LIBOR’s Poker: Interbank Borrowing Costs and Strategic Reporting *

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Abstract

The recent LIBOR scandals have demonstrated that the panel banks did not report in good faith, leading some to question whether LIBOR was ever accurate. This article derives the equilibrium LIBOR reporting strategy and quantifies the LIBOR bias. It finds that the current trimming mechanism cannot prevent LIBOR rigging, although the LIBOR gets less biased when the cross-sectional dispersion of the panel banks’ borrowing costs becomes lower. This explains why LIBOR earned wide adoption before the financial crisis. Additionally, signaling caps LIBOR. However, hiding individual banks’ reports to the market blocks signaling altogether. Finally, there exists a mechanism that induces truthful reporting.

Keywords: LIBOR; Interbank lending; Strategic reporting; Auction theory; Mechanism design

JEL classification: D44, D47, G10, G21

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1 Introduction

The London Interbank Offered Rate (LIBOR) is supposed to measure the average interest rate at which major banks can borrow short-term unsecured funds. Currently, it is produced on a daily basis for ten currencies, each with fifteen maturities ranging from overnight to twelve months. Since it is unlikely that all panel banks borrow from the interbank lending market each day in each currency and maturity, LIBOR is not calculated based on actual interbank lending transactions. Instead, LIBOR fixing, the procedure that determines LIBOR, relies on the panel banks’ honest estimate of their daily perceived borrowing costs from the interbank lending market.1 Thomson Reuters, the designated calculation agent for LIBOR, collects each panel bank’s estimate and calculates the interquartile mean as the LIBOR. Thomson Reuters then releases the LIBOR rate, together with submissions from all the panel banks. Since the British Banker’s Association (BBA) introduced LIBOR on January 1st, 1986, LIBOR has become a pivotal benchmark for not only interbank loans, but also financial contracts of less creditworthy borrowers, such as corporations and home buyers.2

Recent scandals shattered the reputation of LIBOR.3 A series of investigations revealed that some of the panel banks rigged LIBOR to benefit their own positions, while other banks reported low interbank borrowing costs to hide their credit risk. The investigations proved the panel banks behaved strategically, even though the BBA requires them to give honest estimates.

1The BBA website, http://www.bbalibor.com/, describes the process as follows: “Every contributor bank is asked to base their bbalibor submissions on the following question: ‘At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?’ Therefore, submissions are based upon the lowest perceived rate that a bank on a certain currency panel could go into the inter-bank money market and obtain sizable funding, for a given maturity.”

2According to Wheatley (2012), more than 300 trillion dollars of corporate bonds, residential mortgages, and other financial contracts are quoted as spreads over LIBOR nowadays.

This paper studies how the LIBOR fixing mechanism performs when panel banks behave strategically. In particular, what is a panel bank’s equilibrium LIBOR submission strategy? I construct a game-theoretic auction model of LIBOR fixing, in which panel banks can profit from LIBOR manipulation. The banks may also submit low bids to signal their credit quality. A regulator, such as the BBA or the Securities and Exchange Commission (SEC), imposes a fine on the LIBOR-rigging banks once their strategic behavior is discovered. The model yields a closed-form result for the symmetric Bayesian Nash equilibrium, which shows that the current trimming mechanism cannot block the impact of LIBOR manipulation attempts, contrary to Gyntelberg and Wooldridge (2008). Signaling efforts from panel banks inherently limit LIBOR in capturing high interbank borrowing cost. The equilibrium also suggests that panel banks lose their incentive to signal if their submissions are revealed not to public, but only to the regulator. Increasing regulatory monitoring or putting more member banks in the LIBOR panel also helps to reduce the magnitude of LIBOR bias.

The LIBOR scandal called into question the veracity of the past LIBOR rates, and raised an important question: Given these limitations, how could LIBOR managed to earn ubiquitous adoption and solid reputation in the last two decades? It is therefore important to study the magnitude of LIBOR bias under various historical economic conditions. Although it seems straightforward that banks will manipulate LIBOR as long as their bottom line is sensitive to the rate, my equilibrium reveals that banks’ submissions, which are driven by their incentive to profit from manipulating LIBOR, are moderated by the dispersion of borrowing costs across panel banks. When the banks have little incentive to signal their credit quality, the equilibrium reporting strategy only leads to distorted LIBOR when both factors exist. If the borrowing

4For instance, Whitehouse (2008), a Wall Street Journal article, cited BBA chief Ms. Knight: “I see no reason suddenly to up sticks and change a process that has actually served the financial community world-wide extremely well for a very considerable number of years.”
costs among banks are identical, the current LIBOR fixing mechanism induces truthful revelation even if panel banks’ incentive to manipulate LIBOR is large. This finding helps to explain why LIBOR has served the global financial market for more than two decades. It also provides helpful information for some recent disputes over historical LIBOR rates.\(^5\)

A number of empirical researchers have looked into the LIBOR fixing. Hartheiser and Spieser (2010) look at panel banks’ submissions, and estimate the clustering of LIBOR submissions. Snider and Youle (2010) study the relationship between LIBOR submissions and banks’ CDS spreads, in various currencies. Abrantes-Metz et al. (2011) compare individual bank quotes to CDS spreads and market capitalization data from early 2007 to mid-2008, and find the data are inconsistent with a material manipulation of the US dollar 1-month LIBOR rate, though anomalies in individual quotes do exist. Kuo et al. (2012) show that during the financial crisis banks were willing to borrow from the Fed at a higher interest rate on the collateral than what they reported in their LIBOR fixing process. My research contributes to the existing literature by providing a theoretical model which describes the equilibrium LIBOR reporting strategy. Comparative statics results suggest further empirical tests that could be implemented to check how the LIBOR bias varies with different economic factors.

The LIBOR scandal has provoked calls for reforms to build a better benchmark. In this paper, the equilibrium result suggests the existence of a LIBOR fixing mechanism that induces the strategically behaving banks to report honestly. Specifically, I propose a direct, \textit{ex ante} individually rational and \textit{ex ante} budget-balanced d’Aspremont, Gerard and Varet (AGV) mechanism for LIBOR fixing.

Finally, the analysis in this paper goes beyond the LIBOR fixing process. For many financial contracts without a liquid market, prices are usually determined by a trimmed average of players’ surveyed results. For instance, the daily closing prices of many derivatives traded in the over-the-counter market are formed in a similar manner. The Baltic Dry Index is another example. The equilibrium result in this paper can also be applied to those markets.

The paper is organized as follows: section 2 presents the model that describes banks’ optimal response. Section 3 discusses the policy and economic implications of banks’ equilibrium strategy. Section 4 proposes a direct, \textit{ex ante} individually rational and \textit{ex ante} budget-balanced LIBOR fixing mechanism. Concluding remarks appear in Section 6. All proofs are relegated to the Appendix.

2 The model

2.1 Setup

Consider an economy with $N$ risk neutral LIBOR panel banks. The banks’ \textit{ex ante} true borrowing costs are spreads $s = \{s_1, s_2, ..., s_N\}$ over the risk-free rate $r_f$, where each $s_i$ is a random variable independently and identically distributed on $\Theta \equiv [0, s_{\text{max}}]$. Each individual bank’s borrowing cost has an absolutely continuous cumulative density function $F(\cdot)$, and a unimodal and positive probability density function $f(\cdot)$ over the support $\Theta$. The distribution is common knowledge across banks. The probability density function of the distribution of the $n$th order statistics out of $N - 1$ borrowing costs is $f^{(N-1)}_n(\cdot)$, and the probability density function of $N - n$th order statistics is $f^{(N-1)}_{N-n}(\cdot)$. 

5
Allegedly LIBOR is the trimmed average of panel banks’ borrowing costs

\[ L_0 = r_f + \frac{1}{N - 2n} \int_{\Theta_N} \sum_{j=n+1}^{N-n} s_j^{(N)} dF(s) \]  \hspace{1cm} (1)

where \( \bar{F}(s) \) is the joint cumulative density function of \( s \). In addition, \( s_j^{(N)} \) refers to the \( j \)th order statistic of \( s_1, s_2, ..., s_N \).

Since \( s \) is not easily observable, the current LIBOR fixing mechanism requires banks to report their borrowing costs. Given banks’ submissions \( b = \{b_1, b_2, ..., b_N\} \in \Theta^N \), which are also spreads over the risk-free rate, LIBOR is the trimmed average:

\[ L(b) = r_f + \frac{1}{N - 2n} \sum_{j=n+1}^{N-n} b_j^{(N)} \]  \hspace{1cm} (2)

where \( b_j^{(N)} \) is the \( j \)th order statistic among \( b \). From bank \( i \)’s perspective, given its report \( b_i \) and all other banks’ reports \( b_{-i} = \{b_j\}_{j \neq i} \), the LIBOR rate is

\[ L(b_i, b_{-i}) = r_f + \begin{cases} \frac{1}{N-2n} \sum_{j=n}^{N-1-n} b_j^{(N-1)} & b_i \leq b_n^{(N-1)} \\ \frac{1}{N-2n} [b_i + \sum_{j=n+1}^{N-n} b_j^{(N-1)}] & b_n^{(N-1)} < b_i < b_n^{(N-1)} \\ \frac{1}{N-2n} \sum_{j=n+1}^{N-n} b_j^{(N-1)} & b_n^{(N-1)} \leq b_i \end{cases} \]  \hspace{1cm} (3)

where \( b_j^{(N-1)} \) is the \( j \)th order statistics among \( b_{-i} \). Clearly, \( L(b_i, b_{-i}) \) is uniformly continuous, and is increasing in \( b_i \) given any \( b_{-i} \).

Given the evident breadth of the LIBOR scandal, I assume that all the LIBOR panel banks are strategic. Knowing its own borrowing cost but not other banks’ borrowing costs and submissions, bank \( i \) submits \( b_i \), such that it maximizes the expected payoff conditional on its information set \( \mathcal{I}_i = \{s_i, b_i\} \).
2.1.1 Payoff function of LIBOR panel banks

A bank may benefit from both an income channel and a signaling channel by submitting a strategically chosen $b_i$. In this section, I model only the income channel, and defer the analysis of the signaling channel to the next section.

As Diamond and Dybvig (1983) suggests, a regular commercial bank has maturity mismatch on its balance sheets due to its maturity transformation function. The effective duration of a bank’s assets, which usually contains longer-term illiquid assets, is higher than the effective duration of its liabilities, and common sources of a bank’s short-term funding are usually related to LIBOR. Therefore, a low LIBOR rate reduces the bank’s funding costs. Table 1 shows the sensitivity of Bank of America’s projected annual non-trading net interest income to the short end of the yield curve. A lower short-term rate has a significant impact on net income and helps the financial performance of banks in general.

