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A Theory and Test of Credit Rationing

By DWIGHT M. JAFFEE AND FRANCO MODIGLIANI*

Nonprice credit rationing by commercial banks and other intermediaries has attracted a good deal of attention in recent years, in part because of the role assigned to this phenomenon by the Availability Doctrine as developed by Robert Roosa [9] and others in the years immediately following World War II¹. It is by now generally agreed that credit rationing, if it could be shown to be empirically widespread, would have important implications for an assessment of the effectiveness and timeliness of monetary policy as well as for our understanding of its *modus operandi*. But significant disagreement still exists whether credit rationing is consistent with rational bank behavior and whether it is an important empirical phenomenon. These two issues essentially define the goal of this study; namely, to provide affirmative answers to the following questions:

- (1) Is it rational for commercial banks to ration credit by means other than price?
- (2) Can credit rationing be measured? If so, are there significant variations

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¹ Credit rationing is discussed in this study only within the institutional framework of the commercial banking industry. Nonetheless, large sections of the paper, and particularly the theoretical model, could be applied equally well to other financial intermediaries.

in rationing over time and can these variations be accounted for?

With respect to the second question, empirical research has been hampered by the almost insoluble problem of directly measuring credit rationing. This has led to the use of proxy variables in the form of indicators of tight money such as interest rate levels or changes in interest rates. It is difficult to obtain conclusive results with such variables, however, since one cannot then really differentiate credit rationing from other symptoms of tight money. In this study, by contrast, we are able to derive and exhibit an operational proxy for credit rationing based explicitly on a theory of rational lender behavior.

The major contributions concerned with the first question have been provided in a series of complementary studies by Donald Hodgman [3], Merton Miller [8], and Marshall Freimer and Myron Gordon [1].² Although criticism of a technical nature has been raised with respect to the first two studies, the most recent of these works by Freimer and Gordon, provides a complete statement of the model. Consequently, it may seem surprising that even the rationality of credit rationing is still considered a debatable issue. The source of this paradox, we believe, is essentially that the authors have not addressed themselves to the relevant question. Before elaborating on this point, however, it is important to precisely define credit rationing.

In line with the generally accepted terminology, we propose to define credit ra-

² This list is not inclusive and references to other works can be found in the studies cited. A second aspect of the mechanism of credit rationing based on "customer relationships" is developed by Hodgman [3] and Kane and Malkiel [6] and is discussed further below.

tioning as a situation in which the demand for commercial loans exceeds the supply of these loans at the commercial loan rate quoted by the banks. Thus credit rationing is an excess demand for commercial loans at the ruling commercial loan rate. In addition, it is helpful to distinguish two forms of credit rationing depending on the status of the commercial loan interest rate. *Equilibrium rationing* is defined as credit rationing which occurs when the loan rate is set at its long-run equilibrium level. *Dynamic rationing* is defined as credit rationing which may occur in the short run when the loan rate has not been fully adjusted to the long-run optimal level.

This definition serves to bring into focus the basic challenge which must be met in order to establish the rationality of equilibrium rationing, namely: can it ever be rational for the bank to limit the loan to less than the amount demanded by the borrower when both the loan rate and the loan granted are chosen optimally, that is, to maximize profits. The problem of dynamic rationing differs only in that the commercial loan rate is set at levels consistent with short-run profit maximization.³ In either case, three elements enter into the problem: the demand for loans, the supply of loans, and the determinants of the commercial loan rate. The shortcoming of the earlier studies already cited is that they concentrate on the determinants of the quantity supplied by lenders while neglecting the other two elements.⁴

³ Previous discussions of credit rationing have studied the two forms of rationing independently; the theoretical justification of credit rationing was directed only at the equilibrium form; the empirical significance was considered only with respect to dynamic rationing. In our analysis we endeavor to integrate the two because our theory and the related empirical tests stress their common origin.

⁴ The papers by Hodgman [3] and Miller [8] and the discussion of "weak credit rationing" in Freimer and Gordon [1] essentially just omit any reference to demand and the determinants of the rate. The discussion of "strong credit rationing" by Freimer and Gordon states the question properly but then assumes the answer, as shown in fn. 12.

It should be clear that information on the supply curve alone will generally not be sufficient to derive implications about credit rationing. For this reason, the development of our theoretical model of credit rationing integrates the demand for loans and the determinants of the loan rate with the supply of loans. The development of this model is given in Sections I and II of the paper. In Section I, analytic propositions concerning rationing are derived under alternative assumptions about market competition. In Section II, these results are then interpreted in the light of the institutional structure of competition in the commercial banking industry of the United States to provide the complete theory. Section III describes how our theory of credit rationing can be used to derive an operational credit rationing proxy. Section IV contains empirical tests of the theory as set out in Sections I and II using the credit rationing proxy derived in Section III.

I. Some Analytic Propositions

1. The Bank's Optimal Loan Offer Curve

The first set of propositions to be developed are concerned only with the bank's supply curve or offer curve for commercial loans. This offer curve is derived by generalizing the results obtained by Freimer and Gordon [1] for a rectangular density function of possible returns to the bank on commercial loan contracts. We shall then proceed to include a demand function and derive the implications of alternative forms of competition.

To start, consider a banker facing a large number of customers each wishing to finance its investment projects. As in the earlier literature, we define the outcome of the customer's projects as the firm's end of period value, denoted by x . We further assume that the bank views x as a random variable and summarize in the density function $f_1(x)$ the bank's subjective evalua-

tion of the probability of different outcomes. In general, the density function $f_i[x]$ will be affected by the size of the customer's investment which, in turn, may be expected to depend on the size of the loan granted. For expository convenience, our formal analysis here will proceed under the restrictive assumption that the size of the project is fixed and therefore $f_i[x]$ can be taken as independent of the loan size granted. This assumption implies that the firm has alternative means of finance which can be used to complement the bank credit. It can be shown, however, that all the major conclusions reached here can be generalized to the case where the size of the project is not independent of the loan granted, provided the investment opportunity of the customer is subject to decreasing (expected) returns.⁵

The expected profit of the bank from the i th customer loan, P_i , is a function of the size of the loan made L_i , the loan rate r_i , and the density function f_i . It is helpful to assume the existence of a sure minimum outcome k_i and a maximum possible outcome K_i for the projects such that:

$$f_i[x] = 0 \quad \text{for } x < k_i \quad \text{or} \quad x > K_i$$

Now let $R_i = 1 + r_i$ be the interest rate factor and thus $R_i L_i$ will be the total amount of the contract repayment (interest plus principal). Then the contribution of L_i to the bank's expected profits can be written as the difference between the total expected repayment by the firm and the total opportunity cost, i.e.:

$$(1) \quad P_i = P_i[R_i L_i] = R_i L_i \int_{R_i L_i}^{K_i} f_i[x] dx + \int_{k_i}^{R_i L_i} x f_i[x] dx - I L_i$$

⁵ Freimer and Gordon [1] conclude that there is a significant difference between the fixed size investment and variable size investment assumptions. However, this conclusion is based on an inconsistent definition of opportunity cost. In Jaffee [5] the essential equivalence of these two assumptions, using a proper definition of opportunity cost, is derived. See also fn. 11.

The first term in this expression represents the gross receipts of the bank if the outcome x is sufficiently favorable to enable the firm to repay the agreed amount $R_i L_i$ in full. The second term denotes the receipts if the outcome of the project falls short of the contracted amount. In this case, we assume the bank receives the entire outcome x , whatever it might be.⁶ The last term represents the bank's opportunity cost, where $I = 1 + j$ and j is the opportunity rate. The rate is for the moment assumed constant and independent of the loan contract on the premise that the bank has unlimited access to a perfect capital market.⁷

The expected profit function (1) can be further simplified by adding and subtracting

$$R_i L_i \int_{k_i}^{R_i L_i} f_i[x] dx$$

and then integrating the second term of that expression by parts. This yields:

$$(2) \quad P_i = P_i[R_i L_i] = (R_i - I) L_i - \int_{k_i}^{R_i L_i} F_i[x] dx$$

where

$$F_i[A] = \int_{k_i}^A f_i[x] dx$$

⁶ In such cases of at least partial default, it is reasonable that the bank would incur some collections which should be deducted from the outcome x . While this refinement is neglected here, it can be shown that the character of our conclusions would not change if it were taken into account. See Jaffee [5] and Müller [8]. A similar remark applies to the cost of making and servicing a loan.

⁷ The existence of well-developed markets in Federal Funds and Certificates of Deposits (CDs) make this assumption reasonable in normal periods. If these markets cease to operate in tight money periods, for example when the Regulation Q ceiling hinders the issue of new CDs, then the shadow price of funds would have to replace the market indicator as the opportunity cost. This may lead to dynamic effects, and these are considered below in the discussion of dynamic rationing (Section II.3) and in the empirical tests.

is the probability that x will be less than a .

