Unobserved Investment, Endogenous Quality, and Trade*

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Abstract

A seller can make investments that affect a tradable asset's future returns. The potential buyer of the asset cannot observe the seller's investment prior to trade, nor does he receive any signal of it, nor can he verify it in anyway after trade. Despite this severe moral hazard problem, this paper shows the seller will invest with positive probability in equilibrium and that trade will occur with positive probability. The outcome of the game is sensitive to the distribution of bargaining power between the parties, with a holdup problem existing if the buyer has the bargaining power. A consequence of the holdup problem is surplus-reducing distortions in investment level. Perhaps counterintuitively, in many situations, this distortion involves an *increase* in the expected amount invested *vis-à-vis* the situation without holdup.

Keywords: unobserved investment, endogenous quality, endogenous "lemons problem," trade, holdup

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Contents

1 Introduction

Google has acquired over 80 companies, most of them small startups.[1] Microsoft has acquired over 130 firms, again mostly small startups.[2] In fact, acquisition by a large firm is a common "exit strategy" for startup firms. This phenomenon, in which a startup firm is created with the possible objective of being acquired later, is an example of a more general economic situation in which one actor (*e.g.,* a homebuilder, an inventor, or the owner/entrepreneur of a small firm) invests in an asset (a new house, a new product, the firm, respectively) that she may ultimately wish to sell to another actor (*e.g.,* a homeowner, a large firm). Trade, if it occurs, does so before the value of the asset is fully known.[3] Indeed, in this paper, the limiting case of no information being revealed is considered.[4] Moreover, the return to the asset is greater in the potential buyer's hands than it is if it remains in the seller's hands. Critically, the distribution of returns to either actor depends on the seller's initial investment. Because the buyer cannot observe the seller's investment, a moral hazard problem exists; in particular, an equilibrium in which the seller invests a fixed amount and trade always occurs cannot exist because sure-to-happen trade destroys the seller's incentives to invest *ex ante*. On the other hand, if no trade is anticipated, the seller will invest for her own benefit; but then it would be incredible that trade wouldn't occur if the opportunity to trade later arose. The questions are then to what extent does the seller invest, with what frequency is there exchange, and what mechanism might the parties employ to facilitate exchange? That is, how is this endogenous lemons problem resolved?[5]

In some cases, a possible mechanism might be a revenue-sharing contract. However, one can conceive of contexts in which the returns generated by the asset are unverifiable, indeed even unobservable to the seller after trade. For instance, a large firm could use accounting tricks to obscure the returns ultimately generated by the purchased invention or small firm.[6] Or the returns could be private benefits (*e.g.,* the seller is a home builder, the buyer a would-be home owner). Furthermore, the reason the asset is more valuable in the buyer's

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[1] Author's count based on publicly available information.

[2] Author's count based on publicly available information.

[3] Many startups have negligible track records as mature, established firms. For example, like$_\star$com was acquired by Google just three years, nine months, and 15 days after its founding.

[4] An earlier version of this paper considered extensions that allowed for partial information revelation; details available from author upon request.

[5] The assumption that the quality of the good/asset to be traded depends on the seller's investment distinguishes the analysis here from the traditional analysis of lemons markets (Akerlof, 1970) in which the quality of the seller's good/asset is determined exogenously.

[6] This, for example, was alleged in the case of Celador versus Disney. Celador claimed that Disney employed various accounting devices to cheat it out of revenues due it under a revenue-sharing contract connected with Celador's "Who Wants to Be a Millionaire" show. At the time of this writing, Celador has been awarded $270 million after six years of litigation, but Disney plans to appeal. Source: *New York Times*, July 8, 2010, "Disney Is Told to Pay $270 million in 'Millionaire' Suit" by Brian Stelter and Brooks Barnes.

hands could be because of subsequent investments made by the buyer and, thus, a revenue-sharing agreement may be sub-optimal because of the disincentive effect it has on the buyer's investments. Hence, it seems worth considering contexts in which revenue-sharing contracts are infeasible; that is, in which the seller's payment cannot be tied to the realized return from the asset. This is the situation considered in this paper.[7]

This paper is concerned primarily with the case in which the buyer and seller meet only once the seller has invested. Consequently, no contract is in force at the time the seller invests. Despite the absence of any contract, the lemons problem, an inability to contract on returns, and, in some circumstances, a holdup problem, it is shown that equilibria exist in which there is both seller investment and trade with positive probability (Propositions 2, 5, and 7). Moreover, fixing the relevant circumstances, these equilibria are essentially unique (Propositions 6 and 7).

When no contract governs the relation between buyer and seller prior to the seller's investment, the outcome of the game between them depends on the allocation of bargaining power. Here, two extremes are considered: either the buyer can make a take-it-or-leave-it offer to the seller or the seller can make such an offer to the buyer. When the buyer has all the bargaining power, a holdup problem also arises.[8] As will be shown, the threat of holdup can lead to either more investment in expectation or less investment in expectation *vis-à-vis* the situation without holdup; that is, the situation when the seller has the bargaining power. Even though investment can be greater when holdup is an issue, the combined welfare of the two parties will be lower.

The reason welfare is lower is that, to avoid being held up, the seller must randomize over her investment levels. In contrast, when she has the bargaining power, she can just play the optimal level of investment given the constraints imposed by the moral hazard problem. That the first equilibrium involves mixing by the investing party is reminiscent of the results in Gul (2001), González (2004), and Lau (2008), which also consider holdup in the context of unobservable investment. There are, however, a number of differences between this paper and those earlier ones. The principal difference being that in the Gul, Lau, and González papers, the investing party invests solely for her own benefit, whereas

[7]It is also assumed that the parties cannot agree to a sale in which the buyer has the right to return the asset to the seller in exchange for his money back. In many of the motivating examples listed above, such money-back guarantees would be problematic because of seller limited liability (*e.g.,* the inventor spends her payout before the acquiring firm determines whether it wishes to invoke the money-back guarantee) or opportunism by the buyer (*e.g.,* the buyer can strategically delay invocation until he learns additional information about expected returns or the buyer can use the money-back guarantee to effectively steal the seller's intellectual property). More subtly, nothing in the following analysis actually requires the buyer to observe the seller's investment even after taking possession because, on the equilibrium path, he perfectly infers what it must have been. Hence, being able to put the asset back to the seller could be worthless *per se* because the buyer never actually observes the seller's investment even after taking possession.

[8]For classic analyses of the holdup problem, see Williamson (1976), Tirole (1986), and Klein (1988).

here the non-investing party is a direct beneficiary and, in a first-best world, would be the sole beneficiary.[9] Beyond being a situation of direct interest, considering cooperative investment—and hence the moral-hazard problem that ensues—makes possible the conclusion that the problem of holdup is not that it necessarily reduces investment, but that it *distorts* investment away from the optimal level. In this earlier literature, the upper limit of investment levels over which the investing party mixes proves to be the optimal investment level absent a holdup problem; hence, the *expected* amount of investment is necessarily less than this optimal amount. In contrast, as noted above, here the possibility arises that the investing party invests more than the optimal amount; not only with positive probability, but also possibly in expectation. In essence, here the problem of holdup is shown to possibly be one of over-investment rather than underinvestment *vis-à-vis* the optimal level.

Introducing a moral hazard problem (making investment cooperative) has the added benefit of making the situation in which the investing party has the bargaining power interesting. In those earlier articles, that case was of little interest (and, hence, not considered) because the first best would clearly be achieved. Here, in contrast, granting the investing party the bargaining power will never lead to the first best. Indeed, potentially, even the second best might not be achieved, despite the investing party's having the bargaining power, unless her strategy space is defined broadly.

Because the buyer is a direct beneficiary of the seller's investment, which he cannot observe, the situation considered here is also reminiscent of a hidden-action principal-agent problem. In particular, there is a connection between the problem considered here and renegotiation in agency as considered by Fudenberg and Tirole (1990) and Ma (1994). Similar to Fudenberg and Tirole's results, this paper finds that when the principal (buyer) has the bargaining power, the agent (seller) must mix in equilibrium; and, similar to Ma's results, this paper finds that when the agent (seller) has the bargaining power, the agent need not mix in equilibrium. Again, differences exist between this paper and this earlier literature, an obvious one being the contracting technology. In the principal-agent setting, it is possible to fix endogenously how the agent's payoff varies with some verifiable measure equal to or correlated with the return the agent's action generates. That is impossible here because there is no verifiable measure of the returns generated; that is, here, the contract space is much more limited.[10]

As the analysis below shows, the seller will have incentives to invest only if

[9] In Gul and Lau, a buyer makes investments that increase the value of a good to him if he should subsequently acquire it from the seller. In González, a seller makes investments that lower her cost of producing the units the buyer acquires from her. Observe, in this paper, the investment by the investing party is, in part, "cooperative" in the sense of Che and Hausch (1999).

[10] Another difference with the earlier agency literature is that, there, the parties could contract prior to the agent's action. Here, in essence, the parties are restricted to a prior "contract" in which the agent's compensation is simply the return the agent generates. Because this prior "contract" is set exogenously, the renegotiation-proofness principle does not apply.

there is a positive probability along the equilibrium path that she will end up in possession of the asset; that is, there must be a positive probability of no trade. That a possibility of no trade is needed as an incentive device bears similarity to the solution of double moral hazard problems considered by Demski and Sappington (1991) and subsequent authors. In Demski and Sappington's paper, it is impossible to contract directly on the seller's investment, so the threat of leaving her with the asset should she invest too little can provide her incentives. For these incentives to work, however, the buyer must observe the seller's investment. As noted, here the buyer cannot observe the seller's investment. Hence, here, the incentive to invest comes from the fact the seller may wind up with the asset, whereas, in this other literature, the seller's incentive to invest comes from a desire to avoid winding up with the asset. Indeed, in this other literature, the seller never winds up with the asset on the equilibrium path.

The model is introduced in the next section. Section 3 verifies that no pure-strategy equilibrium can exist when the buyer has the bargaining power. It is also shown there that there cannot be a pure-strategy equilibrium in which the seller invests a positive amount. The second-best solution is described. In Section 4, it is shown that the second best is an equilibrium of a game in which the seller can propose exchange, on a take-it-or-leave-it basis, at a price of her choosing. Within the context of such games, however, the second-best outcome is not the unique equilibrium. It can be made the unique equilibrium, however, if the concept of an offer is expanded to encompass the offering of a trading mechanism. Section 5 characterizes the properties that a mechanism offered post investment must satisfy in equilibrium. Section 6 solves the game for the situation in which the buyer makes a take-it-or-leave-it offer of a mechanism after investment and the one in which the seller does. The two situations are contrasted. Because the former situation means a holdup problem exists, one might expect there to be less investment when the buyer has the bargaining power than when the seller does. Counterintuitively, this proves not to be true in many circumstances. Section 7 briefly considers contracting in advance of the seller's investment. Section 8 provides additional discussion and conclusions.