Table 1: BofA’s net income interest rate sensitivity, in million USD. Source: Bank of America

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<td>Non-trading net interest income change</td>
<td>453</td>
<td>1255</td>
<td>971</td>
<td>536</td>
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<td>Net Income</td>
<td>6276</td>
<td>4008</td>
<td>14982</td>
<td>21133</td>
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Note: The first line shows Bank of America’s projected change in non-trading net interest income to a 100 basis points (bps) drop at the short-end of the yield curve from 2005 to 2009. The second line shows its net income during the same period. Clearly, a drop in short-end of the yield curve helps to boost the bank’s net income. If the short term yield dropped 100 bps in 2008, Bank of America’s net income could have increased about 31%.

A bank may also benefit from manipulating LIBOR upwards using derivative positions. However, this can only happen on certain trading days, such as the maturity date of the derivative contract. In addition, there is no clear direction of LIBOR
manipulation in derivative trading. Therefore, I focus on the balance sheet effect of LIBOR, and subsequently assume that banks benefit from a low LIBOR.

Since a bank’s assets and liabilities contain many contracts and positions that relate to the LIBOR, it is reasonable to assume that the bank’s income is a continuous function of LIBOR. I approximate the effect of LIBOR locally using a linear function of the LIBOR rate. Let $A_i$ refer to the size of bank $i$’s assets, and $\delta$ measure the proportion of the bank’s assets that are financed through rolling over short-term LIBOR-linked funding channels. I write the income effect of LIBOR submission as $\delta A_i [L_0 - L (b_i, b_{-i})].$

Clearly, $\delta$ is a measurement of the maturity mismatch of bank’s balance sheet: A bank with higher maturity mismatch is going to gain more from rigging LIBOR downwards. I assume that the all the banks share the same $\delta$.

Regulators have a non-trivial monitoring task to oversee the LIBOR fixing process. Like many other monitoring tasks, investigating whether a panel bank has misreported is costly, because it involves auditing the bank’s balance sheet. It is not feasible for regulators to inspect every panel bank in each period. As a result, regulators may select some banks to audit according to certain audit triggers.$^6$ I assume a regulator audits bank $i$ with a small probability $\alpha \cdot |\hat{s}_i - b_i|$, where $\hat{s}_i = s_i + \varepsilon_i$ is the regulator’s noisy estimation of the bank’s borrowing cost $s_i$. The noise $\varepsilon_i$ is independent to $s_i$ and $b_i$, and $\mathbb{E} [\varepsilon_i] = 0$. The constant coefficient $\alpha > 0$ is common knowledge and is small enough such that $\alpha \cdot |\hat{s}_i - b_i| \in [0, 1]$.

The BBA’s LIBOR fixing protocol demands that panel banks disclose their true borrowing costs or face regulatory penalty. This motivates the setup that, once the regulator audits a panel bank and finds that $b_i$ does not equal the actual cost $s_i$, it will

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$^6$The IRS tax audit is another example about how regulators deal with costly monitoring. In fiscal year 2008, IRS audits about 1% of 137 million tax returns, while certain audit triggers boost the chance of winning the audit lottery.
impose a penalty $\psi A_i \cdot |s_i - b_i|$ on the bank. I assume the constant coefficient $\psi > 0$ is common knowledge among banks. As the cases of UBS$^7$ and Barclays$^8$ suggest, $\psi$ is usually many times higher than the income a bank can earn from a one-period LIBOR manipulation.

Thus, the payoff of bank $i$ given the borrowing cost $s$ and the bank’s bid $b$ is

$$\delta A_i [L_0 - L (b_i, b_{-i})] - \alpha \psi A_i (s_i - b_i)^2 - \alpha \psi A_i \varepsilon_i |s_i - b_i|.$$ 

Since scaling the payoff by $A_i$ does not affect the equilibrium outcome, and $\varepsilon_i$ is independent of $s_i$ and $b_i$, I focus on the bank’s scaled payoff function $\Pi_i : \Theta^N \times \Theta^N \to \mathbb{R}$ as

$$\Pi_i (s, b) = \delta [L_0 - L (b_i, b_{-i})] - \alpha \psi (s_i - b_i)^2$$

(4)

where $L_0$ is defined in equation 1. The parameters $\delta$, $\alpha$, and $\psi$ are the same across panel banks. In addition, since scaling the payoff $\Pi$ by $\delta$ does not change the preference ordering, I can focus on the penalty ratio $\alpha \psi / \delta$ instead. Since the penalty coefficient $\alpha \psi$ is not observed, $\alpha \psi / \delta$ can be treated as the implied penalty coefficient, which banks use to estimate the penalty while submitting LIBOR quotes. $\Pi_i (s, b)$ is random for bank $i$ since $b_{-i} \notin I_i$.

### 2.2 Equilibrium Analysis

A LIBOR panel bank chooses the LIBOR submission that maximizes its expected payoff as defined in the symmetric pure strategy Bayesian Nash equilibrium. The definition of the bank’s value function follows:


Definition. A symmetric pure strategy profile \( \beta(s) = (\beta(s_1), \beta(s_2), ..., \beta(s_N)) \) is a Bayesian Nash equilibrium if, for each bank \( i \), \( \beta(\cdot) \) is the bank’s best response for its LIBOR submission such that for any other \( \beta'(\cdot) \):

\[
\int_{\Theta^N} \Pi_i(s, b^*_i, b^*_{-i}) \, d\bar{F}(s) \geq \int_{\Theta^N} \Pi_i(s, b'_i, b^*_{-i}) \, d\bar{F}(s)
\]

in which \( b^*_i \in \beta(s_i) \), \( b'_i \in \beta'(s_i) \), and \( b^*_{-i} \in \beta(s_{-i}) \).

Corollary 1. Since the probability density function \( f(s_i) \) is positive over the domain \( \Theta \), bank \( i \) maximizes its expected payoff conditional on \( s_i \) for each \( s_i \):

\[
\int_{\Theta^{N-1}} \Pi_i(s_i, s_{-i}, b^*_i, b^*_{-i}) \, d\bar{F}(s_{-i}|s_i) \geq \int_{\Theta^{N-1}} \Pi_i(s_i, s_{-i}, b'_i, b^*_{-i}) \, d\bar{F}(s_{-i}|s_i)
\]

which implies

\[
\int_{\Theta^N} \Pi_i(s_i, s_{-i}, b^*_i, b^*_{-i}) \, d\bar{F}(s) \geq \int_{\Theta^N} \Pi_i(s_i, s_{-i}, b'_i, b^*_{-i}) \, d\bar{F}(s).
\]

Definition. The value function for bank \( i \) is

\[
V_i(s_i) = \int_{\Theta^{N-1}} \Pi_i(s_i, s_{-i}, b^*_i, b^*_{-i}) \, d\bar{F}(s_{-i}|s_i)
\]

where \( b^*_i \in \beta(s_i) \) and \( b^*_{-i} \in \beta(s_{-i}) \) which is defined above.

After the definition of the Bayesian Nash Equilibrium, the next section discusses the property of the equilibrium submission strategies.

2.2.1 Equilibrium

Notice that the payoff function \( \Pi_i(s_i, s_{-i}, b_i, b_{-i}) \) satisfies the strictly increasing difference condition in \( (s_i, b_i) \) for all \( i \), i.e., given \( s_i < s'_i \), and \( b_i < b'_i \), and for any \( s_{-i} \) and
In other words, a bank’s marginal payoff from submitting a lower LIBOR bid goes down when its borrowing cost becomes lower. This guarantees the existence and monotonicity of the symmetric pure strategy Bayesian Nash equilibrium, as proved by Lemma 2.

**Lemma 2.** If the payoff function \( \Pi_i(s_i, s_{-i}, b_i, b_{-i}) \) satisfies the supermodularity condition and \( \int_{\Theta_{N-i}} \Pi_i(s_i, s_{-i}, b_i, b_{-i}) \, d\tilde{F}(s_{-i}|s_i) \) exists for all \( i \), then there exists a weakly increasing pure strategy symmetric Bayesian Nash equilibrium, i.e., for \( \forall s_i < s'_i \) and \( \forall b_i \in \beta(s_i) \), \( b'_i \in \beta(s'_i) \), we have \( b_i \leq b'_i \).

The monotonicity of the best response function is intuitive to see: bank \( i \)'s best response \( \beta^*_i \) only depends on its own \( s_i \), and other bank’s report \( b_{-i} \). Therefore, even if all the other banks have non-monotone reporting strategies, bank \( i \) should still stick to a monotone reporting strategy.

Based on the value of parameters, the equilibrium strategy \( \beta(s_i) \) could be strictly increasing over the support \( \Theta \) or constant over some region \( \theta \subset \Theta \). The next two propositions describe both equilibrium strategies. Proposition 3 gives the condition for having a strictly increasing equilibrium.

**Proposition 3.** If the probability density function of the \( n \)th order statistics \( f^{(N-1)}_n(\cdot) \) and \( N-\)nth order statistics \( f^{(N-1)}_{N-n}(\cdot) \) satisfies the condition for strictly increasing equilibrium

\[
\max_{x \in \Theta} \left[ f^{(N-1)}_n(x) - f^{(N-1)}_{N-n}(x) \right] < (N - 2n) \frac{2\alpha \psi}{\delta},
\]

(5)
then the equilibrium strategy $\beta^*: \Theta \to \Theta$ for each bank $i$ is strictly increasing and

$$\beta^*(s_i) = s_i - \frac{\delta}{2\alpha\psi(N-2n)}\Delta(s_i)$$

where $\Delta(s_i)$ is

$$\Delta(s_i) = \frac{(N-1)!}{n!(N-n-1)!} \int_0^{F(s_i)} \frac{(1-y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy.$$ 

Figure 1 plots a strictly increasing equilibrium. There are a few things to note. First, $\Delta(s_i)$ is the probability that bank $i$’s equilibrium submission gets into the L-I-BOR calculation, where $s_i - \frac{\delta}{2\alpha\psi(N-2n)}$ is the “first-best” report of the bank if there exists a central planner who determines banks’ submissions. Second, $F(s_i) \in [0,1]$ suggests that the integral $\int_0^{F(s_i)} \frac{(1-y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy$ is non-negative, so in equilibrium a bank always reports $\beta^*(s) \leq s$. Finally, when a bank has the lowest possible borrowing cost, it reports its true borrowing cost $\beta^*(0) = 0$.

A careful look at the profit, $\delta \left[L_0 - L(b_i, b_{-i})\right]$, of bank $i$ from manipulating LIBOR and the penalty, $\alpha\psi(s_i - b_i)^2$, helps to understand equation (5). Given $b_i \in [0,s_i]$, a lower $b_i$ always leads to weakly higher profit and a lower penalty. When the marginal effect of $b_i$ on profit becomes larger than its marginal effect on the penalty at some $s_i$, the monotonicity constraint binds and $\partial\beta^*(s_i)/\partial s_i = 0$ for some $s$.