An optimal loan to a customer is defined as the loan size which maximizes the bank's expected profits from that customer for a given loan interest rate.⁸ The bank's offer curve for that customer is then the set of optimal loans corresponding to alternative possible loan rates. This offer curve can be derived from the first-order condition for the maximization of expected profit:

$$(3) \quad \frac{\partial P_i[R_i L_i]}{\partial L_i} = R_i(1 - F[R_i L_i]) - I = 0$$

This condition can be usefully rewritten in the form:

$$F_i[R_i L_i] = 1 - \frac{I}{R_i} = \frac{r_i - j}{1 + r_i}$$

Since the quantity $F_i[R_i L_i]$ is precisely the probability of default, (3) admits the following simple interpretation: the optimal loan is such that the probability of default is equal to the excess of the loan rate over the opportunity cost, normalized by the loan rate factor $R_i = 1 + r_i$.⁹

The offer curve can now be defined as the implicit solution to (3) for L_i in terms of R_i subject to the nonnegativity condition $L_i \geq 0$. We will denote this solution by $\hat{L}_i = \hat{L}_i[R_i]$. From (3) we can deduce several properties of this offer curve which will be used in developing our argument.

⁸ The formulation in terms of expected profits assumes, of course, a linear utility function for the bank. This makes it unnecessary to take into account higher order moments of the distribution of outcomes for the firm's projects and covariance of profits between customers. The rationale for this assumption is that banks service a large number of relatively diverse customers. In addition, a utility function with explicit risk aversion would leave unclear whether the existence of credit rationing in the model arises from the structure of the model or simply from the utility function.

⁹ Since the empirically observed spread between the loan rate and the opportunity rate at which banks can secure or invest funds is typically small, this condition implies that banks should tend to assume quite modest default risk. This conclusion seems to be in agreement with bank loss experience on commercial loans.

PROPOSITION 1. The optimal loan offer curve defined by condition (3) (and drawn in Figure 1 has the following properties:

$$(1.1) \quad \hat{L}_i = 0 \quad \text{for } R_i < I$$

$$(1.2) \quad 0 \leq \hat{L}_i \leq k_i/I \quad \text{for } R_i = I$$

$$(1.3) \quad R_i \hat{L}_i \leq K_i \quad \text{for all } R_i$$

$$(1.4) \quad \lim_{R_i \rightarrow \infty} \hat{L}_i = 0$$

Proposition (1.1) follows from the non-negativity condition and condition (3). Since the marginal expected profit of an additional loan ($\partial P_i / \partial L_i$) is negative for all positive loan sizes whenever $R_i < I$, the best the bank can do under this condition is to extend no loan at all. Similarly, when $R_i = I$, condition (3) can hold only if $F_i[R_i L_i] = 0$; this implies $R_i L_i = I L_i \leq k_i$ which is equivalent to proposition (1.2). This means that the offer curve is a vertical line segment when $R_i = I$. In fact, in the case with no uncertainty in which F_i is identically zero, the offer curve is nothing more than this vertical line.

The logic of proposition (1.3) is that for any given interest rate factor R_i , the bank will not receive additional income from extending loans in amounts which exceed the solution of $R_i L_i = K_i$, because K_i is the maximum amount the firm could conceivably earn. Furthermore, loans cost the bank the opportunity rate I , and hence all solutions must satisfy the condition of proposition (1.3). Thus the optimal loan is finite no matter how high the interest rate offered.¹⁰ From proposition (1.3) follows immediately proposition (1.4). It implies that as the loan rate grows larger and larger, the optimal loan does not follow course; to the contrary, at least after some point, the optimal loan will begin to decline as the rate grows and will eventually

¹⁰ Although the maximum is finite, there may be multiple local maxima for the loan offer curve. In the discussion which follows, we abstract from this complication since it does not affect the results.

approach zero as the rate grows beyond bounds. The common sense of this surprising implication can be understood from the following considerations: (i) by making the size of the contracted repayment $R_i L_i$ sufficiently large, default becomes virtually certain, and hence the bank can count on becoming the owner of all the net activities of the firm; (ii) as R_i rises, the amount L_i that the bank needs to invest to achieve this result grows smaller and smaller and approaches zero as R_i tends to infinity.¹¹

Two other properties of the offer curve are also worth developing.

PROPOSITION 2. For a given interest factor R_i , expected profits decrease monotonically as the loan size varies from the optimal size in either direction.

The proof follows directly from (3), since

$$(4) \quad \frac{\partial^2 P_i[R_i L_i]}{\partial L_i^2} = -R_i^2 f_i[R_i L_i] < 0$$

for $k_i < R_i L_i < K_i$

This also shows that the solution given by (3) is, indeed, a global maximum.

PROPOSITION 3. Expected profits increase along the offer curve for successively higher interest rate factors.

To derive this result, first substitute (3) into (1), which allows us to write the profit function along the offer curve as:

$$(5) \quad P_i[R_i \hat{L}_i] = \int_{k_i}^{R_i \hat{L}_i} x f[x] dx$$

¹¹ A similar result holds for the variable size investment case. Although the optimal loan offer remains positive as the interest rate approaches infinity, the loan size approaches a finite asymptote. In contrast, Freimer and Gordon [1, p. 407] conclude that the optimal loan approaches infinity as the interest rate goes to infinity, because they only consider projects with an expected value exceeding the opportunity cost; but this contradicts the meaning of opportunity cost; see fn. 5.

But then,

$$\frac{dP_i[R_i \hat{L}_i]}{dR_i} = R_i \hat{L}_i f_i[R_i \hat{L}_i] \left(\hat{L}_i + R_i \frac{d\hat{L}_i}{dR_i} \right)$$

and by implicitly differentiating (3)

$$\hat{L}_i + R_i \frac{d\hat{L}_i}{dR_i} = \frac{1 - F_i[R_i \hat{L}_i]}{R_i f_i[R_i \hat{L}_i]}$$

and thus

$$(6) \quad \frac{dP_i[R_i \hat{L}_i]}{dR_i} = \hat{L}_i (1 - F_i[R_i \hat{L}_i]) > 0$$

for $k_i < R_i \hat{L}_i < K_i$

which proves the proposition. An obvious and reasonable implication of this proposition is that the bank will obtain its maximum potential profits when allowed to charge an infinite interest rate. Of course, this only serves to emphasize that the offer curve is defined independently of the demand for loans and thus many points on the offer locus may not prove feasible for the lender.

2. The Banker as a Discriminating Monopolist

The optimal loan offer curve just derived can be interpreted as the bank's supply curve for the bank customer. It is interesting that this supply curve is "backward bending" as shown in Figure 1. In fact, Freimer and Gordon [1] base part of their argument for credit rationing on this characteristic of the supply curve alone.¹² Our

¹² Reference is to the Freimer and Gordon [1] case of weak credit rationing. The terminology is misleading because one would normally not equate a backward bending supply curve with rationing. Freimer and Gordon also consider what they call "strong credit rationing," which conceptually is closely akin to our equilibrium rationing. For this case they argue that a bank sets a conventional interest rate (6 percent to be exact) and then grants loans to all customers up to the amount indicated by their respective bank offer curves. This analysis is inadequate for two reasons. First, the optimality of a 6 percent rate is essentially just assumed; banks do not charge rates below 6 percent because of convention; banks do not charge rates above 6 percent because their numerical examples suggest

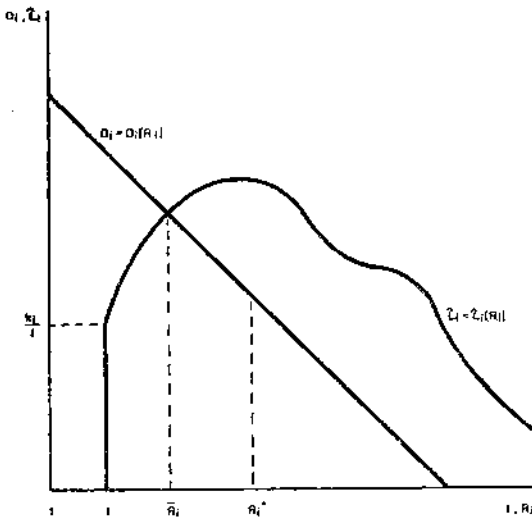


FIGURE 1

definition of credit rationing, however, obviously necessitates taking into account the demand for loans by the bank customer. Such a demand curve for the *i*th bank customer, denoted hereafter by $D_i = D_i [R_i]$, has been drawn in Figure 1. We assume a downward sloping demand curve, which vanishes for a sufficiently high interest rate factor and is finite even for a zero interest rate (or interest factor equal to unity). The boundary conditions for the demand curve are based on the characteristics of the underlying investment projects. At a zero interest rate the firm's demand for loans is bounded by the size of the investment projects, while the existence of some alternative means of financing suggests that at some sufficiently high interest rate the demand will become zero. The negative slope of the demand curve can be derived from the assumption that the firm has only limited access to these alternative means of finance. The implied limit on competition between

that customer utility maximization occurs at lower rates. Secondly, even if the 6 percent rate is accepted as optimal, we have no way of knowing from their analysis whether rationing actually occurs, because the demand curve is never shown.

banks for the customer's business is discussed further in Section II.

