Abstract

We consider how parties’ formal contracts are underpinned by their ongoing relationship and how welfare changes as the legal system improves. Regardless of impatience, the parties write formal contracts they wouldn’t honor—despite stipulated penalties—were interaction one shot. The change in welfare with an improvement in the legal system can be ambiguous and even non-monotonic.

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1 Introduction

Evidence from bankruptcies suggests that there is a tendency for firms to breach their formal (written) contracts when their bilateral relationship suddenly seems at an end (see, e.g., Triantis, 1993). This suggests that contracts are honored not only because of the contractually stipulated damages, but also because of the effect of breach on the ongoing relationship. As we show, an ongoing relationship permits parties to write formal contracts that would not be worth the paper on which they were written absent the relationship. Moreover, we find that no matter how impatient the parties are, they always write contracts they would fail to honor if they interacted only once.

We also provide a framework in which to study how an improving legal system affects welfare. Our analysis offers a more nuanced assessment of the issue than the previous literature: improving formal contracting (i.e., contracts enforced by courts) can be welfare reducing or enhancing; moreover, welfare need not vary monotonically with improvements in the legal system.

2 Model

2.1 Basic Assumptions

There are two parties. In each period, they agree to a contract for that period; each party then chooses an action, \( q \in \mathbb{R}_+ \); and, finally, payoffs are realized.

Let party \( i \)'s payoff be \( \beta(q_i, q_j) - c(q_i) \), where benefit \( \beta : \mathbb{R}_+^2 \to \mathbb{R} \) and cost \( c : \mathbb{R}_+ \to \mathbb{R}_+ \) are both twice continuously differentiable. Assume:

- Marginal cost is increasing in action (i.e., \( c''(\cdot) > 0 \)). To ensure interior maxima, assume \( c'(0) = 0 \).
- The benefit of no actions is normalized to zero (i.e., \( \beta(0, 0) = 0 \)).
- On some margins, at least, benefit increases in action: \( \partial \beta(q_i, q_j) / \partial q_i > 0 \) whenever \( q_i \leq q_j \).

To keep matters straightforward, we assume a very symmetric setting: \( \beta(q, q') = \beta(q', q) \) and symmetry of action is desirable; that is, \( q_1 + q_2 = q'_1 + q'_2 \) and

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1 Macaulay (1963) suggested that informal ties between parties could strengthen their formal contracting, although he did not explore this via an economic model.

2 A few articles (e.g., Sobel, 2006; Battigalli and Maggi, 2008; and Kvaløy and Olsen, 2009) study how an ongoing relationship affects the cost of establishing formal contracts. Here, in contrast, we assume the use of formal contracts is costless.

3 There is evidence (Paley, 1984) that firms write contracts that they would find difficult to enforce in court (in Paley’s study, the contracts were inconsistent with existing regulations).

4 Schmidt and Schnitzer (1995) present a model in which improved formal contracting is welfare reducing because it undermines relational contracts. Other relevant articles in this vein include Kranton (1996), Kranton and Swamy (1999), and McMillan and Woodruff (1999a,b). Like us, Baker et al. (1994) show that improved formal contracting can have ambiguous effects on the ability to sustain relational contracting. As discussed below, their result arises for different reasons than here.

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$|q_1 - q_2| > |q'_1 - q'_2|$ imply $\beta(q_1, q_2) \leq \beta(q'_1, q'_2)$. Two examples: $\beta(q_1, q_2) = f(q_1) + f(q_2)$, $f(\cdot)$ concave; and

$$\beta(q_1, q_2) = (q_1 + q_2)^{3/2} + \exp\left(- (q_1 - q_2)^2\right).$$

Note the latter is not a concave function.

Assume a positive and finite value $q_M$ such that

$$2 \frac{\partial \beta(q, q)}{\partial q} - c'(q) < 0$$

for all $q > q_M$. Consequently, the parties always wish to choose a finite action.

Assume the two parties are the “only game in town,” insofar as neither can trade with a third party. A relational breakdown is, thus, not punished by terminating the relationship, but by reversion to the equilibrium of the one-shot contracting game.

