Feedback Trading (Preliminary version)

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ABSTRACT

In the microstructure literature, it has been demonstrated that order flow, one way buying or selling pressure, carries information to the market. When assessing *how* informative order flow is, the VAR methodology is typically employed, using impulse response functions, following Hasbrouck (1991). However, in such analyses, the direction of causality runs explicitly from order flow to asset return. If data are sampled at anything other than at the highest frequencies then any feedback trading may well appear contemporaneous; trading in period t depends on the asset return in that interval. The implications of contemporaneous feedback trading are examined in the spot USD/EUR currency market and we find that when data are sampled at the five minute frequency, such trading strategies cause the price impact of order flow to be significantly larger than when feedback trading is ruled out.

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In the microstructure literature it has been demonstrated that ‘order flow’, one way buying or selling pressure, carries information from more informed participants to the market as a whole. In this way, information is aggregated via the trading process and so order flow should have permanent effects on prices. Such ideas are not new; see for example Kyle (1985) and Glosten and Milgrom (1985). In order to assess and quantify the informativeness of order flow, the VAR model of Hasbrouck (1991) has frequently been used. However, in this framework the direction of causality runs explicitly from order flow to asset price returns. This intuition is sensible if the data are sampled at ultra high frequencies, such as tick-by-tick. But as soon as one starts aggregating the data, possibly even at the one minute frequency, the problem of ‘contemporaneous feedback trading’ may appear, with serious repercussions when trying to infer the informativeness of trades. Feedback trading, whereby traders react to previous asset price returns, will appear contemporaneous when aggregated at lower, although also possibly high, frequencies. In this case, asset returns will depend on contemporaneous order flows[1] but order flows will also depend on contemporaneous asset returns. The causality, when considering these aggregated time series, may then run in *both* directions. It is the purpose of this paper to examine the importance of feedback trading in VAR models, done by comparing the feedback models with the common restricted VARs where contemporaneous feedback trading is ruled out.

Hasbrouck (1991) introduces methods to evaluate the informativeness of trades in microstructure models. If trades carry private information then one way of analysing the information content of trades is to consider their price impact. Hasbrouck (1991) introduces a shock to the trading process, representing private information, and computes the cumulated effect on the asset return. The greater the cumulated effect, or impulse response, the more information trades are argued to carry. These vector autoregressive models have become standard in the microstructure literature; recent examples include Dufour and Engle (2000) and Engle and Patton (2004) for stocks, Evans (2002), Evans and Lyons (2002b) and Payne (2003) for currencies and Cohen

[1]Models that generate such relationships include Kyle (1985), Glosten and Milgrom (1985), Easley and O’Hara (1987), Madhavan and Smidt (1991) and Evans and Lyons (2002b).

and Shin (2003) and Green (2003) for treasuries. However, in the standard VAR models, the asset return in period t is regressed on contemporaneous order flow (date t) as well as lagged returns and order flows (dated $t-1$ or earlier), whereas order flows are only regressed on lagged returns and flows; order flows at date t do not depend on contemporaneous asset returns. Even though such models allow for some feedback trading (order flow can still depend on previous asset returns, as analysed in Cohen and Shin (2003)), they rule out contemporaneous feedback trading, an assumption which appears overly restrictive when the data are sampled at anything other than at the highest frequencies. Following a shock to order flows within period t, usually taken to represent private information, this will cause a change in the asset price within that interval. If other traders react to this price change by buying or selling the asset themselves in that period, perhaps because they expect a wave of trading activity that will push the price in one direction or another, this will have serious implications on market dynamics and on estimates of trade informativeness. Possible reasons for the existence of feedback trading will be discussed in Section 4. If a positive order flow shock causes an increase in the asset price, which in turn causes an *increase* in order flows via feedback trading within that period, the total effect/price impact of the order flow shock will be higher than when feedback trading is ruled out. Alternatively, if there exists negative feedback trading, perhaps because of expected return reversals of the initial asset price change, the ultimate price impact of the trade will be smaller than the non-feedback case. The consequence of ignoring contemporaneous feedback trading when feedback trading exists will be to bias any estimate of the price impact/informativeness of trades.

The reason for imposing the restriction of no feedback trading, other than the intuition in the theoretical literature that causality runs from flows to returns, which, as explained above, only seems reasonable at the ultra high/tick-by-tick frequencies, is because without such a restriction, the VAR becomes unidentified. By allowing returns to depend on flows but ruling out the converse, the simple two equation VAR (in returns and flows) becomes a recursively ordered structural VAR, which is just identified when the variance/covariance matrix of the residuals is restricted to be diagonal. The structural parameter commonly estimated, the contemporaneous effect of order flow on returns, can only be estimated at the expense of a restriction made elsewhere, namely that on the distribution of the errors. If one wanted to estimate another structural parameter, i.e. the feedback trading parameter (the effect of contemporaneous asset returns on order flows), another restriction must be made but there are none left available.[2] By making the assumption that causality runs explicitly from order flows to prices, one need not worry about the second structural (feedback) parameter. However, this assumption, no matter how convenient for estimation purposes, has to be tested but thus far, it has escaped the econometrician's examination.