When the condition for strictly increasing equilibrium does not hold, the monotonicity of the best response function suggests that there exists $s^1_i \neq s^2_i$ such that $\beta^*(s^1_i) = \beta^*(s^2_i)$. Together with the fact that the probability density function of $s_i$ is unimodal, I can write a convex set $\theta \subseteq \Theta$ in which the $\beta^*(s_i) = \bar{b}$ for all $s_i \in \theta$. Figure 2 shows a sample equilibrium strategy with a bunching region $\theta$. The equilibrium
Note: Equilibrium strategy when the LIBOR panel contains \( N = 16 \) banks with i.i.d. normally distributed true borrowing costs. The mean borrowing cost \( \mu = 5\% \) and standard deviation \( \sigma = 1.5\% \). The penalty ratio \( \frac{\alpha \psi}{\delta} = 0.05 \).

The left subplot shows the equilibrium strategy \( \beta^*(s) \) as a function of borrowing cost \( s \). The right subplot shows the distribution of borrowing costs and equilibrium submissions. The optimal strategy causes the submissions of banks to cluster around the lower half of the support.

Figure 1: Equilibrium strategy with non-binding monotonicity constraint
strategy when the condition does not hold is:

Note: Equilibrium strategy when the LIBOR panel contains $N = 16$ banks with i.i.d. normally distributed true borrowing costs. The mean borrowing cost $\mu = 5\%$ and standard deviation $\sigma = 1.5\%$. The penalty ratio $\alpha\psi/\delta = 0.02$. Similar to the unconstrained case, the left subplot shows the equilibrium strategy $\beta^*(s)$ as a function of borrowing cost $s$. The right subplot shows the distribution of borrowing costs and equilibrium submissions. It shows the bunching level is around the lowest quartile of the support. When the monotonicity constraint binds, the dispersion of banks’ LIBOR submissions is even smaller.

Figure 2: Equilibrium strategy with binding monotonicity constraint

Proposition 4. If the probability density function of the $n$th order statistics $f_{n}^{(N-1)}(\cdot)$ and $N-n$th order statistics $f_{N-n}^{(N-1)}(\cdot)$ satisfies

$$
\max_{x \in \Theta} \left[ f_{n}^{(N-1)}(x) - f_{N-n}^{(N-1)}(x) \right] \geq (N - 2n) \frac{2\alpha\psi}{\delta}
$$

so the monotonicity constraint is binding when $s_i \in \theta$, the equilibrium strategy $\beta^* : \Theta \to \Theta$ for bank $i$ with a realized borrowing cost $s_i \notin \theta$ is the same as the case in Proposition 3

$$
\beta^*(s_i) = s_i - \frac{\delta}{2\alpha\psi (N - 2n)} \Delta (s_i)
$$
with
\[ \Delta (s_i) = \frac{(N-1)!}{n!(N-n-1)!} \int_0^{F(s_i)} \frac{(1-y)^{N-2n}-y^{N-2n}}{[y(1-y)]^{1-n}} dy \]

For any \( s_i \in \theta = [\underline{\theta}, \overline{\theta}] \subset [0, s_{\text{max}}] \), \( \beta^*(s_i) = \bar{b} \) which is a “bunching” level that maximizes \( \int_{\Theta^N} \Pi_i(s, b^*_i, b^*_{-i}) d\tilde{F}(s) \), where \( b^*_i \in \beta^*(s_i) \).

Intuitively, suppose that bank \( i \) with borrowing cost \( s_i \in [\underline{\theta}, \overline{\theta}] \) is determining its optimal LIBOR submission, while all the other banks follow a strategy with a binding monotonic constraint as in Proposition 3. The “bunching” region creates a probability mass for the distribution of other banks’ LIBOR submissions at \( \bar{b} \). The probability mass causes the probability of having bid \( b \) picked for calculating LIBOR to jump when \( b \) crosses the \( \bar{b} \) from below, and leads to a jump in the marginal profit of LIBOR manipulation. Since any other bank \( j \) with \( s_j \in [\underline{\theta}, \overline{\theta}] \) reports \( \bar{b} \) in equilibrium, the marginal profit at \( \bar{b} - \varepsilon \) is lower than the marginal penalty, whereas the marginal profit at \( \bar{b} + \varepsilon \) is higher than the marginal penalty. As a result, bank \( i \)’s submission should also be between \( b - \varepsilon \) and \( b + \varepsilon \), for arbitrarily small \( \varepsilon \). In other words, when other banks’ best responses have a “bunching” level, it is optimal for bank \( i \) to “join the bunch” given its cost \( s_i \in [\underline{\theta}, \overline{\theta}] \).

As Proposition 3 and Proposition 4 have shown, a bank’s best responses under two cases are the same for \( s_i \notin \theta \). This suggests a numerical search method to find the level \( \bar{b} \). I start with
\[ \tilde{\beta}(s_i) = s_i - \frac{\delta}{2\alpha \psi (N-2n)n!(N-n-1)!} \int_0^{F(s_i)} \frac{(1-y)^{N-2n}-y^{N-2n}}{[y(1-y)]^{1-n}} dy \]
which is non-monotonic but continuous, as Figure 3 shows. I first notice that \( \bar{b} \) should be between two stationary points of \( \tilde{\beta}(s) \), i.e., \( \bar{b} \in [\bar{b}_{\text{min}}, \bar{b}_{\text{max}}] \). With this range, the
equilibrium $\bar{b}$ maximizes

$$\int_{\mathcal{G}^N} \Pi_i \left( s_i, s_{-i}, \beta^* (s_i), b^*_{-i} \right) d\bar{F}(s)$$

where $\bar{\theta}$ and $\tilde{\theta}$ are the intersection of $\bar{b}$ with $\tilde{\beta}(s)$.

Figure 3: Determining the bunching level

2.3 Extension with Signaling

Besides the income channel, banks also care about the signaling properties of their LIBOR submissions. When a bank releases a high interbank borrowing cost to the public, it signals its weak balance sheet. Such signals may invite costly bank runs, as in Diamond and Dybvig (1983). Banks’ repo counter-parties, who provides short term funding, may also close positions abruptly, leading to costly fire sales of long term assets (see Shleifer and Vishny (2011)). This section extends the previous model with this signaling effect.
Suppose there are two unobservable states $e \in \{g, r\}$. Each bank is either in a good state $e = g$ or in a “failing” or bank-run state $e = r$, and the bank’s borrowing cost follow different distributions in each state. Each bank also has measure 1 of naive customers, such as demand depositors or repo trading counterparties, who are unaware of the bank’s strategic LIBOR submission. With a prior belief that the bank is in a failing state with probability $p$, customers update their belief about the bank based on its LIBOR report $b_i$. I note the posterior belief as $\pi(b_i)$. Subsequently, a share of $\pi(b_i)$ of customers decide not to roll over their position with the bank. This forces the bank to fire sale its long-term assets with a discount $1/(1 + \rho)$ per unit.

I denote the probability density function of borrowing costs $s_i$ under the failing state as $f_r(x) = \Pr(s_i = x|e = r)$, and under the good state $f_g(x) = \Pr(s_i = x|e = g)$. Bayes’ rule suggests

$$\pi(b_i) = \frac{p \cdot f_r(b_i)}{1 - p \cdot f_g(b_i)} \cdot \frac{1}{1 + \frac{p \cdot f_r(b_i)}{1 - p \cdot f_g(b_i)}}.$$

A bank run usually leads to a much more severe loss than the expected penalty of LIBOR manipulation. Therefore, banks should limit their LIBOR submissions $b$ to the level that $\frac{p \cdot f_r(b_i)}{1 - p \cdot f_g(b_i)}$ close to zero. This suggests a first order approximation for $\pi(b_i)$ as

$$\pi(b_i) \approx \frac{p \cdot f_r(b_i)}{1 - p \cdot f_g(b_i)}.$$

After $\pi(b_i)$ share of the counterparties run, the bank needs to sell $(1 + \rho)\delta A_i \pi(b_i)$ of long term assets to meet the payment obligation $\delta A_i \pi(b_i)$, and therefore suffers a loss $\rho\delta A_i \pi(b_i)$. The payoff for bank $i$ with the effect of signaling is then

$$\delta A_i [L_0 - L(b_i, b_{-i})] - \alpha \psi A_i (s_i - b_i)^2 - \alpha \psi A_i \varepsilon_i |s_i - b_i| + \rho \delta A_i [\pi(s_i) - \pi(b_i)].$$
Similarly, I focus on the payoff function

\[
\Pi_i(s_i, b_i, b_{-i}) = \delta [L_0 - L(b_i, b_{-i})] - \alpha \psi (s_i - b_i)^2 + \rho \delta [\pi(s_i) - \pi(b_i)]
\]

by scaling the payoff by \( A_i \), and taking expectations with respect to its information set \( I_i \).

As long as \( f_r(x) \) stochastically dominates \( f_g(x) \) in terms of likelihood ratio, the new payoff function satisfies the supermodularity in \( s_i \) and \( b_i \)

\[
\Pi_i(s', s_{-i}, b', b_{-i}) - \Pi_i(s, s_{-i}, b, b_{-i}) > \Pi_i(s, s_{-i}, b', b_{-i}) - \Pi_i(s, s_{-i}, b, b_{-i})
\]

so that Proposition 2 still suggests the existence and monotonicity of the equilibrium.

In order to solve for the equilibrium strategy with signaling, I assume that each bank’s borrowing cost follows an i.i.d. normal distribution \( N(\mu_g, \sigma^2) \) in the good state and \( N(\mu_r, \sigma^2) \) in the failing state. Clearly, \( f_r(x) \) stochastically dominates \( f_g(x) \) in terms of likelihood ratio and

\[
\pi(b_i) = \frac{p}{1-p} \phi \exp\{\eta b_i\}
\]

where \( \phi = \exp\left\{\frac{\mu_g^2 - \mu_r^2}{2\sigma^2}\right\} \) and \( \eta = -\frac{\mu_g - \mu_r}{\sigma^2} \).

