The Risk-Based Capital Test for Fannie Mae and Freddie Mac

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ABSTRACT

In recent years, Fannie Mae and Freddie Mac have enlarged their retained mortgage portfolios, increasing their exposure to interest rate and prepayment risk. In principle, this risk is mitigated by a risk-based capital (RBC) test that they are required to pass. We investigate (a) how common the interest rate shocks embedded in the test are, and (b) given stylized company operations, how likely default might be. This leads to suggestions for improvements in the RBC-test.

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I. Introduction

Congress chooses to subsidize private home-ownership. Principally, this subsidy occurs via the asymmetric federal tax treatment of home-owners and landlords,[1] while smaller, more targeted programs are provided by the FHA and VA.[2] A third vehicle used to transfer housing subsidies are the Federal Housing Enterprises,[3] mainly the for-profit Federal National Mortgage Association (Fannie Mae) and Federal Home Loan Mortgage Corporation (Freddie Mac), which we shall refer to collectively as F&F (see Frame and Wall (2002)).

F&F serve as a conduit from the mortgage market to the capital markets by pooling whole mortgages as mortgage backed securities (MBS) and selling them to capital market investors. F&F also serve as intermediaries by purchasing mortgage related securities (including repurchase of their own MBS) and holding them in the firms' portfolios, funding the positions by issuing F&F corporate debt. The MBS and corporate debt issued by F&F trade at small spreads above Treasury yields, because investors believe that there is at least an implicit commitment to protect these securities against F&F default. It is as if investors have a free option to put their F&F debt to the Treasury if it appears that F&F will not be able to meet their obligations. This means that F&F can take on substantial amounts of interest rate and credit risk, but not face the rising cost of debt that would otherwise constrain private market firms. The implicit guarantee thus provides an incentive for risk taking, and Jaffee (2003) has presented evidence that F&F are taking on extensive interest rate risk.

In principle, the government regulates excessive risk-taking by F&F through the Federal Housing Enterprises Financial Safety and Soundness Act (1992).[4] This Act prescribes that core capital must exceed 2.5% of assets plus 0.45% of off-balance-sheet obligations (mainly outstanding MBS), and that F&F pass a risk-based capital (RBC) test measured against the core capital. We outline the test in Section II. It is argued that if F&F pass this test, the possibility of bankruptcy is "remote."[5] Further, Stiglitz, Orszag, and Orszag (2002) find that "... the probability of a shock as severe as embodied in the risk-based capital standard is substantially less than one in 500,000 - and may be smaller than one in three million.[6]" Put together, these findings suggest that the free

[1]The Office of Management and Budget (OMB 2001) estimates this subsidy to have been $114 billion in 2000.

[2]See, for example, Vigdor (2003).

[3]Or *G*overnment *S*ponsored housing *E*nterprises, housing GSE's.

[4]The Act is an outgrowth of the late 70's and early 80's, when Fannie Mae was technically insolvent.

[5]Attributed to former FRB Chairman Paul Volcker in Stiglitz, Orszag, and Orszag (2002).

[6]Meaning that in 500,000 trials no path met or exceeded the severity of the RBC-test shocks. More precisely, they require that CMT_{10} fall (rise) by at least the amount stipulated in the test by $t = 12$ months, and then remain above (below) this value for 9 years. Additionally, the term structure in the last 9 years may not vary by more than $\pm 10\%$ from the values prescribed by the RBC-test. See Section II.

put F&F receive is so far out-the-money that it does not represent a significant cost to the Treasury or taxpayers.

In this note, we revisit these questions. To do so, in Section III we consider and estimate several interest rate models for the Treasury term structure: a simple bootstrap, a 3-factor no-arbitrage model, and statistical models (multivariate GARCH and ARCH-Poisson jump processes).[7] We find in Section IV that the prescribed RBC-shocks are actually relatively frequent, depending on the initial term structure. Further, a stylized Monte Carlo simulation of F&Fs' operations shows that interest rate shocks not counted in these results might be equally detrimental. This leads us to the conclusion that a RBC-test should perhaps consider a larger set of interest rate paths, with the requirement (a) that F&F fail only in the very "worst" paths in some ranking system, or (b) F&F demonstrate an acceptable maximum insolvency rate/VaR loss. We suggest a heuristic severity index for interest rate paths and discuss the VaR methodology in Section V. Section VI concludes and discusses future work.

II. The OFHEO Risk-based Capital Test

The Office of Federal Housing Enterprise Oversight (OFHEO) is an independent agency within HUD. It administers the risk-based capital (RBC) test and has issued a detailed document outlining its methodology (OFHEO 2003). The test was established to simulate the most damaging interest rate and credit risk conditions experienced in the US post-WWII. More specifically, F&F must survive[8] a 120 month period subject to (1) two prescribed (severely rising and falling) interest rate paths; (2) a severe prepayment/default loss experience; and (3) a stylized, arguably conservative set of rules on how the firm is to be run during this period.[9]

A. Interest Rate Paths

Figure 1 summarizes the up-rate and down-rate scenarios (URS and DRS, respectively). During the first year, the 10-year constant maturity Treasury rate (CMT_{10}) rises (falls) linearly to the larger (lesser) of its prior 9-month average plus (minus) 600bps or 1.6 (0.6) $\times$ its prior 36-month average. The rise (drop) is capped (floored) at 1.75 (0.5) $\times$ its prior 9-month average. In the URS,

[7] Work in progress.

[8] A 30% provision is made for operating risk.

[9] In this work we are less concerned about the potential losses arising from F&F's guarantee business, even though they may be severe. Nonetheless, we shall need an early mortgage termination model to obtain the impact on the retained portfolio.

the term structure (TS) becomes flat by $t = 12$ months and remains so for the remainder of the test. For the DRS, the regulation specifies an upward-sloping yield curve. The URS is “bad” for F&F because its long-term assets become less valuable, while rolling over short term debt becomes more expensive. In the DRS, F&F experience increased prepayment which can only be re-invested at lower long-term rates. Non-Treasury rates are obtained from CMT rates by adding a constant proportional spread, equal to the prior 24-month average; similarly, mortgage rates follow from a constant absolute spread. The Enterprise cost-of-funds are assumed to be a fixed 10bps above CMT rates of equal maturity.

B. Default/Prepayment Experience.

While locally realized growth rates are used to obtain the starting home values, during the test, all pools are subject to the housing price inflation (HPI) and loss experience for 1984-93 of the west south-central census district. This district includes Texas which experienced one of the worst housing recessions on record in the US during this period. Similarly, rent growth is based on the CPI-U for three MSAs in this district. There is additional inflation adjustment for the final 60 months of the URS shock.

C. Firm Operating Rules.

During the test, the firms conduct no “new business.” This means that they may not create or purchase new MBS, nor actively reset their derivatives portfolios.[10] Each month, cash flow is allocated depending on its sign:

1. If net cash flow is an inflow (profit), the firm must invest it in 6-month Treasuries. Also, for the first four quarters, it pays a common divided based on prior quarter payout, provided core capital remains intact. Similar provisions apply to preferred stock.
2. If net cash flow is an outflow (loss), the firm issues debt at a mix of 6-month and 5-year (no-call 1-year) enterprise cost-of-funds, such that its debt duration remains fixed at its $t = 0$ value.

Other regulations specify rules for stock repurchase, credit loss recognition, charge-offs, *etc.*

[10] There seems to be disagreement on whether this hurts or helps F&F survive the test. On the one hand, dynamic, true hedging helps the cause, but if the firm follows a trend-reversing strategy, new business may compound losses.

III. Interest Rate Models

If we intend to ascertain the riskiness of a financial firm such as F&F, it is clearly critical to have a good model of interest rates and their term structure.[11] As with any quantative analysis of catastrophic events, the difficulty is that the events of interest occur with low frequency so that their probability distribution is least well-known. We now outline salient features of the interest rate data that we believe are important to our work.

A. Data

We ultimately need constant maturity zero coupon Treasury rates at a high frequency. In practice, this means monthly data. We shall variously use

1. the Federal Reserve Board's (FRB 2003) constant maturity, average-of-business-day par rates, and bootstrap the zero coupon Treasury (ZCT) rates using the spline technique of Fisher, Nychka, and Zervos (1995). The problem with this approach is that (a) the series published by the FRB are of varying length[12] across maturities, with intermediate periods of missing data; and (b) it is unclear how the Treasury interpolates the data to obtain CMT rates. In principle, neither of these objections is particularly serious.
2. Alternatively, we use the McCulloch and Kwan (1993) ZCT rates, available monthly for 3 month, 6 month, 1 year, 2 year, 5 year, and 10 year maturities, for 1953.01–1998.08. We have not yet addressed the issue of 20 and 30 year rates. For intermediate maturities, we use the Fisher, Nychka, and Zervos (1995) spline technique and then standard bond pricing to find the par-CMT rates.

The empirical feature of the data that are important to us include:

1. *Moments:* Particularly for short maturities, the monthly change in rates is non-normal, with negative skewness and high excess kurtosis.[13] The largest Kolmogorov-Smirnov p-value for normality is only 0.6%, for the 10-year ZCT. Table I shows various moments $\Delta_m ZCT$.[14] The full data set contains 552 observations. Notice that the values are sensitive to removal

[11]While the spreads of other bonds to Treasuries also deserves considerable attention, in this preliminary version, we shall concentrate on Treasuries, and assume benchmark average spreads to obtain other rates.

[12]The full time span is 1954.01–present.

[13]Standard Edgeworth expansions with respect to a normal distribution fail, because the kurtosis is too high. The resulting approximation distributions are non-positive and multi-nodal.

[14]Note: The variables $\Delta_m \mathrm{ZCT}/\mathrm{ZCT}$ and $\Delta_m \mathrm{ZCT}/\sqrt{\mathrm{ZCT}}$ (*c.f.* CIR Model.) are also non-normal, but considerably less so.

of the largest observations. The second set of columns illustrate the values of the moments if we remove the largest and smallest ΔZCT observation. This clearly illustrates the difficulty in trying to fix the distribution for extreme tail events.