2.2 The Legal System

A contract specifies the parties’ actions. Party $i$ has breached contract $(\bar{q}_1, \bar{q}_2)$ if $q_i < \bar{q}_i$. The legal system is imperfect: in the event of breach, a court awards damages to the injured party with probability $\theta \in [0,1)$ only. Note $\theta = 0$ is equivalent to no court system and greater values of $\theta$ represent better legal systems. The parameter $\theta$ is common knowledge at the time of contracting.

In keeping with the law’s abhorrence of penalties in private contracts (see e.g., Hermelin et al., 2007, §5.3), assume damages, $D$, cannot exceed the actual loss suffered; hence,

$$D \leq \beta(\bar{q}_1, \bar{q}_2) - \beta(q_i, \bar{q}_j).$$

The courts will dismiss greater damage claims as punitive (consistent with practice in many legal systems).

In equilibrium, the parties honor their contract. It is, therefore, without loss to assume the parties seek maximum deterrence: (2) binds. The expected payoffs to the breaching party (without loss of generality, 1) and to the injured party (here, 2) are, respectively,

$$\Pi_1 = (\theta + 1)\beta(q_1, \bar{q}_2) - \theta \beta(\bar{q}_1, \bar{q}_2) - c(q_1) \quad \text{and}$$

$$\Pi_2 = (1 - \theta)\beta(q_1, \bar{q}_2) + \theta \beta(\bar{q}_1, \bar{q}_2) - c(\bar{q}_2).$$

5Formally, the function $\beta$ is Schur concave.

6The analysis can be extended to allow the parties to terminate their relationship and search for new partners: if (i) reputation is public; or (ii) a newly partnerless party is mistrusted, so limited to playing the one-shot equilibrium with new partners. In either alternative interpretation, the breaching party’s continuation payoffs considered below would simply be the payoffs from contracts it signs with a new partner (or partners).

7The possibility of “breach” in which $q_i > \bar{q}_i$ can be ignored: in equilibrium, the parties don’t write contracts that give incentives to “over” do it.

8Similar results would attain if there were no limits on damages, but the breacher could escape overly large damages via bankruptcy.
3 First Best and the One-Shot Game

The following lemma is critical to the subsequent analysis.

Lemma 1. For
\[ \zeta \beta(q_1, q_2) - c(q_1) - c(q_2), \] (4)
\[ \zeta \in (0, 2], \] the following hold:

(i) If \( q_1' + q_2' = q_1'' + q_2'' \), but \( |q_1' - q_2'| > |q_1'' - q_2''| \), then (4) is greater given \((q_1', q_2')\) than given \((q_1'', q_2'')\).

(ii) There exists a finite action, \( q^*(\zeta) \), such that (4) is maximized if each party chooses it.

(iii) \( \zeta > \zeta' \) implies \( q^*(\zeta) > q^*(\zeta') \).

(iv) Holding one party’s action fixed, a finite action exists for the other that maximizes (4).

(v) The action in part (iv) is increasing in \( \zeta \).

Joint payoffs are
\[ \beta(q_1, q_2) - c(q_1) + \beta(q_1, q_2) - c(q_2) = 2\beta(q_1, q_2) - c(q_1) - c(q_2). \] (5)

From the lemma, there exists a finite \( q^*(2) \) that, if each party chooses it, maximizes joint payoffs.

Define
\[ q^{BR}(q, \zeta) = \arg\max_x \zeta \beta(x, q) - c(x). \] (6)

Lemma 2. \( q^{BR}(q^*(\zeta), \zeta) = q^*(\zeta) \).

Lemma 2 implies that a Nash equilibrium of the game played once, absent any contract, is for both parties to choose \( q^*(1) \).