In this paper we examine the spot USD/EUR (US dollar per euro) foreign exchange market and compare the price impact/informativeness of order flow shocks with and without feedback trading. The data are taken from the Reuters D2000-2 electronic trading system, one of the two dominant brokered trading platforms used in the interdealer spot FX markets and cover the eight month period from 1st December 1999 to 24th July 2000.[3] In order to estimate the structural, feedback VAR for the USD/EUR market we use instrumental variables, where the instruments are statistics obtained from the closely linked markets of USD/GBP (US dollar per pound sterling) and GBP/EUR (pound sterling per euro).[4] It is clear that the USD/EUR rate should, in the absence of arbitrage, equal the USD/GBP rate multiplied by the GBP/EUR rate. Statistics obtained from these markets, in particular returns and order flows, may then be correlated with the endogenous variables for which we are trying to instrument, i.e. USD/EUR returns and flows. The usefulness of these statistics as

[2] Blanchard and Quah (1989) suggest another restriction for identification; that one type of shock has no *long run* effect on one of the other variables. This assumption cannot be used in this setting. Order flow shocks are likely to have permanent effects on the asset price, as in the models of Kyle (1985), etc., and asset return shocks are likely to have non-zero long run effects on cumulative order flow due to reasons of portfolio rebalancing for example.

[3] The other electronic trading system is that of EBS and together they account for between 85% and 95% of all interdealer trading. See Bank for International Settlements (2001).

[4] When both structural parameters are allowed to vary freely, OLS will induce the familiar simultaneity bias since each of the endogenous explanatory variables will be correlated with the error in each equation, i.e. flows (returns) will be correlated with the error in the structural return (flow) equation.

instruments will be discussed and tested in Section 2.2.

In order to evaluate the importance of the feedback trading parameter, we consider two sampling frequencies; one minute and five minutes.[5] From both frequencies we estimated the VARs with and without feedback trading and calculated impulse response functions following an order flow shock, representing private information, in order to assess the informativeness of trades.

We find that for the one minute frequency, the feedback trading parameter in the structural VAR is positive and significant at the 1% level and the impulse response function following an order flow shock is larger when feedback trading is permitted. However the difference between the restricted and unrestricted IRFs is not significantly different, at the 5% level. At the five minute frequency, the feedback trading parameter in the VAR is quantitatively large and significant at the 5% level, and the impulse response without feedback trading is significantly below that when feedback trading is permitted. This suggests that in the case of spot FX markets, feedback trading is prevalent and has significant implications when examining the price impact/informativeness of order flow, especially when the data are sampled at the lower/five minute frequency. For the spot FX market considered in this exercise, feedback trading is positive, i.e. order flow in one period depends positively on the asset return in that period. This results in the price impact of an order flow shock being significantly greater than when one imposes a recursive ordering of the VAR. Private information, in the form of unanticipated order flow shocks, then has a larger impact on returns than previously believed, i.e. trades carry more information than previous estimates suggest.

This paper examines the effect of feedback trading on measures of trade informativeness and offers a simple solution as to how one can estimate an otherwise unidentified model. The use of instrumental variables is standard in this estimation exercise and the only real obstacle to estimating a non restricted VAR, i.e. that allows con-

[5]When the data were sampled at lower frequencies the abilities of other market statistics to instrument for USD/EUR endogenous variables deteriorated to such an extent that such analysis became pointless.

temporaneous feedback trading, is the need to find good/strong instruments. The instruments used for the endogenous USD/EUR variables are variables obtained from the related USD/GBP and GBP/EUR markets. In so far as many markets, whether they are currency, stocks or bonds, have a number of markets related to them, it would appear that instruments should be readily available to the econometrician in order to allow for contemporaneous feedback trading.[6] The question of *which* instruments to use is simply an empirical one. One may simply choose those instruments found to be strongly correlated with the endogenous right hand side regressors.

The rest of the paper is organised as follows. Section 1 motivates the need to model contemporaneous feedback trading when data are aggregated at any level. Section 2 introduces the model to be estimated and describes the standard techniques to be employed as well as explanations as to how to obtain analytical confidence bounds for the impulse response functions. Section 2 also discusses the choice of instruments used. Section 3 presents the regression results and reports the impulse response functions. Section 4 discusses our findings, placing them within the existing finance literature and Section 5 concludes.