2 Model

There are two risk-neutral parties, a seller and a buyer. The seller owns an asset (an invention, small firm, etc.) in which she can invest. Let $I \in [0, \infty)$ denote her investment. After investing, an opportunity arises in which the seller can sell the asset to a buyer. To motivate trade, assume the buyer, if he acquires the asset, can take a subsequent action, $b \in \mathcal{B} \subset \mathbb{R}_+$, that affects the asset's return (to him at least). Assume $0 \in \mathcal{B}$ (the buyer has the option "not to act") and $\mathcal{B} \setminus \{0\} \neq \emptyset$ (not acting is not the buyer's only option). Finally, the asset yields a return, r, to its then owner, where the return depends on investments made in it. Note the realization of r occurs after the point at which exchange can occur. The payoffs to the buyer and seller—*ignoring* transfers—are, respectively,

$$U_B = \begin{cases} 0\,, & \text{if no exchange} \\ r - b\,, & \text{if exchange} \end{cases} \text{ and } U_S = \begin{cases} r - I\,, & \text{if no exchange} \\ -I\,, & \text{if exchange} \end{cases},$$

where "exchange" means ownership of the asset passes to the buyer. Not surprisingly, the buyer takes no action if there isn't exchange.

Given investment I and action b, the return r has an expected value $R(I,b)$. The following assumption is maintained throughout the analysis:

{ass:basic}

Assumption 1. *The expected return function, $R : \mathbb{R}_+ \times \mathcal{B} \to \mathbb{R}$, has the following properties:*

(i) For all $b \in \mathcal{B}$, $R(\cdot,b) : \mathbb{R}_+ \to \mathbb{R}$ is a twice continuously differentiable, strictly increasing, and strictly concave function (the latter two conditions state that expected return is increasing in the seller's investment, but that there is diminishing marginal return to that investment);

(ii) Infinite investment by the seller is never collectively optimal: For any $b \in \mathcal{B}$, there exists an $\bar{I}(b) < \infty$ such that $\partial R(I,b)/\partial I < 1$ if $I > \bar{I}(b)$;

(iii) Zero investment is not privately optimal for the seller if she is certain to retain ownership: $\partial R(0,0)/\partial I > 1$; and

(iv) The buyer strictly prefers to take action if he obtains ownership of an asset in which the seller has invested a positive amount: For any $I \in \mathbb{R}_+$, $\operatorname{argmax}_{b\in\mathcal{B}} R(I,b) - b$ exists and, if $I > 0$, it is a subset of $\mathcal{B} \setminus \{0\}$.

Define $V(I) = \max_{b\in\mathcal{B}} R(I,b) - b$. Finally, assume:

(v) The parties' collective welfare is never maximized by zero investment by the seller and there exists a positive investment level that maximizes their collective welfare: An $I^* > 0$ exists such that $V(I^*) - I^* \geq V(I) - I$ for all $I \geq 0$ and $V(I^*) - I^* > V(0)$.

Among other implications, Assumption 1(v) rules out a situation wherein it is more efficient to have the seller skip investing and to let the buyer make all the enhancements to return.

Observe the buyer's value for the asset is increasing in the seller's investment:

{lemma:V-increasing}

Lemma 1. *The function $V(\cdot)$ is an increasing function.*

Proof: Let $b(I)$ denote a solution to $\max_{b\in\mathcal{B}} R(I,b) - b$. Consider $I' > I''$. By revealed preference and Assumption 1(i):

$$V(I') = R\big(I', b(I')\big) - b(I') \geq R\big(I', b(I'')\big) - b(I'') > R\big(I'', b(I'')\big) - b(I'') = V(I'')\,.$$

■

The expected return to the seller if she retains ownership is $R(I,0)$. From Assumption 1(iv), $V(I) > R(I,0)$ for all $I > 0$; that is, the asset is always more valuable in the buyer's hands than the seller's given seller investment. No assumption is made here as to whether the asset is or isn't more valuable in the buyer's hands absent such investment.[11]

Were there no possibility of later trade (*i.e.,* under autarky), the seller would choose a level of investment to solve

$$\max_I R(I,0) - I\,.$$

Assumptions 1(i)–(iii) ensure that this program has a unique, interior solution. Let $\hat{I}$ denote the solution; that is, the level of investment the seller would choose were later trade infeasible.

Throughout, the solution concept is perfect Bayesian equilibrium.

3 Preliminary Analysis

{sect:Prelim}

Consider a situation in which no contract exists between buyer and seller at the time the seller invests. Buyer and seller meet after the seller's investment and a means of exchange is then established. At the time they meet, the buyer knows or learns only that expected return is generated by the function $R(\cdot,\cdot)$ (the seller of course knows that function when she makes her investment decision). As noted above, the buyer does not observe the seller's investment, nor does he receive any signal about it.

Although there are many possible bargaining games that could be considered, attention is limited here to take-it-or-leave-it (tioli) bargaining. That is, either the buyer or the seller has the ability to make a tioli offer to the other, where an offer consists of a contract or mechanism for the parties to play.

Similar to other settings of this nature, essentially no pure-strategy equilibrium can exist (see, *e.g.,* Gul for a discussion):

{prop:NoEfficiency}

Proposition 1. *If the buyer has the ability to make a take-it-or-leave-it offer to the seller, then no pure-strategy equilibrium exists. The same is true if the seller can make a take-it-or-leave-it offer unless welfare given exchange and no investment exceeds the maximum possible welfare given no exchange (i.e., unless* $V(0) \geq R(\hat{I},0) - \hat{I}$*).*

Proof: The proof is by contradiction; that is, the supposition of a pure-strategy equilibrium is shown to lead to a contradiction. If the seller is playing the strategy of investing I, then the asset is worth $R(I,0)$ to her and $V(I)$ to the buyer. These, respectively, are the price, p, set when the buyer or seller is the one able to make a tioli offer. Suppose $I > 0$. Because $p - I < p$, it follows that the seller would do better to deviate to $I = 0$. Suppose $I = 0$. If the buyer makes the tioli offer, the seller's profit is $R(0,0) - 0$. But, because $\hat{I} > 0$ uniquely maximizes $R(I,0) - I$, the seller would do better to deviate (*i.e.,* invest and

[11] This represents a minor difference between the situation of Gul (2001) and Lau (2008) and the model here: As Gul noted, the existence of an equilibrium with a positive probability of investment in the Gul or Lau setting depends on trade being valuable even in the absence of investment. Here, in contrast, investment occurs with positive probability in equilibrium regardless of whether trade is valuable absent investment or is valueless absent investment.

retain ownership). If the seller makes the TIOLI offer, the price is $V(0)$. It is readily seen that a pure-strategy equilibrium exists in this case if and only if $V(0) \geq R(\hat{I}, 0) - \hat{I}$. ■

An immediate corollary is

{corr:NoFirstBest}

Corollary 1. *The first-best outcome is not attainable as an equilibrium.*

Proof: The first best requires $I^* > 0$ be played as a pure strategy and trade always occur in equilibrium. No such pure-strategy equilibrium exists. ■

What about the second best? As demonstrated by Proposition 1, there is a tradeoff between trading efficiently and providing the seller investment incentives. Letting x denote the probability of trade and p the price paid the seller if trade occurs,[12] the problem of welfare maximization can be written as

$$\max_{\{x,I,p\}} xV(I) + (1-x)R(I,0) - I \tag{1}$$ {eq:2ndBest}

subject to

$$I \in \operatorname*{argmax}_I xp + (1-x)R(I,0) - I\,, \tag{2}$$ {eq:2ndBestIC}

$$V(I) \geq p\,, \tag{3}$$ {eq:2ndBestBIR}

and

$$R(\hat{I},0) - \hat{I} \leq xp + (1-x)R(I,0) - I\,. \tag{4}$$ {eq:2ndBestSIR}

Constraint (2) follows from the moral-hazard problem and reflects that the seller's investment choice must be incentive compatible. Constraints (3) and (4) are the participation constraints of the buyer and seller, respectively.

Assumption 1 implies that (2) is globally concave in I with an interior solution. Hence, we are free to replace it with the corresponding first-order condition $(1-x)\partial R(I,0)/\partial I - 1 = 0$. This, in turn, implicitly defines the probability of trade as

$$1 - \left(\frac{\partial R(I,0)}{\partial I}\right)^{-1}\,. \tag{5}$$ {eq:2ndBestTradeProb}

Note that probability is zero if $I = \hat{I}$. Because probabilities must lie in $[0,1]$, there is no loss in restricting attention to $I \in [0, \hat{I}]$. Substituting this function for x in the expression of welfare, expression (1), the constrained welfare-maximization program becomes

$$\max_{I\in[0,\hat{I}]} \left(1 - \left(\frac{\partial R(I,0)}{\partial I}\right)^{-1}\right) V(I) + \left(\frac{\partial R(I,0)}{\partial I}\right)^{-1} R(I,0) - I\,. \tag{6}$$ {eq:2ndBest-sub}

[12] Because, recall, I is the seller's hidden information, neither p nor x can be directly contingent on it.

Because the domain is compact and the function to be maximized continuous, (6) must have at least one solution. Let $\mathcal{M}$ equal the maximized value of (6).

By Assumption 1(iii), the probability of trade given by (5) is bounded away from 1. As such the analysis has, to this point, ignored the possibility of having the seller invest nothing and trading with certainty. This course of action is collectively optimal if $V(0) \geq \mathcal{M}$. Reflecting this, define

$$\mathcal{S} = \begin{cases} \mathcal{M}\,, & \text{if } \mathcal{M} > V(0) \\ V(0)\,, & \text{if } \mathcal{M} \leq V(0) \end{cases}$$

as maximum second-best surplus. Let $\mathcal{I}_{\mathcal{M}}$ denote the set of positive investment levels that could be constrained welfare maximizing (*i.e.,* that maximize (6)). A second-best level of investment can, then, be defined as

$$I^{\mathrm{s}} = \begin{cases} I \in \mathcal{I}_{\mathcal{M}}\,, & \text{if } \mathcal{M} > V(0) \\ 0\,, & \text{if } \mathcal{M} \leq V(0) \end{cases} .$$

We have so far ignored the participation constraints. This is without loss: Together the participation constraints imply that a necessary condition for a second-best optimum with a positive probability of trade is that maximized social welfare with trade exceed social welfare under autarky. Because the analysis allows for the possibility that $\hat{I} \in \mathcal{I}_{\mathcal{M}}$, this condition is met. Furthermore, it is readily seen that if $\mathcal{S} > R(\hat{I},0) - \hat{I}$, then (3) and (4) will be satisfied by $p = V(I^{\mathrm{s}})$ (among, possibly, other prices).