If party \( i \) will honor the contract \( \langle \bar{q}_i, \bar{q}_j \rangle \), then
\[ \beta(\bar{q}_i, \bar{q}_j) - c(\bar{q}_i) \geq \max_q (\theta + 1)\beta(q, \bar{q}_j) - \theta \beta(\bar{q}_i, \bar{q}_j) - c(q), \] (7)
where the right-hand side (RHS) follows from (3). Observe
\[ (\theta + 1)\beta(\bar{q}_i, \bar{q}_j) - c(\bar{q}_i) \geq \max_q (\theta + 1)\beta(q, \bar{q}_j) - c(q) \] (8)
is equivalent to (7); hence,

Lemma 3. A contract \( \langle \bar{q}_i, \bar{q}_2 \rangle \) will be honored in equilibrium only if \( \bar{q}_i = q^{BR}(\bar{q}_j, \theta + 1) \) and \( \bar{q}_j = q^{BR}(\bar{q}_i, \theta + 1) \).

Although not essential, it speeds the analysis if \( q^*(\zeta) \) is unique \( \forall \zeta \in [1, 2] \):

Assumption 1. The univariate function \( \mathbb{R}_+ \to \mathbb{R} \) defined by \( q \mapsto \zeta \beta(q, q) - 2c(q) \) is a strictly concave function (of \( q \)) for all \( \zeta \in [1, 2] \).
We can now establish:

**Proposition 1.** If the quality of the legal system is $\theta$, $\theta \in [0, 1)$, then the only formal contract that will be honored as a pure-strategy equilibrium in the one-shot game is one that has each party play $q^*(\theta + 1)$.

Proposition 1 has a few implications. Given Lemma 1(v), it implies that the equilibrium action is greater the better is the legal system. Second, if the court has minimum quality or is non-existent (i.e., $\theta = 0$), then the outcome is identical to one in which no contract is written. Third, a perfect court (i.e., $\theta = 1$) results in the first-best outcome. Finally, given Assumption 1, the closer $q^*(\theta + 1)$ is to $q^*(2)$, the greater is welfare: absent repeated play, a better legal system enhances welfare.

## 4 The Repeated Game

Consider an infinitely repeated game. Let $\delta \in (0, 1)$ denote the common discount factor. In what follows, the parties will write a contract $\langle \hat{q}, \hat{q} \rangle$ that is better than the one-shot (Proposition 1) contract; that is, $\hat{q} \in [q^*(\theta + 1), q^*(2)]$. Such a contract is supported by both the penalty for breach and the threat of reversion to the equilibrium of the one-shot game. Let

$$\pi_{\text{ONE}}(\theta) \equiv \beta(q^*(\theta + 1), q^*(\theta + 1)) - c(q^*(\theta + 1))$$

be a party’s payoff in the equilibrium of the one-shot game. Hence, the parties will honor $\langle \hat{q}, \hat{q} \rangle$ provided

$$\left(\frac{\beta(q, \hat{q}) - c(q)}{1 - \delta}\right) \geq \left(\frac{\max(q + 1, \beta(q, \hat{q}) - \theta \beta(q, \hat{q}) - c(q))}{\delta}\right) + \frac{\delta}{1 - \delta} \pi_{\text{ONE}}(\theta). \quad (9)$$

**Proposition 2.** Suppose a positive discount factor. Then, in a repeated-game equilibrium, the parties’ contract will stipulate a greater and more efficient level of action than could be sustained by the legal system alone (i.e., than would prevail in the one-shot equilibrium).

Not surprisingly, if the parties are sufficiently patient, then full efficiency is attainable. Let $\delta^*(\theta) \in (0, 1)$ solve (9) as an equality when $\hat{q} = q^*(2)$. Provided $\delta \geq \delta^*$, a fully efficient equilibrium exists.\(^9\)

How does equilibrium welfare depend on the quality of the legal system (i.e., $\theta$)? Unlike some articles that find a higher quality legal system can have a deleterious effect on welfare (see, e.g., Schmidt and Schnitzer, 1995), here the effect is ambiguous:

**Proposition 3.** Suppose $c(q) = q^2/2$. The discount-factor cutoff, $\delta^*(\theta)$, for which full efficiency can be supported is

\(^9\)The proof, which is straightforward, is omitted.
Figure 1: Welfare can be non-monotonic in the quality of the legal system. Figure drawn assuming $\beta(q_1, q_2) = q_1 + q_2 - (q_1 - q_2)^2$, $c(q) = q^2/2$, and $\delta = 3/10$. Scales of horizontal and vertical axes are not the same.