	$N = 552$			$N = 550$		
Maturity	S.D.	Skewness	Excess Kurt.	S.D.	Skewness	Excess Kurt.
3M	0.54	-1.8	15.9	0.49	-1.1	9.5
6M	0.53	-1.7	17.6	0.47	-0.8	7.5
1Y	0.52	-0.8	14.0	0.46	-0.2	5.2
2Y	0.47	-0.4	11.9	0.42	-0.1	3.3
5Y	0.37	0.1	4.9	0.35	-0.0	2.5
10Y	0.31	-0.2	2.9	0.29	-0.2	2.3

Table I

Moments for the monthly change in ZCT rates for 1953.01–1998.12. $N = 552$is the full data set. The columns under $N = 550$ show results if the largest and smallest changes are removed.

2. Augmented Dickey-Fuller tests show that all ZCT yields are integrated of order 1, at a 10% level or better.
3. Johansen tests show that there is at least one co-integrating relationship between the six rates. In fact, a principal components analysis yields eigenvalues of $\lambda = [3.432, 7.2 \times 10^{-2}, 5 \times 10^{-3}, 3 \times 10^{-4}, 1 \times 10^{-4}]'$ – a sharp drop-off after the first factor. Kenz, Littermann and Scheinkman (1994) identify the first three factors with the level, slope and hump-structure of the term structure, respectively, and this is one reason 3-factor models are popular.
4. The spreads between adjacent rates are, however, quite stationary: the largest ADF-statistic is -4.1, which is significantly less than the 10% critical value of -2.6.
5. The residuals obtained upon fitting a VAR(1) to the term structure are uncorrelated. In fact, the rational expectations hypothesis suggests that this should be so: information contained in today's yield curve should be a sufficient predictor of tomorrow's yield curve. Put in another way, yesterday's information is fully contained in today's term structure and plays no role for tomorrow once today's effect has been factored in. Figure 2 shows the sample partial auto-correlation for the residuals of the ZCT_{10}: there is little predictive power left. The horizontal lines represent two standard deviations for the PACFs, if the series were white noise.
6. The squared residuals, on the other hand, are significantly auto-correlated (see Figure 3). This suggests a GARCH-structure.

We now proceed to describe in turn the various interest rate models that we have considered, estimated and used.

B. "Bootstrap" Models

Define sample interest rate changes

$$dx_{t,\tau} \;=\; (r_{t,\tau} - r_{t-1,\tau})/r^{\gamma}_{t-1,\tau}\;, \qquad \tau = 3\mathrm{M}, 6\mathrm{M}, \ldots \tag{1}$$

and $t = 2, \ldots, T$. We have considered $\gamma = 0$ (motivated by the level dependence of the conditional volatility in a Vasicek model), $1/2$ (similarly, a CIR model), and 1. A series of random dates t_i are drawn, generating the fictitious interest rates through time

$$r_{t,\tau} \;=\; r_{t-1,\tau} \;+\; dx_{t_i,\tau}\, r^{\gamma}_{t-1,\tau} \tag{2}$$

where a single date is used for the change across the whole yield curve. This model is simple and computationally inexpensive. It gets the moments right, as well as the cross-correlation between different maturities. Unfortunately, there is no mean reversion, nor any conditional heteroscedasticity. Even more worrisome is the fact that the interest rate paths for different maturities are not co-integrated, even though the changes are correctly correlated. Nonetheless, for shortish time horizons, the model should serve as a useful tool.

C. Three-Factor Affine Model

The affine model setup is (Duffie and Kan 1996)

$$\begin{aligned} dx &= \Phi(\Theta \;-\; x)\, dt \;+\; \Sigma\, dw && (3)\\ r &= \delta_0 \;+\; \delta' x && (4)\\ \frac{d\Pi}{\Pi} &= -r\, dt \;-\; \sigma'_{\Pi}\, dw && (5)\\ dw_i &= [\alpha_i \;+\; \beta'_i x]^{1/2}\, dz_i && (6) \end{aligned}$$

where x is a set of factors (typically at most three) whose structure resembles a CIR-specification: a VAR(1) means model, with a square-root level dependence of the conditional volatility. Σ is the covariance matrix for the Wiener processes dw, r is the short (risk free) interest rate, and Π is the stochastic discount factor. Nominal interest rates then follow for the basic pricing equation $1 = E[\Pi R]$, which has a simple solution under the affine setup Eq. (3)–(6):

$$P \;=\; \exp(A(\tau) - B(\tau)' x) \tag{7}$$

where A and B are solutions to a system of easily solved ODE's (see Cochrane (2001)) Notice that this model admits no arbitrage, since Π is a positive process.

Duffee (2002) has fitted a three-factor affine model of the type outlined here to ZCTs. We use his parameters.[15] While the model does considerably better than a simple Vasicek or CIR model for the short rate, it has no skewness and underestimates the kurtosis. The no-arbitrage (NA) and tractability conditions imposed on it make it a poor choice for our purposes. Of course, it is free of the co-integration issues noted for the bootstrap model. We use this model for preliminary calculations in later sections, but turn next to statistical models that do admit arbitrage opportunities.

D. Purely Statistical Models

As mentioned above, we are not primarily interested in enforcing exact NA. For this reason, we next consider purely statistical models, with stochastic volatility (in an ARCH/GARCH approximation) and, possibly, added jump-processes.

D.1. Multivariate GARCH

We have noted that the ZCT are co-integrated, but that the spreads seem to be stationary. A VAR(1) model with level-dependent volatility (Andersen and Lund 1997) is a parsimonious and probably sufficient description for the conditional mean:

$$dr_t \;=\; K\,[\Theta \;-\; r_{t-1}]\,dt \;+\; r_{t-1}^{\gamma}\,u_t \tag{8}$$

where all variables are $N\times 1$ or $N\times N$. Further, PAC-analysis suggests a (multivariate) GARCH(p,q) specification for the conditional volatility of u_t. Any model along these lines implies an explosion in the number of parameters to be estimated. For this reasons, we simplify things and choose

1. $p = q = 1$;
2. We reduce the number of processes to be estimated to three, *viz.*, the 1-year ZCT, and the spreads of the the 3-month and 10-year ZCT's to the 1-year rate.[16] In this way we have one co-integrating factor;
3. We fix $\gamma = 1/2$ (Andersen and Lund 1997) for the 1-year rate, and assume that the spread volatility is driven by the 1-year level; and

[15]Specifically, the parameter set referred to as "preferred completely affine A2(3)" in his paper.
[16]We simply interpolate the other rates.

4. We further use the Bollerslev (1990) GARCH specification of constant correlations. The diagonal elements conditional covariance matrix, H_t, are modelled as GARCH(1,1)

$$h_{ii,t} \;=\; \omega_i \;+\; \alpha_1\, \epsilon^2_{i,t-1} \;+\; \beta_1\, h_{ii,t-1}\;, \tag{9}$$

but the correlations $\rho_{i \neq j}$ remain fixed in though time

$$h_{ij,t} \;=\; \rho_{ij}\, \sqrt{h_{ii,t}\, h_{jj,t}} \quad \forall i \neq j\;. \tag{10}$$

The ω_i are chosen to match the unconditional variance of the residuals.[17]

The total parameter set and computation time are thus considerably reduced - we have a total of 21 parameters. The dashed line in Figure 4 show the distribution of the monthly changes in ZCT$_{1Y}$, along with the historical data (bars). Compared to a pure Gaussian fit (dotted line), the process has longer tails due to the persistence and level dependence of the volatility. The estimation does not produce very satisfactory skew and kurtosis, but we have already noted that these are strongly influenced by "outliers." More extreme moments can be obtained by adding jumps to this model.

D.2. Jump Model

Fat tails in interest rate distributions have prompted many investigators to add jump processes to the underlying diffusion equation. Das (2002), for example, uses jumps to show day-of-the-week effects in the Federal Funds Rate, while other have looked for effects related to announcements from FOMC-meetings. In a univariate setting, the specification we estimate is (Das 2002)

$$dr \;=\; \phi\,(\theta \;-\; r)\,dt \;+\; \sigma\,dz \;+\; J(\mu,\gamma)d\pi \tag{11}$$

where dz is a standard Wiener process for the diffusion, and $J \sim N(\mu, \gamma^2)$ is the jump distribution. π is a Poisson process, for which we use the Bernoulli approximation considered by Ball and Torous (1983): if the arrival rate for two jumps within one period (in our case, a month) is negligible, then the Poisson process is well approximated by a simple binary probability $q = \lambda \Delta t$ of a jump (and $(1 - q)$ for no jump). This allows us to estimate the the process using a simple linear combination of the standard ML-function

$$\max_{\{\phi,\theta,\sigma,\mu,\gamma,q\}} \;\sum_{t=1}^{T}\; \ln H\Big[r_{t+1} \mid r_t\Big] \tag{12}$$

[17] We force the correlation matrix to be positive definite by estimating its Choleski decomposition.,

where

$$H(r_{t+1} \mid r_t) \;=\; q\, N\Big[\Delta r - \phi(\theta - r_t)\Delta t - \mu,\, \sigma^2\Delta t + \gamma^2\Big] \\ +\;\; (1-q)\, N\Big[\Delta r - \phi(\theta - r_t)\Delta t,\, \sigma^2\Delta t\Big] \tag{13}$$

and N is the standard normal distribution. Craine, Lochstoer, and Syrtveit (2000) have recently estimated a jump-diffusion model for exchange rates. Their conclusion from Monte Carlo experiments is that unless one includes stochastic volatility as well, jumps alone may give a bad estimate of the true process. For this reason we have also considered an ARCH specification for σ in Eq. (11) (Das 2002):

$$\sigma^2_{t+1} \;=\; a_0 \;+\; a_1 \Big\{ r_t - E[r_t \mid r_{t-1}] \Big\}^2 . \tag{14}$$

In an estimation of the $\mathrm{ZCT_{3M}}$, we find a jump probability of $\sim\, 0.2 - 0.3$, depending on the error specification, with a very slight negative mean jump with large variance. This lead to a minimal improvement in the skewness estimates, but a significant increase in kurtosis. For the 10-year rates, the jump probability is very small. We have not yet computed the S.E. for our estimates, nor yet done a LRT to see if any improvement in fit is statistically significant. Table II shows the moments of $\Delta\mathrm{ZCT}_{3M}$ for three models: pure gaussian diffusion (Gauss), Gaussian diffusion + Jump (Poisson-Gauss) process, and an ARCH-Gauss + Jump (ARCH-Jump) model. The solid line in Figure 4 shows the estimated ARCH-Jump distribution of $\Delta\mathrm{ZCT_{1Y}}$. The slightly fatter tails relative to the MV-GARCH estimation lead to the improved estimate of the kurtosis. The framework needs to be extended to a multivariate setting, work on which is in progress.

