be shown that *ex post* efficiency can be smaller or greater when the seller cannot condition price on  $s$  than when it can condition price on  $s$ .

## 6 Price Discrimination

One interpretation of our analysis is that it demonstrates the effects of exogenously given differences in the information structure (*e.g.*, human capital investments may be harder to observe than investments in specialized machinery). Another interpretation is that the analysis sheds light on the

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<sup>30</sup>Although it is outside of our model, observe that, if the buyer were risk averse, pricing based on  $s$  could allow the seller to provide a form of insurance in those cases where a low realization of  $r$  was likely to be correlated with a low realization of  $s$ .

effects of price discrimination.<sup>31</sup> The investment effects of discrimination are of considerable interest for public policy.<sup>32</sup> For example, arguments about the effects of price discrimination on investment lie at the heart of much of the current debate over “network neutrality” regulation. In particular, one aspect of the debate is whether to ban Internet access providers from discriminating among applications providers that rely on the Internet access providers to reach household customers. This policy decision is sometimes framed as a choice between: (a) allowing discrimination in order to generate profits and investment incentives for Internet service providers,<sup>33</sup> and (b) banning discrimination in order to raise application provider’s profits and investment incentives.<sup>34</sup> Our analysis indicates that this framing could well be incorrect when the seller is imperfectly informed about the returns to the buyer’s investment; in particular, it could well be application providers who would benefit (and, in some cases, invest more) and ISPs who would suffer if discrimination were permitted.<sup>35</sup>

Our finding that discrimination can raise buyer investment stands in contrast to the main thrust of the relatively small literature on price discrimina-

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<sup>31</sup>Although modeling discrimination is straightforward, modeling non-discrimination is a bit delicate. A natural interpretation of a non-discrimination requirement is that the seller can observe the value of each buyer’s signal but must charge the same price to all buyers. For analytical tractability, we model non-discrimination as a perfectly uninformative signal. When discrimination corresponds to basing price on the buyer’s investment level and there is a continuum of *ex ante* identical buyers, the two approaches are equivalent. When discrimination is based on a general, noisy signal of the buyer’s returns, however, our approach yields a constant price while the seller’s non-discriminatory price under the other interpretation typically varies with the sample distribution of the signals of various buyers’ returns.

<sup>32</sup>At the U.S. federal level, the Robinson-Patman Act prohibits a supplier from engaging in price discrimination that harms competition. There are also numerous state laws that limit the franchisors, manufacturers, or wholesalers of various products from discriminating among their retail distributors or franchisees.

<sup>33</sup>See FTC Report, page 10 and Section III.B.6.

<sup>34</sup>See Hemphill (2008, p. 164).

<sup>35</sup>One might therefore ask why ISPs, which control the “last mile” connecting households to the Internet, lobby to preserve their—as yet unused—right to discriminate. One possible answer is that ISPs may be concerned that any initial limitations on their pricing behavior may lead to stronger forms of price regulation in the future. Of course, another possibility is that the Monotone Hazard Condition is not satisfied and future discrimination might be profitable.



tion and investment. Katz (1987, Proposition A.4) showed that, when buyers cannot backward integrate and the seller is perfectly informed about the cost and demand conditions that the buyers face, a discriminating upstream monopolist selling to downstream Cournot competitors charges higher prices to those firms whose production costs excluding the cost of the monopolized input are lower. Several authors have since shown that this pattern of input pricing dampens the downstream firms' incentives to make cost-reducing investments when the results of those investments are observable. DeGraba (1990) examined an input producer, Haucap and Wey (2004) considered a labor union, and Choi (1995) examined tariff setting, where the government can be interpreted as a monopoly seller of sales licenses.

Like us, Kim and Nahm (2007) and Inderst and Valletti (2006) find that discrimination can raise downstream investment incentives in some circumstances. The forces at work are very different, however. Specifically, buyer interdependency is essential to the result of Kim and Nahm, and they find that discrimination lowers downstream R&D investment incentives when the buyers are local monopolists.<sup>36</sup> Turning to Inderst and Valletti, the seller in their model observes a perfect signal of  $r$ . However, the seller's ability to extract a buyer's quasi-rents is limited by the buyer's threat to switch to an alternative source of supply that can be accessed only by incurring a fixed cost. A buyer with lower marginal costs of production excluding the cost of the input under examination has a stronger threat of switching. Hence, a discriminating upstream supplier reduces the price charged to a buyer that invests more, which increases investment incentives.