(i) greater, the better is the legal system if $\beta(q_1, q_2) = \log(q_1 + q_2 + 1)$;
(ii) constant with respect to the legal system if $\beta(q_1, q_2) = q_1 + q_2$; and
(iii) less, the better is the legal system if $\beta(q_1, q_2) = q_1 + q_2 - (q_1 - q_2)^2$.

A better legal system has two, conflicting, effects on welfare: (i) it makes formal contracts more effective, which increases welfare; but (ii), it makes a breakdown of the repeated relationship less dire, which makes cooperation harder to sustain, which reduces welfare.\(^{10}\) Moreover, a non-monotonic relation between the quality of the legal system and welfare can exist: in Figure 1, the maximum $\hat{q}$ that can be supported in equilibrium was calculated given $\theta$ and the corresponding per-period welfare plotted.

5 Conclusion

We have presented a model in which parties, no matter how much they discount the future, write formal contracts they would have no intention of honoring on a one-time basis. That the parties do honor them is a consequence of both the

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\(^{10}\)In Baker et al. (1994), if the parties will still trade after breakdown of the relational contract, then improving the formal contracting technology reduces welfare due to reason (ii). If the parties would cease to trade altogether, then improving the technology can enhance welfare because it makes the value of the ongoing relation more valuable. Here, the parties always trade, so the ambiguous effects of better formal contracting on welfare operate through a different mechanism than in Baker et al., as detailed in the text.
stipulated damages in the contract and a wish not to erode their ongoing relationship. In such an environment, an improved legal system has two conflicting effects: it reduces the value of the ongoing relationship, because the fallback position of relying on contracts that would be honored even on a one-time basis becomes better; at the same time, it raises the expected payment a contract breacher pays to the injured party. The first effect encourages breach, the second discourages it. This leads to a more nuanced and ambiguous view of how improvements in legal systems affect welfare.

Appendix A: Proofs and Additional Material

A well-known revealed-preference result (proof available upon request) is

**Theorem 1.** Let \( f(\cdot, \cdot) : \mathbb{R}^2 \to \mathbb{R} \) be twice differentiable with a cross-partial derivative of constant sign. Let \( \hat{x} \) maximize \( f(x, z) \) and \( \hat{x}' \) maximize \( f(x, z') \), \( z > z' \). Then \( \hat{x} \geq \hat{x}' \) if the cross partial is positive and \( \hat{x} \leq \hat{x}' \) if it is negative. The inequalities are strict if \( \hat{x} \) or \( \hat{x}' \) is an interior maximum.

**Proof of Lemma 1:** The strict convexity of \( c(\cdot) \) implies \(-c(q_1) - c(q_2)\) is strictly Schur concave (Marshall and Olkin, 1979, p. 64). \(^{11}\) Schur concavity is maintained under addition and positive scalar multiplication, so

\[
\zeta \beta(q_1, q_2) - c(q_1) - c(q_2)
\]

is strictly Schur concave. Result (i) follows.

Consequently, \((q_1, q_2)\) maximizes (4) only if \(q_1 = q_2\). Action profile \((0, 0)\) doesn’t maximize (4) because \( \partial \beta(0, 0)/\partial q_i > 0 \). Consider

\[
\max_{(q_1, q_2)} 2\beta(q_1, q_2) - c(q_1) - c(q_2),
\]

which, given part (i), is equivalent to

\[
\max_q 2\beta(q, q) - 2c(q) \quad (10)
\]

Expression (10) is smaller if \(q > q_M\) than if \(q = q_M\). Attention can be limited to \(q \in [0, q_M]\), a compact domain. A maximizer of (10), \(q^*(2)\), therefore exists. From Theorem 1, for any \(q > q^*(2)\),

\[
\zeta \beta(q, q) - 2c(q) < \zeta \beta(q^*(2), q^*(2)) - 2c(q^*(2)) \quad (11)
\]

A maximizer, \(q^*(\zeta)\), of the left-hand side (LHS) of (11) is thus in \([0, q^*(2)]\).

Results (iii) and (v) follow immediately from Theorem 1.