$\mathrm{ZCT_{3M}}$	S.D. [%]	Skewness	Kurtosis
Empirical	0.54	-1.8	15.9
Gaussian	0.57	-0.00	0.01
Poisson-Gauss	0.57	-0.25	8.5
ARCH-Jump	0.56	-0.03	11.5

Table II

Moments of the monthly change in the $\mathrm{ZCT_{3M}}$ rates from a pure Gaussian diffusion, a Jump-diffusion, and an ARCH-Jump estimation over the period 1953.01–1998.12 ($N = 552.$). The moments are obtained by simulating the respective processes with random historical initial term structures. See Text.

IV. The Frequency of RBC-Test Shocks

One goal for our term structure models is to estimate the expected frequency of the URS and DRS shocks embedded in the prescribed RBC-test. We present the frequency of these “OFHEO-like” interest rate paths in two ways: (1) How likely is such a path, starting with a given initial yield curve, over the next ten years (which is the length of the test) and (2) how likely is it that such a path will start at least once over, say, a 40 year interval?

Table III shows the results for 25,000 simulations of the multivariate GARCH model of Section D.1 over 120 months (an additional 36 months are appended to obtain the required 9 and 36 month prior averages). Case (a) starts the test at random initial yield curves, drawn from the historically realized set. From the simulation, the CMT_{10}-paths are constructed. The first line gives the fraction of CMT_{10}-paths that rise (or fall) by the amount required by the RBC-test, within 12 months. Only ~ 4 in a hundred do this. Next, we require that these path remains within 10% of this new level for the balance of the 120 months of the test. The 10% cushion is an arbitrary buffer introduced to prevent paths that briefly dip below (rise above) r_{max} (r_{min}) from being rejected. About 3 in 1,000 paths survive the combined requirements in the URS. Interestingly, the DRS has far fewer occurrences (0.02% of paths qualify as dropping sufficiently and staying there). We believe that this is possibly due to the “repulsion” at zero interest rates. Note that we have not required that the yield curve assume the specific shape of the RBC-test (becomes flat in the URS, and upward-sloping in the DRS). Obviously, this could significantly further reduce the frequency depending on precisely how it is implemented (Stiglitz, Orszag, and Orszag 2002). However, one could make the case that, for example, a downward-sloping yield curve is worse than a flat one in the URS, so that our final numbers should perhaps be reduced by a factor of two or so.

Case (b) in Table III considers a low slightly upward-sloping initial interest rate environment (as was the case 1953.01). The URS-frequency has increased dramatically. Fully 1.4% of paths qualify as URS-shocks. Once again, DRS-paths seem to be kicked out by the non-negativity restriction. Finally, case (c) of a very high and downward-sloping initial yield curve (1981.09), produces a greatly reduced frequency of the RBC-shocks.

We note that similar results are obtained using the 3-factor affine model described in Section C, and searching over a 40 year window. We find that at least one occurrence of an URS or DRS path happens in 1% of such windows (see Table IV). The paths were started out in the 5th decile of CMT_{10} levels.

We conclude that, especially under the right initial conditions, the frequency of the RBC-shock is not negligible, but on the order of 0.5%.

Initial TS	CMT_{10}	URS	DRS
(a) Uncond.	path rises/drops to $r_{\max}/r_{\min}$ within 12 months	1.62%	2.24%
	path remains above/below $0.9 \times r_{\max}/1.1 \times r_{\min}$ for balance of 120 mnts	0.30%	0.02%
(b) 1953.01	path rises/drops to $r_{\max}/r_{\min}$ within 12 mnts	5.41%	3.68%
	path remains above/below $0.9 \times r_{\max}/1.1 \times r_{\min}$ for balance of 120 mnts	1.38%	0.01%
(c) 1981.09	path rises/drops to $r_{\max}/r_{\min}$ within 12 mnts	0.20%	1.08%
	path remains above/below $0.9 \times r_{\max}/1.1 \times r_{\min}$ for balance of 120 mnts	0.01%	0.17%

Table III

RBC-Path Frequency for the CMT_{10} for 25,000 simulations of 120+37 month paths. The first 36 months are used to compute r_9 and r_{36}. The interest rate model is the CIR-GARCH specification of Section D.1. The various initial term structures are: (a) Unconditionally chosen from all historically realized values (1953.01-1998.12). (b) The values on 1953.01: (ZCT_{3M}, ZCT_{1Y}, ZCT_{10Y}) = (2.0%, 2.1%, 2.7%). (c) The values on 1981.09: (ZCT_{3M}, ZCT_{1Y}, ZCT_{10Y}) = (14.8%, 15.9%, 15.1%).

V. Alternative RBC Specifications

Clearly, the paths that the RBC-test prescribes are not the only ones that may lead to F&F insolvency. For example: Using the 3-factor affine model of Section C, Figure 5 shows the fraction of ZCT_{10} paths that exceed $Y_{\max}$ *and* drop below $Y_{\min}$ at least once in a 50 year time-frame. If such see-saw behavior is relatively rapid, the effect on F&F financial health could be severe. Equally troublesome is that prescribing one (unlikely) path of interest rates allows one to cheaply insure against it (window-dress to pass the test). In the next Section, we suggest a simple ordering scheme that ranks paths according to how severe they may be for a firm like F&F. Section B describes a more detailed experiment in which we simulate a stylistic F&F firm and determine the probability of insolvency and VaR losses.

CMT_{10}	URS	DRS
path rises/drops to $r_{\max}/r_{\min}$ within 12 mnts	4.7%	2.7%
path remains above/below $0.9 \times r_{\max}/1.1 \times r_{\min}$ for balance of 120 mnts	0.65%	0.50%

Table IV
RBC-Path Frequency for the CMT_{10} rate over a 40 year period, using the 3-factor affine model of Section C with 2,000 simulations. See Text.

A. Severity Index

It is not particularly difficult to construct (realistic) "non-RBC" paths that would bankrupt the GSEs as surely as the RBC-shocks. For example, consider a company that owns 100 units of an asset that pays 5% coupon for the next ten years. The firm undertakes no new business, does not raise new equity, pays no dividends, and there is no asset prepay or default. It finances the assets with 3% equity, and raises the rest by rolling over 1-year debt, less cash flow from the asset. Consider two hypothetical paths for the cost-of-funds. Path I is OFHEO-like, starting at 4%, rising by 75% in the first year, and remaining there for the next two. Path II would not be considered "as severe as the one embodied in the RBC-test" in Stiglitz, Orszag, and Orszag (2002) or Section IV. It falls back to its starting point by year 2, after rising to 11% in year 1. A simple income statement/balance calculation shows that under both scenarios, the firm is insolvent at some point in time.

Roughly speaking, the firm fails on Path II because the cumulative changes in the cost of funds from r_0 are as bad for the firm as the changes on Path I. This suggests a simple severity ranking for a general path:

$$\text{SI}_\gamma \;=\; \sum_{t=1}^{T} \mid r_t \,-\, r_0 \mid^\gamma \;, \tag{15}$$

where γ is a positive constant. We consider $\gamma = 1$, though if debt is rolled over, and/or convexity plays a role, a quadratic form may be more appropriate. Using this heuristic severity index, how often would one rank a historically realized 10-year window of CMT_{10} rates as more severe than the contemporaneous RBC-test shocks? The solid line in Figure 6 shows the realized SI_1 for the rolling next 10 years of CMT_{10} path, during the period 1965-1989. The dotted (dashed) line is the corresponding index for the URS-shock (DRS-shock), as would have been calculated at that date.

The historical series exceed the DRS-shock in severity several times in the period 1965-1977 and came close in the early 1980's. The URS-shock was less severe than the realized path for a period in the early 1980's. While the severity index SI_1 is *ad hoc*, it does captures some of what drives losses in the firm, and clearly there have been episodes in the past 40 years that, in hindsight, were as severe as the shocks embodied in the RBC-test.

How might one put the SI on a more firm footing? One simple example is to consider a firm with one unit of asset that continuously pays $r_A(t)$, and which continuously rolls over debt at cost $r(t)$. Then the book value of debt evolves according to

$$\dot{D}(t) - r(t)\, D(t) \;=\; -r_A(t)\;. \tag{16}$$

The solution to this ODE is standard. Insolvency happens when debt exceeds asset value, so a suitable severity index would be of the form

$$\mathrm{SI}_{\mathrm{DR}} \;=\; \sum_{t=0}^{T} f(1 - D(t)) \tag{17}$$

where $f > 0$, $f' < 0$, and f becomes rapidly larger as D exceeds 1. An interesting exercise would be to expand the solution to Eq. (16) in $r - r_0$ and find an appropriate f and γ for this mode of firm operation. A risk based capital test based on this or any other severity index would specify that the firm fail only, say, in the 0.1% highest severity paths. Clearly, while one could find severity indices for other simple financing schemes (*e.g.*, keeping debt-to-equity ratio fixed), the difficulty of this approach is choosing a realistic ranking system.