4 The Seller Has All Bargaining Power: Achieving the Second Best

{sect:SBP-2ndBest}

Suppose the seller possesses all the bargaining power; that is, she can make the buyer a TIOLI offer. In this case, an equilibrium in which the second-best outcome is reached exists:

{prop:SellerTIOLIprice}

Proposition 2. *If the seller makes take-it-or-leave-it offers to the buyer, then there exists a perfect Bayesian equilibrium that achieves the second best. Specifically, the seller invests at a second-best level (i.e., I^{s}) with certainty and offers the buyer, on a take-it-or-leave-it basis, the asset for a price just equal to the buyer's willingness to pay (i.e., equal to $V(I^{\mathrm{s}})$). The buyer plays the mixed strategy by which he accepts the seller's offer with probability x, given by*

$$x = \begin{cases} 1\,, & \textit{if } I^{\mathrm{s}} = 0 \\ 1 - \left(\frac{\partial R(I^{\mathrm{s}},0)}{\partial I}\right)^{-1}\,, & \textit{if } I^{\mathrm{s}} > 0 \end{cases} ,$$

and rejects it with probability $1-x$. The buyer believes an offer at any price less than $V(I^{\mathrm{s}})$ means the seller has invested nothing, he believes a price of $V(I^{\mathrm{s}})$ means the seller has invested I^{s}, and he believes an offer at any price greater than $V(I^{\mathrm{s}})$ means the seller has invested no more than I^{s}.

Proof: Given his beliefs, the buyer is indifferent between accepting and rejecting an offer at price $V(I^{\mathrm{s}})$. Hence, he is willing to mix. Given the probabilities with which the buyer mixes if offered the asset at price $V(I^{\mathrm{s}})$, the seller's unique profit-maximizing level of investment given she intends to offer the asset at price $V(I^{\mathrm{s}})$ is I^{s}: Assumption 1(i) guarantees that (2) has a unique solution for any given x. Hence, the unique I that solves the first-order condition

$$\left(\frac{\partial R(I^{\mathrm{s}},0)}{\partial I}\right)^{-1}\frac{\partial R(I,0)}{\partial I}-1=0$$

is $I=I^{\mathrm{s}}$. It remains to be verified that the seller does not wish to deviate with respect to investment *and price* given the buyer's beliefs. From Lemma 1, the buyer, given his beliefs, will reject any price greater than $V(I^{\mathrm{s}})$ or in the interval $\big(V(0),V(I^{\mathrm{s}})\big)$. Because the seller's expected payoff is $\mathcal{S}$ if she invests I^{s} and sets a price of $V(I^{\mathrm{s}})$, she prefers, at least weakly, that course of action to autarky. Hence, she cannot gain by offering a price in $\big(V(0),V(I^{\mathrm{s}})\big)\cup\big(V(I^{\mathrm{s}}),\infty\big)$ regardless of what she invests. By construction, $xV(0)+(1-x)R(I,0)-I\leq\mathcal{S}$ for all $x\in[0,1]$ and I; so the seller cannot gain *vis-à-vis* her equilibrium payoff by setting a price of $V(0)$ regardless of what she invests. ■

It is worth commenting on the similarity between the equilibrium in Proposition 2 and that derived by Fong (2005) for a credence-good problem. Both have the feature that, along some paths, the buyer mixes between accepting and rejecting and this plays a role in inducing the desired behavior from the seller. In Fong's paper, the desired behavior is truthful revelation of the seller's exogenously determined type (her diagnosis of the problem suffered by the buyer). Here, the desired behavior is inducing investment from the seller. Given the buyer's beliefs, there is no scope for the seller to misrepresent her investment; the sole purpose of the buyer's mixing is to provide investment incentives.

Although it is the second-best equilibrium, the Proposition 2 equilibrium is not unique: Other equilibria can be constructed by varying the buyer's beliefs. For example, if there is a second-best level of investment that lies strictly between no investment and the autarky level (*i.e.,* $0<I^{\mathrm{s}}<\hat{I}$) and welfare given that level of investment is strictly greater than under autarky (*i.e.,* $\mathcal{S}>R(\hat{I},0)-\hat{I}$), then, by continuity, there exists an $\widetilde{I}$ near I^{s} such that welfare if the seller invests $\widetilde{I}$ and trade occurs with probability $1-\big(\partial R(\widetilde{I},0)/\partial I\big)^{-1}$ also exceeds welfare given autarky. Along the lines of Proposition 2, one can construct a perfect Bayesian equilibrium in which the buyer expects to be offered the asset for $V(\widetilde{I})$, accepts such an offer with probability $1-\big(\partial R(\widetilde{I},0)/\partial I\big)^{-1}$, and believes a lower price indicates $I=0$ and a higher price indicates $I\leq\widetilde{I}$. Given these beliefs and the buyer's resulting best responses, it is readily shown that the seller's best response is to invest $\widetilde{I}$ and offer the asset at price $V(\widetilde{I})$.

The possibility of such sub-optimal equilibria can be eliminated, however, by allowing the seller a richer strategy space. In particular, one can conceive of the seller offering not just a price, but an entire mechanism. Because a mechanism is also what the buyer would offer if he had the bargaining power, the next section is given to investigating mechanisms in this context.

5 Mechanism Design

{sect:MechDesign}

After the seller has sunk her investment, a means of arranging trade is a mechanism.

5.1 Characterization

In light of the revelation principle, attention can be restricted to direct-revelation mechanisms. Because the seller is the only actor with private information, a mechanism must induce her to reveal that information.

Although it is natural to think of the seller's information as the amount she has invested, I, it proves easier to work with the transformation of investment, $R(I,0)$; that is, in what follows, the seller's type is her expected return if she retains ownership. By Assumption 1(i), $R(\cdot,0)$ is a strictly increasing function. It hence has an inverse: Let $\iota(\cdot)$ denote that inverse; that is, $\iota\big(R(I,0)\big) \equiv I$. Assumption 1(i) entails that $\iota(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly *convex*. Observe we are, therefore, free to act as if the seller chooses her expected return, R, should no trade occur, since this is equivalent to assuming she chooses investment $\iota(R)$.

The seller's type could be any R in the range of $R(\cdot,0)$. As will be seen, however, there is no loss in restricting attention to

$$\mathcal{R} \equiv [\underline{R}, \bar{R}] \subseteq [R(0,0), R(\hat{I},0)] \,.$$

To economize on notation, henceforth, let $R^\circ = R(0,0)$ and $\widehat{R} = R(\hat{I},0)$. A mechanism is, then, a pair $\big\langle x(\cdot), t(\cdot)\big\rangle$, where $x : \mathcal{R} \to [0,1]$ is the probability that ownership of the asset is transferred to the buyer; and $t : \mathcal{R} \to \mathbb{R}$ is the transfer (payment) to the seller.

Let

$$U(R) = \big(1 - x(R)\big)R + t(R) \tag{7}$$

{eq:defUR}

denote the seller's utility if she truthfully announces her type (note, at this point, her investment is sunk). Following standard methods (see Appendix for details), it can be shown that

{prop:MechNec_and_Suff}

Proposition 3. *Necessary conditions for a mechanism to induce truth-telling are (i) that the probability of trade, $x(\cdot)$, be non-increasing in seller type and (ii) that the seller's utility as a function of her type be given by*

$$U(R) = \bar{U} - \int_R^{\bar{R}} \big(1 - x(z)\big)dz \,, \tag{8}$$

{eq:SolvedDiffEQ}

where $\bar{U}$ is a constant ($\bar{R}$, recall, is $\sup \mathcal{R}$).

Moreover, any mechanism in which $x(\cdot)$ is non-increasing and expression (8) *holds induces truth-telling (i.e., conditions (i) and (ii) are also sufficient).*

5.2 CONSEQUENCES

The analysis of the previous subsection establishes the following results.

Anticipating the mechanism to be played, the seller is willing to invest $\iota(R)$ if and only if it maximizes $U(R) - \iota(R)$. It follows:

{prop:EqProbTrade}

Proposition 4. *If $\iota(R) > 0$ is a level of investment chosen by the seller with positive probability in equilibrium, then the subsequent probability of trade given that investment is $1 - \iota'(R)$.*

Proof: By supposition, the seller chooses R with positive probability in equilibrium, hence $R \in \mathcal{R}$. Consequently, R must satisfy the first-order condition

$$0 = U'(R) - \iota'(R) = 1 - x(R) - \iota'(R)\,. \tag{9}$$ {eq:R-FOC}

The result follows. ■

Immediate corollaries are

Corollary 2. *If trade always occurs in equilibrium, then the seller invests nothing.*

Corollary 3. *There is no equilibrium in which the seller invests more than her autarky level of investment, $\hat{I}$.*

The analysis to this point implies that there is no loss of generality in assuming the space of seller types, $\mathcal{R}$, is $[R^\circ, \widehat{R}]$. This does not mean that the seller necessarily plays all R in $[R^\circ, \widehat{R}]$ with positive probability, rather that the mechanism can accommodate all such R.

6 EQUILIBRIUM

{sect:Equilibrium}

6.1 THE BUYER HAS ALL THE BARGAINING POWER

Suppose the buyer offers a mechanism to the seller on TIOLI basis. In equilibrium, any mechanism offered by the buyer and acceptable to the seller must yield the seller at least what she would have achieved under autarky.

{lemma:Boundary}

Lemma 2. *On the equilibrium path, the seller's expected utility must be at least her autarky level of utility, $\widehat{R} - \iota(\widehat{R})$.*

Proof: A course of action available to the seller is to invest $\iota(\widehat{R})$ and decline to trade. This would yield her $\widehat{R} - \iota(\widehat{R})$. Hence, any strategy played in equilibrium by the seller other than this must do at least as well. ■

In what follows, assume the seller's strategy, $F : (\underline{R}, \bar{R}]$, is differentiable. Denote the derivative by $f(R)$. Assume $f(R) > 0$ for all $R \in (\underline{R}, \bar{R}]$. I show these assumptions are consistent with an equilibrium below. Because the seller never invests more than $\widehat{R}$, $\bar{R} \leq \widehat{R}$.

Expression (9) must hold for any $R > R^{\circ}$ that the seller chooses with positive probability. In light of Lemma 2 and $f(R) > 0$, it further follows that

$$U(R) - \iota(R) = \bar{U} - \iota(\bar{R}) \geq \widehat{R} - \iota(\widehat{R}) \tag{10}$$

{eq:SellerIR}

for all $R \in (\underline{R}, \bar{R}]$, where the equality follows from (8) and Proposition 4. As it is the buyer who makes the TIOLI offer, (10) is binding.