**Proposition 5.** With the signaling effect, if the probability density function of the distribution of the \( n \)th and \( N - n \)th order statistics satisfies the condition,

\[
\max_{x \in \Theta} \left[ \tilde{f}_n^{(N-1)}(x) - \tilde{f}_{N-n}^{(N-1)}(x) \right] < 2\alpha \psi (N - 2n)
\]

the strictly increasing equilibrium strategy \( \beta^*_i: \Theta \rightarrow \Theta \) for bank \( i \) with realized borrowing
cost $s_i$ is

$$
\beta^* (s_i) = s_i - \frac{\delta}{2\alpha\psi (N - 2n)} \Delta (s_i) - \frac{1}{\eta} W \left( \frac{\rho \delta \eta^2}{2 \alpha \psi} \frac{p}{1 - p} \exp \left\{ \eta \left( s_i - \frac{\delta}{2\alpha\psi (N - 2n)} \Delta (s_i) \right) \right\} \right)
$$

where $W$ is the Lambert-W function,$^9$ and

$$
\Delta (s_i) = \frac{(N - 1)!}{n! (N - n - 1)!} \int_0^{F(s_i)} \frac{(1 - y)^{N - 2n} - y^{N - 2n}}{[y (1 - y)]^{1-n}} dy
$$

as in the previous section while $\tilde{F} (s_i) = pF_r (s_i) + (1 - p) F_g (s_i)$.

The proof is in the Appendix.

**Remark.** Given the distribution of borrowing costs, adding the signaling effect does not change the condition for the strictly increasing bidding strategy.

Figure 4 shows the equilibrium strategy when the LIBOR panel contains $N = 16$ banks with i.i.d. normally distributed true borrowing costs in each state. The borrowing costs are normally distributed with mean $\mu = 3\%$ in the good state, and $\mu = 13\%$ in the bad state. The standard deviation $\sigma = 1.819\%$ is the same in both states. The prior belief assigns $p = 0.2$ probability that banks are in the bad state. Therefore, the ex ante distribution is normal with $\mu = 5\%$ and $\sigma = 1.5\%$, similar to Figure 1. The penalty ratio is also the same $\alpha \psi / \delta = 0.05$, with the implied bank run cost $\rho / \delta = 100$. The left subplot shows the equilibrium strategy $\beta^* (s_i)$ as a function of borrowing cost $s$. The right subplots shows the cumulative distribution function and probability density function of the borrowing cost, as well as equilibrium submissions.

---

$^9$The Lambert-W function, also called the omega function, is the inverse function of $f(W) = We^W$. It is implemented in Mathematica as ProductLog[x], and in Matlab as lambertw(x). The Lambert-W function $W (x)$ is strictly increasing in $x$ when $x > 0$. Hence higher $p$ leads to a more biased LIBOR.
A comparison between Figure 1 and Figure 4 shows how the signaling effect changes the behavior of banks with high borrowing costs. The difference between $\beta^*(s_i)$ in these two cases shows that the signaling effect introduces a negative extra term

$$\frac{-1}{\eta} W \left( \frac{\rho\delta\eta^2}{2\alpha\psi} \frac{p}{1-p} \exp \left\{ \eta \left( s_i - \frac{\delta}{2\alpha\psi(N-2n)} \Delta (s_i) \right) \right\} \right)$$

to a bank's LIBOR submission.

Note: Equilibrium strategy when the LIBOR panel contains $N = 16$ banks with i.i.d. normally distributed true borrowing cost and banks signal for credit quality. The mean borrowing cost is $\mu = 3\%$ in the good state, and $\mu = 13\%$ in the failing state. The standard deviation of borrowing costs is $\sigma = 1.819\%$. The penalty ratio $\alpha\psi/\delta = 0.05$. The implied bank run cost $\rho/\delta = 100$. The left subplot shows the equilibrium strategy $\beta^*(s)$ as a function of borrowing cost $s$. The right subplot shows the distribution of borrowing costs and equilibrium submissions. The optimal strategy causes the submissions of banks to cluster around the lower half of the support.

Figure 4: Strictly increasing equilibrium strategy with signaling

Similar results can be obtained for the bunching case with signaling:

**Proposition 6.** With the signaling effect, if the probability density function of the
distribution of the \( n \)th and \( N - n \)th order statistics satisfies the condition,

\[
\max_{s \in \Theta} \left[ \tilde{f}_n^{(N-1)}(s) - \tilde{f}_{N-n}^{(N-1)}(s) \right] \geq - (N - 2n) \frac{2\alpha \psi}{\delta}
\]

so the monotonicity constraint is binding when \( s \in \theta \), the equilibrium strategy \( \beta^* : \Theta \to \Theta \) for a bank with realized borrowing cost \( s \notin \theta \) is the same as the previous case

\[
\beta^* (s_i) = s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \Delta (s_i) - \frac{1}{\eta} \frac{\rho \delta \phi \eta^2}{2\alpha \psi} \frac{p}{1 - p} \exp \left\{ \eta \left( s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \Delta (s_i) \right) \right\}
\]

where \( W \) is Lambert-W function, and for \( s \in \theta = [\underline{\theta}, \bar{\theta}] \subset [0, \max] \), \( \beta^* (s) = \bar{b} \) which is a “bunching” level that maximizes \( \int_{\Theta} \Pi_i (s, b_i^*, b_{-i}) dF (s) \), where \( b_i \in \beta^* (s_i) \). Similarly, \( \Delta (s) \) is

\[
\Delta (s_i) = \frac{(N - 1)!}{n! (N - n - 1)!} \int_0^{F(s_i)} \frac{(1 - y)^{N - 2n} - y^{N - 2n}}{[y (1 - y)]^{1-n}} dy
\]

as in the previous section while \( \tilde{F} (s_i) = p F_r (s_i) + (1 - p) F_g (s_i) \).

The proof is in Appendix.

Figure 5 shows the equilibrium strategy when the LIBOR panel contains \( N = 16 \) banks with i.i.d. normally distributed true borrowing costs. The borrowing cost has mean \( \mu = 3\% \) in the good state, and \( \mu = 13\% \) in the bad state. The standard deviation \( \sigma = 1.819\% \) is the same in both states. The prior belief assigns \( p = 0.2 \) probability that banks are in the bad state. Therefore, the ex ante distribution is normal with \( \mu = 5\% \) and \( \sigma = 1.5\% \), similar to Figure 1. The left subplot shows the equilibrium strategy \( \beta^* (s) \) as a function of the borrowing cost \( s \). The right subplot shows the cumulative distribution function and probability density function of borrowing cost, as well as equilibrium submissions. The effect of signaling banks’ LIBOR submissions in this case is similar to that in Figure 4.
Equilibrium Strategy When $N = 16$, $n = 4$, $\alpha \psi/\delta = 0.02$, $\rho/\delta = 100$, and $S \sim N(3.000, 1.819)$ in good state, $S \sim N(13.000, 1.819)$ in failing state.

Note: Equilibrium strategy when the LIBOR panel contains $N = 16$ banks with i.i.d. normally distributed true borrowing cost and banks signal for credit quality. The mean borrowing cost is $\mu = 3\%$ in good state, and $\mu = 13\%$ in failing state. The standard deviation of borrowing costs is $\sigma = 1.819\%$. The penalty ratio $\alpha \psi/\delta = 0.02$. The implied bank run cost $\rho/\delta = 100$. The left subplot shows the equilibrium strategy $\beta^*(s)$ as a function of borrowing cost $s$. The right subplot shows the distribution of borrowing costs and equilibrium submissions. The optimal strategy causes the submissions of banks to cluster around the lower half of the support.

Figure 5: Equilibrium strategy with binding monotonicity constraint with signaling.
This section shows that the symmetric equilibrium strategy is not truth-telling: LIBOR panel banks may not always submit their true borrowing cost, so the rates are error prone. Although it might not be surprising that banks will submit low quotes given that they prefer a lower level of LIBOR, it is still necessary to decompose the LIBOR bias into different components, and study quantitatively how each component varies. The next section focuses on the economic implications of the model according to the comparative statics of banks’ equilibrium submissions.

3 Economic implications

3.1 Signaling

Signaling causes a bank to report low borrowing costs to avoid a bank run. As a result, it creates an upper bound that is lower than the \( s_{\text{max}} \), for LIBOR reporting.

**Corollary 7.** The signaling creates an upper bound \( b_{\text{max}} \), which is lower than \( s_{\text{max}} \), for LIBOR. The upper bound is

\[
b_{\text{max}} = s_{\text{max}} - \frac{1}{\eta} W \left( \frac{\rho^2 \eta^2}{2 \alpha \psi} \frac{p}{1 - p} \exp \{ \eta s_{\text{max}} \} \right).
\]

This result suggests that the LIBOR fixing mechanism has an inherent limit in capturing the actual interbank borrowing costs: a bank never reports borrowing cost higher than the threshold \( b_{\text{max}} \). In other words, when the panel banks’ borrowing costs become sufficiently high, the current LIBOR fixing mechanism will not generate meaningful results.

Besides creating an upper bound of LIBOR, the signaling also causes LIBOR to fluctuate not only with the borrowing cost \( s \), but also with market sentiments.
Corollary 8. **Signaling becomes more effective when counterparties are less confident of the panel banks’ creditworthiness.** Therefore, a higher prior belief with regard to the panel banks’ insolvency induces them to report lower costs.

When the investors’ sentiment about banks’ creditworthiness deteriorates, the magnitude of LIBOR bias goes up even the actual fundamentals of the banks are unchanged. Figure 6 shows the equilibrium strategies under various prior beliefs. In general, a panel bank reports lower borrowing cost when market participants become more pessimistic about the panel bank’s creditworthiness.

![Equilibrium strategy with various prior belief](image)

*Figure 6: Equilibrium strategy with various prior belief $p$*

Note: In general, a panel bank reports lower borrowing cost when market participants become more pessimistic about the panel bank’s creditworthiness, even when its borrowing cost is unchanged.

However, the signaling problem could be easily fixed by restricting access to the panel bank’s LIBOR reports to regulators only. Without the information of panel banks’ borrowing costs, the market participants will not be able to infer panel banks’ creditworthiness. In addition, since the reference rate is based on the trimmed average rather than a single bank’s report, restricting access to each banks’ report does not
lead to the disruption of market function. Therefore, after simply withholding certain
information from the market participants, the panel bank’s signaling attempt can be
blocked. Nevertheless, LIBOR bias also depends on banks’ fundamentals, as the next
section discusses.

3.2 Bank’s fundamentals

How does LIBOR bias vary with panel banks’ fundamentals? I evaluate two aspects of
banks’ fundamentals. The first is the degree of maturity mismatch on banks’ balance
sheets, and the second is the dispersion of borrowing costs among banks. Interestingly,
LIBOR bias only exists when both aspects exist.

Corollary 9. When the LIBOR panel banks’ degree of maturity mismatch goes up, the
equilibrium submissions of banks move away from the truth due to both the income and
the signaling channel.