## 7 Concluding Remarks

In our baseline model, only the buyer makes an investment decision. We now briefly examine the seller's investment incentives. Suppose that the seller sinks its investment before the buyer arrives. If the seller's potential investment projects have the following pattern of returns, then there is a

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<sup>36</sup>When the buyers are Hotelling duopolists that charge two-part tariffs to final customers, the downstream firm with lower costs sells more units of output per customer than does the higher cost firm. Hence, the upstream supplier has an incentive to steer downstream customers to the lower-cost firm and does so by raising the input price charged to the higher-cost firm, which strengthens the downstream incentives to invest in lower costs.

simple mapping from the seller's expected profits in the continuation game to the investment levels. Suppose that all projects have only two possible states, *success* and *failure*, and the seller cannot operate absent success. Moreover, suppose that increased amounts of seller investment raise the probability of success. Then, the seller's equilibrium investment level is a weakly increasing function of its profits in the continuation game. The analysis in the text thus demonstrates, for example, that, if the buyer's investment returns satisfy the Monotone Hazard Condition, then the seller's investment incentives are *lower* when it can later observe the buyer's investment level than when the seller's signal is perfectly uninformative. Of course, in a more complex model of seller investment or buyer-seller bargaining, additional effects could arise.<sup>37</sup>

As we have shown, in general "anything can happen" when the seller's information is imperfect and the only regularity condition imposed is that an increase in buyer investment raises the unconditional expected value of the buyer's gross value of the good. The reason, in part, is that the shape of the distribution of the buyer's gross value influences the share of it that the seller can appropriate. An increase in buyer investment can change the distribution in ways such that this share may rise or fall. When the seller's share falls sufficiently fast, the buyer can face socially excessive investment incentives. In other cases, however, the distortion runs in the opposite direction.

When the buyer's investment level is the signal, one can say much more under the assumption that an increase in investment leads to an improvement in the distribution of gross value in the sense of first-order stochastic dominance, and still more under the Monotone Hazard Condition. Indeed, making the latter assumption, we established conditions under which the seller's ability to observe the buyer's investment harms the seller, benefits the buyer, and reduces equilibrium investment and total surplus.

However, we also found that, even under an extended version of the Monotone Hazard Condition, definitive results are scarce in the noisy-signal-of-returns case. What should one make of this finding? One implication is that it is necessary to look in great detail at the buyer's technology and the

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<sup>37</sup>When both buyer and seller can invest, the problem is similar to a two-sided agency problem (see, *e.g.*, Demski and Sappington, 1991; Rogerson, 1992; and Hermalin and Katz, 1993). These analyses all presume an ability to contract prior to investment. Although not completely analogous to the case of investment preceding contracting, the extensions of Demski and Sappington that consider contract renegotiation (*e.g.*, Nöldeke and Schmidt, 1995, 1998; and Edlin and Hermalin, 2000) may provide some insight at least with respect to the observable-investment case.

market's information structure before reaching conclusions about the investment effects of increased seller information or an ability to engage in price discrimination. For example, as discussed above, claims about the effects of discrimination on buyer and seller investment incentives play a major role in the network neutrality debate. Our analysis suggests that it is not evident that basing the prices charged applications providers for Internet carriage on various signals of willingness to pay would either adversely affect application providers' investments or promote Internet access providers' investments.

## Appendix

**Proof of Proposition 2:** The buyer's expected profit given  $p$  is

$$\int_p^\infty D(r|I)dr - I.$$

The derivative of profit with respect to investment is

$$\int_p^\infty \frac{-\partial F_r(r|I)}{\partial I} dr - 1. \quad (17)$$

The FOSD Improvement Condition implies  $\partial F_r(r|I)/\partial I < 0$  and, hence, that the derivative of profit with respect to investment is decreasing in  $p$  for any given  $I > 0$ . Utilizing a standard revealed-preference argument, it follows that  $I_1 \geq I_2$ .

Suppose, counterfactually, that  $I_1 = I_2 > 0$ . Rationality implies  $I_1$  is finite. Consequently, a necessary condition for both  $I_1$  and  $I_2$  is that (17) equal 0, which clearly cannot hold for  $I_1 = I_2$  when  $p_1 < p_2$ . ■

**Proof of Proposition 3:** Consider the program

$$\max_I \int_p^\infty (r - q)f_r(r|I)dr - I. \quad (18)$$

Observe that (18) is the buyer's optimization program if  $q = p$  and is the social planner's second-best program if  $q = 0$ . The derivative of the marginal return to investment with respect to  $q$  is

$$\frac{\partial F_r(p|I)}{\partial I} < 0, \quad (19)$$

where the inequality follows from first-order stochastic dominance. The result follows because, as just shown, an increase in  $q$  lowers the maximizer's marginal return to investment. ■

**Proof of Proposition 4:** If the buyer played a pure strategy, then the seller's best response would be  $p^*(I) = r(I)$  and the buyer's payoff would be  $-I \leq 0$ . Hence, the only pure-strategy equilibria are of the form  $I = 0$  and  $p \geq \max_I r(I) - I$ .