The buyer chooses $x(\cdot)$ and $\bar{U}$ to maximize his expected net return,

$$\int_{\underline{R}}^{\bar{R}} \left(x(R)V\big(\iota(R)\big) + \underbrace{\big(1 - x(R)\big)R - \bar{U} + \int_{R}^{\bar{R}} \big(1 - x(z)\big)dz}_{=-t(R)} \right) dF(R)\,. \tag{11}$$

{eq:BuyerEUtility1}

Using integration by parts, this expression becomes:

$$\begin{aligned}
&- \bar{U} + \left(x(\underline{R})V\big(\iota(\underline{R})\big) + \big(1 - x(\underline{R})\big)\underline{R} + \int_{\underline{R}}^{\bar{R}} \big(1 - x(z)\big)dz \right) F(\underline{R}) \\
&+ \int_{\underline{R}}^{\bar{R}} \left(x(R)V\big(\iota(R)\big) + \big(1 - x(R)\big)R + \frac{F(R) - F(\underline{R})}{f(R)}\big(1 - x(R)\big) \right) f(R)dR \\
&\qquad = -\bar{U} + \Big(x(\underline{R})V\big(\iota(\underline{R})\big) + \big(1 - x(\underline{R})\big)\underline{R} \Big) F(\underline{R}) \\
&\quad + \int_{\underline{R}}^{\bar{R}} \left(x(R)V\big(\iota(R)\big) + \big(1 - x(R)\big)R + \frac{F(R)}{f(R)}\big(1 - x(R)\big) \right) f(R)dR\,.
\end{aligned} \tag{12}$$

{eq:BuyerEqUtility}

From Proposition 4, if the seller plays an $R > \underline{R} \geq R^{\circ}$ with positive probability, then $x(R)$ must be $1 - \iota'(R)$. Differentiating, pointwise, the buyer's expected utility, expression (12), with respect to the probability of trade for $R > \underline{R}$, reveals that consistency with both Proposition 4 (the seller is willing to mix) and optimization by the buyer is met if and only if

$$\frac{F(R)}{f(R)} = V\big(\iota(R)\big) - R\,, \tag{13}$$

{eq:ReverseHazard}

because, then, the buyer is indifferent as to his choice of $x(\cdot)$ and might as well choose $x(\cdot)$ to be consistent with the seller mixing (*i.e.*, such that $x(\cdot) = 1 - \iota'(\cdot)$).

Using (13), we can rewrite the buyer's expected utility, expression (12), as

$$\begin{aligned}
&\Big(x(\underline{R})V\big(\iota(\underline{R})\big) + \big(1 - x(\underline{R})\big)\underline{R} \Big) F(\underline{R}) - \bar{U} + \int_{\underline{R}}^{\bar{R}} \big(F(R) + Rf(R)\big)dR \\
&\qquad\qquad = \bar{R} - \bar{U} + \Big(V\big(\iota(\underline{R})\big) - \underline{R} \Big) F(\underline{R})x(\underline{R})\,,
\end{aligned}$$

where the equality follows by "undoing" the product rule of differentiation. Recalling that the seller's participation constraint binds, this last expression and (10) imply the buyer's expected utility is

$$\Big(V\big(\iota(\underline{R})\big) - \underline{R}\Big)F(\underline{R})x(\underline{R}) + \bar{R} - \iota(\bar{R}) - \big(\widehat{R} - \iota(\widehat{R})\big)\,.$$

Because $\widehat{R}$ uniquely maximizes $R - \iota(R)$, the buyer's expected utility cannot exceed $\big(V(\iota(\underline{R})) - \underline{R}\big)F(\underline{R})x(\underline{R})$. Given the buyer could deviate from offering the mechanism and simply make a TIOLI offer to buy at price $\underline{R}$, which would net him expected profit $\big(V(\iota(\underline{R})) - \underline{R}\big)F(\underline{R})$, it follows that his expected utility cannot be less than $\big(V(\iota(\underline{R})) - \underline{R}\big)F(\underline{R})$. It further follows that he offers this mechanism in equilibrium exists only if $\bar{R} = \widehat{R}$ and, if $F(\underline{R}) > 0$, $x(\underline{R}) = 1$.

We are now in position to establish:

{prop:BuyerTIOLI-continuous}

Proposition 5. *There exists a subgame-perfect equilibrium of the game in which the buyer makes the seller a take-it-or-leave-it offer in which the seller plays a mixed strategy whereby she chooses $R \in [R^\circ, \widehat{R}]$ according to the distribution function*

$$F(R) = \exp\left(-\int_R^{\widehat{R}} \frac{1}{V\big(\iota(z)\big) - z}dz\right) \tag{14}$$

{eq:FR-buyerTIOLI}

and the buyer offers the mechanism $\big\langle x(\cdot), t(\cdot)\big\rangle$ such that

$$x(R) = \begin{cases} 1\,, & \text{if } R = R^\circ \\ 1 - \iota'(R)\,, & \text{if } R > R^\circ \end{cases}$$

and

$$t(R) = \widehat{R} - \iota(\widehat{R}) + \iota(R) - \big(1 - x(R)\big)R\,. \tag{15}$$

{eq:UR-buyerTIOLI}

Proof: Expression (14) solves the differential equation (13). By Assumption 1(iv), $V\big(\iota(z)\big) - z > 0$ for $z > R^\circ$. So if $\underline{R} > R^\circ$, then

$$\int_{\underline{R}}^{\widehat{R}} \frac{1}{V\big(\iota(z)\big) - z}dz < \infty\,;$$

hence, $F(\underline{R}) > 0$. This implies $x(\underline{R}) = 1$. But $x(\underline{R}) = 1$ is inconsistent with Proposition 4 if $\underline{R} > R^\circ$. Consequently, then, $\underline{R} = R^\circ$. Expression (15) follows from Proposition 3 because $1 - x(R) = \iota'(R)$. The remainder of this proposition was established in the text that preceded its statement. ■

As an example, suppose that $R(I, b) = \gamma\big(1 + \sqrt{I}\big)\big(1 + \sqrt{b}\big)$ and $\mathcal{B} = \mathbb{R}_+$, where $\gamma \in (0, 2)$. Straightforward calculations reveal:

$$I^* = \frac{\gamma^2}{(2-\gamma)^2}\,;\ \hat{I} = \frac{\gamma^2}{4}\,;\ \iota(R) = \frac{(R-\gamma)^2}{\gamma^2}\,;\ \text{and } V\big(\iota(R)\big) - R = \frac{R^2}{4}\,.$$

The Proposition 5 equilibrium will thus be characterized by

$$F(R) = \exp\left(-\frac{4}{R}\right)\exp\left(\frac{8}{\gamma(2+\gamma)}\right)\ ;\ x(R) = 1 - \frac{2(R-\gamma)}{\gamma^2}\ ;$$
$$\text{and } t(R) = 1 + \frac{1}{4}\gamma(4+\gamma) - \frac{R^2}{\gamma^2}\,.$$

Expected welfare is

$$\frac{1}{4}\gamma\left(4+\gamma+\gamma\exp\big(-\frac{4}{2+\gamma}\big)\right)\,.$$

This exceeds welfare given autarky by

$$\frac{\gamma^2}{4}\exp\left(-\frac{4}{2+\gamma}\right)\,.$$

First-best welfare is $\frac{2\gamma}{2-\gamma}$. For instance, if $\gamma = 1$, first-best welfare is 2, equilibrium welfare is approximately 1.3159, and welfare under autarky is 1.25. The equilibrium probability of exchange does not have a convenient closed-form solution. If $\gamma = 1$, that probability is approximately .6017.

Is the equilibrium in Proposition 5 unique? Within a broad class of possible strategies, the answer is yes.

{def:PiecewiseAC}

Definition 1. *A mixed strategy for the seller, $F : [\underline{R}, \bar{R}] \subseteq [R^\circ, \widehat{R}] \to [0,1]$, is* piecewise absolutely continuous *if, for a finite sequence $R_1 < \cdots < R_N$, $F(\cdot)$ is absolutely continuous on all segments (R_n, R_{n+1}), $n = 0, \ldots, N$, where $R_0 \equiv \underline{R}$ and $R_{N+1} \equiv \bar{R}$ and discontinuous at each R_n, $n = 1, \ldots, N$.*[13]

Because a constant function is absolutely continuous, observe that mixed strategies in which the seller mixes over a discrete set of investment levels are included in this definition. The strategy in Proposition 5 is also an element of the set of strategies defined in Definition 1 (for it, $N = 1$ and $R_0 = R_1 = R^\circ$). Moreover, it is the only strategy within this set that can be played in equilibrium:

{prop:BuyerTIOLI-unique}

Proposition 6. *For the game in which the buyer makes the seller a take-it-or-leave-it offer and the seller is limited to piecewise absolutely continuous strategies, the equilibrium in Proposition 5 is unique.*

The proof can be found in the appendix.

6.2 The Seller Has All the Bargaining Power

{sect:SellerTIOLI}

Suppose, now, it is the seller who makes a tioli offer. The focus will be on showing there is a unique equilibrium in which the seller invests at a second-best level, I^{s}, with certainty. Because the problem is of little interest if 0 or the

[13]If $F(\cdot)$ were not discontinuous at an R_n, then there is nothing special about that R_n insofar as $F(\cdot)$ will be absolutely continuous on (R_{n-1}, R_{n+1}).

autarky investment level, $\hat{I}$, is second best, attention is restricted to the case in which neither 0 nor $\hat{I}$ is second best (formally, $\mathcal{S} > V(0)$ and $\hat{I} \notin \mathcal{I}_{\mathcal{M}}$).

In equilibrium, the buyer must correctly anticipate the seller's strategy; that is, the distribution of seller types. A potential issue is whether, similar to the situation in Section 4, "undesired" equilibria can be supported by allowing the buyer to hold certain beliefs (*e.g.,* as in that earlier section, where by believing the seller invests $\widetilde{I} \neq I^{\mathrm{s}}$, the buyer "forces" an equilibrium in which an amount other than the second-best level of investment occurs).

When the seller offers a mechanism, the buyer's beliefs are essentially irrelevant given common knowledge of rationality. To establish this—in fact to establish the seller will invest a second-best level I^{s}—consider the mechanism:

$$x(R) = 1 - \iota'(R^{\mathrm{s}}) \text{ and } t(R) = \big(1 - \iota'(R^{\mathrm{s}})\big)V(I^{\mathrm{s}})\,, \tag{16}$$
{eq:SellerMechanism}

where $R^{\mathrm{s}} = R(I^{\mathrm{s}}, 0)$. This mechanism is readily seen to satisfy Proposition 3.

If the seller knew the mechanism in (16) were certain to be played (she will offer it and the buyer will accept), then the seller would choose the second-best investment level with certainty. This is readily verified by considering her utility-maximization program

$$\max_R U(R) - \iota(R) \equiv \max_R \big(1 - \iota'(R^{\mathrm{s}})\big)V(I^{\mathrm{s}}) + \iota'(R^{\mathrm{s}})R - \iota(R)\,.$$

Upon seeing the mechanism given by (16), the buyer can only reason as follows: "The seller expects me to accept, in which case she must have invested I^{s}; she expects me to reject, in which case she must have invested $\hat{I}$; or she expects me to mix, in which case she must have invested

$$\operatorname*{argmax}_R \alpha U(R) + (1 - \alpha)R - \iota(R)\,, \tag{17}$$
{eq:AlphaMax}

where α is the probability with which she expects me to accept."

{lemma:Accept}

Lemma 3. *For all probabilities of acceptance, $\alpha \in [0, 1]$, the program* (17) *has a unique solution and that solution lies in* $[R^{\mathrm{s}}, \widehat{R}]$.