B. Insolvency Rates and VaR

It is quite difficult to capture all the negative (and positive) impact of any given interest rate path (and house price inflation) on the financial health of F&F in a single function. An alternative approach is to build all relevant features into a Monte Carlo simulation of the firm. The primary input, of course, is the interest rate model, which should be further correlated/co-integrated with a house price/rent process. Equally importantly, one can now prescribe detailed operations for the firm (such as in the current RBC-test, outlined in Section II). The weakness of this approach is that interest rate models are not particularly well estimated for extreme events.

Nonetheless, we present here a preliminary calculation that uses the bootstrap interest rate model (with $\gamma = 1$) on the FRB CMT data to simulate the firm for 10 years.[18] Our intent is to

[18] Work is in progress on implementing the VAR-GARCH model.

illustrate the methodology, not provide a completely realistic stress test for F&F. We make the following assumptions:

1. Assume that no debt instrument or mortgage pool has a maturity exceeding 10 years.[19] All instruments make annual coupon payments.
2. At $t = 0$, the firm's assets are chosen to roughly match F&F's asset duration (Jaffee and Vedrashko 2003). Similarly, the debt distribution is chosen to match a desired debt duration. We do not model derivatives directly in this setup, but assume that they are being used "efficiently" to create synthetic long-dated debt that is indistinguishable from actual long-term debt. Any transactions costs are subsumed in the spreads. Thus, the firm starts out with some initial duration gap.[20,21]
3. We specify an initial D/E ratio. Typically F&F are quite close to $E/(D + E) = 2.5\%$, which we choose here.
4. An initial term structure is specified. We consider two cases: a flat and an upward-sloping initial yield curve (see below).
5. In principle, we should specify the initial seasoning and burnout for the pools in the portfolio, and use, for example, the Mattey and Wallace (1998) termination model to determine the prepayment rate. In the calculations here, however, we have turned off prepayment. Also, currently we have not considered default, the cost of which F&F bear through their guarantee business.[22]
6. At each coupon date, the total cash flow corresponds to maturing instruments (and, in principle, prepay cash flows), and coupon payments. For the examples here, we have chosen for the firm to do the following with these cash flows:
    (a) All maturing (and, in principle, prepaying) assets are reinvested in new 1-, 3-, 5- and 10-year assets, and maturing debt is rolled over into new 1-, 3-, 5- and 10-year debt.[23]

[19]While this is obviously not true, the effective duration securities in F&F's portfolio is ~ 7 years (Jaffee and Vedrashko 2003). As a first pass, this may therefore not be too bad an approximation.

[20]In recent times, F&F's duration gap has been as large as one year, but typically they aim to keep it below six months.

[21]Note that the underlying distribution of maturities is not unique for a given duration gap. Also, if rates move far and rapidly, convexity effects kick in because F&F would not be able to adjust their hedges fast enough - their portfolios are only hedged "perfectly" for a ± 25bp move (see Jaffee (2003)). From this point of view our default and VaR estimates will be conservative. On the other hand, our results may overstate insolvency if in some simulation paths F&F are "dying a death of a thousand cuts," which their dynamic derivatives strategy might, in fact, be perfectly hedging.

[22]In fact, we have completely ignored losses from this side of the business, an oversight that deserve attention in a fuller project. Here, we are principally concerned with the interest rate and, eventually, prepayment risk faced by F&F.

[23]We assume that the debt is not callable. Assets are not prepayable.

(b) Profits (asset coupon less debt coupon) may be (a) paid out as dividends, (b) used to retire maturing debt, or (c) used to buy new assets. Corresponding losses are covered by issuing new debt. The firm does not issue new equity.

(c) The new assets and debt are chosen to keep the $t = 0$ duration gap fixed though time. First, try issuing new assets at the longest available maturity (10 years). For each state of the world (interest rate path), this gives the new duration of the asset portfolio, which in turn fixes the required duration of the new debt. Next, issue debt as a mixture of the two maturities whose duration brackets this required duration. If this is not possible (the required debt would exceed 10-year maturity), gradually lower the duration of new assets, and repeat the process until the initial duration gap is restored in all states of the world.

(d) The assets are assumed to yield a fixed 60 or 110bps more than the corresponding CMT, while the firm's cost of funds is similarly fixed at 10bp. The lower asset spread case can be considered to account (roughly) for credit losses.

The firm is now "propagated" forward in time, many times over with the same initial conditions and operating procedure, but with different interest rate environments. Through time, we therefore have the book and market values of F&F's assets, debt and equity. We can now compute two quantities of interest

1. *The insolvency rate* is simply the fraction of paths for which the firm's book (or market value of) equity becomes negative sometime over the course of the 10 year time span.
2. *The VaR*, or *V*alue *a*t *R*isk, is that loss which the firm will not exceed with a given probability, over a given period of time. We consider nominal 10 year VaRs here. So, say one requires a 99%-VaR, then the reported VaR is the 10-year loss L that the firm will only exceed with a probability of 1%. In other words, in 99% of parallel universes, the firm's losses would be less than L, over the next ten years.

Figure 7 shows the fraction of paths for which the market value of equity, E, becomes negative sometime during the ten year time horizon, versus the fixed duration gap. The squares correspond to our baseline case: (i) All profits are paid out as dividends; (ii) the initial term structure for the 1, 3, 5 and 10-year CMT par-yields is 1%, 2%, 3% and 5%, respectively; (iii) the agency cost of funds is 10bps over the corresponding CMT, while the assets spread is 110bps; and (iv) the initial asset mix is 10% per maturity.[24] Each point represent a 3,000 path simulation in which the initial debt mix is changed to vary the duration gap. Out to a gap of about 0.75 years the insolvency

[24]To get extreme duration gaps, we have also increased (decreased) slightly the weight of the long (short) term assets relative to this baseline asset mix.

rate is zero ($< 1/3,000$), and then rises roughly linearly to 70bps at a gap of 2 years.[25] There is no insolvency at (reasonable) negative duration gaps: we have not included prepayment of assets (nor, of course, callable debt). Even though this model is extremely simplified, a 10bp (1 year gap) insolvency rate seems reasonable. Of course, the assumption that all profits are paid out is extreme.

The triangles in Figure 7 similarly show the insolvency rate for case 1, identical to the baseline, except that the term structure starts out flat at 5%. Given that short maturities are more volatile than long, it is not surprising that the insolvency rate starts increasing earlier and at a much faster rate.

Figure 8 shows case 2 (triangles): here, the yield curve starts out flat at 5%, and profits are used to pay down debt. This is a conservative approach, relative to case 1, and we see that the rise in the insolvency is less dramatic, and starts out later than case 1. Nonetheless, the flat term structure makes this case more risky relative to the baseline case.

Figure 9 shows case 3 (triangles), in which is as the baseline case, but with a reduced asset spread of only 60bps (compared to 110bps for the baseline case). This reduction could be considered as a "poor man's" method of incorporating credit losses. In both scenarios the debt spread is 10bps. The insolvency rate rises faster than in the baseline because there is less margin for error, but starts at roughly the same duration gap (0.75 years).

Lastly, we turn to VaR values. Figure 10 shows the VaR curves for our model firm starting with an initial book value of 100, and keeping a fixed duration gap of 1 year. The solid line is the nominal cumulative 10-year loss that the case-2 firm (flat term structure, profits used to pay down debt and a 110bps asset spread) has exceed with frequency given on the x-axis. For example, in 1% of paths, the firm lost more than $\sim 2\%$ of its initial assets, in nominal terms. The horizontal dotted line represents the firm's initial equity of 2.5. Similarly, the dashed line shows the VaR for our case-1 firm (flat term structure, profits paid as dividends and a 110bps asset spread). The VaR at a given probability jumps as the payout ratio increases from zero to unity.

VI. Discussion and Future Work

We have argued that F&F are taking on increasing risk in search of profit growth, and that they can do this because of the taxpayer-provided put on their debt. The Government requires that F&F

[25]The insolvency rate scatters quite a bit, because of the small sample size, but presumably also because the mapping of duration gap to portfolio composition is not unique.

pass a RBC-test, which some argue verifies that F&F impose little risk on the government. Our stylized calculations suggest that this may not be true.

First, the interest rate shocks embodied in the RBC-test are not as uncommon as suggested in the literature. We have considered several interest rate models, and conclude that their frequency is about 0.5% per 40 years. Second, while these shocks are quite severe for highly levered firms such as F&F, we have also argued that they are not the only ones that deserve attention in determining F&F's financial soundness. A heuristic severity index for interest rate paths, based on a stylized notion of losses from interest rate movements, suggests that paths "as severe as" the currently legislated ones have occurred ~ 10 times in the past 40 years. Preliminary simulations for a stylized F&F-like firm suggest that the insolvency rate over ten years might be as large as 10-100bps at a duration gap of 1 year.

Of course, these results are subject to revision. In particular, we are working on (a) incorporating a fully fledged multivariate GARCH model (possibly with jumps) in the simulations; (b) investigating the issue of more realistic spreads for the mortgages and agency debt; and (c) including prepayment in model, and the effect of default (via the guarantee business); A further line of investigation is to consider alternative specifications for the severity ranking of interest rate paths, though this is difficult from first principles. It would seem that a VaR Monte Carlo methodology is quite well suited for a RBC-test. This is the approach that the Basel II Accord has adopted for private banks.

The difficult issue for the simulation based approach is to postulate and estimate a reasonable and realistic term structure model - the events we wish to characterize are by their nature infrequent. If, on the other hand, we specify an interest rate path ranking system, the frequency of severe paths is not an issue. Instead, the problem becomes one of fixing company operations realistically, it i.e., determining the severity of a given path. Either way, we believe that an expanded RBC-test, that considers more than one or two interest rate shocks, is called for to regulate Fannie Mae and Freddie Mac and protect the taxpayers' interest. We hope that this work, once completed, can contribute toward this goal.