\(^{11}\)If function \( \phi : \mathbb{R}^2 \to \mathbb{R} \) is Schur concave (alternatively, strictly so), then \( x_1 + x_2 = x'_1 + x'_2 \) and \(|x_1 - x_2| > |x'_1 - x'_2|\) imply \( \phi(x_1, x_2) \leq \phi(x'_1, x'_2) \) (alternatively, imply \( \phi(x_1, x_2) < \phi(x'_1, x'_2) \)).
Consider (iv). Were it false, then $\zeta$ and $q^2$ would exist such that, for any $\bar{q}^1$, a $q_1 > \bar{q}^1$ exists such that

$$
\zeta \frac{\partial \beta(q_1, q^2)}{\partial q_1} - c'(q_1) > 0.
$$

(12)

Consider $q_1 > \max\{q_M, q^2\}$ satisfying (12). The function $\beta$ is Schur concave if and only if

$$
(x_1 - x_2) \left( \frac{\partial \beta(x_1, x_2)}{\partial x_1} - \frac{\partial \beta(x_1, x_2)}{\partial x_2} \right) \leq 0
$$

(13)

(Marshall and Olkin, 1979, p. 57). Because $c(\cdot)$ is strictly convex, (12) implies, for all $q < q_1$, that

$$
\zeta \frac{\partial \beta(q_1, q)}{\partial q^2} - c'(q) > 0.
$$

Letting $q \to q_1$,

$$
\zeta \frac{\partial \beta(q_1, q_1)}{\partial q^2} - c'(q_1) \geq 0.
$$

Given symmetry, this, in turn, implies

$$
0 \leq 2 \frac{\partial \beta(q_1, q_1)}{\partial q_1} - c'(q_1).
$$

(14)

Because $q_1 > q_M$, (14) contradicts (1). The result follows.

Proof of Lemma 2: By definition of an optimum:

$$
\zeta \beta \left( q^{BR}(q^*(\zeta), \zeta), q^*(\zeta) \right) - c \left( q^{BR}(q^*(\zeta), \zeta) \right) \geq \zeta \beta \left( q^*(\zeta), q^*(\zeta) \right) - c(q^*(\zeta)).
$$

Subtracting $c(q^*(\zeta))$ from both sides:

$$
\zeta \beta \left( q^{BR}(q^*(\zeta), \zeta), q^*(\zeta) \right) - c \left( q^{BR}(q^*(\zeta), \zeta) \right) - c(q^*(\zeta))
$$

\[ \geq \zeta \beta \left( q^*(\zeta), q^*(\zeta) \right) - c(q^*(\zeta)) - c(q^*(\zeta)). \]

(15)

But $q^*(\zeta)$ maximizes (4) and moreover, by Schur concavity, no pair $(q, q^*(\zeta))$, $q \neq q^*(\zeta)$ can: (15) can hold only if $q^{BR}(q^*(\zeta), \zeta) = q^*(\zeta)$.

Proof of Proposition 1: Lemmas 2 and 3 imply that both parties will honor the contract $\langle q^*(\theta + 1), q^*(\theta + 1) \rangle$ in equilibrium.

Uniqueness: we now show no contract $\langle \bar{q}_1, \bar{q}_2 \rangle$ will be honored in equilibrium if $\bar{q}_1 \neq \bar{q}_2$. Suppose $\bar{q}_2 > \bar{q}_1$. Given (13):

$$
\frac{\partial \beta(\bar{q}_1, \bar{q}_2)}{\partial q_2} \leq \frac{\partial \beta(\bar{q}_1, \bar{q}_2)}{\partial q_1}.
$$

(16)
Appendix A: Proofs and Additional Material

If \( \langle \bar{q}_1, \bar{q}_2 \rangle \) is honored, the actions are best responses and, so, solve the first-order conditions
\[
(\theta + 1) \frac{\partial \beta(\bar{q}_1, \bar{q}_2)}{\partial \bar{q}_1} - c'(\bar{q}_1) = 0 \quad \text{and} \quad (\theta + 1) \frac{\partial \beta(\bar{q}_1, \bar{q}_2)}{\partial \bar{q}_2} - c'(\bar{q}_2) = 0.
\] (17)