Proof: Substituting, the program can be written as

$$\max_R \alpha\big(1 - \iota'(R^{\mathrm{s}})\big)V(I^{\mathrm{s}}) + \Big(1 - \alpha\big(1 - \iota'(R^{\mathrm{s}})\big)\Big)R - \iota(R)\,. \tag{18}$$
{eq:AlphaMax2}

Given $1 - \alpha\big(1 - \iota'(R^{\mathrm{s}})\big) > 0$, uniqueness follows from Assumption 1(i). If $\alpha = 0$, the solution is $\widehat{R}$. If $\alpha = 1$, the solution is R^{s}. Because the cross-partial derivative of (18) with respect to R and α is $-\big(1 - \iota'(R^{\mathrm{s}})\big) < 0$, it follows from usual comparative statics that the solution to (18) (equivalently, (17)) is decreasing in α, from which it follows the solution to (17) lies in $[R^{\mathrm{s}}, \widehat{R}]$ for $\alpha \in [0, 1]$. ■

The buyer's expected payoff if he accepts the mechanism and the seller has chosen R is

$$\underbrace{\big(1 - \iota'(R^{\mathrm{s}})\big)}_{x(R)} V\big(\iota(R)\big) - \underbrace{\big(1 - \iota'(R^{\mathrm{s}})\big)V(I^{\mathrm{s}})\big)}_{t(R)}\,.$$

By Lemma 1 this payoff is non-negative for all $R \in [R^{\text{s}}, \widehat{R}]$. Hence, invoking Lemma 3, it follows that regardless of what the buyer believes the seller's expectations are about his accepting the mechanism, the buyer does best to accept. The following can now be established:

{prop:SellerTIOLImech}

Proposition 7. *If the seller can offer the buyer a mechanism on a take-it-or-leave-it basis, then, in a perfect Bayesian equilibrium, the seller must invest at a second-best level (I^{s}) and she must capture all the expected surplus.*

Proof: The preceding text showed that if the seller offers (16), the buyer will accept. Given acceptance, it was shown that the seller does best to invest I^{s}. Consequently, the seller's expected payoff is

$$\big(1 - \iota'(R^{\text{s}})\big)V(I^{\text{s}}) + \iota'(R^{\text{s}})R^{\text{s}} - \iota(R^{\text{s}}) = \mathcal{S}\,.$$

Because the seller is capturing all the possible surplus, there cannot be an equilibrium in which she does better. Since she can capture all the surplus by investing I^{s} and offering the mechanism given by (16), she would never be willing to offer a mechanism that gave her less. ■

6.3 Comparison of Equilibria

When the seller has the ability to make a TIOLI offer, the second best is achieved. When it is the buyer who makes a TIOLI offer, the seller must mix on the equilibrium path. Consider an incentive compatible choice of R under the mechanism offered by the buyer. Welfare given that R is

$$\big(1 - \iota'(R)\big)V\big(\iota(R)\big) + \iota'(R)R - \iota(R)\,.$$

This is the same maximand as in (6). It follows, therefore, that unless almost every $\iota(R) \in [0, \hat{I}]$ is a second-best level of investment, mixing by the seller will result in her investing a non-second-best level with positive probability. Consequently, expected welfare when the buyer has the bargaining power will be less than when the seller has it.

Intuitively, regardless of who makes the offer, there is a moral hazard problem. Hence, the first best is never possible (see Corollary 1). The situation is further exacerbated when the buyer has the bargaining power because of holdup: If the buyer knew precisely how much the seller had invested, he would capture all gains to trade. Hence, the only way the seller can retain some of the gains to trade is if the buyer is uncertain about how much she has invested; that is, the seller must play a mixed strategy. Because the seller mixes over investment levels other than those that are second-best optimal, the holdup problem further reduces welfare.

Given this discussion, one might expect that the seller's investment would be lower, on average, when the buyer has the bargaining power than when the seller does. This, however, need not be the case. Recall the previously used example with $\gamma = 1$. Calculations reveal that expected investment is approximately .0687 under the mechanism the buyer offers, which exceeds the value of second-best investment, which is approximately .0662. These calculations do not, though, reveal a universal truth: Using the same example, but with $\gamma = 3/2$, $\iota(R^{\mathrm{s}}) \approx .1551$, whereas expected investment when the buyer has the bargaining power is approximately .1270.

The analysis indicates, therefore, that the problem with holdup is not necessarily that it reduces investment relative to a no-holdup benchmark so much as it *distorts* investment *vis-à-vis* that benchmark.[14] The desire to avoid holdup (to realize some gains from trade) can induce the seller to *over*-invest as well as to underinvest.

A general characterization of when the holdup problem will lead to over versus underinvestment relative to the second-best investment level is not practical give the many "moving parts" in the general setup. However, for an important class of situations, a characterization is manageable and arguably instructive. Specifically, make the assumption—common in the literature—that the buyer's expected return, should he take possession, be additively separable in seller investment and his own action; that is, suppose

$$R(I,b) = R(I,0) + v(b)\,, \tag{19}$$ {eq:AddSeparable}

where $v(\cdot) : \mathcal{B} \to \mathbb{R}_+$. Define

$$g = \max_{b \in \mathcal{B}} v(b) - b\,.$$

Assume that $v(\cdot)$ is such that Assumption 1(iv) still holds. Consequently, g, the gain to trade, is a non-negative constant.[15] Observe that, in this class of situations, $V(I) = R(I,0) + g$ and, hence, $V(0) \geq R^{\circ}$.

Because $\iota(\cdot)$ is strictly convex (Assumption 1), $\iota''(\cdot) \geq 0$. To keep the analysis in the remainder of this section straightforward, it is convenient to strengthen this to assuming this second derivative is bounded away from zero:

{ass:bounded}

Assumption 2. *For all $R \geq R^{\circ}$, $\iota''(R) \geq \eta > 0$, η a positive constant.*

Welfare is

$$R + x(R)g - \iota(R)\,. \tag{20}$$ {eq:2ndBestWelfare-gain}

Where, taking into account the moral hazard problem,

$$x(R) = \begin{cases} 1\,, & \text{if } R = R^{\circ} \\ 1 - \iota'(R)\,, & \text{if } R > R^{\circ} \end{cases}\,.$$

Given that the component of surplus that does not depend on trade, $R - \iota(R)$, is bounded, intuition suggests that, for g large enough, but finite, welfare, expression (20), is maximized by $R = R^{\circ}$; that is, if the gains from trade are

[14] Williamson (1976) made a related observation that holdup could distort the kinds of investment made.

[15] One example would be $v(b) = 2\sqrt{gb}$ and $\mathcal{B} = \mathbb{R}_+$. Note the example introduced after Proposition 5 (*i.e.,* in which $R(I,b) = \gamma\big(1 + \sqrt{I}\,\big)\big(1 + \sqrt{b}\,\big)$) does *not* belong to this class.

great enough, then trade should occur with certainty and the second-best level of investment should, thus, be zero. Formally, we have:

{lemma:IS-continuous-in-g}

Lemma 4. *Suppose that buyer's expected return is additively separable in his action and the seller's investment (i.e., it is given by expression* (19)*). Then there is a finite* $\bar{g} > 0$ *such that the second-best level of investment,* I^{s}*, is positive but decreasing in the gains to trade,* g*, for* $g \in [0, \bar{g})$ *and is zero for* $g \geq \bar{g}$*.*

The proof can be found in the appendix.

For g finite, it follows, from expression (14), that

$$F(R) = \exp\left(\frac{R - \widehat{R}}{g}\right) \implies f(R) = \frac{1}{g}\exp\left(\frac{R - \widehat{R}}{g}\right) > 0 \tag{21}$$ {eq:F-gain}

for all R. Hence,

$$\int_{R^\circ}^{\widehat{R}} \iota(R) f(R) dR > \iota(R^\circ) = 0 \tag{22}$$ {eq:ExpectedInvestment-gain}

given that $\iota(\cdot)$ is increasing. The previous analysis of g large enough and expression (22) together yield:

Proposition 8. *Suppose that buyer's expected return is additively separable in his own action and the seller's investment (i.e., expression* (19) *holds). Then, if the gain from trade,* g*, is sufficiently large, the equilibrium expected level of investment when the buyer has the bargaining power is greater than the equilibrium expected level of investment when the seller has the bargaining power.*

Intuitively, when the gain from trade (*i.e.,* g) is large, efficiency dictates that trade be likely to occur.[16] Given the moral hazard problem, this means the corresponding level of investment, I^{s}, be small. Because the seller will capture the gains from trade when she has the bargaining power, she internalizes this, leading to an equilibrium in which she invests little and trade is likely. When she doesn't have the bargaining power, she doesn't internalize this. Instead, her objective is to capture as much surplus as she can by avoiding, to the extent possible, being held up by the buyer. She does so by mixing over her investment. If I^{s} is small enough, it will necessarily be less than her expected level of investment given that she mixes over all investment levels up to the autarky level (*i.e.,* she mixes over $[0, \hat{I}]$).

What about the opposite extreme, when g is small (near zero)? As $g \downarrow 0$, the solution to (20) tends to $\widehat{R}$; that is, $\lim_{g \downarrow 0} I^{\mathrm{s}} = \hat{I}$, the maximum possible level of investment given the moral hazard problem. Intuitively, when the gain to

[16] The astute reader might object that the analysis has only really dealt with g so large that efficiency dictates that trade occur with certainty and, thus, $I^{\mathrm{s}} = 0$. Appealing to continuous functions could be problematic because there is no guarantee that I^{s} is continuous in g at $g = \bar{g}$. If, however, $\iota'(R^\circ) = 0$—as would be true if $\iota(R) = R^2/2$—then it is continuous at $g = \bar{g}$ and one can appeal to continuity to conclude that, for g large enough, expected investment when the buyer has the bargaining power is greater than when the seller does even when, in the latter, $I^{\mathrm{s}} > 0$.

trade is small, inducing investment is more important than trading efficiently, so the second-best level of investment tends toward the level that maximizes welfare when trade is infeasible. Given the seller captures all the surplus when she has the bargaining power, she internalizes all this, leading to an equilibrium in which she invests a lot and trade is unlikely. As before, when she doesn't have the bargaining power, she doesn't internalize this; her investment strategy is motivated by her desire to avoid being held-up. This leads her to mix, which means putting positive weight on relatively low investment levels. This would seem to suggest that when the gains to trade are small, investment is greater when the seller has the bargaining power than it is (in expectation) when the buyer has the bargaining power.

There is, however, a critical caveat to that last intuitive argument. Expression (21) entails $\partial F(R)/\partial g > 0$; hence, the distribution over R when g is small first-order stochastically dominates the distribution when g is larger. Because $\iota(\cdot)$ is an increasing function, the expected investment level when the buyer has the bargaining power increases as the gains to trade decrease. Two questions arise: Do we know this expected investment level either (i) does not converge to $\hat{I}$ as $g \downarrow 0$ or (ii), if it does so converge, does it do so sufficiently slowly as g falls that it does not always exceed I^{s}? Unfortunately for the intuitive argument, the expected level does converge to $\hat{I}$ as $g \downarrow 0$.

{lemma:converge}

Lemma 5. *Suppose that buyer's expected return is additively separable in his own action and the seller's investment (i.e., it is given by expression* (19)*). The expected level of investment when the buyer can make a* TIOLI *offer converges to the autarky level, $\hat{I}$, as the gains to trade, g, go to zero.*

The proof can be found in the appendix.