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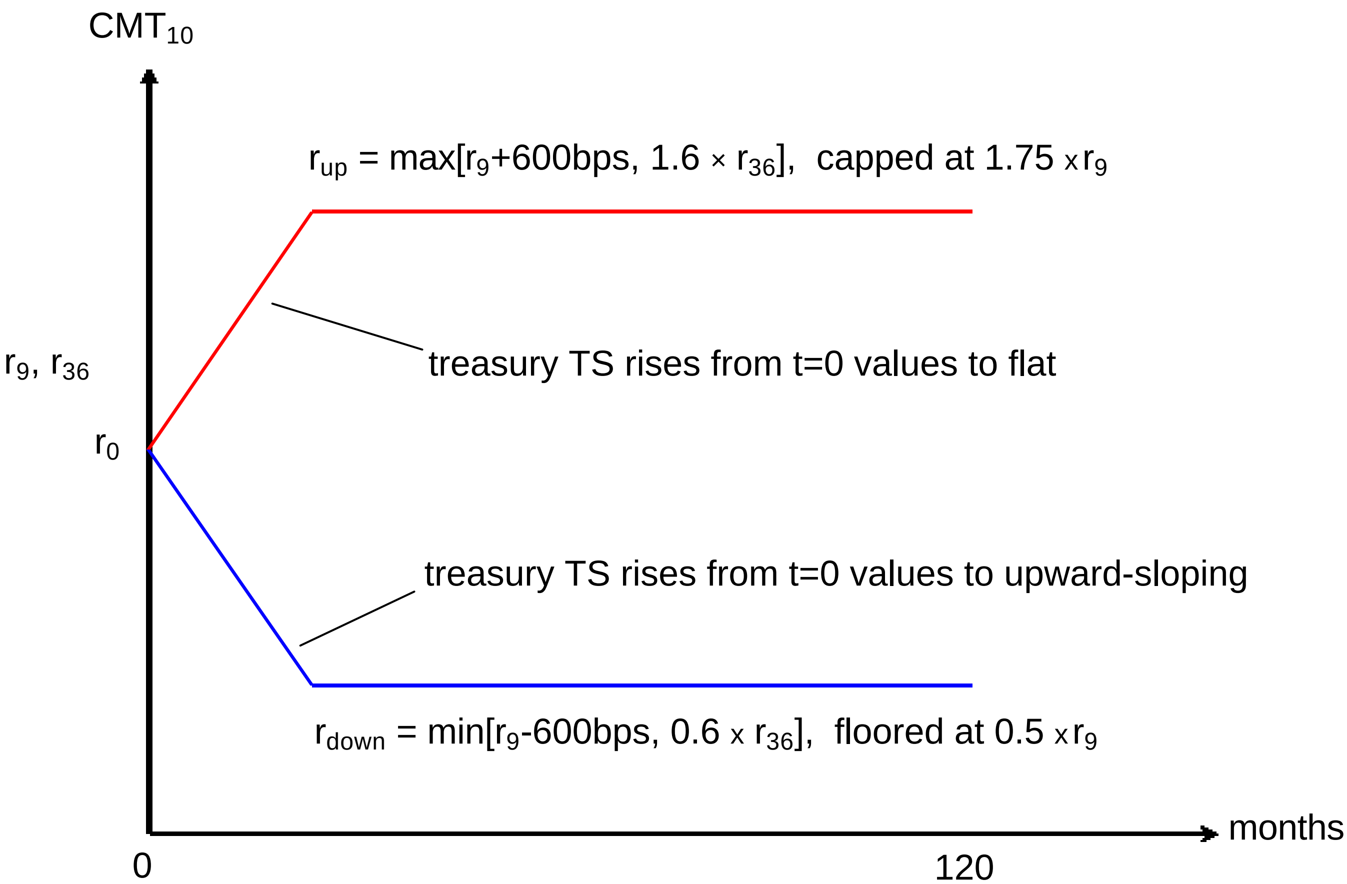

Figure 1. The two 120-month interest rate shocks for the constant maturity ten year treasury rate, CMT_{10}, prescribed for the RBC test under the Financial Soundness Act (1992). r_9 and r_{36} are the prior 9 and 36 month average rates of CMT_{10}, respectively.

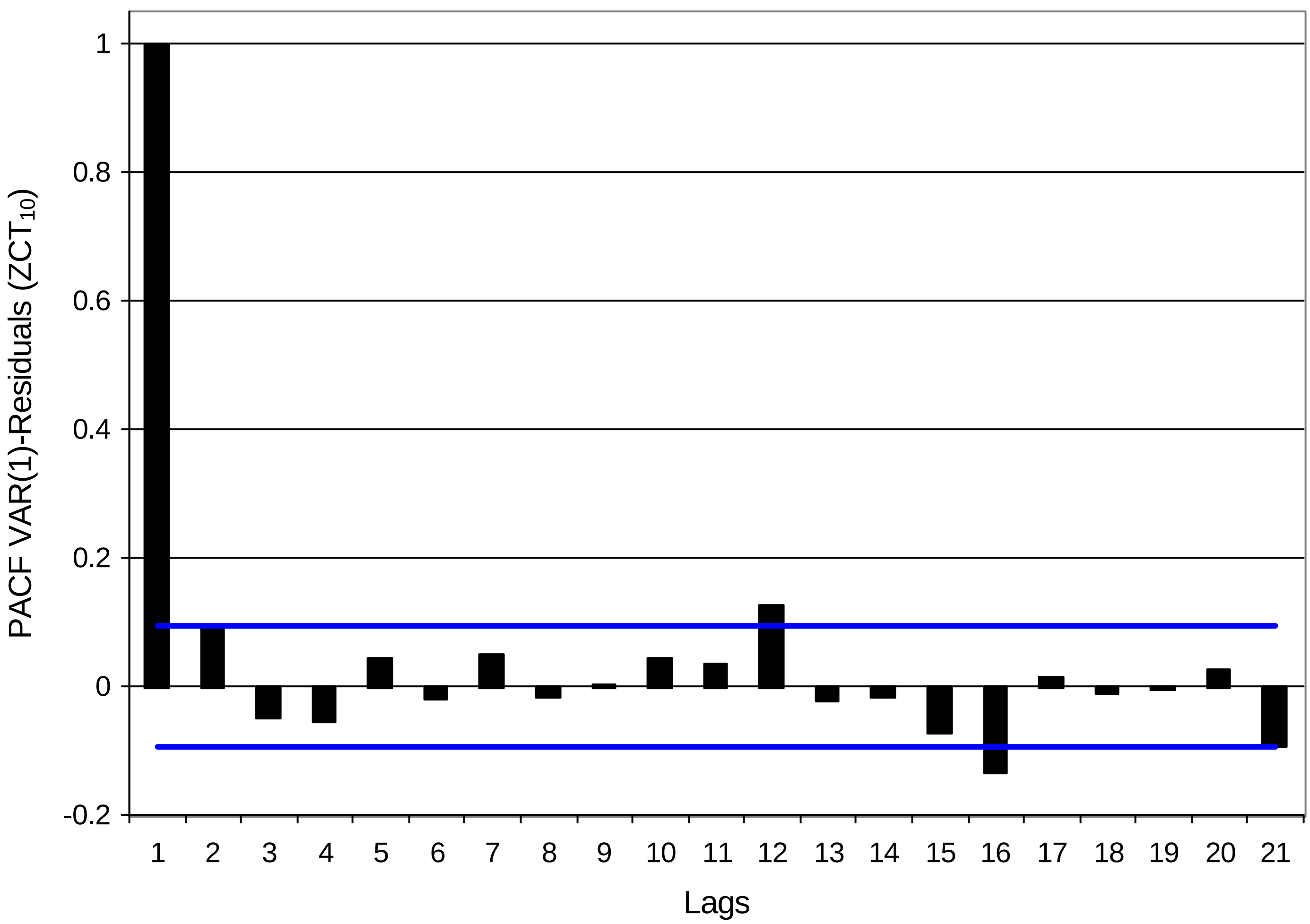

Figure 2. The partial auto-correlation function for the residuals of a VAR(1) estimate of the historical (1961.06-1998.12) ZCT_{10}. The horizontal lines show two standard deviations for a white noise series.