The higher the maturity mismatch of panel banks’ balance sheets, the more those
banks rely on short term funding. As an important reference rate with less than one
year maturity, a lower LIBOR rate reduces short-term funding costs. Therefore, a
higher degree of maturity mismatch gives banks more incentive to manipulate LIBOR.
Besides the magnitude of maturity mismatch, the dispersion of borrowing costs across
panel banks is also a key determinant of LIBOR bias.

Corollary 10. The income and signaling effect of LIBOR manipulation also depends
on the dispersion of borrowing cost S. When S becomes more dispersed, each bank’s
equilibrium submission moves from its true borrowing cost.
In good state $s \sim N(3.0, 3.31)$ with prob. 0.8
in failing state $s \sim N(13.0, 3.31)$ with prob. 0.2

Borrowing Cost $s$ with $\delta = 0.5$
Borrowing Cost $s$ with $\delta = 1$
Borrowing Cost $s$ with $\delta = 2$

Note: The LIBOR bias is also modulated by the dispersion of borrowing cost $s$. A larger dispersion in $s$ leads to a more biased LIBOR.

Figure 7: Equilibrium strategy with various $\delta$ and $\sigma^2$

Figure 7 shows the effect of both aspects. It is tempting to believe that as long as panel banks benefit from lower LIBOR, they report lower borrowing costs. However, the equilibrium suggests that the income channel becomes significant only when there is dispersion in banks’ borrowing costs. In other words, when banks’ borrowing costs become very close, the equilibrium strategy is to report the cost almost truthfully, even though banks have a large incentive to manipulate LIBOR. Therefore, the fact that banks prefer a lower rate does not automatically lead to the conclusion that the number generated by the LIBOR fixing mechanism is wrong. This also explains why LIBOR was a reasonably reliable measure before the financial crisis, when the banks were not highly leveraged.

In addition, as a crucial benchmark of the overall health of the financial system, LIBOR rates get more attention during a financial crisis than during normal times. Unfortunately, since the dispersion of banks’ borrowing costs is higher during the financial crisis, the current LIBOR fixing process causes a more biased LIBOR during
the turmoil, exactly when a solid benchmark is needed the most. Clearly, in order to minimize the size of LIBOR bias, proper regulation is indispensable.

3.3 Regulation and policy implications

Next, I evaluate how regulation shapes banks’ submission strategies. Intuitively, when banks can report untruthfully without any consequence, they will push the LIBOR to the lowest level. Therefore, banks’ LIBOR submissions contain no information about their interbank borrowing cost. This will not happen when there is a penalty, which could either be an explicit audit to check whether a bank reports its borrowing cost truthfully or the threat of an audit. The following proposition confirms the intuition.

**Corollary 11.** If regulators do not monitor the LIBOR fixing, then strategically behaved banks will report the lowest possible borrowing cost. If there is a penalty for misreporting, then strategically behaved banks will not report the lowest possible borrowing cost as long as their borrowing cost is higher than the riskless rate. This is true even with arbitrarily small positive $\alpha\psi$.

Since the LIBOR manipulation scandal broke, regulators have started to discuss possible options for strengthening the LIBOR mechanism, as documented in Wheatley (2012). One option is to increase participation in LIBOR panels. An alternative way is to calculate by taking the average of a random submission from the central quartiles. The next two corollaries analyze each option.

**Corollary 12.** When the number of banks in the pool increases, the income effect of LIBOR manipulation fades away and causes each bank’s equilibrium submission to
converge to its true borrowing cost from below. However, the signaling channel cannot be removed by adding more banks.

Adding more banks into the LIBOR panel helps to reduce the size of LIBOR bias. The effect of penalty coefficient $\alpha\psi$ and the number of panel banks to LIBOR bias are illustrated in the Figure 8.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{Equilibrium strategy under various regulation environments}
\end{figure}

Note: More stringent regulation and a larger panel help to reduce the magnitude of LIBOR bias.

**Corollary 13.** Calculating LIBOR by taking the average of a random selection of submissions from the central quartiles has no effect.

Suppose that each bank in the central quartile has a probability $q$ of getting selected. This reduces each bank’s probability of changing LIBOR by a factor of $q$. On the other hand, if bank $i$ does get selected in the averaging, its submission now has $1/q$ times more weight. Therefore, the conditional expectation of the bank’s payoff $\int_{\Theta_{N-1}} \Pi_i (s_i, s_{-i}, b^*_i, b^*_{-i}) \ d\bar{F} (s_{-i}|s_i)$ is unchanged, as is the equilibrium.
This section discussed three measures that improve the LIBOR fixing mechanism. Hiding individual banks’ reports to market participants blocks the panel banks’ incentive to signal creditworthiness. Enhancing regulatory monitoring makes LIBOR manipulation more costly to the panel banks. Increasing the number of banks in the panel makes it harder for banks to move LIBOR. All three measures help to reduce the size of LIBOR bias. The next section discusses the LIBOR fixing procedure from the perspective of mechanism design.

4 Mechanism design

The previous section shows that the LIBOR rate is not what it is supposed to be - a trimmed arithmetic average of banks’ true borrowing costs - when strategic panel banks with different borrowing costs can benefit from manipulating LIBOR. This motivates a LIBOR fixing mechanism in which banks truthfully report. I note the direct mechanism, in which the banks report true borrowing cost, as \((L^*, M^*)\), in which \(L^* : \Theta \rightarrow \mathbb{R}^+\) defines how to calculate the LIBOR, and \(M^*_i : \Theta^N \rightarrow \mathbb{R}\) is the payment rule which sets the amount every bank \(i\) pays BBA to join the LIBOR panel. I define the mechanism as efficient if it generates a LIBOR equal to

\[
rf + \frac{1}{N - 2n} \sum_{j=n+1}^{N-n} s_j^{(N)}.
\]

Clearly, a direct LIBOR fixing mechanism \((L^*, M^*)\) is efficient.

The existence of the payment rule motivates a direct LIBOR fixing mechanism \((L^*, M^*)\) to be \textit{ex ante} individually rational and \textit{ex ante} budget balanced, as defined below.
Definition. The LIBOR fixing mechanism \((L^*, M^*)\) is ex ante individually rational, if given \(z = \{z_1, ..., z_N\}\) as banks’ equilibrium reports under the mechanism \((L^*, M^*)\), the unconditional expectation of the sum of the banks’ payoffs and payments to the BBA over the realization of a borrowing cost \(s_i\) is zero for all \(i\):

\[
\int_{\Theta^N} \Pi_i(s, z) - M_i^*(z) \, d\bar{F}(s) = 0.
\]

Definition. A direct LIBOR fixing mechanism is ex ante budget balanced if the sum of banks’ expected payments to the BBA is zero.

\[
\sum_{i=1}^{N} \int_{\Theta^N} M_i^*(s) \, d\bar{F}(s) = 0.
\]

An ex ante individual rationality defined above is different from traditional individual rationality conditions. The traditional individual rationality condition requires a bank to receive higher value \(\Pi_i(s, z)\) from LIBOR manipulation than \(M_i^*(z)\), the amount it has to remit to the regulator, under any realization of borrowing costs \(s\). Therefore, the ex ante individual rationality is a weaker condition than traditional individual rationality. However, this does not imply that an ex ante individually rational mechanism is not feasible. When the mechanism satisfies the ex ante individual rationality, a bank’s expected payoff from participating in the LIBOR fixing is zero. Therefore, the regulator can sign a contract with the panel banks to have them commit to participate in LIBOR fixing mechanism \((L^*, M^*)\) at the beginning of an extended period.

Similarly, an ex ante budget-balanced LIBOR fixing mechanism means that the expected transfer to the regulator is zero, even if the total transfer to the regulator is not zero in each day. The following section shows the existence of an efficient, ex ante
individually rational and \textit{ex ante} budget-balanced mechanism.

4.1 AGV mechanism

The signaling channel encourages LIBOR panel banks to misrepresent their borrowing costs to signal a strong balance sheet. This channel can be muted by publishing the banks’ submissions only to the regulator, not to the general public. By turning off the signaling channel, a bank’s equilibrium reporting strategy becomes the case in Proposition 3 and Proposition 4.

The income channel can also be turned off under the direct, efficient, \textit{ex ante} budget-balanced and individually rational LIBOR fixing mechanism, along the line of the AGV mechanism by d’Aspremont and Gérard-Varet (1979).

\textbf{Proposition 14.} There is a direct, efficient, \textit{ex ante} budget balanced, and individually rational LIBOR fixing mechanism \((L^*, M^*)\) in which banks report \(z = \{z_1, ..., z_N\}\), where \(z_i = s_i\) for all \(i\). The LIBOR is calculated as

\[
L^* (z) = r_f + \frac{1}{N - 2n} \sum_{j=n+1}^{N-n} z_j^{(N)}
\]
and every bank $i$ pays the BBA

\[
M_i^*(z) = -\frac{\delta}{N - 2n} \sum_{j=n+1}^{N-n} \left[ z_j^{(N)} - \beta^* \left( z_j^{(N)} \right) \right] + \alpha \psi \left( z_i - \beta^* \left( z_i \right) \right)^2 + \rho \delta \left[ \pi \left( z_i \right) - \pi \left( \beta^* \left( z_i \right) \right) \right]
\]

\[
+ \frac{\delta}{N - 2n} \frac{N}{N - 1} \sum_{j=n+1, z_j^{(N)} \neq z_i}^{N-n} \left[ z_j^{(N)} - \beta^* \left( z_j^{(N)} \right) \right]
\]

\[
- \frac{1}{N - 1} \sum_{k=1, k \neq i}^{N} \alpha \psi \left( z_k - \beta^* \left( z_k \right) \right)^2 - \frac{1}{N - 1} \sum_{k=1, k \neq i}^{N} \rho \delta \left[ \pi \left( z_k \right) - \pi \left( \beta^* \left( z_k \right) \right) \right].
\]

Despite the long formulae of $M_i^*(z)$, its intuition is quite clear. Part (1) of $M_i^*(z)$ measures how much bank $i$ can obtain through LIBOR manipulation. This is exactly the intuition behind the revelation principle. Part (2) is the average of the BBA’s transfer to the rest of the panel: it ensures the \textit{ex ante} budget balance of the BBA. The detailed proof is in the Appendix.

## 5 Conclusion

Since its inception in 1986, LIBOR has become a widely referenced pivotal benchmark rate. However, recent LIBOR manipulation scandals have proved that the system collapsed during the recent financial crisis. Investigation into LIBOR fraud suggests that banks report their borrowing costs strategically. I have addressed the following questions in this research: First, what is the equilibrium submission strategy under the current LIBOR fixing mechanism? Second, how much has LIBOR been rigged since its inception? Finally, how could LIBOR manage to obtain such wide adoption over the last two decades? I develop an auction-theoretic model and solve for the closed-
form solution of equilibrium LIBOR reporting strategy. I show that, when there is dispersion among the borrowing costs of banks, the equilibrium strategy of banks under the current mechanism leads to an understated LIBOR rate. Thus, trimming does not block the LIBOR rigging. Only when the borrowing costs among banks are identical does the current trimming mechanism induce truthful reporting even if the panel bank’s incentive to manipulate LIBOR is large. Releasing banks’ LIBOR submissions to the public also causes a signaling effect, which leads to even more LIBOR distortion. In addition, the LIBOR submissions cluster together and show a much smaller dispersion than do other indicators of borrowing costs.