What about the second question, labeled (ii) above? Consider the following example: $\iota(R) = R^2/2$. It is readily verified that $R^\circ = 0$, $\widehat{R} = 1$, $\bar{g} = 1$ and, for $g \leq \bar{g}$, $R^{\mathrm{s}} = 1 - g$. Expected R in the equilibrium in which the buyer makes a TIOLI offer is readily shown to be

$$\mathbb{E}\{R\} = 1 - \big(1 - \exp(-1/g)\big)g > 1 - g = R^{\mathrm{s}}$$

for all $g \in (0, \bar{g}]$. Because $\iota(\cdot)$ is convex, it follows that

$$\mathbb{E}\big\{\iota(R)\big\} \geq \iota\big(\mathbb{E}\{R\}\big) > \iota(R^{\mathrm{s}})$$

for all $g \in (0, \bar{g}]$. That is, if $\iota(R) = R^2/2$, then the expected level of investment when the buyer has the bargaining power always exceeds the level of investment when the seller has the bargaining power, except in the limit when there are no gains to trade, at which point they are equal.[17]

In fact, it is an open question whether this questionable intuition is ever correct. For instance, it can be shown that if $\iota'''(\cdot)$ exists and is everywhere

[17] Of course when there are no gains to trade, there is no point to either party offering to trade; that is, regardless of who has the bargaining power, the equilibrium corresponds to the autarky outcome.

non-positive, then $\mathbb{E}\{R\} > R^{\mathrm{s}}$ for all $g > 0$ (details available upon request), from which it again follows that the expected level of investment must be greater when the buyer makes a TIOLI offer than when the seller does. Although, more generally, I have been unable to prove that the intuition is always false (*i.e.,* I have been unable to prove that investment is always greater in expectation when the buyer can make a TIOLI offer), I have similarly been unable to find a counter-example.[18]

Summary. *Suppose that buyer's expected return is additively separable in his own action and the seller's investment (i.e., it is given by expression* (19)*). For a wide array of investment functions, $\iota(\cdot)$, the equilibrium expected level of investment is greater, despite the holdup problem, when the buyer has the bargaining power than when the seller has it.*

It bears remembering that this last conclusion assumes additive separability. When, as was true of the example following Proposition 5, there is complementarity between the seller's investment and the buyer's action, it is clearly possible, as was shown at the beginning of this subsection, for investment to be greater in expectation when the seller possesses the bargaining power.

7 Pre-Trade Contracting

{sect:Contracts}

In contrast to the situation considered so far, suppose now the parties could contract *prior* to the seller's investment. In essence, a principal (the buyer) is hiring an agent (the seller) to invest on his behalf. The agent could, for example, be an independent contractor or a research scientist.

The moral hazard problem remains, so ultimate ownership of returns must still be stochastic for the investing party to have incentives to invest. But because the parties bargain over the contract under full information, it is reasonable to presume that they agree to a contract that achieves the second best:

{prop:FixedTcontract}

Proposition 9. *Suppose buyer and seller can enter into a contract prior to the seller's investing. Then they will agree to a second-best contract that fixes the probability of exchange at x, with x being given by*

$$x = \begin{cases} 1\,, & \text{if } I^{\mathrm{s}} = 0 \text{ (equivalently, if } R^{\mathrm{s}} = R^{\circ}\text{)} \\ 1 - \iota'\big(R(I^{\mathrm{s}},0)\big)\,, & \text{if } I^{\mathrm{s}} > 0 \text{ (equivalently, if } R^{\mathrm{s}} > R^{\circ}\text{)} \end{cases},$$

and that fixes a non-contingent transfer from buyer to seller of T,

$$I^{\mathrm{s}} - (1-x)R(I^{\mathrm{s}},0) + R(\hat{I},0) - \hat{I} \leq T \leq xV(I^{\mathrm{s}})\,. \qquad (23)$$ {eq:bigT}

[18] Among examples considered, expected investment is greater when the buyer makes a TIOLI offer if $\iota(R) = \zeta R^3$, $\zeta > 0$, or if $\iota(R) = \beta\big(\exp(\xi R) - \kappa\big)$, $\beta \in (0,1)$ and $\frac{1}{\beta\xi} > \kappa > 1$ (details available upon request).

That T exist that satisfy (23) follows because, as was shown earlier in Section 3, p exist that satisfy expressions (3) and (4).[19]

Observe the second-best contract requires the principal to leave the asset (*e.g.,* research project) in the agent's hands with positive probability. Possible real-world examples of such a situation are when engineers from Xerox Parc left to found companies (*e.g.,* Metaphor Computing Systems and Adobe) that built on research done at Xerox Parc. Although *ex post* such departures may have raised questions about the wisdom of Xerox's management, *ex ante* committing to the *possibility* of such departures could have been necessary to induce the engineers to expend effort in the first place.[20]

Unlike some similar settings (*e.g.,* Che and Hausch, 1999), the level of efficiency that can be achieved does not depend on whether the parties are or aren't able to commit not to renegotiate the contract. The effect of any lack of commitment is solely on the contractually set transfer price. Specifically, if the agent has invested I^{s}, then the principal is tempted to renegotiate by offering the agent $R(I^{\mathrm{s}})$ in return for certain exchange. Were the agent to anticipate this, then the agent's investment incentives would evaporate. It follows, therefore, if the principal cannot commit not to make a subsequent offer, then the parties are limited to the following version of the Proposition 9 contract:

{prop:RP-FixedTContract}

Proposition 10. *Suppose buyer and seller can enter into a contract prior to the seller's investing. Suppose, however, that the buyer cannot commit not to make subsequent offers to the agent. Then they will agree to a second-best contract that gives the buyer the right to take possession of the asset in exchange for paying the agent $V(I^{\mathrm{s}})$. This contract fixes a non-contingent transfer from buyer to seller of T',*

$$I^{\mathrm{s}} - \iota'(R^{\mathrm{s}})R^{\mathrm{s}} - \big(1-\iota'(R^{\mathrm{s}})\big)V(I^{\mathrm{s}}) + R(\hat{I},0) - \hat{I} \leq T' \leq 0\,. \tag{24}$$ {eq:bigTprime}

In equilibrium, the buyer will choose to take possession with probability $1-\iota'(R^{\mathrm{s}})$.

(Recall $R^{\mathrm{s}} = R(I^{\mathrm{s}},0)$.) The proof is similar to that of Proposition 2 and, so, omitted.

8 Discussion and Conclusions

{sect:Conclusion}

This paper has shown that it is possible to induce a seller to invest, with positive probability, in an asset to be traded, with positive probability, even when the potential buyer cannot observe the seller's investment. The critical assumption

[19] This analysis is predicated on the ability of the seller, should the parties not agree to a contract, to invest subsequently on her own behalf. If, instead, an agreement with the buyer is necessary for any investment to occur, then the range of possible Ts is

$$I^{\mathrm{s}} - (1-x)R(I^{\mathrm{s}},0) \leq T \leq xV(I^{\mathrm{s}})\,.$$

A similar caveat applies to Proposition 10.

[20] I do not wish to suggest this the only possible—or even the most plausible—explanation for what occurred at Xerox Parc.

is that the seller expects some return from the asset if trade fails to occur. For this reason, it is possible to overcome, partially, the moral hazard problem that exists given that the seller is otherwise investing on behalf of the buyer. The seller has the maximum incentive to invest when trade is certain *not* to occur. The provision of this maximum incentive is not, in general, optimal because, given investment by the seller, there are gains to trade: the tradable asset is more valuable in the buyer's hands than the seller's. A tradeoff thus exists between the provision of incentives *ex ante* and achieving efficiency *ex post*. This paper has shown how this tradeoff can be managed to achieve a second-best outcome.

In addition to the applications noted above (*e.g.,* the sale of startups to established firms), the model has bearing on various used-goods markets. For example, if the quality of a used car is largely a function of how well its current owner has maintained it (a form of investment), then this paper offers insights into the used-car market beyond those found in Akerlof (1970). Such insights could, in turn, shed light on certain institutional practices; for instance, when the "seller" (lessee) has the right to force a "sale" (return a leased car), then the "buyer" (lessor) will have to ensure proper "investment" (require regularly scheduled maintenance). Conversely, if the seller (owner) is less certain of her ability to sell her car quickly (a consequence, perhaps, of a change in macroeconomic conditions), her efforts at maintenance could increase. It is possible that an analysis of a used-good market, in which maintenance investment is important, could serve as means to test, in part, the model presented here.[21]

Despite the model's complexity, it is possible that a variant of it could lend itself to experimental testing. In particular, the analysis of Section 4 may be testable, although the experimentalist would need to be sensitive to psychological factors such as concerns for fairness (*e.g.,* buyers may view an attempt by a seller to leave them with no surplus—setting the price at $V(I^{\mathrm{s}})$—as unfair and, thus, always refuse to trade). Nonetheless, if the experimentalist found evidence that sellers systematically invested less than the autarky level and the probability of sale was significantly less than one, then such findings could be seen as consistent with the model.

There are also open theoretical questions. One stems from the fact that the solutions proposed above rely on there being a positive probability of an inefficient allocation *ex post*; that is, unless there is a positive probability of the asset remaining in the seller's hands, which is *ex post* inefficient, there is no means of inducing the seller to invest. One question, then, is why, when the mechanism has left the seller in possession of the asset, don't the parties renegotiate to an efficient allocation?

This is a question that applies to much of the literature on mechanism design and trade under asymmetric information.[22] In settings in which the uninformed player (and only the uninformed player) can make repeated offers and the un-

[21] As a possible test, if automobile resale possibilities fall for some exogenous reason (*e.g.,* change in licensing fees), do expenditures on maintenance supplies (oil, filters, etc.) rise?

[22] Among the earlier cited papers, for instance, this question applies to both Fudenberg and Tirole (1990) and Demski and Sappington (1991).

informed player's value of trade is independent of investment (as in Gul, 2001, or Lau, 2008), a Coase-Conjecture-like result can be shown to hold, with trade occurring with probability one on the equilibrium path. Moreover, in Gul and Lau, the investing party retains incentives to invest. In contrast, for the problem considered here, if the bargaining game were to yield trade with probability one, then the seller's investment will never be directly beneficial to her and she can, thus, have no incentive to invest.

On the other hand, it may be possible for the parties to commit to their take-it-or-leave-it offers; for instance, by developing reputations *not* to continue negotiations. For example, some divorce lawyers—known as "bombers"—have developed reputations for sticking to their take-it-or-leave-it offers. A company that sought to provide its engineers and scientists incentives along the lines of Proposition 9 would necessarily have to develop a reputation to let the engineers and scientists walk away with positive probability.

It could also be that there is a relatively narrow window in which exchange can occur. If each round of negotiation takes non-negligible time, then the seller could end up with the asset because the parties simply run out of time to bargain. Unlike bargaining under symmetric information, in which bargaining is typically reached in a single round, with asymmetric information there can be multiple rounds of bargaining on the equilibrium path (see, *e.g.,* Spier, 1992). Modeling such bargaining games is beyond the scope of the present paper, but it seems reasonable to predict that such models will again find a tradeoff between *ex ante* incentives and *ex post* efficiency.