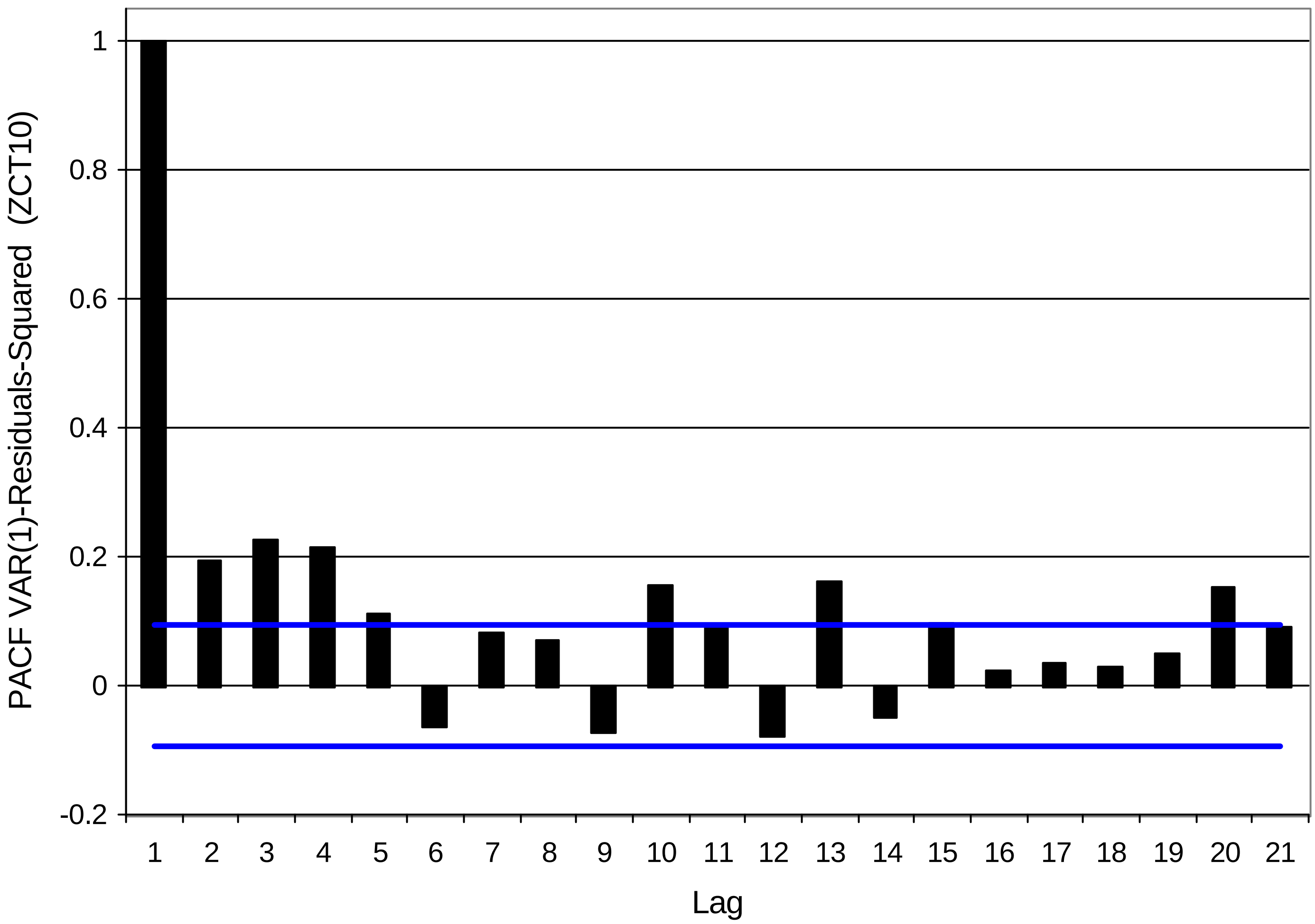

Figure 3. The partial auto-correlation function for the squared-residuals from a VAR(1) estimate of the historical (1961.06-1998.12) ZCT_{10}. The horizontal lines show two standard deviations for a white noise series.

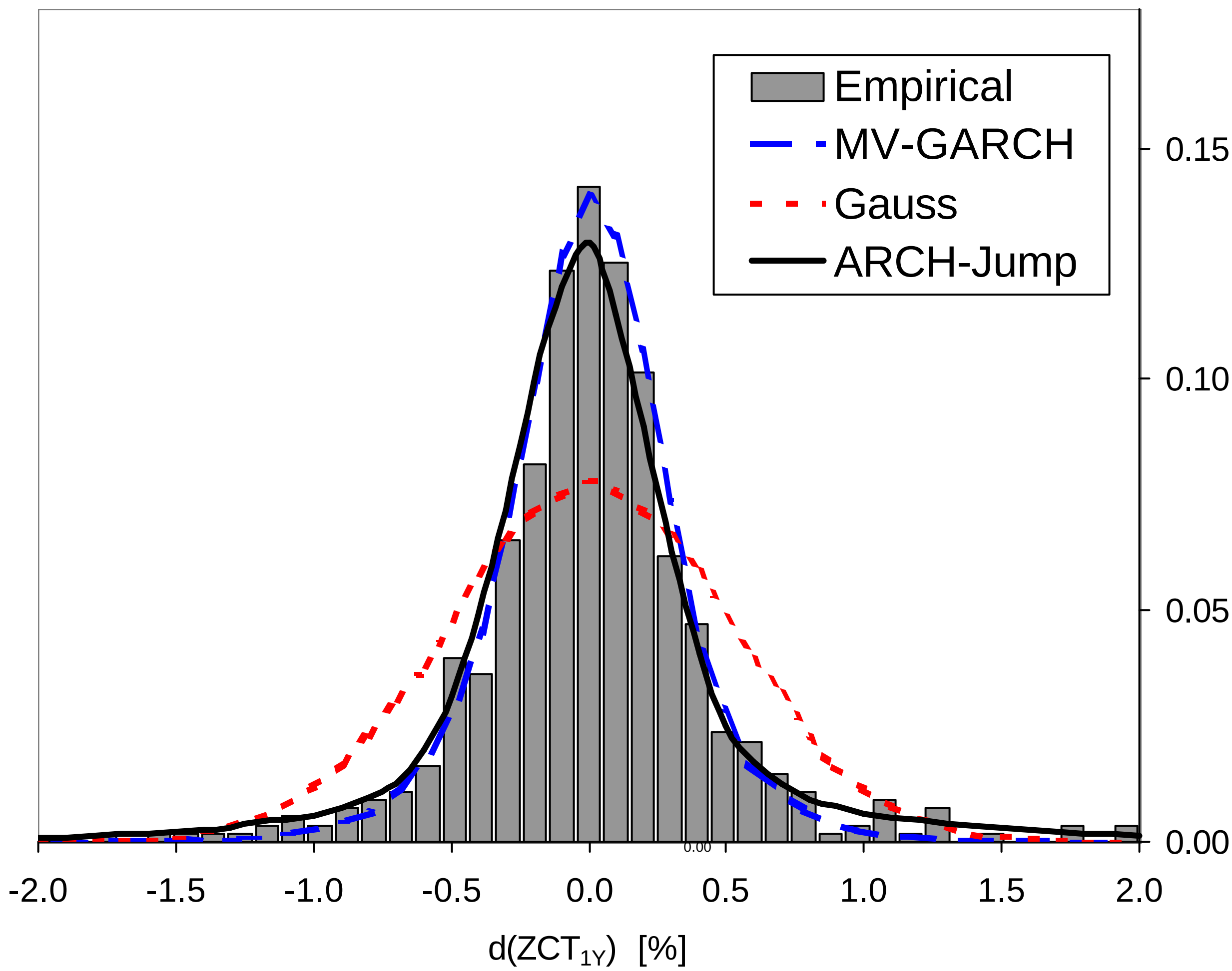

Figure 4. Frequency distribution for ΔZCT_{1Y} of a Gauss (dotted), and an ARCH-Jump (solid), and a MV-GARCH (dashed) estimation. The bars are the empirical distribution for 1953.01-1998.12. See Section III.

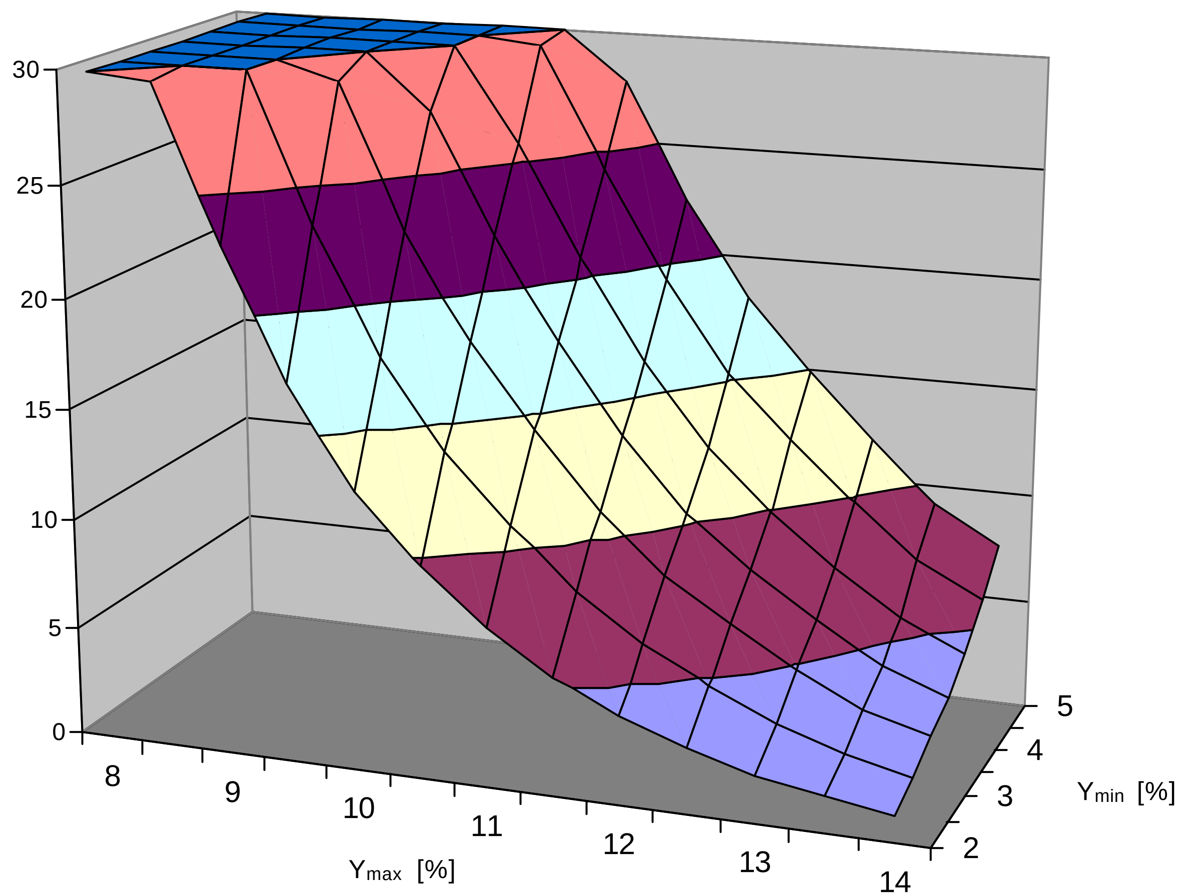

Figure 5. The frequency with which the AF3 ZTC_{10} pierces at least once both $Y_{\max}$ and $Y_{\min}$ within a 50 year time-frame. The initial yield curve was chosen to be the mean of rates between 1961.08 and 1998.08.

