Mock Exam

Economics 201B

1. There is a population of N consumers. Each consumer wants, at most, one unit of a good sold by a monopolist. A consumer's utility function is $v\delta - p$, where $\delta \in \{0,1\}$ indicates whether the consumer purchases the good, v is his valuation for the good, and p is the price he pays for the good if he chooses to purchase it. A consumer's utility if he doesn't purchase the good is 0. Assume that $v \sim U[v_0, v_1]$; that is, the probability that $v \leq \hat{v} = (\hat{v} - v_0)/(v_1 - v_0)$. Assume each of the consumers' valuations are drawn independently. For future reference, define $\Delta_v = v_1 - v_0$. For convenience, assume the marginal cost of production is 0.

   (a) What is expected demand as a function of price, p? What is expected *inverse* demand as function of quantity, x? [***Hint:*** be careful.]

   (b) As a function of the parameters of this problem, what is the price that maximizes expected profit (assuming linear pricing)?

   (c) Assume that N consumers are non-students. Assume that there is also a population of S students. Each student's utility function is $w\delta - p$, where the w are distributed independently according to the distribution $U[w_0, w_1]$. Assume $w_1 < v_0$. Derive sufficient conditions for third-degree price discrimination to be welfare improving relative to simple monopoly pricing.

2. Consider a society with I citizens and a benevolent social planner. The I citizens have preferences $v_i(x, \theta_i) + y_i$, where i indexes the citizen, x is the level of some public good, θ_i is citizen i's type, and y_i is citizen's i's allocation of a transferable good (money). Although the social planner is benevolent, she likes some citizens more than others. Hence, she seeks to maximize the following function

$$\sum_{i=1}^{I} \lambda_i v_i(x, \theta_i)\,,$$

   where the λ_i vary across citizens but are all positive. Assume that there exists a finite $x^*(\boldsymbol{\theta})$ that maximizes that sum for all $\boldsymbol{\theta} \in \Theta \equiv \Theta_1 \times \cdots \times \Theta_I$, where Θ_i is the possible type space for citizen i. Find a dominant strategy mechanism that implements $x^*(\cdot)$.

3. Consider a standard hidden-action agency model. Let the action space be $\{0,1\}$. Let the agent's utility be $u(y) - aC$, where $u(\cdot)$ is strictly increasing and concave, a denotes the action, and $C > 0$. Assume the

agent's reservation utility is zero. Let the performance measure, x, be drawn from the interval $[0,1]$. Assume that the distribution of x given a is $qaF_1(x) + (1-qa)F_0(x)$, where $0 < q < 1$. Define $F_i(\cdot)$ on $[0,1]$ as follows

$$F_1(x) = \begin{cases} 2(x - \frac{1}{2}), & \text{if } x \geq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases} \quad \text{and } F_0(x) = \begin{cases} 1, & \text{if } x \geq k \\ \frac{x}{k}, & \text{if } x \leq k \end{cases}$$

where $k \in \left(\frac{1}{2}, 1\right)$.

(a) What is the optimal contract for the principal to offer that implements $a = 1$?

(b) Consider an alternative model in which $k = 1/2$. Prove that the expected cost of implementing $a = 1$ is less in this alternative model than in the model presented above.