Furthermore, the model suggests that the LIBOR bias is not a constant. Regulators can play a significant role in reducing the distortion: when a regulator imposes a higher penalty on fraudulent activity, banks’ strategic behavior become less significant, and LIBOR will be closer to its true level. Adding more banks into the panel also helps. The state of the financial market also impacts the LIBOR bias. When the market is concerned about the creditworthiness of banks, banks are more willing to signal their credit quality through reduced LIBOR submissions. So the LIBOR bias could change even if bank fundamentals remain the same, though the signaling effect can be removed by simply not sharing individual banks’ reports with the market. Finally, the fundamentals of panel banks determine LIBOR distortion. When banks’ bottom line becomes more sensitive to the LIBOR rate, the size of LIBOR bias becomes bigger.

More interestingly, the size of LIBOR bias increases with the cross-sectional dispersion of banks’ borrowing costs, since higher dispersion gives a bank more space to misreport. The signaling effect is also stronger. This explains why LIBOR can secure wide acceptance during a period when banks are solid. During a financial crisis, banks’ credit spreads are more volatile, and the cross-sectional dispersion of borrowing costs
become higher: the strategic behavior of panel banks causes the LIBOR to be more biased. As a result, LIBOR - arguably the most fundamental benchmark in financial markets - becomes least reliable when the market needs a measure of funding stress most.

This article also gives suggestions on the mechanism design of the LIBOR fixing process. Increasing the number of banks in the panel will send the LIBOR rate closer to the correct level. Most importantly, there exists a direct and ex-ante budget-balanced mechanism under which each bank reports its true borrowing costs.

A Appendix

This section contains the proof for the Propositions and Lemmas. To simplify notation, I write $\mathbb{E}[\cdot] = \int_{\Theta} \cdot dF(s)$, and $\mathbb{E}[\cdot|\mathcal{I}_i] = \int_{\Theta^{-1}} \cdot dF(s_{-i}|s_i)$.

A.1 Proofs

I first prove the existence of pure strategy symmetric Bayesian Nash equilibrium, using the results from Athey (2001), then show the monotonicity of equilibrium.

Proof of Lemma 2

Proof. The single crossing property holds thanks to the supermodularity condition. In addition, $\mathbb{E}[\Pi_i (s_i, b'_i, b^*_{-i}) | \mathcal{I}_i]$ exists, so the A1 assumption in Athey (2001) also holds. Therefore, by Theorem 1 of of Athey (2001), for any finite type space $\Theta' \subset \Theta$, there is a pure strategy Nash equilibrium in nondecreasing strategies.
Now let me extend the proof to the continuum type space Θ. Since (i) for all bank \( i \), \( \Theta = [0, \bar{s}] \), and (ii) for all bank \( i \), \( \mathbb{E} [L (b_i, b_{-i})] \) is continuous in \( b_i \), so does \( \Pi_i (s_i, b_i, b_{-i}) \). Together with the existence of pure strategy Nash equilibrium in nondecreasing strategies in any finite type space \( \Theta' \), Theorem 2 of Athey (2001) suggests the existence of a pure strategy Nash equilibrium in nondecreasing strategies in continuum type space \( \Theta \).

Using monotone selection theorem in Milgrom and Shannon (1994), an alternative for Topkis’ theorem when supermodularity, aka, the strictly increasing difference (SID) holds. The function satisfies SID since given \( s_i < s'_i \), and \( b_i < b'_i \), I have \( \Pi_i (s'_i, b'_i, b^*_{-i}) - \Pi_i (s'_i, b_i, b^*_{-i}) > \Pi_i (s_i, b'_i, b^*_{-i}) - \Pi_i (s_i, b_i, b^*_{-i}) \). Then for \( b^*_i \in \beta (s'_i) = \arg \max_{b_i} \Pi_i (s'_i, b_i, b^*_{-i}) \), and \( b^*_i \in \beta (s_i) \), monotone selection theorem says \( b^*_i \leq b^*_i \).

Before I proceed to Proposition 3, I need the following Lemma.

**Lemma A.1.** With the assumption of strict monotonicity in \( \beta (\cdot) \), the expected value of LIBOR \( \mathbb{E} [L (b_i, b_{-i}) | I_i] \), is twice differentiable in \( b_i \).

**Proof.** First, \( \left\{ b_{n+1}^{(N-1)}, ..., b_{N-1-n}^{(N-1)} \right\} \) are always in the calculation of LIBOR, regardless the choice of \( b_i \). With the cumulative distribution function of \( n \)th order statistics of \( N - 1 \) quotes as \( G_j^{(N-1)} (x) \equiv \Pr \left\{ b_j^{(N-1)} \leq x \right\} \). When the cumulative distribution function \( G (x) \) is absolute continuous, \( G_j^{(N-1)} (x) \) is also absolute continuous, therefore
probability density function \( g_j^{(N-1)}(x) \) exists and I can write

\[
\mathbb{E}[L(b_i, b_{-i})|I_i] = r_f + \frac{1}{N - 2n} \sum_{j=n+1}^{N-1-n} \mathbb{E}[b_j^{(N-1)}] \\
+ \frac{1}{N - 2n} \int_{b_i}^{s} x g_n^{(N-1)}(x) \, dx \\
+ \frac{1}{N - 2n} \int_{0}^{b_i} x g_{N-n}^{(N-1)}(x) \, dx \\
+ \frac{1}{N - 2n} b_i \left[ 1 - \int_{b_i}^{s} g_n^{(N-1)}(x) \, dx - \int_{0}^{b_i} g_{N-n}^{(N-1)}(x) \, dx \right].
\]

(6)

It is easy to show \( \mathbb{E}[L(b_i, b_{-i})|I_i] \) is once and twice differentiable in \( b_i \), since \( g_j^{(N-1)}(x) \) is continuous. In addition,

\[
\frac{\partial}{\partial b_i} \mathbb{E}[L(b_i, b_{-i})|I_i] = \frac{1}{N - 2n} \left[ 1 - \int_{b_i}^{s} g_n^{(N-1)}(x) \, dx - \int_{0}^{b_i} g_{N-n}^{(N-1)}(x) \, dx \right] \\
= \frac{1}{N - 2n} \left[ G_n^{(N-1)}(b_i) - G_{N-n}^{(N-1)}(b_i) \right].
\]

\( \square \)

With the Lemma A.1, I now prove Proposition 3.

**Proof of Proposition 3**

*Proof.* First let us assume \( \beta(\cdot) \) is strictly increasing. The assumption allows the banks’ submissions maintain the same order as banks’ realization of borrowing cost \( b_i^{(N)} = \beta(s_i^{(N)}) \ \forall i \in \{1, \ldots, N\} \). Hence, instead of working with the cumulative distribution function of \( n \)th order statistics of \( N - 1 \) submissions as \( G_j^{(N-1)}(x) \equiv \Pr \{b_j^{(N-1)} \leq x\} \), I can focus on the realization of borrowing cost \( s \) among banks. I write the cumulative distribution function of \( j \)th order statistic of \( N - 1 \) realization of \( s \) as \( F_j^{(N-1)}(x) \equiv \Pr \{s_j^{(N-1)} \leq x\} \).
\[ \Pr \left\{ s_j^{(N-1)} \leq x \right\}. \] Strictly increasing \( \beta (\cdot) \) suggests \( G_j^{(N-1)} (\beta (x)) = F_j^{(N-1)} (x) \), as well as the PDFs \( g_j^{(N-1)} (\beta (x)) = f_j^{(N-1)} (x) \).

Therefore, for bank \( i \) with borrowing cost as \( r_f + s_i \), and submitting \( b_i \), replace the \( g_j^{(N-1)} (\cdot) \) with \( f_j^{(N-1)} (\cdot) \) in equation 6 yields

\[
\mathbb{E} [L (b_i, b_{-i}) | I_i] = r_f + \frac{1}{N - 2n} \sum_{j=n+1}^{N-1-n} \mathbb{E} [b_j^{(N-1)}] \\
+ \frac{1}{N - 2n} \int_{s_n(b_i)}^8 \beta (t) f_n^{(N-1)} (t) dt \\
+ \frac{1}{N - 2n} \int_0^{s_n(b_i)} \beta (t) f_{N-n}^{(N-1)} (t) dt \\
+ \frac{1}{N - 2n} b_i \left[ 1 - \int_{s_n(b_i)}^8 f_n^{(N-1)} (t) dt - \int_0^{s_n(b_i)} f_{N-n}^{(N-1)} (t) dt \right].
\]

Since \( \mathbb{E} [L (b_i, b_{-i})] \) is twice differentiable in \( b_i \),

\[
\frac{\partial}{\partial b_i} \mathbb{E} [L (b_i, b_{-i}) | I_i] = \frac{1}{N - 2n} \left[ F_n^{(N-1)} (s_i) - F_{N-n}^{(N-1)} (s_i) \right]
\]

and clearly \( \frac{\partial}{\partial b_i} \mathbb{E} [L (b_i, b_{-i}) | I_i] \) is bounded in \( [0, \frac{1}{N-2n}] \).

Let us get back to the bank \( i \)'s expected payoff \( \mathbb{E} [\Pi_i (s_i, b_i, b_{-i})] \). FOC suggests

\[
\beta^* (s_i) = s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \left[ F_n^{(N-1)} (s_i) - F_{N-n}^{(N-1)} (s_i) \right]
\]

therefore the condition for strictly increasing \( \beta (s_i) \) is \( \max_{x \in \Theta} \left[ f_n^{(N-1)} (x) - f_{N-n}^{(N-1)} (x) \right] < (N - 2n) \frac{2\alpha \psi}{\delta} \).

Writing \( \Delta (s_i) = F_n^{(N-1)} (s_i) - F_{N-n}^{(N-1)} (s_i) \), and by the property of order statistics,
\[ F^{(N-1)}_n(s_i) - F^{(N-1)}_{N-n}(s_i) \geq 0, \text{ therefore} \]

\[ \beta^*(s_i) = s_i - \frac{\delta}{2\alpha\psi(N-2n)} \Delta(s_i) \leq s_i \]

the best response is weakly lower than the actual borrowing cost, and \( \beta(0) = 0 \) regardless of the distribution of borrowing costs \( s \).