The conditions under which rational credit rationing will occur can now be seen with the aid of Figure 1. The *i*th firm's demand curve and the bank's offer curve to that customer are assumed to intersect at the interest rate factor \bar{R}_i .¹³ If the bank chooses to charge an interest factor greater than \bar{R}_i , rationing will not occur since the loan demand is less than the loan offer at such an interest factor. In fact, if the firm were willing to accept a larger loan (which it is not by definition of the demand curve), the bank would increase its expected profits by providing such a loan (see proposition 2). On the other hand, if the bank chooses to charge an interest factor less than \bar{R}_i , then rationing will occur since in this region the bank's optimal loan offer is less than the amount demanded by the customer. The bank would only reduce its expected profits by increasing the loan offer to meet the demand. Thus the question of the rationality of credit rationing can be reduced to a consideration of the optimal rate factor to be charged by the bank, and its relation to \bar{R}_i .

Two critical variables enter into the bank's selection of the loan rate. First there is the question of the time horizon: in the long run, a rational banker would select the rate which maximizes his expected profits; but other constraints may preclude immediate full adjustment in the short run. This leads, of course, to our distinction between the cases of equilibrium rationing and dynamic rationing as already defined. We start by considering only the equilibrium case since it has been

¹³ The existence of at least one point of intersection is assured by the boundary conditions on the demand curve and proposition (1.4). Throughout the remainder of the paper we shall assume that, in fact, only one such point exists. Equivalent results can be obtained for the case of multiple intersections, but only at the cost of more complexity in the analysis.

the center of the theoretical discussion and since the short-run dynamic case is easily derived from the equilibrium case.

The second important variable influencing the bank's choice of rate is the nature of market competition. Because the degree of competition turns out to be a critical factor in determining the existence of rationing, it is worthwhile considering several different regimes from a purely analytic standpoint.

The first regime considered is the simple case of a discriminating monopolist. It is assumed that the bank maximizes its expected profits with respect to each customer separately and is free to charge each customer a different interest rate. Thus, we can take the bank's solution for the i th customer as typical:

PROPOSITION 4. Let R_i^* be the rate factor which maximizes the bank's expected profits when the bank is acting as a discriminating monopolist. Then:

- (4.1) $R_i^* \geq \bar{R}_i$, which implies
- (4.2) Credit rationing is not profitable for a banker acting as a discriminating monopolist.

The proof of this proposition is easily derived from Figure 1 and the properties of the offer curve. The bank will always charge an interest factor at least as high as \bar{R}_i since; (i) expected profits increase along the offer curve for successively higher interest rates (proposition 3) and (ii) the loan $L_i(\bar{R}_i)$ is feasible by the definition of \bar{R}_i . We next observe that, in view of proposition 2, for any rate equal to, or larger than \bar{R}_i , the bank's profit will be higher the closer the loan is to the corresponding point on the offer curve. But since the demand curve places a ceiling on the feasible loan size, the bank will find the optimal rate R_i^* by maximizing profit along the demand curve constraint, which means that credit rationing will not be

profitable. Indeed, as in the standard theory of monopoly under certainty, at the rate R_i^* the bank would be glad to lend more than the customer is prepared to take.

3. *The Banker Must Charge All Customers a Uniform Rate of Interest*

Now suppose that the banker is constrained to charge all customers the same rate though he can choose that rate freely and can also decide on the size of the loan to be granted each customer. We will show that under these conditions credit rationing may (and very frequently will) be profitable, i.e. at the common optimal interest rate, for some customers the most profitable loan for the bank to supply is less than the amount demanded.

To establish this proposition it is convenient to deal first with a subsidiary problem. Suppose that the bank faces only two customers and, for the moment, rule out credit rationing by requiring that the bank *must* satisfy the customers' demand at the chosen common rate.

The bank's expected profits under these conditions can be written as:

$$(7) \quad P = P_1[RD_1] + P_2[RD_2]$$

where R is the common rate factor charged both customers and the constraint of satisfying both demand functions is implicit in the notation. By differentiating this profit function with respect to the interest factor, we can obtain the first-order condition for the optimal interest factor, say R^* .

An important property of R^* can be obtained, however, without explicit reference to this first order condition. Let R_1^* and R_2^* be the optimal rate factors a discriminating monopolist banker would charge customers 1 and 2, respectively, and assume, without loss of generality, that $R_1^* < R_2^*$. It follows from this definition that the expected profit from each cus-

tomers must be a concave function of R in the neighborhood of R_1^* and R_2^* , respectively. We shall go somewhat further and assume that the concavity of each expected profit function holds for all $R > 1$.¹⁴ We can then establish:

PROPOSITION 5. If R^* is the common rate factor that maximizes the bank's expected profit, we must have:

$$R_1^* \leq R^* \leq R_2^*$$

This result can be verified through use of a proof by contradiction. The assumption of concave expected profit functions implies that expected profits decrease monotonically as the absolute value of the spread between the actual rate to a customer and the discrimination monopolist rate increases. Thus, if the bank chose a common rate factor R^* that was less than both R_1^* and R_2^* , it would find that expected profits could be increased by increasing the rate factor at least to the level of R_1^* , thus contradicting the assumption that the original rate factor was optimal. Essentially the same argument shows that $R^* > R_2^*$ also leads to a contradiction.

We can now relax the restriction that the bank must satisfy both customers' demand function and show that, under these conditions, it will never pay to ration customer 1 but it may very well pay to ration customer 2. Specifically, we can establish:

PROPOSITION 6. For the common rate regime

$$(6.1) \quad R^* \geq R_1^* \geq \bar{R}_1 \text{ implying it is not profitable to ration customer 1.}$$

¹⁴ The assumption of global concavity is not a necessary condition for proposition 6, to be proven below; the proof can be carried out with weaker assumptions. We have chosen to assume global concavity, however, because it leads, via proposition 5, to a particularly interesting demonstration of proposition 6.

$$(6.2) \quad R_2^* \geq R^* \geq \bar{R}_2, \text{ implying that } R^* < \bar{R}_2 \text{ is possible, in which case, credit rationing is profitable.}$$

Proposition (6.1) follows directly from propositions 5 and 4. As for establishing (6.2), we need only exhibit a concrete example in which the condition holds. That it is, in fact, quite easy to construct such examples can be seen from the following considerations. First, going back to proposition 5, one can readily establish that the position of R^* within the range R_1^* to R_2^* depends on the relative size of the two customers (as measured, say, by the size of the loan demanded for any R in the critical range), and on the elasticity of the two demand curves. In particular, R^* can be made arbitrarily close to R_1^* by assuming that customer 1 is sufficiently larger than customer 2, and/or by assuming that the demand curve for customer 1 is sufficiently inelastic in the range of rates above R_1^* . By the same token, \bar{R}_2 can be made arbitrarily close to R_2^* by assuming a sufficiently elastic demand curve for customer 2. Thus, by appropriate choice of these functions, one can readily construct situations where R^* is lower than \bar{R}_2 . The above construction also provides an interesting interpretation of proposition 6. The constraint of charging both customers the same rate R^* forces the bank to charge customer 1 a rate which is too high relative to R_1^* and hence, the customer is not rationed. On the other hand, the bank is forced to charge customer 2 a rate which is too low relative to R_2^* . If the rate is sufficiently low relative to \bar{R}_2 , that is $R^* < \bar{R}_2$, then the second customer will be rationed.

The possibility of credit rationing also adds another dimension to the problem. In the case in which rationing does not occur, R^* is the optimal common rate charged by the bank and the customers receive loans

of $D_1 [R^*]$ and $D_2 [R^*]$, respectively. But when customer 2 can be profitably rationed at R^* , the very existence of rationing changes the conditions of the problem and R^* need no longer be the optimal rate. Instead, there will exist a more general optimum optimum rate, say \hat{R} , which yields the maximum expected profits after allowing for credit rationing of customer 2. \hat{R} will just equal R^* in cases in which rationing is not profitable, but will generally differ from R^* when rationing is profitable. The fact that \hat{R} may differ from R^* creates some difficulties in deriving the comparative static properties of the model, as will be apparent below. However, it does not affect our basic conclusion; proposition 6 remains valid even if R^* is replaced with the true optimum \hat{R} . For, as one can readily verify, $R^* \leq \bar{R}_2$ implies $\hat{R} \leq \bar{R}_2$, and conversely.¹⁵

Before proceeding to the generalizations of the propositions developed so far, it is worthwhile considering the special case of a risk free firm, that is a customer for whom the bank's subjective evaluation of default risk is zero.