Expressions (16) and (17) yield
\[
c'(\bar{q}_1) = (\theta + 1) \frac{\partial \beta(\bar{q}_1, \bar{q}_2)}{\partial \bar{q}_1} \geq (\theta + 1) \frac{\partial \beta(\bar{q}_1, \bar{q}_2)}{\partial \bar{q}_2} = c'(\bar{q}_2) > c'(\bar{q}_1)
\]
(recall \( c(\cdot) \) is strictly convex). This is a contradiction: no contract with \( \bar{q}_1 \neq \bar{q}_2 \) will be honored in equilibrium.

Finally, we rule out \( \langle \bar{q}, \bar{q}_2 \rangle, \bar{q}_1 \neq q^*(\theta + 1) \). The first-order conditions, (17), imply \( \bar{q} \) maximizes \( (\theta + 1) \beta(q, q) - 2c(q) \). Assumption 1 entails that function has only one maximum, \( q^*(\theta + 1) \).

Proof of Proposition 2: Define
\[
\pi_{\text{dev}}(\theta, \hat{q}) \equiv \max_q (\theta + 1) \beta(q, \hat{q}) - \theta \beta(\hat{q}, \hat{q}) - c(q).
\]
From Lemma 2, \( \pi_{\text{dev}}(\theta, q^*(\theta + 1)) = \pi_{\text{one}}(\theta) \). Hence, (9) is an equality when \( \hat{q} = q^*(\theta) \). The result follows, therefore, if the LHS of
\[
\beta(\hat{q}, \hat{q}) - c(\hat{q}) = \delta \pi_{\text{one}}(\theta) + (1 - \delta) \pi_{\text{dev}}(\theta, \hat{q})
\] (18)
increases in \( \hat{q} \) faster than the RHS starting from \( \hat{q} = q^*(\theta + 1) \). The derivative of the LHS at \( \hat{q} = q^*(\theta + 1) \) is
\[
\frac{\partial \beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q_1} + \frac{\partial \beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q_2} - c'(q^*(\theta + 1))
\]
\[
= 2 \frac{\partial \beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q} - c'(q^*(\theta + 1))
\] (19)
\[
= (1 - \theta) \frac{\partial \beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q}, \quad (20)
\]
where (19) follows from symmetry and (20) from Proposition 1. The derivative of the RHS of (18) reduces, using the envelope theorem and evaluated at \( \hat{q} = q^*(\theta + 1) \), to
\[
(1 - \theta)(1 - \delta) \frac{\partial \beta(q^*(\theta + 1), q^*(\theta + 1))}{\partial q}.
\] (21)
Because \( \delta > 0 \), (20) exceeds (21), as was to be shown.

Proof of Proposition 3: The proof is by construction. Let \( \pi_{\text{dev}}(\theta) = \pi_{\text{dev}}(\theta, q^*(2)) \). Observe \( \delta^*(\theta) \) is the solution to
\[
\beta(q^*(2), q^*(2)) - c(q^*(2)) = \delta \pi_{\text{one}}(\theta) + (1 - \delta) \pi_{\text{dev}}(\theta).
\]
Hence,
\[
\delta^*(\theta) = \frac{\beta(q^*(2), q^*(2)) - c(q^*(2)) - \pi_{\text{dev}}(\theta)}{\pi_{\text{one}}(\theta) - \pi_{\text{dev}}(\theta)}.
\]

It follows that
\[
\text{sign} (\delta^*(\theta)) = \text{sign} \left( \left( \beta(q^*(2), q^*(2)) - c(q^*(2)) \right) \left( \pi'_{\text{dev}}(\theta) - \pi'_{\text{one}}(\theta) \right) \right). 
\]

For case (i), (22) is positive. For case (ii), calculations (available upon request) reveal that \(\delta^*(\theta) \equiv 1/2\), which is constant. Finally, for case (iii), calculations (available upon request) reveal that
\[
\delta^*(\theta) = \frac{-1}{(2 + \theta)^2} < 0.
\]

References


12The Mathematica program that verifies this is available from the authors upon request.