Plug in the order statistics, I have

\[ \Delta(s_i) = \frac{(N-1)!}{n! (N-n-1)!} \int_0^{F(s_i)} \frac{(1-y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} \, dy. \]

\[ \square \]

Let us now look at the case in which the monotonicity constraint is binding. Again, I prove a Lemma about the twice differentiability first.

**Lemma A.2.** When constraint \( \partial \beta(x)/\partial x \geq 0 \) binds for \( x \) in a convex set \( \theta \subset [0, \bar{s}] \), \( \mathbb{E}[L(b_i, b_{-i}) | I_i] \) is twice differentiable at \( b_i \neq \bar{b} \), where \( \bar{b} = \beta(x) \) for \( x \in \theta \). At \( b_i = \bar{b} \), \( \mathbb{E}[L(b_i, b_{-i}) | I_i] \) is continuous but not differentiable. In fact, I have \( \forall b_i \in [0, s_i] \)

\[ \frac{\partial}{\partial b_i} \mathbb{E}[L(b_i, b_{-i}) | I_i] = \frac{1}{N-2n} \left[ G^{(N-1)}_n(b_i) - G^{(N-1)}_{N-n}(b_i) \right] \]

so \( \frac{\partial}{\partial b_i} \mathbb{E}[L(b_i, b_{-i}) | I_i] \) is right continuous with left limits (RCLL) at \( b_i = \bar{b} \).

**Proof.** In this case, cumulative distribution function \( G(x) \) is RCLL, and is continuous everywhere except at \( \bar{b} \). In this case, cumulative distribution function \( G^{(N-1)}_j(x) \) is also RCLL, and is continuous everywhere except at \( \bar{b} \). Define

\[ g^{(N-1)}_j(x) = \begin{cases} \frac{\partial}{\partial x} G^{(N-1)}_j(x) & x \neq \bar{b} \\ 0 & x = \bar{b} \end{cases} \]
Suppose that \( \Pr \left\{ b_j^{(N-1)} = \bar{b} \right\} = p_j = 1 - \int_{x \neq \bar{b}} \hat{g}_j^{(N-1)} (x) \, dx \). Using Dirac function \( \delta \), the probability density function is

\[
g_j^{(N-1)} (x) = p_j \delta (x - \bar{b}) + \hat{g}_j^{(N-1)} (x) .
\]

In this case, I can write \( \mathbb{E} \left[ L (b_i, b_{-i}) \right] \) similarly as 6, and I know it is twice differentiable when \( b_i \neq \bar{b} \).

While for \( b_i = \bar{b} + \varepsilon \), and \( \varepsilon \downarrow 0 \), I have

\[
\lim_{\varepsilon \downarrow 0} \frac{\mathbb{E} \left[ L \left( \bar{b} + \varepsilon, b_{-i} \right) \mid \mathcal{I}_i \right] - \mathbb{E} \left[ L \left( \bar{b}, b_{-i} \right) \mid \mathcal{I}_i \right]}{\varepsilon} = \frac{1}{N - 2n} \left[ G_n^{(N-1)} (\bar{b}) - G_{N-n}^{(N-1)} (\bar{b}) \right]
\]

at the same time

\[
\lim_{\varepsilon \uparrow 0} \mathbb{E} \left[ L \left( \bar{b} + \varepsilon, b_{-i} \right) \mid \mathcal{I}_i \right] = \mathbb{E} \left[ L \left( \bar{b}, b_{-i} \right) \mid \mathcal{I}_i \right]
\]

and

\[
\lim_{\varepsilon \uparrow 0} \frac{\mathbb{E} \left[ L \left( \bar{b}, b_{-i} \right) \mid \mathcal{I}_i \right] - \mathbb{E} \left[ L \left( \bar{b} + \varepsilon, b_{-i} \right) \mid \mathcal{I}_i \right]}{\varepsilon} = \frac{1}{N - 2n} \left[ G_n^{(N-1)} (\bar{b}_{-}) - G_{N-n}^{(N-1)} (\bar{b}_{-}) \right].
\]

Hence I have shown that at \( \bar{b} \), \( \mathbb{E} \left[ L (b_i, b_{-i}) \mid \mathcal{I}_i \right] \) is continuous and \( \frac{\partial}{\partial b_i} \mathbb{E} \left[ L (b_i, b_{-i}) \mid \mathcal{I}_i \right] \) is RCLL.

Before working on the equilibrium strategy under monotonicity constraint, let’s look at the relationship between the \( j \)th order statistic cumulative distribution function of borrowing cost \( F_j^{(N-1)} (\cdot) \), and that of banks’ submissions \( G_j^{(N-1)} (\cdot) \).
Corollary A.3. For any borrowing cost $s$ such that the monotonicity constraint is not binding, there is a one-to-one mapping between the cumulative distribution function of borrowing cost $F(\cdot)$ to the cumulative distribution function of submissions $G(\cdot)$. Similarly, there is a one-to-one mapping between the $j$th order statistic cumulative distribution function of borrowing cost $F_j^{(N-1)}(\cdot)$ to that of submissions $G_j^{(N-1)}(\cdot)$.

Notice in the proof of Proposition 3, I need the strict monotonicity of $\beta(s)$ to get a one-to-one mapping from the cumulative distribution function of borrowing cost $F(\cdot)$ to the cumulative distribution function of submissions $G(\cdot)$. The region $\theta$ in which the monotone constraint is binding makes the one-to-one mapping not hold everywhere. However, as shown in Figure 3, I still have $G(\beta(x)) = \Pr\{\beta(y) \leq \beta(x)\} = \Pr\{y \leq x\} = F(x)$, for the area $\Theta \setminus [s_1, s_2]$. I can proceed to the proof of Proposition 4.

**Proof of Proposition 4**

Proof. The constrained problem is to find $\beta(\cdot)$, using its derivative $\mu(\cdot)$ as a constrained control, and

$$\max_{\beta} \int_0^s \left\{ \delta [L_0 - \mathbb{E} \{L(\beta(s_i), b_{-i}) | I_i]\} - \alpha \psi(s_i - \beta(s_i))^2 \right\} f(s_i) ds_i$$

s.t.

$$\partial \beta(s_i) / \partial s_i = \mu(s_i) \geq 0.$$

According to Gelfand and Fomin (1963), with the continuity proved in Lemma A.2
the Hamiltonian is

\[
H(s_i, \beta(s_i), \lambda(s_i), \mu(s_i)) = \{ \delta [L_0 - \mathbb{E} [L(\beta(s_i), b_{-i}) | \mathcal{I}_i]] - \alpha \psi (s_i - \beta(s_i))^2 \} f(s_i) \\
+ \lambda(s_i), \mu(s_i)
\]

and Pontryagin’s principle suggests an optimum \(\beta^*(s_i)\) and \(\mu^*(s_i)\) satisfies

\[
H(s_i, \beta^*(s_i), \lambda(s_i), \mu^*(s_i)) \geq H(s_i, \beta(s_i), \lambda(s_i), \mu(s_i))
\]

and in the region that derivative exists,

\[
\frac{\partial \lambda(s_i)}{\partial s_i} = -\frac{\partial H(s_i, \beta(s_i), \lambda(s_i), \mu(s_i))}{\partial \beta(s_i)}.
\]

From complementary slackness, when constraint is not binding, I have \(\lambda(s_i) = 0\), so \(\partial \lambda(s_i) / \partial s_i = 0\). Therefore, for \(s \notin \theta\), the FOC is the same as before:

\[
\beta^*(s_i) = s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \left[ G_n^{(N-1)} (\beta^*(s_i)) - G_{N-n}^{(N-1)} (\beta^*(s_i)) \right].
\]

From the Corollary A.3, for \(s_i \in \Theta \setminus \theta\), there is a one-to-one mapping from \(F(s_i)\) to \(G(\beta(s_i))\), therefore,

\[
\beta^*(s_i) = s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \left[ F_n^{(N-1)} (s_i) - F_{N-n}^{(N-1)} (s_i) \right] \quad \forall s_i \in \Theta \setminus \theta
\]

which is the same as the unconstrained optimization problem.

I now prove that \(\beta^*(\cdot)\) is continuous. It is straightforward to see \(\beta^*(s_i)\) is continuous for \(s_i \in (\underline{s}, \bar{s})\), as well as \(s_i \in [0, \underline{s}) \cup (\bar{s}, \bar{s}]\). The continuity of Hamiltonian \(H(s_i, \beta^*(s_i), \lambda(s_i), \mu^*(s_i))\) as well as the costate variable \(\lambda(s_i)\) when \(s_i = \{\underline{s}, \bar{s}\}\)
suggests $\beta^*(s_i)$ is also continuous at point $\theta$ and $\bar{\theta}$.

I now focus on how to determine $\bar{b}$, as shown in Figure 3. The continuity of $\beta(\cdot)$ together with the constraint suggests that $\beta^*(\theta) = \beta^*(\bar{\theta}) = \bar{b}$, the bunching level. In order to determine the bunching level $\bar{b}$, I write

$$J(\bar{b}) = \int_{\theta}^{\bar{b}} \left\{ \delta [L_0 - \mathbb{E} [L (\beta^*(s_i), b_{-i}) | \mathcal{I}_i]] - \alpha \psi (s_i - \beta^* (s_i))^2 \right\} f (s_i) \, ds_i$$

$$+ \int_{\bar{b}}^{\bar{\theta}} \left\{ \delta [L_0 - \mathbb{E} [L (\bar{b}, b_{-i}) | \mathcal{I}_i]] - \alpha \psi (s_i - \bar{b})^2 \right\} f (s_i) \, ds_i$$

$$+ \int_{\bar{\theta}}^{\bar{s}} \left\{ \delta [L_0 - \mathbb{E} [L (\beta^*(s_i), b_{-i}) | \mathcal{I}_i]] - \alpha \psi (s_i - \beta^* (s_i))^2 \right\} f (s_i) \, ds_i$$

in which $\{\theta, \bar{\theta}\}$ are the minimum and maximum values of $\beta^{-1}(\bar{b})$. If $\bar{b}$ is indeed the optimal bunching level, variation of the level should not change $J(\bar{b})$ in the first order. Therefore FOC suggests

$$\frac{\partial}{\partial \bar{b}} J(\bar{b}) = 0$$

which can be used to solve for $\bar{b}$. \qed

**Proof of Propositions 5**

*Proof.* Assume $\beta(s)$ to be strictly increasing, banks’ submissions maintain the same order as their borrowing costs $b_i^{(N)} = \beta(s_i^{(N)}) \forall i \in \{1, ..., N\}$. Similar to the proof of Propositions 5, it is easy to see that $\mathbb{E} [L (b_i, b_{-i})]$ is twice differentiable in $b_i$,

$$\frac{\partial}{\partial b_i} \mathbb{E} [L (b_i, b_{-i}) | \mathcal{I}_i] = \frac{1}{N-2n} \left[ F_n^{(N-1)} (\beta^{-1} (b_i)) - F_{n-n}^{(N-1)} (\beta^{-1} (b_i)) \right]$$

and $\frac{\partial}{\partial b_i} \mathbb{E} [L (b_i, b_{-i}) | \mathcal{I}_i]$ is bounded in $[0, \frac{1}{N-2n}]$. 42
The FOC of bank’s expected payoff \( E[\Pi_i (s_i, b_i, b_{-i})] \) now suggests

\[
\beta^* (s_i) = s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \left[ F_{n}^{(N-1)} (s_i) - F_{N-n}^{(N-1)} (s_i) \right] - \rho \delta \frac{p}{1-p} \phi \exp \{ \eta \beta^* (s_i) \}.
\]