PROPOSITION 7. Neither a banker acting as a discriminating monopolist, nor a banker charging all customers the same rate, will ration a risk free customer.

The result of proposition 7 for the case of a discriminating monopolist is simply a special case of proposition 4 and follows directly from that proposition. To prove the proposition for a banker charging a

¹⁵ We have previously established that $R^* \geq \bar{R}_2$ implies that R^* is already the optimum optimum or $R^* = \hat{R}$; hence $R^* \geq \bar{R}_2 \rightarrow \hat{R} \geq \bar{R}_2$. Similarly $\hat{R} \geq \bar{R}_2$ implies that even when rationing is allowed, it is not profitable and therefore \hat{R} must coincide with R^* , the maximum subject to the constraint that rationing is not permissible; that is $\hat{R} \geq \bar{R}_2 \rightarrow \hat{R} = R^*$ and therefore $\hat{R} \geq \bar{R}_2 \rightarrow R^* \geq \bar{R}_2$. But these two propositions together imply $R^* \geq \bar{R}_2 \leftrightarrow \hat{R} \geq \bar{R}_2$, which in turn implies $R^* \leq \bar{R}_2 \leftrightarrow \hat{R} \leq \bar{R}_2$.

common rate factor, it is important to recall from proposition 1 that the bank's offer curve for a risk free customer is a vertical line at $R=I$. Furthermore, it must be true that R^* (or \hat{R}) is greater than I if the bank's expected profits are to be positive. But this implies that $R^* \geq \bar{R}$ for the risk free customer, which on the basis of proposition 6 rules out the possibility of credit rationing.

We can now proceed to generalizations of the propositions developed in this part. First, consider a banker facing n customers and constrained to charge all n customers the same loan rate. Again, let R_i^* ($i=1, 2, \dots, n$) denote the rate the bank would charge if acting as a discriminating monopolist and let \hat{R} be the common "optimum optimum" rate the bank charges when allowing credit rationing. Let us number customers in ascending order with respect to the monopolist rate; that is $R_i^* \geq R_{i-1}^*$ for $i=2, 3, \dots, n$. Proposition 5 can then be generalized to:

PROPOSITION 5'. The optimal common rate \hat{R} charged all customers must lie between the rate charged customer 1 and the rate charged customer n when the bank is acting as a discriminating monopolist. Formally, there exists an integer j , $2 \leq j \leq n$, such that $R_j^* \geq \hat{R} \geq R_{j-1}^*$.

Similarly, proposition 6 can be generalized to:

PROPOSITION 6'.

(6.1') For any customers i such that $\hat{R} \geq R_i^*$, we have $\hat{R} \geq \bar{R}_i$ implying that rationing of these customers is not profitable.

(6.2') In the case of customers for whom $\hat{R} < R_i^*$, we have $\hat{R} \geq \bar{R}_i$, implying that rationing of some of these customers may be profitable.

The interpretation and proof of these

propositions follows directly from the corresponding propositions for the case of two customers, and accordingly are not repeated here.

4. *Generalization to m Separate Customer Classes*

It is now easy to extend our conclusions about the rationality of credit rationing to the case in which the bank can assign customers to any one of, say, m classes, where within each class the bank must charge a single uniform rate. The principles which govern the assignment of customers to classes and the choice of the rate for each class can be readily inferred from the previous analysis:

(i) If the profit functions are all concave in the relevant range, then the optimal classification will be achieved by dividing the entire range of the set of R_i^* (the rate charged customer i when the bank acts as a discriminating monopolist) into m intervals and assigning to the same class all customers whose R_i^* falls in a given interval. If the profit functions are not concave, the principle for optimal classification becomes more complex; but in any event, it is clear there will exist a set of optimal group rates and a corresponding optimal classification for the customers, and that furthermore, each class will contain customers with different R_i^* s, as long as the number of customers exceeds the number of classes.

(ii) The optimum rate for any given class j , say \bar{R}_j , must fall somewhere between the smallest and the largest R_i^* of the customers in that class.

(iii) It will not be profitable to ration customers whose R^* is smaller than the group rate \bar{R}_j , but it may pay to ration those for whom R_i^* exceeds \bar{R}_j . In particular, rationing will occur whenever $\bar{R}_i > \bar{R}_j$.

(iv) The likelihood that it will be profitable to ration at least some customers

in a class will be positively related to the heterogeneity of the R_i^* of the customers in that class and hence the likelihood of rationing will be inversely related to the size of m . Indeed, if m is allowed to be as large as the number of customers, that is, n , then the bank will be in the position of a discriminating monopolist and credit rationing will not occur.

There remains now to draw the implications of these results by combining the propositions developed so far with a number of considerations arising from the nature of competition in the banking industry.

II. *Competition in Banking and Credit Rationing*

1. *The Nature of Competition in Commercial Banking and its Implications for Credit Rationing.*

We have shown in Section I that a single bank, free to discriminate between borrowers by charging each customer its monopolist rate R_i^* , would not ration credit. A similar conclusion holds even if there are many banks, as long as they act collusively to maximize joint profits, relying if necessary on side payments. If all banks share the identical subjective evaluations of the profitability of borrowers' investment projects, then clearly the optimum rate R_i^* to be charged to the i th customer would be the same no matter which bank served him. Furthermore, even allowing for differences in the subjective evaluation of borrower risk and assuming an arbitrary initial distribution of customers between banks, the device of buying and selling customers would allow each bank and the industry as a whole to maximize profits. In this way a banker's Pareto optimum would be reached with each bank charging its customers the monopolist rate, and thus, again, no credit rationing would occur.

In this section we propose to argue that

this solution is in fact not feasible, at least in the present American economy. We suggest, instead, that banks can best exploit their market power, while remaining within the bounds set by prevailing institutions, by classifying customers into a rather small number of classes within each of which a uniform rate is charged, even though the membership of each class will exhibit considerable heterogeneity in terms of R_i^* .

First, even if there were but a single monopoly bank or a perfectly collusive banking system, the mere existence of usury laws would lead toward the indicated solution. Such laws would prevent the banker from charging any rate R_i^* which is greater than the legal limit. Thus all customers for whom R_i^* is larger than the ceiling would be classified together in the category with the ceiling rate. Since the monopolist rate R_i^* for each customer in this class would equal or exceed the uniform rate set for the class, namely the usury ceiling, it is apparent from the results of Section I that many, if not all, customers in this class would be profitably rationed.

Even aside from usury ceilings, the pressure of legal restrictions and considerations of good will and social mores would make it inadvisable if not impossible for the banker to charge widely different rates to different customers. A banker would tend, instead, to limit the spread between the rates and to justify the remaining differentials in terms of a few objective and verifiable criteria such as industry class, asset size, and other standard financial measures. An effort would no doubt be made to choose the criteria for classification so as to minimize the difference between the optimal classification of customers into rate classes and the categories dictated by the objective criteria, but a close approximation might be difficult to achieve.

The inducement to adopt a classification scheme of the type described is likely to be greatly strengthened when we take into account the fact that banks cannot openly collude, although they share a common desire to maintain rates as close as feasible to the collusive optimum. In order to prevent, or at least minimize, competitive underbidding of rates they would need tacit agreement as to the appropriate rate structure for customers, and thus a classification scheme based on readily verifiable objective criteria would appear as an efficient and effective device. Furthermore, to make the whole arrangement manageable, the number of different rate classes would have to be reasonably small. Finally, one can also readily understand how such tacit agreement on the structure of class rates could be facilitated by tying these rates through fairly rigid differentials, to a prime rate set through price leadership.

If we now superimpose the impact of usury ceilings along with the other legal and social constraints, it is clear that the entire structure of rates would tend to be compressed within narrower limits than would otherwise be optimal. This means, in particular, that the rate for each class would tend to the lower limit of the R_i^* spread appropriate for the customers in that class, with the possible exception of the lowest class rate reserved for the riskless or nearly riskless prime customers. The result is that widespread rationing would occur, particularly in the higher rate classes.

Finally, we may observe that the oligopolistic price setting pattern outlined above is likely to lead to a very sluggish and somewhat jerky adjustment of the entire rate structure as changes in underlying conditions generate changes in the optimal level and structure of rates. The considerations relevant here are well known from the literature on oligopolistic

market structure and price leadership. It follows that when conditions are changing rapidly we might expect to find that the entire structure of rates would lag behind, and thus, for a while, would tend to be higher or lower than the optimal structure, depending on the direction of the change. These considerations provide the key to dynamic rationing to be elaborated below.

2. Long-Run Equilibrium Credit Rationing

We are now in a position to assemble the complete theory of credit rationing. We first take up equilibrium credit rationing which occurs when the loan rate is set at its optimal level. In Section I it was shown that credit rationing will be profitable, even in long-run equilibrium, as long as there is uncertainty of loan repayment and banks cannot discriminate perfectly between customers. In Section II.1 both of these conditions were verified as features of the commercial banking industry, and thus we conclude that equilibrium rationing is consistent with rational economic behavior.