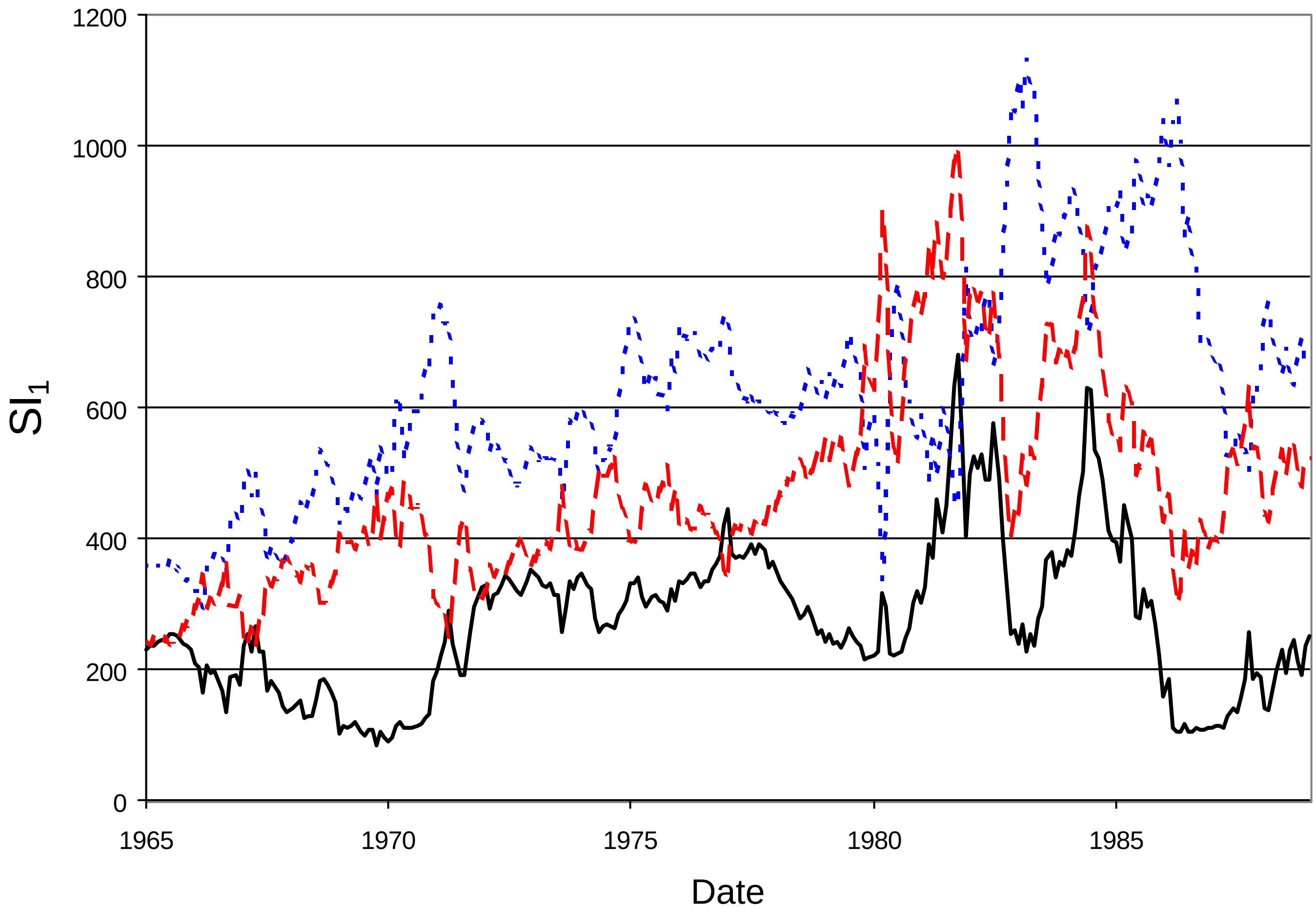

Figure 6. The severity index SI_1 for the historical CMT_{10} rates (solid line), along with the corresponding index for the RBC-URS (dotted) and the DRS (dashed) shocks.

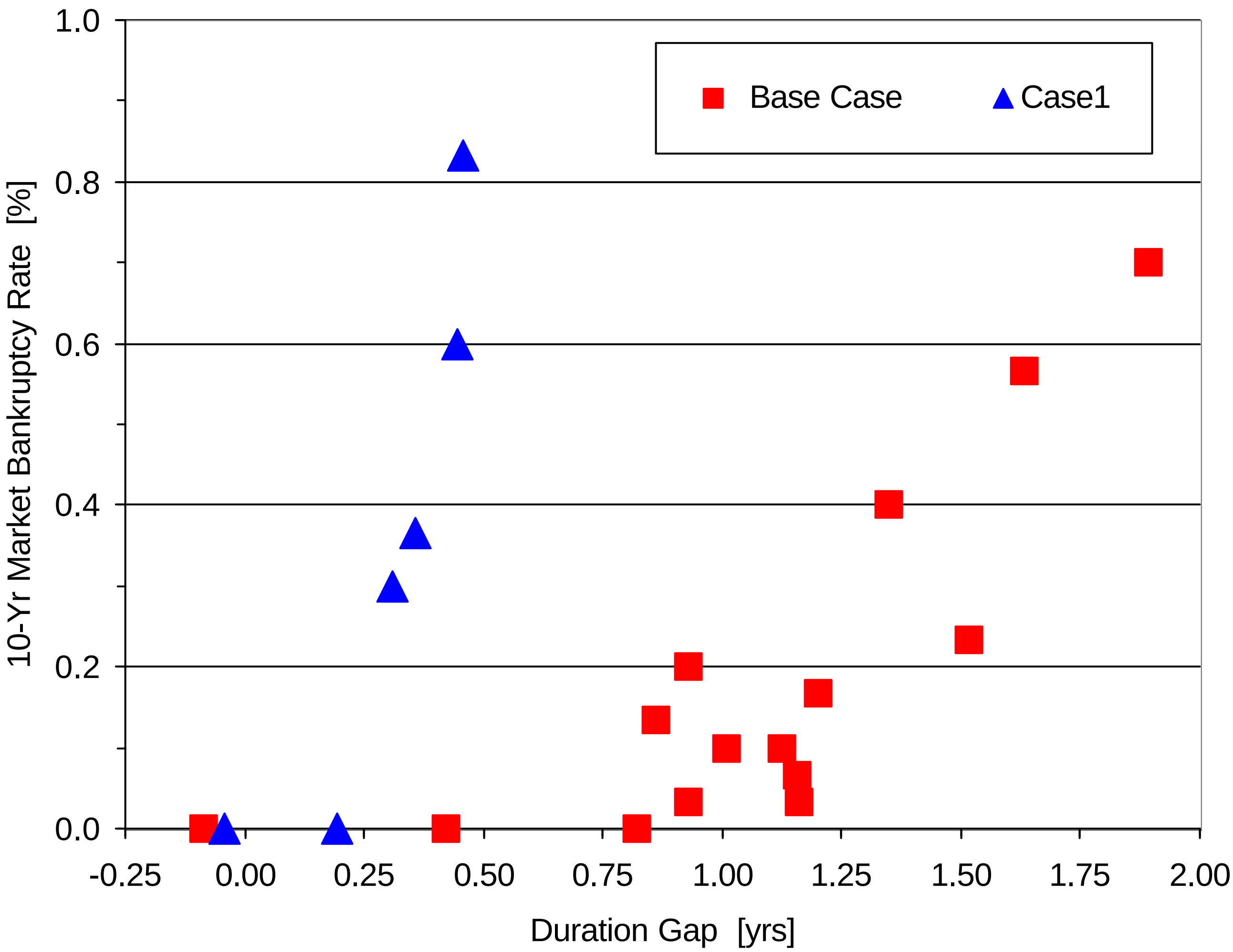

Figure 7. The 10-year probability of market insolvency versus duration gap. See Section V.