Writing \( \Delta (s_i) = F_{n}^{(N-1)} (s_i) - F_{N-n}^{(N-1)} (s_i) \), and introduce the Lambert-W function \( W(z) \) as the solution of the equation \( z = W(z) \exp \{ W(z) \} \), I can solve the \( \beta^* (s_i) \) as

\[
\beta^* (s_i) = s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \Delta (s_i) - \frac{1}{\eta} W \left( \frac{\rho \delta \phi \eta^2}{2\alpha \psi} \frac{p}{1-p} \exp \left\{ \eta \left( s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \Delta (s_i) \right) \right\} \right).
\]

Similarly, the best response is weakly lower than the actual borrowing cost.

Plugging in the order statistics, I have

\[
\Delta (s_i) = \frac{(N - 1)!}{n! (N - n - 1)!} \int_{0}^{F(s_i)} \frac{(1-y)^{N-2n} - y^{N-2n}}{[y(1-y)]^{1-n}} dy
\]

where \( \bar{F}(s_i) = pF_r (s_i) + (1 - p) F_g (s_i) \). Notice in this case I have assumed the borrowing costs across banks are normally distributed.

I then check the condition for strictly increasing bidding strategy. It is easy to show the derivative of Lambert-W function

\[
\frac{dW(z)}{dz} = \frac{W(z)}{z (1 + W(z))}
\]

for \( z \notin \{0, -\frac{1}{e}\} \). Therefore,

\[
\frac{\partial \beta^* (s_i)}{\partial s_i} = \frac{1}{1+W(Z)} \left( 1 - \frac{\delta}{2\alpha \psi (N - 2n)} \Delta' (s_i) \right)
\]

where \( Z = \frac{\rho \delta \phi \eta^2}{2\alpha \psi} \frac{p}{1-p} \exp \left\{ \eta \left( s_i - \frac{\delta}{2\alpha \psi (N - 2n)} \Delta (s_i) \right) \right\} \).

Since \( W(z) > 0 \) for all \( z > 0 \), \( \frac{\partial \beta^* (s_i)}{\partial s_i} > 0 \) as long as \( 1 - \frac{\delta}{2\alpha \psi (N - 2n)} \Delta' (s_i) > 0 \).
for all $s_i$. Therefore the same condition for the strictly increasing equilibrium applies to the signaling case. \hfill $\square$

**Proof of Propositions 6**

*Proof.* The Proof of Proposition 4 also works in the signaling case as long as I replace the payoff function of banks. \hfill $\square$

**Proof of Proposition 11**

*Proof.* Suppose banks adopt symmetric strategy. When there is no penalty, $\alpha \psi = 0$,

$$\frac{\partial}{\partial \beta (s_i)} \mathbb{E} [\Pi_i (s_i, \beta (s_i)) | \mathcal{L}_i] = -\frac{\delta}{N-2n} \left[ F_{n}^{(N-1)} (s_i) - F_{N-n}^{(N-1)} (s_i) \right] < 0$$

so for any $\beta (s_i) > 0$, there is always another $\tilde{\beta} (s_i) = \beta (s_i) - \varepsilon < \beta (s_i)$ that Pareto dominates $\beta (s_i)$. In other words, banks will report the lowest possible borrowing costs, aka $b = \beta (s_i) = 0$, and the LIBOR rate will converge to the riskless rate.

In case $\alpha \psi > 0$, assume $\beta (s_i) = 0$ for some $s_i > 0$, then

$$\frac{\partial}{\partial \beta (s_i)} \mathbb{E} [\Pi_i (s_i, \beta (s_i)) | \mathcal{L}_i] \bigg|_{\beta (s_i) = 0} = \alpha \psi s_i > 0$$

so submitting $\tilde{\beta} (s_i) = \varepsilon > 0$ Pareto dominates submitting $\beta (s_i) = 0$. Hence banks’ submissions will be higher than the riskless rate. \hfill $\square$

Finally I prove Proposition 14.
Proof of Proposition 14

Proof. I first prove the mechanism is incentive compatible. Assume bank \( i \) reports \( z_i \) when all the other banks report their true borrowing cost \( z_{-i} = s_{-i} \). Bank \( i \)'s payoff function is

\[
\delta [L_0 - L (z_i, z_{-i})] + \rho \delta [\pi (s_i) - \pi (z_i)] - M^*_i (z_i, z_{-i})
\]

therefore the optimal reporting value \( z^*_i \) is

\[
z^*_i = \arg \max_{z_i} \delta [L_0 - L (z_i, z_{-i})] + \rho \delta [\pi (z_i) - \pi (\beta^* (z_i))] - M^*_i (z_i, z_{-i})
\]

Notice that

\[
L (z_i, z_{-i}) = r_f + \frac{1}{N-2n} \sum_{j=n+1}^{N-n} z_j^{(N)}
\]

plug in the \( M^*_i (z_i, z_{-i}) \) and the \( \frac{1}{N-2n} \sum_{j=n+1}^{N-n} z_j^{(N)} \) term cancels out. Removing all the terms that do not contain the choice variable \( z_i \) gives us

\[
z^*_i = \arg \max_{z_i} \mathbb{E} \left[ \delta \left[ L_0 - \frac{1}{N-2n} \sum_{j=n+1}^{N-n} \beta^* \left( z_j^{(N)} \right) \right] - \alpha \psi (z_i - \beta^* (z_i))^2 + \rho \delta [\pi (z_i) - \pi (\beta^* (z_i))] \right].
\]

The construction of \( \beta^* \) suggests

\[
z^*_i = s_i
\]

hence \( (L^*, M^*) \) is direct and efficient.
In addition,

\[
\sum_{i=1}^{N} \mathbb{E} [M_i^* (z)] = \sum_{i=1}^{N} \mathbb{E} [M_i^* (s)]
\]

\[
= -\frac{\delta}{N - 2n} \sum_{i=1}^{N} \sum_{j=n+1}^{N-n} \mathbb{E} \left[ s_j^{(N)} - \beta^* \left( s_j^{(N)} \right) \right]
\]

\[
+ \frac{\delta}{N - 2n} \sum_{i=1}^{N} \sum_{j=n+1, s_j^{(N)} \neq s_i}^{N-n} \mathbb{E} \left[ s_j^{(N)} - \beta^* \left( s_j^{(N)} \right) \right]
\]

\[
+ \sum_{i=1}^{N} \left\{ \alpha \psi \mathbb{E} \left[ (s_i - \beta^* (s_i))^2 \right] - \frac{1}{N - 1} \sum_{k=1, k \neq i}^{N} \alpha \psi \mathbb{E} \left[ (s_k - \beta^* (s_k))^2 \right] \right\}
\]

\[
+ \sum_{i=1}^{N} \left\{ \rho \delta \mathbb{E} \left[ \pi (s_i) - \pi (\beta^* (s_i)) \right] - \frac{1}{N - 1} \sum_{k=1, k \neq i}^{N} \rho \delta \mathbb{E} \left[ \pi (s_k) - \pi (\beta^* (s_k)) \right] \right\}
\]

\[
= 0
\]

suggests \((L^*, M^*)\) is \textit{ex ante} budget balanced.

About individual rationality: since the mechanism is direct, so \(z_i^* = s_i\), the \textit{ex ante} payoff of bank under the mechanism \((L^*, M^*)\) is

\[
\mathbb{E} \left[ \delta \left( L_0 - L (z_i^*, z_{-i}^*) \right) + \rho \delta \left[ \pi (z_i) - \pi (\beta^* (z_i)) \right] - M_i^* (z^*) \right] = -\mathbb{E} [M_i^* (s)]
\]

since \(L_0 = L (s_i, s_{-i})\).
By the equation of $M_i^*(s)$,

$$
\mathbb{E}[M_i^*(s)] = \frac{\delta}{N-2n} \mathbb{E}
\left[
- \sum_{j=n+1}^{N-n} \left[ s_j^{(N)} - \beta^* \left( s_j^{(N)} \right) \right] + \frac{N}{N-1} \sum_{j=n+1, z_j^{(N)} \neq z_i}^{N-n} \left[ s_j^{(N)} - \beta^* \left( s_j^{(N)} \right) \right]
\right] + \alpha \psi \mathbb{E}
\left[
(s_i - \beta^*(s_i))^2 - \frac{1}{N-1} \sum_{k=1, k \neq i}^{N} (s_k - \beta^*(s_k))^2
\right] + \rho \delta \mathbb{E}
\left[
\pi(s_i) - \pi(\beta^*(s_i)) - \frac{1}{N-1} \sum_{k=1, k \neq i}^{N} (\pi(s_k) - \pi(\beta^*(s_k)))
\right].
$$

Notice the expectation is over the distribution of $s = \{s_1, ..., s_N\}$, so I have

$$
\mathbb{E}
\left[
- \sum_{j=n+1}^{N-n} \left[ s_j^{(N)} - \beta^* \left( s_j^{(N)} \right) \right] + \frac{N}{N-1} \sum_{j=n+1, z_j^{(N)} \neq z_i}^{N-n} \left[ s_j^{(N)} - \beta^* \left( s_j^{(N)} \right) \right]
\right] = 0
$$

$$
\mathbb{E}
\left[
(s_i - \beta^*(s_i))^2 - \frac{1}{N-1} \sum_{k=1, k \neq i}^{N} (s_k - \beta^*(s_k))^2
\right] = 0
$$

$$
\mathbb{E}
\left[
\pi(s_i) - \pi(\beta^*(s_i)) - \frac{1}{N-1} \sum_{k=1, k \neq i}^{N} (\pi(s_k) - \pi(\beta^*(s_k)))
\right] = 0
$$

therefore the mechanism is \textit{ex ante} individually rational. \hfill \Box

**References**