This conclusion would remain basically unchanged if one recognizes that the bank's expected return and cost is affected by other contract terms such as loan maturity and compensating balance requirements. To be sure, if a bank could discriminate freely between customers with respect to such factors, there might be no occasion for equilibrium rationing. But while the empirical evidence is scanty, one would surmise that banks are limited in their power to discriminate with respect to these terms as well as the nominal interest charge. Similarly, our analysis can incorporate the benefits of maintaining long-run "customer relationships" (cf. Hodgman [4], Edward Kane and Burton Malkiel [6]) without invalidating our basic conclusions about the rationality of rationing.¹⁶

¹⁶ Although the customer relationship and nonprice

It is worthwhile to consider briefly the comparative static properties of equilibrium rationing in our model. The total amount of equilibrium rationing, E , can be written as the difference between loan demand and loan supply for those customers of the bank experiencing rationing:

$$(8) \quad E = \sum_{i=1}^n \max [D_i(\hat{R}) - \hat{L}_i(\hat{R}_i), 0]$$

The excess supply of loans to those customers to whom the bank would like to extend additional loans does not, of course, offset the rationing to other customers. The offer curve to a firm depends on the opportunity cost of the bank, I , and the firm's density function of possible outcomes, f_i , and hence E depends on these two factors and the firms' demand functions. Consequently, for purposes of comparative static analysis, one may consider the impact of changes in demand, risk, and opportunity cost on the amount of equilibrium rationing. Unfortunately, the evaluation of the impact of these changes turns out to be a very difficult process. In particular, the rationed or nonrationed status of customers may change because of the change in the underlying parameter. To at least illustrate the possible outcomes, however, we shall outline the case of a change in the opportunity cost I .

Two important results for the comparative static analysis are given in the following propositions:

PROPOSITION 8. *A ceteris paribus increase in the opportunity cost causes the optimal loan offer curve to shift downward at every interest rate. Indeed, by implicit differentiation of equation*

contract terms may serve to modify the importance of uncertainty in our theory of credit rationing, the bank's inability to discriminate perfectly in setting these terms is still critical. The need for such an element can be seen clearly in the attempts by Hodgman [4, p. 265] and Kane and Malkiel [6, p. 123] to rationalize credit rationing without appeal to such imperfect discrimination.

(3) we obtain:

$$(9) \quad \frac{\partial \bar{L}_i}{\partial I} = \frac{-1}{f_i R^2} < 0$$

PROPOSITION 9. The common optimal rate factor R^* (applicable if the bank were constrained to satisfy all demand functions) is positively related to the opportunity cost I .

Proposition 8 is self-evident. Proposition 9 is in line with the standard theory of the firm in that an increase in marginal cost necessitates an increase in marginal revenue which takes the form of an increase in R^* . Because the very act of changing the parameter I may lead to a change in the rationed or nonrationed status of customers, as demonstrated below, special cases can arise in which \hat{R} (the common optimal rate allowing for rationing when profitable) and I are not positively related, contrary to the relationship between R^* and I . Since such cases are distinctly abnormal, the obvious and reasonable situation being a positive relationship between I and \hat{R} , we shall proceed on the premise that proposition 9, is equally applicable to \hat{R} .¹⁷

With the aid of Figure 2, we can survey the impact of an increase in the opportunity cost from I_0 to I_1 . From proposition 8, it is known that the offer curve will shift downward from the locus \bar{L}_0 to \bar{L}_1 ; and from proposition 9 it is known that the common optimal rate factor will increase, say from \hat{R}_0 to \hat{R}_1 . Thus the net impact of a change in the opportunity cost on the maximum loan offered is the result of a shift in the offer curve and a movement along this curve, and the amount of equilibrium rationing may either rise or fall. These two alternative outcomes are illustrated in Figure 2. In the case of the

demand curve D_1 , the customer would move from a rationed to a nonrationed status, while for the case illustrated by the demand curve D_2 , the opposite conclusion holds. Cases in which the rationing status remains unchanged are, of course, equally possible. Thus the effect of an increase in I on the size of rationing must remain uncertain. This result, however, should not be regarded as disturbing. What it implies is that in the long run, *no systematic relationship exists between the extent of equilibrium rationing and the absolute level of interest rates.*

3. Dynamic Credit Rationing

We define dynamic rationing as the difference between equilibrium rationing and the volume of rationing that arises when the actual rate charged customers, R , differs from the long-run equilibrium rate, \hat{R} . By this definition, dynamic rationing can be positive or negative, and we shall show that its magnitude will be positively associated with the spread, $\hat{R}-R$. Furthermore, we have already suggested, in view of the oligopolistic structure of the banking industry, that R is likely to adjust slowly to changes in \hat{R} , thus lending

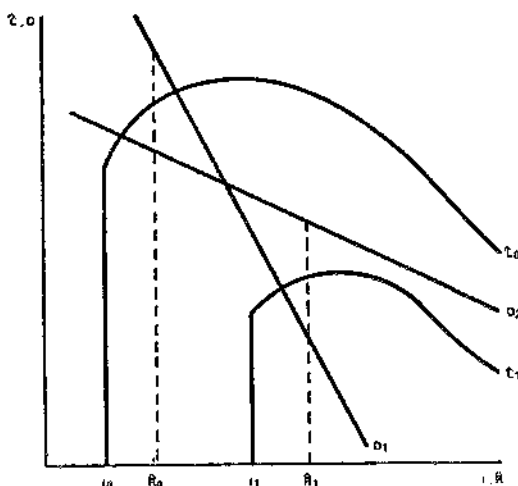


FIGURE 2

¹⁷ The abnormal conditions which lead to this abnormal result are discussed further in [5, p. 56 and p. 86].

credence to the empirical importance of dynamic credit rationing. In fact, there is evidence that banks may tend to rely on some objective signal, such as changes in the Federal Reserve's discount rate, in determining the timing of loan rate changes, and hence, dynamic credit rationing would vary significantly depending upon Federal Reserve policy.¹⁸

It is helpful to begin by considering a system that starts in long-run equilibrium with a rate, \hat{R}_0 , and then receives a shock that changes \hat{R}_0 to \hat{R}_1 while the quoted rate remains unchanged at \hat{R}_0 . As we have seen, there are three main market forces that can change \hat{R} : a change in market interest rates leading to a change in the opportunity cost I ; a shift in customer demand schedules; and a change in risk as may be indicated by a shift in $F[x]$. We shall first consider the effects of changing each of these factors, one at a time. Then by combining the results of these *ceteris paribus* experiments, we can examine more realistic situations in which several factors change simultaneously.

1. Consider first a change in market rates of return which, for convenience, may be summarized by some representative rate, r_M . a rise in r_M will directly increase the opportunity rate for funds invested in the loan portfolio. From propositions 8 and 9, we know that such an in-

¹⁸ Empirical tests performed in an earlier work by one of the authors [5, Ch. 4] confirms the hypothesis that, in the period covered by our data, changes in the Federal Reserve discount rate were a major factor influencing the timing of changes in the commercial loan rate. It should be stressed, though, that such a relationship might not continue to hold in the future unless the Federal Reserve continues to operate the discount window in the customary fashion. Should the recent Federal Reserve proposal [11] to keep the discount rate closely in line with market rates be enacted, it is likely that the commercial loan rate would adjust more quickly toward its desired level. In this case, dynamic rationing in response to changes in market conditions would tend to die out faster than in the past. Thus discount rate policy is at least one way in which the Federal Reserve can influence the amount of credit rationing.

crease in I will lower all offer curves and also increase \hat{R} . Since the quoted rate remains at \hat{R}_0 and all the demand curves are unchanged, *there must be an aggregate increase in rationing*. Our model also provides some information about the incidence of this increase. At one extreme, the risk free firms will still not experience any rationing because the offer curve for them is a vertical line.¹⁹ At the other extreme, firms already rationed in the initial equilibrium will be rationed even more as the offer curve shifts down. Finally, among the risky firms initially not rationed, some will remain unrationed while others will experience new rationing, with the amount depending upon the extent of the shift.

Thus on the whole, the loan portfolio will shrink and the funds released by rationing will be shifted into other assets whose yield has increased (or be used to repay the now more costly borrowed funds). However, the opportunity cost I is now likely to fall relative to the market rate r_M because loans will be a smaller percentage of the total portfolio. This will tend to moderate, though not eliminate, the initial increase in rationing.