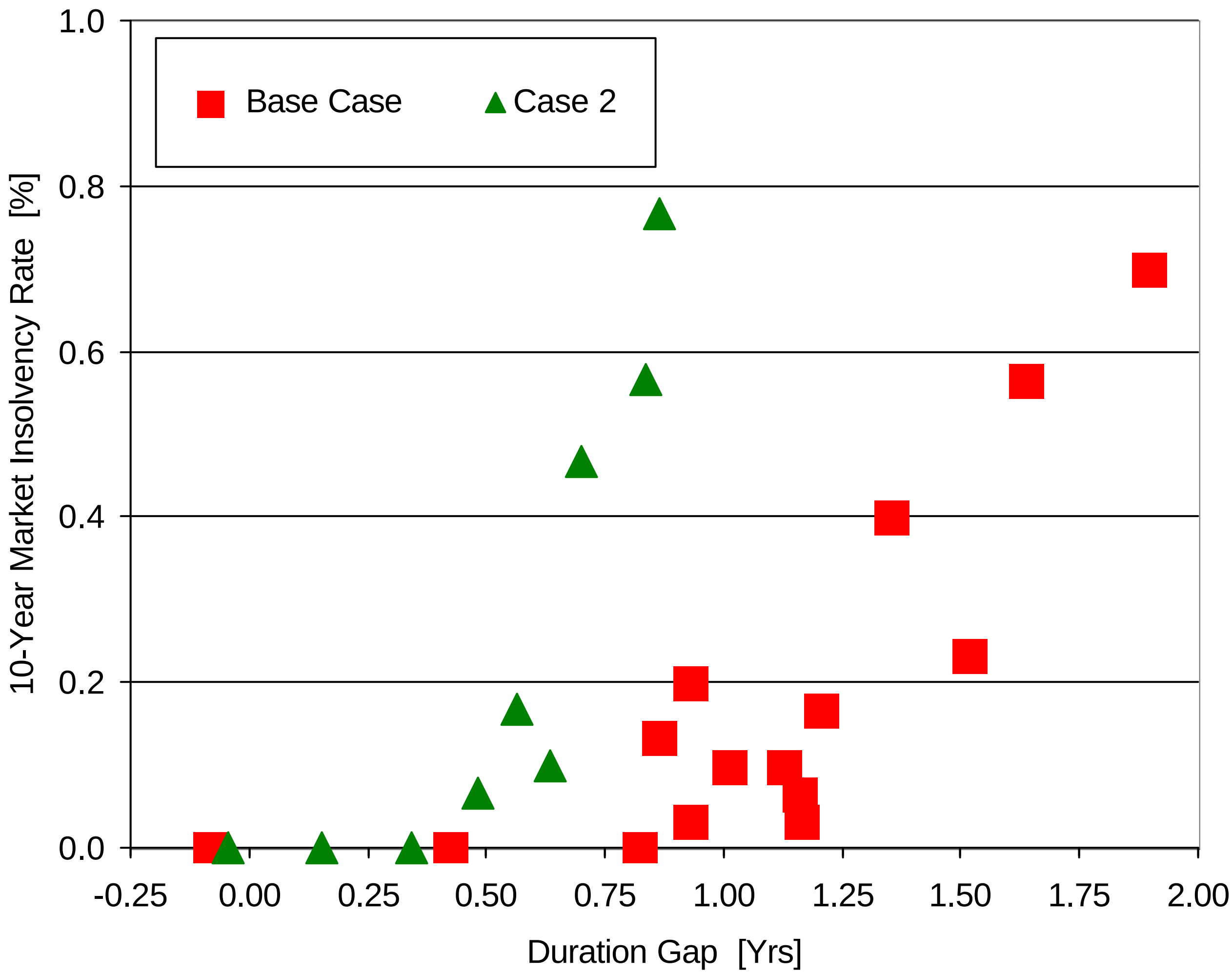

Figure 8. The 10-year probability of market insolvency versus duration gap. See Section V.

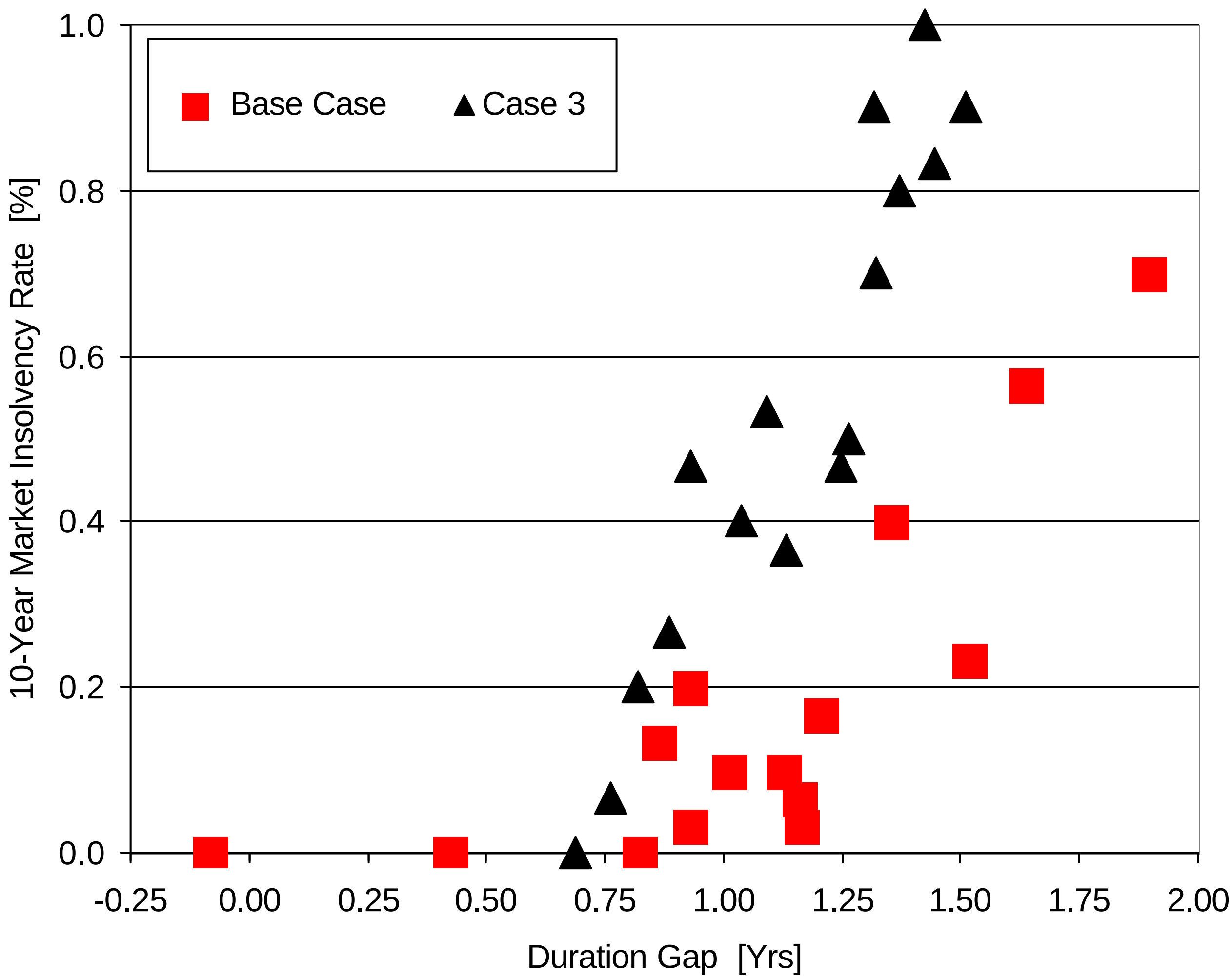

Figure 9. The 10-year probability of market insolvency versus duration gap. See Section V.

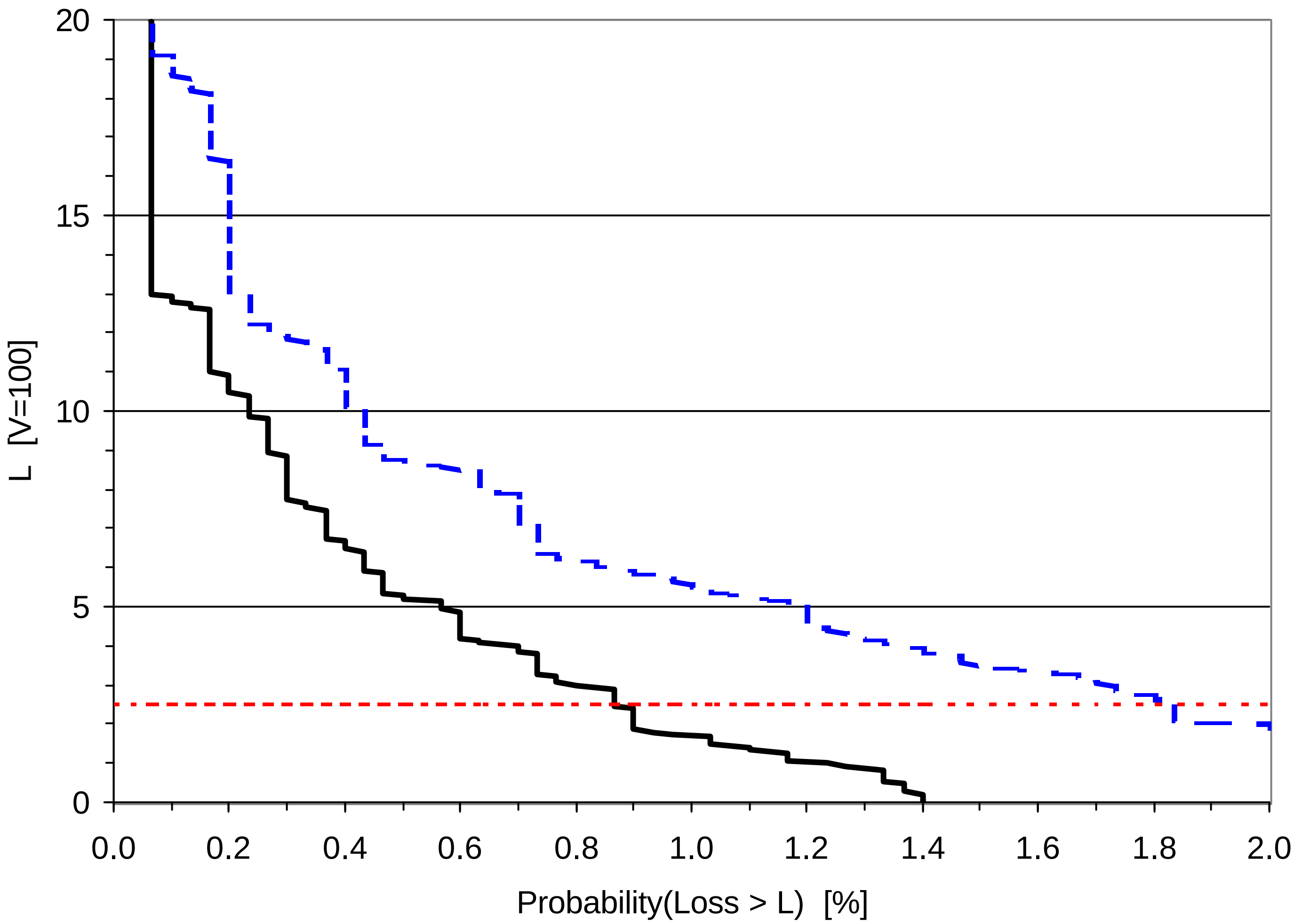

Figure 10. The 10-year VaR loss curves for two stylized firms as described in Section V. The firm starts with a 2.5% equity/asset ratio and maintains a fixed duration gap of 1 year. Solid line: case 2, dashed line: case 1.