In the model considered here, the motive for trade is that the buyer is able to take a further action that raises the return from the asset. Other rationales for trade could also exist: For instance, the returns generated by the asset are idiosyncratic to the owner and the buyer's distribution of returns given any investment level dominate the seller's distribution according to a stochastic order such as first-order stochastic dominance. In fact, this motivation is simply an alternative interpretation of the analysis above: Let the buyer's "action," b, be 0 or 1, with the latter simply indicating possession.[23]

Another motivation for trade could be different risk tolerances. For instance, the seller, as an individual entrepreneur, could be risk averse, whereas the large company that might buy her out could be risk neutral. Assuming a risk-averse seller complicates the analysis because the units of the parties' payoffs are no longer the same (the buyer's remains money, but the seller's is now utils). In addition, the seller will now care about the riskiness of the returns as well as their expected value, which means she could be making investment decisions on two margins: risk and return (*e.g.,* if returns were distributed normally, she would be concerned with both mean and variance). An analysis of the problem with differing attitudes toward risk remains a topic for future research.

Another open questions is when will investment given a holdup problem exceed, in expectation, investment absent a holdup problem? When the gains

[23]Under this alternative interpretation, the buyer's expected utility would be $R(I, 1)$ rather than $R(I, 1) - 1$; it is readily seen that change has no effect on the analysis above.

to trade are independent of the amount invested, one is led to conjecture that expected investment is greater with holdup than without. In contrast, at least from examples, the existence of strong complementarities between the seller's investment and the buyer's subsequent action could lead to settings in which investment is greater absent a holdup problem. {A}

Appendix A: Proofs not Given in Text

{app:A}

Proof of Proposition 3: The proof builds on the following two lemmas:

{lemma:x-nonincreasing}

Lemma A.1. *If the mechanism induces truth-telling (is incentive compatible), then the probability of trade, $x(\cdot)$, is non-increasing in the seller's type (level of investment).*

Proof: By the revelation principle, there is no loss in restricting attention to equilibria in which the seller's best response is to announce her type truthfully. Hence, for $R' \neq R$,

$$U(R) \geq \big(1 - x(R')\big)R + t(R') = U(R') + \big(1 - x(R')\big)(R - R')\,, \qquad (25)$$ {eq:RP-RnotRprime}

where the equality follows from (7). The same logic implies (25) also holds with R and R' interchanged. Expression (25) and its "interchanged version" together imply:

$$\big(1 - x(R)\big)(R - R') \geq U(R) - U(R') \geq \big(1 - x(R')\big)(R - R') \qquad (26)$$ {eq:Pinch}

By considering $R > R'$, the lemma is immediate from (26). □

{lemma:U-convex}

Lemma A.2. *If the mechanism induces truth-telling, then $U(\cdot)$ is a convex function.*

Proof: Pick R and R' in $\mathcal{R}$ and a $\lambda \in (0, 1)$. Define $R_\lambda = \lambda R + (1 - \lambda)R'$. Truth-telling implies

$$\lambda U(R) \geq \lambda U(R_\lambda) + \lambda\big(1 - x(R_\lambda)\big)(R - R_\lambda) \ \text{ and} \qquad (27)$$ {eq:UR-lambda}

$$(1 - \lambda)U(R') \geq (1 - \lambda)U(R_\lambda) + (1 - \lambda)\big(1 - x(R_\lambda)\big)(R' - R_\lambda)\,. \qquad (28)$$ {eq:URp-lambda}

Adding (27) and (28) yields:

$$\lambda U(R) + (1 - \lambda)U(R') \geq U(R_\lambda) + \big(1 - x(R_\lambda)\big)\underbrace{\big(\lambda R + (1 - \lambda)R' - R_\lambda\big)}_{=0}\,.$$

The result follows. □

Condition (i) of Proposition 3 follows from Lemma A.1. Convex functions are absolutely continuous (see, *e.g.,* van Tiel, 1984, p. 5). Every absolutely continuous function is the integral of its derivative (see, *e.g.,* Yeh, 2006, Theorem

13.17, p. 283). By dividing (26) by $R-R'$ and taking the limit as that difference goes to zero, it follows that $U'(R) = 1 - x(R)$ almost everywhere. Expression (8) follows.

Finally, to establish the sufficiency of conditions (i) and (ii), suppose the seller's type is R and consider any $R' < R$. We wish to verify (25):

$$U(R) - U(R') = \int_{R'}^{R} \big(1 - x(z)\big)dz \geq \int_{R'}^{R} \big(1 - x(R')\big)dz = \big(1 - x(R')\big)(R - R'),$$

where the first equality follows from (8) and the inequality follows because $x(\cdot)$ is non-increasing. Expression (25) follows. The case $R' > R$ is proved similarly and, so, omitted for the sake of brevity. ■

Proof of Proposition 6: As a distribution, $F(\cdot)$ is right continuous (*i.e.,* $\lim_{R\downarrow R_n} F(R) = F(R_n)$). Define $F(R_n^-) = \lim_{R\uparrow R_n} F(R)$. Because $F(\cdot)$ is non-decreasing, each point of discontinuity is a jump up. Note $F(R_0^-) = 0$ and $F(R_n) - F(R_n^-) > 0$ for all R_n. This last point implies that the seller plays each R_n with positive probability; hence, by Proposition 4, $x(R_n) = 1 - \iota'(R_n)$.

Using Fubini's Theorem expression (11) can be rewritten as

$$\begin{aligned} -\bar{U} + \int_{R_0}^{\bar{R}} \Big(x(R)V\big(\iota(R)\big) + \big(1 - x(R)\big)R\Big)dF(R) \\ + \sum_{n=0}^{N} \Bigg(\big(F(R_n) - F(R_n^-)\big) \int_{R_n}^{\bar{R}} \big(1 - x(z)\big)dz \\ + \big(F(R_{n+1}^-) - F(R_n)\big) \int_{R_{n+1}}^{\bar{R}} \big(1 - x(z)\big)dz \\ + \int_{R_n}^{R_{n+1}} \big(F(R) - F(R_n)\big)\big(1 - x(R)\big)dR \Bigg). \end{aligned}$$

Canceling like terms, this expression becomes

$$\begin{aligned} -\bar{U} + \int_{R_0}^{\bar{R}} \Big(x(R)V\big(\iota(R)\big) + \big(1 - x(R)\big)R\Big)dF(R) \\ + \sum_{n=0}^{N} \int_{R_n}^{R_{n+1}} F(R)\big(1 - x(R)\big)dR. \end{aligned} \tag{29}$$

{eq:AppBuyUtil}

Because $F(\cdot)$ is absolutely continuous on each segment, it is differentiable almost everywhere on each segment; moreover, it is the integral of its derivative. Denote its derivative by $f(\cdot)$. Expression (29) can thus be rewritten as

$$-\bar{U}+\sum_{n=0}^{N}\Bigg(\big(F(R_n)-F(R_n^-)\big)\Big(x(R_n)V\big(\iota(R_n)\big)+\big(1-x(R_n)\big)R_n\Big)$$
$$+\int_{R_n}^{R_{n+1}}\Big(x(R)V\big(\iota(R)\big)+\big(1-x(R)\big)R\Big)f(R)dR+\int_{R_n}^{R_{n+1}}F(R)\big(1-x(R)\big)dR\Bigg)\,.$$

Some consequences of this last expression are:

1. If $F(R_0)>0$, then $x(R_0)=1$ because the buyer offers a mechanism that maximizes his expected utility. If $x(R_0)=1$, then $R_0=R^\circ$ by Proposition 4.

{step:no-flats}

2. Suppose $\iota(R')$ and $\iota(R'')$ are two possible investment levels in equilibrium, $R'<R''$. I claim $F(R')<F(R'')$. Proof: note it must hold if $R'<R_n\leq R''$ for some R_n; hence, suppose $R_n\leq R'<R''<R_{n+1}$ for some $n=0,\dots,N$. If $F(R')=F(R'')$, then $f(\cdot)$ equals zero almost everywhere. Consequently, $x(R)$, $R\in(R',R'')$, enters the buyer's expected utility expression only in the integral

$$\int_{R'}^{R''}F(R)\big(1-x(R)\big)dR=F(R')\int_{R'}^{R''}\big(1-x(R)\big)dR\,.$$

Buyer expected utility maximization then implies $x(R)=x(R'')$ for $R\in(R',R'']$ (recall $x(\cdot)$ must be non-increasing). Because $\iota(R'')$ is in the set of investments over which the seller mixes, Proposition 4 implies $x(R'')=1-\iota'(R'')$. But then

$$U'(R)-\iota'(R)=1-x'(R'')-\iota'(R)=\iota'(R'')-\iota'(R)>0$$

for all $R\in(R',R'')$ (recall $\iota(\cdot)$ is convex). Hence,

$$\big(U(\tilde{R})-\iota(\tilde{R})\big)-\big(U(R')-\iota(R')\big)=\int_{R'}^{\tilde{R}}\big(U'(R)-\iota'(R)\big)dR>0$$

for any $\tilde{R}\in(R',R'']$; therefore, the seller would never play R', a contradiction. Observe, *inter alia*, this proof extends to show there is no equilibrium in which the seller mixes over a finite number of investment levels (let $R'=R_n$ and $R''=R_{n+1}$, but work with $F(R_{n+1}^-)$ to show $F(R_n)\neq F(R_{n+1}^-)$).

3. Suppose $\iota(R')$ and $\iota(R'')$ are two possible investment levels in equilibrium, $R'<R''$. I claim $F(R')<F(\tilde{R})<F(R'')$ for all $\tilde{R}\in(R',R'')$. The claim is true, from the previous result, for all $\tilde{R}\in[R_m,R_n)$ if

$$R'<R_m<R_n<R''\,.$$

Hence, without loss, restrict attention to $R_n \leq R' < R'' < R_{n+1}$ for some $n = 0, \ldots, N$. Suppose, first, that $F(R') = F(\tilde{R})$. Let $\mathcal{Z} = \sup\left\{R \middle| F(R) = F(R')\right\}$. Because $\mathcal{Z} < R'' < R_{n+1}$, $F(\cdot)$ is continuous at $\mathcal{Z}$, hence $F(\mathcal{Z}) = F(R')$. Because $F(\cdot)$ increases at $\mathcal{Z}$, $\mathcal{Z}$ must be a possible play of the seller. But then we have a contradiction of step 2. Hence, $F(R') < F(\tilde{R})$. Suppose $F(R'') = F(\tilde{R})$. Let $\mathcal{I} = \inf\left\{R \middle| F(R) = F(R'')\right\}$. It follows that $F(\cdot)$ is continuous at $\mathcal{I}$, hence $F(\mathcal{I}) = F(R'')$. Because $F(\cdot)$ increases at $\mathcal{I}$, $\mathcal{I}$ must be a possible play of the seller. But then we have a contradiction of step 2.