2. Consider next the effect of a downward revision of the anticipated distribution of outcomes; operationally, we may think of the initial distribution $F[x]$ being replaced by a new one $G[x]$ with $G[x] \leq F[x]$. As can be verified from (3), such a re-

¹⁹ This conclusion would not hold if the shift in I were so large as to exceed \hat{R}_0 . In this case, the best course of action would be to cut off loans even to the prime firms, and a fortiori to all customers. But this case can be disregarded for the banks could be counted upon to respond promptly by raising their quoted rate, i.e., \hat{R}_0 would not remain unchanged in these circumstances. More generally in the usual theory of monopoly or oligopoly, if the equilibrium price rose because of a shift in either the demand or the cost function, and, for some reason, the market price was prevented from rising, rationing would occur only if the shift were such that the marginal cost would exceed the price. It appears, therefore, that dynamic rationing, just as equilibrium rationing, is intimately related to the uncertainty about the outcome of the loan.

vision results again in a downward shift in the offer curve, and in an increase in the R_1^* and hence in \hat{R} . Since the quoted rate has not changed, the maximum loan offered at that rate must tend to decline. With the demand unchanged, rationing must therefore increase for some of the customers, and the incidence of the increased rationing is entirely analogous to that of case 1. Once more, as loans shift out of the now less remunerative loan portfolio, the opportunity cost I may decline somewhat, mitigating the initial effects.

3. Much the same conclusion can be seen to hold for the case of a shift in all demand schedules, r_M and $F[x]$ constant, except that in this case, it is the demand curve which shifts while the offer curve remains unchanged. If I remains unchanged, then the initially rationed customers will experience more rationing because their increased demand is not satisfied at all; the risk free customers, in contrast, will receive larger loans and remain unrationed; and the initially unrationed risky customers will also obtain larger loans, but possibly not enough to match the increase in their demand. Furthermore, as funds flow into the loan portfolio (because some of the increased demand is satisfied), the opportunity cost of loans will tend to increase giving rise to additional rationing of the type under (1).

Normally, an increase in loan demand will tend to occur in periods of buoyant economic activity and hence will be associated with a rise in r_M (partly reflecting the higher demand in all markets and partly causing, in turn, a higher demand for bank funds), and also with an increase in the anticipated profitability of the investment projects, the latter producing a decrease in risk as measured by $F[x]$. The outcome is then a combination of the *ceteris paribus* results under (1), (2), and (3). The shift in $F[x]$ tends to raise the loan

offered at the unchanged rate, \hat{R}_0 ; at the same time, the quantity demanded at that rate rises, and if I were unchanged, the effect on \hat{R} , as well as on rationing, would depend on the relative amount of the two shifts. Even on this basis, one would anticipate an increase in rationing because the demand curve is likely to shift further than the offer curve, reflecting an increase in the optimism of firms relative to that of the banks. But more fundamentally, I must rise, both because with I constant, more funds would flow into the loan portfolio causing I to rise relative to r_M , and because r_M itself will be rising. Hence the final outcome will tend to be the same as in the previous three cases. The extent of increased rationing will depend on the relative shift in $F[x]$, in the quantity demanded at the unchanged rate, and in r_M , and on whether, on balance, these shifts will cause funds to flow in or out of the loan portfolio. It is apparent, however, that the rise in I and in rationing will tend to be greater the smaller the elasticity of supply of funds to the banking system. The effect might be particularly severe if the ability of banks to attract funds were actually reduced, as happened in some recent episodes in which the Certificates of Deposit rates in secondary markets pierced the Regulation Q ceiling.

*We may thus conclude quite generally that as \hat{R} rises relative to \hat{R}_0 , rationing will tend to increase; and the incidence of the increased rationing will tend to fall most heavily on customers who would be rationed in equilibrium, and will tend to affect the least, if at all, the riskless, or nearly riskless, prime customers.*²⁰ One implication of this

²⁰ As we have argued in the text, the amount of dynamic rationing tends to be positively related to the spread between the equilibrium rate and the quoted rate. Actually, though this relationship must hold in the large, in some special circumstances it may fail to hold in the small. The special cases arise only when the slope of the demand curve exceeds the slope of the offer

result is worth stressing since it provides the key to our operational measurement of credit rationing set forth in the next section. Suppose that we were to classify all customers into two broad classes, the prime customers and all others. We should then expect that as the gap between \hat{R} and R widens and dynamic rationing becomes more severe, loans to the riskless customers will tend to represent a growing share of the total loan portfolio.

This result can be given the following useful interpretation, which also serves to bring to light the common nature of equilibrium and dynamic rationing. In the presence of risk as to the outcome of the loan, reducing the size of the loan will increase the expected rate of return, by reducing the expected loss from insolvency of the firm. It is therefore quite understandable that a bank faced with a higher opportunity cost (whether from a rise in the market rate or in lending opportunities) and unable to raise the return by raising rates, will find it profitable to raise its return at least by upgrading the quality of its portfolio through a reduction in risk; the upgrading may take the form of shifting funds toward less risky customers, and/or of reducing loans made to risky customers, depending on the nature of the shift in underlying conditions.

III. A Measure of Credit Rationing

In this section we shall develop an operational measure of credit rationing which is based on the theory as developed in Sections I and II and which is used in the test of the theory presented in Section IV below. In principle, the volume of credit rationing should be measured by the difference between the loan demand and bank supply for rationed customers as

defined by E in equation (8). The degree or relative incidence of credit rationing could then be measured by the ratio of the volume of rationing to the potential demand of rationed customers, or:

$$(10) \quad \hat{H} = \frac{E}{E + L_2} = \frac{D_2 - L_2}{D_2}$$

where D_2 denotes the demand of rationed customers and L_2 the volume of loans actually granted to them.

Unfortunately, the direct measurement of E and L_2 , the components of \hat{H} , requires information on the *ex ante* customer demand and bank supply which is unlikely to be available, even in the future. The analysis of Section II.3, however, points to a possible, operational, proxy measure of the degree of dynamic credit rationing. As shown there, our model suggests that there should be a positive association between variations in dynamic credit rationing and variations in the proportion of the total loan portfolio accounted for by the risk free prime customers. Let us then denote by $L_1 \approx D_1$, the volume of loans granted to these customers, and by L_2 and D_2 , respectively, the loan granted and the loan demanded by all other customers. Our proposed operational proxy for the non-observable \hat{H} is either of the following two:

$$(11.a) \quad H_1 = \frac{L_1}{L_1 + L_2}$$

or

$$(11.b) \quad H_2 = \frac{1}{H_1} = \frac{L_1 + L_2}{L_1}$$

The first measure, H_1 , is simply the percentage of total loans which are granted to the risk free customers, and hence is positively related to the degree of rationing. The alternative proxy is its reciprocal and hence, is negatively related to the degree of rationing.

To see the relation between either proxy

curve. Since this can occur only at relatively high interest rates, indicating default risks well above levels bankers would consider acceptable, the case would seem to have little empirical relevance.

and the ideal measure \hat{H} , let

$$B = \frac{D_2}{D_1}$$

Then from (11) we obtain:

$$(12.a) \quad H_1 = \frac{1}{[1 + B(1 - \hat{H})]}$$

$$(12.b) \quad H_2 = 1 + B(1 - \hat{H})$$

since $L_1 = D_1$.

Differentiating (12) with respect to \hat{H} , yields:

$$(13.a) \quad \frac{\partial H_1}{\partial \hat{H}} = \frac{B}{[1 + B(1 - \hat{H})]^2} > 0$$

$$(13.b) \quad \frac{\partial H_2}{\partial \hat{H}} = -B < 0$$

The equations (12) show that for a given value of B , our proxies are monotonic functions of the ideal measure \hat{H} , and (13) confirms that the relation is in the direction expected. The functions relating either proxy to \hat{H} involve as a parameter the relative demand factor, B , essentially because we have replaced the nonobservable D_2 with the observable D_1 . This of course implies that any change in B will give rise to a variation in H_1 or H_2 which does not correspond to variations in \hat{H} . Hence, if there were sizable variation in B over time, our proxy measures of \hat{H} could be subject to appreciable errors in measurement. Note, however, that even in this event, as long as H_1 or H_2 are used as dependent variables in a statistical test as we shall do below, these errors will not tend to generate bias in the estimated coefficients unless B happens to correlate with the behavioral determinants of \hat{H} .²¹ Fortunately, the conclusions of Section II

²¹ Because (12a) is nonlinear in \hat{H} , this statement will be true only for a linear approximation of H_1 . If either proxy were being used as an independent variable, the problem of bias would be more serious. See, for example, Malinvaud [7, Ch. 10].

concerning the comparative static properties of the model and the classification of customers suggest that, at least in principle, these variables would not be correlated.