Now, suppose the buyer mixed and its expected investment level was positive. Then there would exist some  $I_0 > 0$  such that  $\Pr\{I \geq I_0\} > 0$ . Any price,  $p$ , that the seller charges with positive probability in equilibrium must yield as least as much expected profit as  $\Pr\{I \geq I_0\}r(I_0) \equiv \underline{p} > 0$ , which implies  $p \geq \underline{p}$ .

Because the buyer is willing to play  $I > 0$  with positive probability, there must exist  $I > 0$  such that  $r(I) - I - \underline{p} \geq 0$ . The continuity of  $r(\cdot)$  implies there is a smallest such  $I$ , call it  $I_1$ .  $r(0) = 0$  and  $\underline{p} > 0$  imply  $I_1 > 0$ . Observe  $I \in (0, I_1)$  is strictly dominated by 0 or  $I_1$ ; hence, the buyer never plays an  $I$  in that interval.

Let  $\mathcal{I}$  denote the support of the buyer's strategy. By supposition,  $\mathcal{I} \setminus \{0\} \neq \emptyset$ . Any  $I \in \mathcal{I} \setminus \{0\}$  must satisfy  $r(I) - I - \underline{p} \geq 0$ , which implies  $\underline{I} \equiv \inf \mathcal{I} \setminus \{0\} \geq I_1 > 0$ . Clearly, the seller will always charge at least  $r(\underline{I})$ . Hence,  $r(\underline{I}) - \underline{I} - p < 0$  for any  $p$  charged with positive probability. By the definition of  $\underline{I}$  as the greatest lower bound on the buyer's investment, and the continuity of  $r(\cdot)$ , this is a contradiction. ■

**Proof of Proposition 5:** The buyer chooses  $I$  to maximize the buyer's profits. The second-best program seeks to maximize the sum of the buyer's profits and the seller's profits. By the envelope theorem,  $d\pi^S(p^*(I), I)/dI = \partial\pi^S(p^*(I), I)/\partial I$ , which is positive by Lemma 1. Therefore, for any value of  $I$  that maximizes  $\pi^B(p^*(I), I)$ , there is a larger value of  $I$  that maximizes  $\pi^B(p^*(I), I) + \pi^S(p^*(I), I)$ . ■

**Proof of Proposition 10:** Consider two values of  $\zeta$ ,  $\zeta_H$  and  $\zeta_L$ , with  $\zeta_H > \zeta_L$ . Let  $I_H$  and  $I_L$  be the corresponding equilibrium levels of investment. Define

$$\Pi^B(I) \equiv \pi^B(p^*(I), I) - I.$$

Along the equilibrium path, the seller charges the same price whether or not the buyer's investment is revealed; hence,  $\Pi^B(I_t)$  is the buyer's equilibrium expected payoff given  $\zeta_t$ . Suppose, counterfactually, that  $\Pi^B(I_H) < \Pi^B(I_L)$ . By revealed preference

$$\Pi^B(I_H) \geq \zeta_H \Pi^B(I_L) + (1 - \zeta_H) \left\{ \pi^B(p^*(I_H), I_L) - I_L \right\}$$

and

$$\Pi^B(I_L) \geq \zeta_L \Pi^B(I_H) + (1 - \zeta_L) \left\{ \pi^B(p^*(I_L), I_H) - I_H \right\}. \quad (20)$$

These inequalities can be rearranged, respectively, to yield

$$\Pi^B(I_H) - \left\{ \pi^B(p^*(I_H), I_L) - I_L \right\} \geq \frac{\zeta_H}{1 - \zeta_H} \{ \Pi^B(I_L) - \Pi^B(I_H) \} \quad (21)$$

and

$$\frac{\zeta_L}{1 - \zeta_L} \{ \Pi^B(I_L) - \Pi^B(I_H) \} \geq \left\{ \pi^B(p^*(I_L), I_H) - I_H \right\} - \Pi(I_L). \quad (22)$$

Because  $\zeta_H > \zeta_L$  by assumption and  $\Pi^B(I_H) < \Pi^B(I_L)$  by supposition, expressions (21) and (22) imply

$$\Pi^B(I_H) - \left\{ \pi^B(p^*(I_H), I_L) - I_L \right\} > \left\{ \pi^B(p^*(I_L), I_H) - I_H \right\} - \Pi(I_L).$$

Rearranging, we have

$$\pi^B(p^*(I_H), I_H) - \pi^B(p^*(I_H), I_L) > \pi^B(p^*(I_L), I_H) - \pi^B(p^*(I_L), I_L). \quad (23)$$

The FOSD Improvement Condition, which is implied by the Monotone Hazard Condition, implies that  $\pi^B(p, I)$  has decreasing differences in  $p$  and  $I$ .<sup>38</sup> Consequently (23) can hold only if (i)  $p^*(I_H) < p^*(I_L)$  and  $I_H > I_L$  or (ii)  $p^*(I_H) > p^*(I_L)$  and  $I_H < I_L$ . But neither pairing is possible given Proposition 1. By contradiction, it must be that  $\Pi^B(I_H) \geq \Pi^B(I_L)$ .