{step:1-iotaprime}

4. The previous two steps establish that $F(\cdot)$ is everywhere increasing, thus $f(R) > 0$ for almost every R. Hence, almost every R is possible, thus $x(R) = 1 - \iota'(R)$ almost everywhere. In fact, because $x(\cdot)$ is non-increasing and $\iota'(\cdot)$ continuous, $x(R) = 1 - \iota'(R)$ everywhere on $(\underline{R}, \bar{R}]$.

5. I claim $F(R_n) > F(R_n^-)$ is impossible in equilibrium for $R_n > \underline{R}$. Proof: suppose not and pick ε such that $R_{n-1} < R_n - \varepsilon < R_n$. Define $R_\varepsilon = R_n - \varepsilon$. Suppose the buyer deviated from offering a mechanism with $x(R) = 1 - \iota'(R)$ to offering the following

$$\tilde{x}(R) = \begin{cases} x(R)\,, & \text{if } R \notin [R_\varepsilon, R_n] \\ x(R_\varepsilon)\,, & \text{if } R \in [R_\varepsilon, R_n] \end{cases} .$$

The change in the buyer's expected utility would be

$$\big(F(R_n) - F(R_n^-)\big)\big(x(R_\varepsilon) - x(R_n)\big)\Big(V\big(\iota(R_n)\big) - R_n\Big) + \int_{R_\varepsilon}^{R_n} \big(x(R_\varepsilon) - x(R)\big)\left(V\big(\iota(R)\big) - R - \frac{F(R)}{f(R)}\right) f(R)dR\,. \quad (30)$$

{eq:NoJumps1}

Because the buyer cannot wish to deviate, (30) must be negative for all such ε. The first line of (30) is positive. Hence, the integral is negative for all ε. This implies

$$V\big(\iota(R)\big) - R - \frac{F(R)}{f(R)} < 0 \quad (31)$$

{eq:NoJumps2}

almost everywhere in some neighborhood $(R_n - \delta, R_n)$. Define $R_\delta = R_n - \delta$. Consider, instead, a deviation in which the buyer offers

$$\hat{x}(R) = \begin{cases} x(R)\,, & \text{if } R \notin [R_\delta, R_n] \\ x(R_n)\,, & \text{if } R \in [R_\delta, R_n] \end{cases} .$$

The change in the buyer's expected utility would be

$$\int_{R_\delta}^{R_n} \big(x(R_n) - x(R)\big)\left(V\big(\iota(R)\big) - R - \frac{F(R)}{f(R)}\right) f(R)dR\,.$$

But this is positive by (31), so the buyer *would* wish to deviate. *Reductio ad absurdum*, there is no equilibrium in which $F(R_n) > F(R_n^-)$ for $R_n > \underline{R}$.

6. It has been established that the buyer's equilibrium expected utility can be written as (12). Rewrite that expression as

$$\bar{R} - \bar{U} + \Big(x(\underline{R})V\big(\iota(\underline{R})\big) + \big(1 - x(\underline{R})\big)\underline{R}\Big)F(\underline{R})$$
$$+ \int_{\underline{R}}^{\bar{R}} x(R)\left(\Big(V\big(\iota(R)\big) - R\Big)f(R) - F(R)\right) dR\,. \tag{32}$$

{eq:X-factored}

7. Following step 4, let $x(R) = 1 - \iota'(R)$ for $R \in (\underline{R}, \bar{R}]$. Because the buyer must offer $x(\cdot)$ in equilibrium, the buyer cannot prefer to offer $x(R) - \varepsilon$, $\varepsilon > 0$, for all $R \in (R', \bar{R})$ for any $R' \in (\underline{R}, \bar{R})$. Hence, from (32),

$$\int_{R'}^{\bar{R}} \left(\Big(V\big(\iota(R)\big) - R\Big)f(R) - F(R)\right) dR \geq 0\,. \tag{33}$$

{eq:NoDownProbs}

Suppose (33) were a strict inequality on a set of $R > R'$ of positive measure for some R'. Then the buyer would do better to deviate to

$$\tilde{x}(R) = \begin{cases} x(R)\,, & \text{if } R < R' \\ x(R')\,, & \text{if } R > R' \end{cases} .$$

This last claim can be established by integrating by parts the change in the buyer's expected utility from this deviation:

$$\big(x(R') - x(R)\big) \int_{R}^{\bar{R}} \left(\Big(V\big(\iota(z)\big) - z\Big)f(z) - F(z)\right) dz \Bigg|_{R'}^{\bar{R}}$$
$$+ \int_{R'}^{\bar{R}} \underbrace{\left(\int_{R}^{\bar{R}} \left(\Big(V\big(\iota(z)\big) - z\Big)f(z) - F(z)\right) dz\right)}_{\mathsf{A}} \underbrace{\big(-x'(R)\big)}_{\mathsf{B}} dR\,. \tag{34}$$

{eq:AB-integrate_by_parts}

The first line of (34) is zero, term A is positive because (33) is positive for a positive measure of $R > R'$, and term B is positive because $-x'(R) = \iota''(R)$ and $\iota(\cdot)$ is convex. Hence, (34) is positive, implying the buyer would wish to deviate. *Reductio ad absurdum*, (33) is zero for almost every R'. Because integrals are continuous, (33) is zero for all R'. Hence, the function of R' defined by the integral in (33) is constant, so its derivative,

$$-\Big(V\big(\iota(R')\big) - R'\Big)f(R) + F(R') = 0\,, \tag{35}$$

{eq:DiffEqAgain}

almost everywhere.

Expression (35) repeats the differential equation (13), so the seller's strategy must satisfy (13). The text preceding Proposition 5 showed that if the seller's strategy satisfied (13), then $\bar{R} = \widehat{R}$ in equilibrium. Because $F(\widehat{R}) = 1$, the differential equation has a unique solution, given by (14). ■

Proof of Lemma 4: Restricting $x(R) = 1 - \iota'(R)$, the first-order condition for maximizing (20) is

$$1 - \iota'(R) - \iota''(R)g \begin{cases} \leq 0\,, & \text{if } R = R^{\circ} \\ = 0\,, & \text{if } R \in (R^{\circ}, \widehat{R}) \\ \geq 0\,, & \text{if } R = \widehat{R} \end{cases} . \tag{36}$$

{eq:IS-cont1}

Consider

$$g \in \left(0, \frac{1 - \iota'(R^{\circ})}{\iota''(R^{\circ})}\right) .$$

Because $1 \geq 1 - \iota'(R^{\circ}) > 0$ and, from Assumption 2, $\iota''(R^{\circ}) > 0$, the upper limit of that interval is a positive, finite amount. It is readily seen that only an $R \in (R^{\circ}, \widehat{R})$ can satisfy (36) if g is in that interval. Moreover, for any such g, the lefthand side of (36) is positive for $R = R^{\circ}$ and negative for $R = \widehat{R}$. Because the functions are continuous, there is thus an $R \in (R^{\circ}, \widehat{R})$ that solves (36) for any g in that interval. Hence, from the implicit function theorem that the second-best welfare-maximizing value of R (*i.e.,* R^{S}) is continuous in g for g in that interval; that is, I^{S} is continuous in g for g in that interval. It is readily seen that continuity extends to the end points. Hence, I^{S} is continuous in g for

$$g \in \left[0, \frac{1 - \iota'(R^{\circ})}{\iota''(R^{\circ})}\right] . \tag{37}$$

{eq:IS-cont2}

{claim:1}

Claim 1. *Restricting* $x(R) = 1 - \iota'(R)$*, the* R *that maximizes* (20)*,* $R^*(g)$*, is decreasing in* g *for* g *in the interval given by* (37)*.*

Proof of Claim: Because the cross-partial derivative of (20) with respect to R and g, $R > R^{\circ}$, is $-\iota''(R) < 0$, the usual comparative statics imply that the R that maximizes (20) is non-increasing in g. To see it is strictly decreasing, consider g_0 and g_1 in the interval (37), where $g_0 < g_1$. The same R cannot satisfy the first-order condition for these two values of g because, otherwise, we would have the contradiction:

$$0 = 1 - \iota'(R) - \iota''(R)g_0 > 1 - \iota'(R) - \iota''(R)g_1 = 0\,.$$

□

Define g^* to be the upper limit of the interval in (37). Observe

$$\widehat{R} - \iota(\widehat{R}) > R^{\circ} + 0$$

and

$$R^{\circ} + \Big(1 - \iota'(R^{\circ})\Big)g^* \leq R^{\circ} + g^*\,.$$

The first expression states that, if $g = 0$, $I = \hat{I}$ is welfare superior to no investment and certain trade. Hence, from earlier analysis and Claim 1, $I^{\mathrm{s}} = \hat{I}$ when $g = 0$. The second expression states that no investment and certain trade is welfare superior to no investment and trade with probability $\Big(1 - \iota'(R^\circ)\Big)$, which, from previous analysis and Claim 1, is in turn superior to any positive investment and corresponding probability of trade. By continuity, there must therefore exist a $\bar{g} \in (0, g^*]$ such that

$$R^*(\bar{g}) + \Big(1 - \iota'\big(R^*(\bar{g})\big)\Big)\bar{g} - \iota\big(R^*(\bar{g})\big) = R^\circ + \bar{g}\,.$$

Invoking the envelope theorem

$$\frac{d}{dg}\left(R^*(g) + \Big(1 - \iota'\big(R^*(g)\big)\Big)g - \iota\big(R^*(g)\big)\right) = \Big(1 - \iota'\big(R^*(g)\big)\Big) < 1\,.$$

Hence, for any $g < \bar{g}$ it must be welfare superior to invest $\iota\big(R^*(g)\big)$ and trade with the corresponding probability than to not invest and trade with certainty.

That I^{s} is decreasing in g for $g \in [0, \bar{g})$ follows from Claim 1 because $\iota(\cdot)$ is a strictly increasing function.

■

Proof of Lemma 5: Let $\mathbb{E}\{\cdot\}$ denote the expectation operator on R given the seller's strategy when the buyer makes a TIOLI offer. Because $R \leq \widehat{R}$ and $\iota(\cdot)$ is increasing,

$$\mathbb{E}\big\{\iota(R)\big\} \leq \iota(\widehat{R}) \tag{38}$$ {eq:Converge1}

for all $g > 0$. Because $\iota(\cdot)$ is a strictly convex function, Jensen's inequality implies

$$\iota\big(\mathbb{E}\{R\}\big) \leq \mathbb{E}\big\{\iota(R)\big\} \tag{39}$$ {eq:Jensens}

with the inequality being strict as long as $F(\cdot)$ is not degenerate. Given (38), (39), and the fact that $\iota(\cdot)$ is continuous, the result follows if $\lim_{g\downarrow 0}\mathbb{E}\{R\} = \widehat{R}$. Observe

$$\mathbb{E}\{R\} = \int_{R^\circ}^{\widehat{R}} R\frac{1}{g}\exp\left(\frac{R - \widehat{R}}{g}\right)dR = \widehat{R} - g + (g - R^\circ)\exp\left(\frac{R^\circ - \widehat{R}}{g}\right),$$

which clearly converges to $\widehat{R}$ as $g \downarrow 0$. ■

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