Some data problems are encountered even in attempting to measure the proxy developed in equations (11). Our source of data is the Federal Reserve's "Quarterly Interest Rate Survey" which records the volume of new loans granted by rate and size class during the first two weeks of the last month of the quarter. With this data, the percentage of loans granted to risk free firms (proxy H_1) can be at least approximated by the percentage of loans granted at the prime rate (and perhaps secondarily by the percentage of loans which were large in size). Unfortunately, in several instances the prime rate changed during the period of the survey, with the effect that one cannot distinguish between loans made at the new prime rate and loans made at this same rate while the old prime rate was still in effect. To circumvent this problem and secure a more reliable measure, we smoothed these quarters as well as possible. Following John Hand [2] who first used the data for this purpose, we combined the smoothed series with three other measures based on the distribution of loans by size through principal components analysis. More specifically, we calculated the first principal component of the following four series:

- a) The proportion of total loans granted at the prime rate.
- b) The proportion of total loans over \$200,000 in size.
- c) The proportion of loans over \$200,000 in size granted at the prime rate.
- d) The proportion of total loans \$1,000-\$10,000 in size.

The first three series should enter positively into the principal component and

the fourth negatively, of course. The principal component has a mean of zero and standard deviation of unity and the four factor loadings were, respectively:

- (a) .988
- (b) .968
- (c) .939
- (d) -.959

This indicates that each series enters prominently and about equally into the principal component.

The principal component thus derived corresponds to the H_1 measure since the series (a) is analagous to H_1 . Exploratory calculations indicated that the results would not be significantly affected by relying on the alternative H_2 measure derived from the reciprocal of series (a) to (d).²² The solid line in Figure 3 is a plot of the seasonally adjusted principal component H to be used in the tests of the following section. The pattern of rationing indicated by the proxy seems quite credible throughout the period. Note in particular how the most recent pattern is consistent with what we might have expected, rising very high in the second and third quarters of 1966 and then falling off somewhat in the fourth quarter. Unfortunately this series cannot be computed beyond 1966 at the present time, since the information produced by the loan survey beginning in the first quarter of 1967 is not strictly comparable with the earlier information.²³

²² We estimated a number of equations using the rationing model developed in Section IV for the specification of the independent variables, and the individual series (a) to (d), the principal component, and the reciprocal of each as separate dependent variables. The principal component yielded the best fit as expected, but comparable results were obtained with the other measures. We are grateful to John Hand for making his data on these series readily available.

²³ The difficulty arises from the fact that with the first quarter of 1967, the period of the survey was changed from the first two weeks of the last month of each quarter to the first two weeks of the second month of each quarter. Because of the strong seasonal com-

IV. *A Test of the Model Using the Credit Rationing Proxy*

In this section we propose to use the credit rationing proxy to test the implications of our model as to the forces controlling variations in time in the extent of credit rationing. This test, relying on quarterly time series data for the years 1952 to 1965, will thus serve to shed light on three aspects of our problem: our theory of credit rationing, the effectiveness of the proxy variable based on the theory, and the existence of rationing as an empirically significant phenomenon.²⁴

The principal implication of our theoretical model is that the main source of systematic variations in credit rationing is to be found in changes in dynamic rationing; and that these changes in turn are positively associated with the spread between the long-run optimal or equilibrium loan rate denoted hereafter by r_L^* , and the rate actually prevailing, r_L .²⁵ If we further assume that, to a first approximation, this association can be formulated as a linear relationship within the empirically relevant range, we are led to

$$(14) \quad H = a_0 + a_1(r_L^* - r_L) + \epsilon$$

Recall that, according to our model, when the commercial loan rate is at its long-run desired level, so that the second term of (14) is zero, we have only equilibrium

ponent in the four series used to compute H , sufficient observations for the new survey period must be obtained before the new seasonal component can be reliably determined.

²⁴ It is important to note that success in this test will confirm the value of the proxy as a variable to be used in testing for the impact of credit rationing on the real sectors of the economy. Indeed, the first uses of the proxy for this purpose have been made in [2] and [5].

²⁵ r_L^* denotes an empirical approximation to the theoretical construct \bar{R} developed above. In particular, r_L^* must stand proxy for the spectrum of optimal rates corresponding to the respective risk classes. Similarly, r_L stands for the spectrum of actual rates and is measured as the average rate on commercial loans compiled from the "Quarterly Interest Rate Survey."

$$(15) \quad r_L^* = c_0 + c_1 \{ r_T + b_1 [(DEP/TB) - 1] - b_2 D62 + b_2' C + b_3 L / (A - L) + b_4 \Delta [L / (A - L)] \}$$

rationing. On the microeconomic level, equilibrium rationing depends on the specific parameter values for the demand functions, density functions, and opportunity cost. But our investigation in Section II.2 indicated that no systematic relationship existed between the degree of rationing and changes in these parameters. Accordingly in (14), the constant term a_0 may be thought of as a measure of equilibrium rationing up to a stochastic error term which is included in the overall error term ϵ . When the quoted rate is above the equilibrium rate, the amount of dynamic rationing can be considered negative in the sense that rationing will be reduced below its equilibrium level, even though the actual amount of rationing can never be less than zero by definition. The dependent variable as measured by our principal component proxy will, in fact, take on negative values because the zero point for H is chosen arbitrarily. The arbitrary choice of origin is, of course, reflected in the constant a_0 , and consequently one cannot distinguish between the level of equilibrium rationing and the scaling effect in the constant.²⁶

We now turn to the important task of specifying the equilibrium commercial loan rate, r_L^* . Our theory indicated that the desired commercial loan rate would be at

²⁶ Equation (14) can be rewritten such that the spread between the desired commercial loan rate and the rate actually quoted is a linear function of the degree of credit rationing. In addition, it has already been suggested that the timing of adjustments in the commercial loan rate depends on changes in the Federal Reserve discount rate. These two factors can be combined into a testable partial adjustment model of the determinants of the commercial loan rate in which the size of the desired adjustment depends on the degree of rationing while the speed of adjustment depends on changes in the discount rate. See fn. 18.

that level at which the marginal proceeds from a commercial loan, after adjustment for risk, would just equal the opportunity cost. We also know that this relationship will be valid for all other assets in the bank's portfolio. This means that the optimal commercial loan rate will tend to equal the market yield on any other asset held in the bank's portfolio after adjustment for risk, maturity, liquidity, and, possibly, any expectations concerning future levels of that rate. We are free, then, to choose as the standard of comparison, any security which is widely held by banks, the obvious criterion being practical expedience. Our choice, on this basis, is the bank's holdings of Treasury bills.

On the basis of the above considerations we are led to equation (15) as the specification for the desired commercial loan rate, where the notation will be defined as we proceed.

Consider first the bank's return on Treasury bills. It may be regarded as consisting essentially of two components. The first component, the Treasury bill rate, r_T , is straightforward. The second component is the liquidity value of Treasury bills, which is more involved and, in fact, accounts for the second, third, and fourth terms in the brackets. Our basic premise is that the liquidity yield of Treasury bills should decrease as the bank's holdings of these bills (TB) rises relative to its deposit liabilities (DEP). Our specification for this liquidity term takes the form $b_1 [(DEP/TB) - 1]$ where $b_1 (>0)$ is an estimated parameter.²⁷

²⁷ The legal requirement that banks must maintain government securities in their portfolio as collateral for government deposits raises one conceptual problem. To the extent that this requirement necessitates holding

The liquidity value of Treasury bills has been reduced since about 1962, however, by the development of a broad and active market in Certificates of Deposits (*CD*'s), which affords the banks an important means for increasing their liquidity at short notice. The effect of the existence of this market may be measured by a simple shift parameter or dummy variable and this is included as the term $-b_2(D62)$ where *D62* is a dummy variable which is unity starting in 1962-I and zero before then.²⁸ The value of the *CD* market is of course severely limited when the Regulation Q ceiling on *CD* interest rates is binding. This countervailing effect may be specified by an additional dummy variable which takes the value one in those quarters, if any, in which the secondary market rate for *CD*'s exceeds the ceiling rate. One would expect this dummy variable, denoted by *C*, to have a coefficient opposite in sign and of the same order of magnitude as the *CD* dummy variable *D62*.

The next to last term in equation (15) measures the share of the loan portfolio in total assets which, as suggested in Section II, should tend to affect the opportunity cost of funds for loans relative to market rates. In addition, this variable may be visualized as an adjustment of the required rate on loans for their relative illiquidity. We anticipate that this illiquidity should increase at an increasing rate as the ratio

Treasury bills of some required amount, the correct variable for our analysis would be the free bills; that is, the bills held above the required amount. It has been suggested that, at least in 1966, such a restriction was a restraining influence on bill holdings. Legally, however, a wide variety of Federal and Local Government securities are acceptable as collateral and it is unlikely that more than a few banks, probably centered in New York City, held Treasury bills only to satisfy collateral requirements, even in 1966.

²⁸ We have also experimented with incorporating the *CD* term in the form of a multiplicative factor operating on the Treasury bill ratio itself. Since the two estimates are nearly identical, only the results for the linear form will be shown.

of loans to assets (or liabilities) grows and thus measure this effect as $b_3(L/(A-L))$ where $b_3 (>0)$ is an estimated coefficient, *A* is total loans and investments, and *L* is the commercial loan portfolio of the banks. Finally, we should also consider changes in the liquidity ratio, since a dynamic short-run increase in loans which is beyond the control of the bank would have additional (although only transitional) liquidity cost, and this effect is accounted for by the last term in (15).