If  $I_H > I_L$ , then  $p^*(I_H) > p^*(I_L)$  by Proposition 1. But then expression (20) implies  $\Pi^B(I_L) > \Pi^B(I_H)$ , a contradiction; hence, it must be that  $I_H \leq I_L$ . It therefore follows from Lemma 1 that the seller's expected profit is non-increasing in  $\zeta$ . ■

Let  $h(r|\mathbf{a})$  be the hazard rate associated with the conditional distribution  $G(r|\mathbf{a})$ , where  $\mathbf{a}$  is a scalar or a vector.

**Lemma A.1** *Suppose for all  $r \in (0, \infty)$  that  $h(r|\mathbf{a}) < h(r|\mathbf{a}')$ . Then  $G(r|\mathbf{a})$  dominates  $G(r|\mathbf{a}')$  in the sense of first-order stochastic dominance.*

**Proof:** The inequality relation between the conditional hazard rates implies

$$\int_0^r h(x|\mathbf{a})dx < \int_0^r h(x|\mathbf{a}')dx.$$

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<sup>38</sup>That is,  $I > I'$  and  $p > p'$  implies  $\pi^B(p, I) - \pi^B(p, I') < \pi^B(p', I) - \pi^B(p', I')$ .

Using the fact that

$$G(r|\mathbf{a}) \equiv 1 - e^{-\int_0^r h(x|\mathbf{a})dx},$$

it follows that

$$G(r|\mathbf{a}) = 1 - e^{-\int_0^r h(x|\mathbf{a})dx} < 1 - e^{-\int_0^r h(x|\mathbf{a}')dx} = G(r|\mathbf{a}').$$

■

Let  $G(r|s, I)$  be the distribution of  $r$  conditional on  $s$  and  $I$ . Let  $h(\cdot|s, I)$  be the corresponding hazard rate. Let  $F_s(s|I)$  be the distribution of the signal  $s$  conditional on investment and  $f_s(\cdot|I)$  denote the corresponding density.

**Proof of Proposition 11:**

Part (i) follows from the fact that  $1 = \epsilon = ph(p|s, I)$ .

To prove part (ii), fix an investment level  $I_0 > 0$  (if  $I_0 = 0$ , then the result is immediate). Consider  $I_1 > I_0$ . Observe that the seller's profit given  $I_1$  is

$$\begin{aligned} \max_{p(\cdot)} \int_0^\infty p(s)(1-G(p(s)|s, I_1))f_s(s|I_1)ds \\ \geq \int_0^\infty p^*(s, I_0)(1-G(p^*(s, I_0)|s, I_1))f_s(s|I_1)ds \end{aligned} \quad (24)$$

$$> \int_0^\infty p^*(s, I_0)(1-G(p^*(s, I_0)|s, I_0))f_s(s|I_1)ds \quad (25)$$

$$\geq \int_0^\infty p^*(s, I_0)(1-G(p^*(s, I_0)|s, I_0))f_s(s|I_0)ds, \quad (26)$$

where the inequality in (25) follows because an increase in  $I$  improves  $G(p|s, I)$  in the sense of first-order stochastic dominance. Given the first-order stochastic dominance assumption on  $F_s(s|I)$ , the inequality in (26) follows if  $p^*(s, I_0) \times (1 - G(p^*(s, I_0)|s, I_0))$  is a non-decreasing function of  $s$ . That it is can be seen by employing the envelope theorem and recalling that an increase in  $s$  improves  $G(p|s, I)$  in the sense of first-order stochastic dominance. Because (26) is the seller's profit given  $I_0$ , an increase in the buyer's investment would raise the seller's profits.

To prove part (iii), suppose that, contrary to (iii), the buyer chooses the investment level that maximizes  $W(p^*(\cdot), I)$ . From the necessary conditions for an optimum, the partial derivative of  $W(p^*(\cdot), I)$  with respect to  $I$  is zero. Because that derivative is the sum of the derivatives of the seller's and buyer's profits with respect to  $I$ , and the first part of the proof showed that

the former is positive, the latter must be negative. But, then, the buyer is not playing a best response to the seller's equilibrium strategy, a contradiction. The result follows *reductio ad absurdum*. ■



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