We have now almost completed the task of specifying the desired commercial loan rate. To obtain more generality we have formulated the desired loan rate as a linear function of the terms just summarized. The need for the linear function arises because we have not yet formally accounted for differences in risk and maturity between commercial loans and Treasury bills. The basic equation to be estimated can now be derived by substituting equation (15) into (14), which yields:

$$\begin{aligned}
 H = & (a_0 + a_1c_0 - d_1b_1) - a_1(r_L) \\
 & + d_1(r_T) + d_1b_1(DEP/TB) \\
 (16) \quad & - d_1b_2(D62) + d_1b_2' C \\
 & + d_1b_3[L/(A - L)] \\
 & + d_1b_4\Delta[L/(A - L)]
 \end{aligned}$$

where $d_1 = a_1c_1$.

The coefficients of (16) were estimated using ordinary least squares from observations for the period 1952-II to 1965-IV.²⁹ (The year 1966 was omitted to enable us to carry out extrapolation tests reported below.) The results are as follows:

²⁹ The source of data for the independent variables is the *Federal Reserve Bulletin* with the exception of commercial loans. Commercial loans are the sum of industrial and commercial loans (from an unpublished Federal Reserve series) and nonresidential mortgage loans of the commercial banks (from the *Federal Reserve Bulletin*). The mortgage loans are included since they are made to the same customers as the shorter maturity commercial loans. All dollar magnitudes are seasonally adjusted and interest rates are measured as a percent.

$$\begin{aligned}
 (17) \quad H = & -3.48 - 1.16r_L + .296r_T \\
 & (-7.0) \quad (-5.3) \quad (2.2) \\
 & + .007DEP/TB - 1.27D62 \\
 & (1.2) \quad (-7.8) \\
 & + .614C + 26.7[L/(A - L)] \\
 & (1.6) \quad (8.0) \\
 & + 9.0\Delta[L/(A - L)] \\
 & (.92)
 \end{aligned}$$

$$S_e = .350 \quad R^2 = .84 \quad D.W. = 2.02$$

(T statistics shown in parentheses)

These estimates confirm the implications of our model in that all coefficients have the correct sign and the goodness of fit is respectable, taking into account the noise in the dependent variable which was discussed above. Note in particular the very significant negative coefficient of the commercial loan rate. This supports one of the most distinctive implications of our model, to wit, that a rise in the commercial loan rate given the optimal level of this rate as measured by the remaining variables in the equation, tends to reduce rationing as it reduces the demand and increases the supply. Similarly, the opportunity cost as measured by the Treasury bill rate, and the loan illiquidity variable, measured by the share of loans in the bank's portfolio, appear as the most significant factors tending to increase rationing, given the commercial loan rate. Although the Treasury bill liquidity term and the change in the commercial loan illiquidity term are not significant, this is probably due to multicollinearity since in the short run, with the level of total assets essentially fixed, banks may have to sell Treasury bills to meet unexpected loan demand.

The remaining variables are the dummies intended to measure the effect of

CD's. The coefficient of the *CD* dummy, *D62*, is both very significant and large (since the dependent variable has by construction unit variance). It suggests that the newly acquired ability of banks to attract and shed funds through *CD*'s contributed appreciably to a reduction of rationing. The contribution of the ceiling rate dummy *C*, on the other hand, is harder to assess because the ceiling was binding, and hence the dummy was one, only in the last quarter of the sample, 1965-IV.³⁰ Its coefficient has the expected positive sign but its magnitude, which we had expected to be roughly equal to that of the coefficient of *D62*, is instead only half as large. Yet this result makes good sense when we recall that the ceiling rate was effective only to December 6, which is just about in the middle of the two week period during which the loan survey is taken. Indeed, it suggests that if the ceiling had been effective throughout the period, then its effect would have come close to offsetting totally the contribution of the *D62* dummy as expected.

The effectiveness of our model in explaining the behavior of the rationing proxy can also be judged from the plot of the values of *H* computed from equation (17) which is shown by the dotted line in Figure 3. The equation appears to track the broad movements of the actual series as well as the major turning points with leads or lags not exceeding one quarter. The largest errors correspond to a number

³⁰ The multiplicity of *CD* maturities and the thinness of the secondary market make it difficult to obtain a reliable indicator of those periods in which the ceiling is binding. In addition, an aggregation problem arises because the ceiling may be binding only for some banks or for some regions. The available information suggests, though, that until the last quarter of 1965 the ceiling rate was promptly raised whenever it threatened to become a significant hindrance to the issuing banks. In October and November of 1965, however, the secondary rate rose unequivocally above the ceiling and remained there until December 6th when the ceiling was again raised. See Willis [10] for further discussion of the secondary market for *CD*'s.

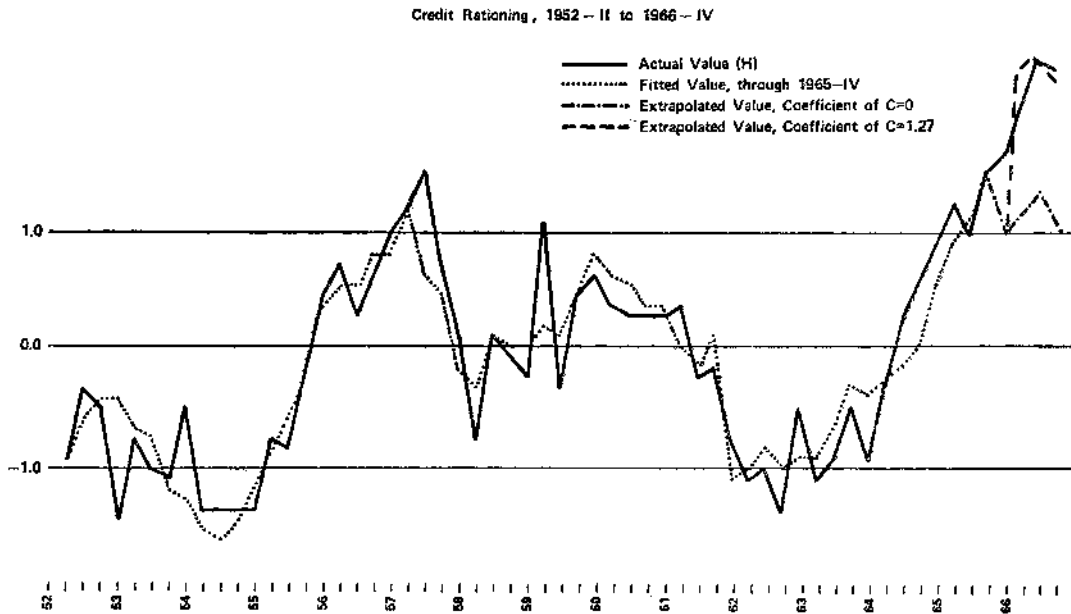


FIGURE 3

of one quarter spikes in H which may well reflect mostly noise in that series.

In Figure 3 we also present some extrapolations of our equation to the year 1966, which marks the end of the period for which usable data on the rationing proxy are presently available.³¹ Unfortunately extrapolations of (17) to 1966 run into rather formidable difficulties because throughout the last three quarters the ceiling rate on CD 's fell short of the secondary market rate. Nonetheless we feel it worthwhile to exhibit these extrapolations because of the tentative light they shed on the working of the ceiling rate. If one extrapolates (17) as though the ceiling rate dummy C were zero, one obtains the computed values represented by the dotted dashed line and as expected, this extrapolation very much underestimates the extent of rationing in the last three quarters. In order to allow for the ceiling effect, an alternative extrapolation of (17) was carried out for the last three quarters,

represented in Figure 3 by the dashed line, in which the dummy variable C was assigned the value of one. In addition, since the ceiling rate was effective throughout these quarters, it was also assumed that the coefficient of C was equal numerically to that of $D62$. Stated differently, the alternative extrapolation assumes that a binding ceiling has the effect of undoing the loosening effect of CD 's measured by the dummy $D62$. It is seen that this alternative fits the observations remarkably well.

These results, if taken at face value, have rather interesting implications for the *modus operandi* of ceiling rates as a tool of monetary policy. In particular they would support the view that, by allowing the ceiling rate to become a real hindrance to the ability of banks to attract new CD funds, the Federal Reserve could reduce significantly the availability of funds to the commercial bank customers of the banking system. This reduction would presumably occur to the benefit of customers of other intermediaries and/or of

³¹ Cf. fn. 23.

those firms able to raise funds directly in the market. It should be recognized, however, that our evidence in support of this inference is at the moment rather limited and hence, quite tentative, until it can be confirmed by further experience under similar circumstances. Of course, by the time the system is again exposed to similar circumstances, it may have learned ways of evading or by-passing, at least partially, the constraint imposed by the ceiling.

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