1 The Inevitability of Contemporaneous Feedback Trading in Aggregated Data

In this section we show how contemporaneous feedback trading can result simply from considering aggregated data. By definition, contemporaneous feedback trading, whereby order flows at date t depend on date t asset returns, cannot occur in tick-by-tick data; a trader can only respond to a price change once the price has indeed changed. Assume that tick-by-tick event time data can be approximated by data

[6] In the case of stocks, when trying to estimate a feedback trading VAR for IBM, one could try using statistics based on Hewlett Packard flows and returns for example, or any stock in the same or related industry.

aggregated at the ten second calendar time frequency.[7] Also assume, for simplicity, that returns and flows can be characterised by a VAR with only 1 lag. Since the possibility of contemporaneous feedback trading is ruled out, the system of equations can be written as:

$$\begin{aligned} R_t &= \alpha_1 + \beta F_t + \phi_{11} R_{t-1} + \phi_{12} F_{t-1} + \epsilon_t^R \\ F_t &= \alpha_2 + \phi_{21} R_{t-1} + \phi_{22} F_{t-1} + \epsilon_t^F \end{aligned} \tag{1}$$

where R_t is the return on the asset in period t, defined as the log first difference of the price, F_t is the order flow in period t; the number of buyer less seller initiated trades and ϵ_t^R and ϵ_t^F are serially uncorrelated, independent errors with variances $\sigma^2_{\epsilon^R}$ and $\sigma^2_{\epsilon^F}$ respectively. The system can be written as a structural VAR:

$$\begin{gathered} \underbrace{\begin{bmatrix} 1 & -\beta \\ 0 & 1 \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} R_t \\ F_t \end{bmatrix}}_{y_t} = \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}}_{\alpha} + \underbrace{\begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}}_{\phi} \underbrace{\begin{bmatrix} R_{t-1} \\ F_{t-1} \end{bmatrix}}_{y_{t-1}} + \underbrace{\begin{bmatrix} \epsilon_t^R \\ \epsilon_t^F \end{bmatrix}}_{\epsilon_t} \\ \Rightarrow Q y_t = \alpha + \phi y_{t-1} + \epsilon_t \qquad\qquad Var\left(\epsilon_t\right) = \Omega = \begin{bmatrix} \sigma^2_{\epsilon^R} & 0 \\ 0 & \sigma^2_{\epsilon^F} \end{bmatrix} \end{gathered} \tag{2}$$

where $y_t = [R_t \; F_t]'$ and $\epsilon_t = \left[\epsilon_t^R \; \epsilon_t^F\right]'$, $t = 1, \ldots, 2T$. What we wish to do is to move from a structural VAR at the ten second frequency to a structural VAR at the twenty second frequency. However, in order to do this, (2) must be converted into a reduced form and put into state space representation. Aggregation can then be performed using the methods in Harvey (1989). The reduced form of (2), at the ten second

[7] The choice of the ten second frequency is arbitrary. Essentially, we are choosing a calendar time frequency whereby contemporaneous feedback trading is impossible/negligible. The assumption that tick-by-tick event time data can be approximated by equi-temporally spaced high-frequency calendar time data is technically incorrect. However, for the purpose of this study we simply wish to examine data at a frequency where no contemporaneous feedback trading can occur and suggest that the ten second frequency is sensible. We then see what happens when we aggregate the data at a lower frequency.

frequency, is clearly

$$y_t = Q^{-1}\alpha + Q^{-1}\phi y_{t-1} + Q^{-1}\epsilon_t \qquad t = 1, \dots, 2T \tag{3}$$

and the corresponding reduced form at the twenty second frequency is shown in Appendix A.1 to be

$$y_\tau = \mu^+ + Ay_{\tau-1} + e_\tau \qquad \tau = 1, \dots, T \tag{4}$$

where $A = Q^{-1}\phi\left(I_2 + Q^{-1}\phi\right)$. Here we use subscript t to denote ten second frequency data and τ to denote data at the twenty second frequency. Since returns and order flows are both flow variables, as opposed to stocks, then the twenty second return is simply the sum of the two ten second returns in that period, and similarly for order flows. The variance of e_τ is given by

$$Var\left(e_\tau\right) = G\Omega G' + Q^{-1}\Omega\left(Q^{-1}\right)' \tag{5}$$

where $G = \left(I_2 + Q^{-1}\phi\right)Q^{-1}$. To convert (4) into a structural VAR, we first factorise the variance of e_τ. This is done in (6).

$$Var\left(e_\tau\right) = \begin{bmatrix} G & Q^{-1} \end{bmatrix} \begin{bmatrix} \Omega & 0 \\ 0 & \Omega \end{bmatrix} \begin{bmatrix} G' \\ \left(Q^{-1}\right)' \end{bmatrix} = P\tilde{\Omega}P' \tag{6}$$

If we premultiply (4) by $\left[I_2\ I_2\right]P^+$, where P^+ is the Moore-Penrose inverse of the 2×4 matrix P, $P^+ = \left(P'P\right)^{-1}P'$, then the structural form can be written as:

$$\left[I_2\ I_2\right]P^+y_\tau = \left[I_2\ I_2\right]P^+\mu^+ + \left[I_2\ I_2\right]P^+Ay_{\tau-1} + \left[I_2\ I_2\right]P^+e_\tau \tag{7}$$

The variance of the error vector in the structural form is now

$$Var\left(\left[I_2\ I_2\right]P^{+}e_\tau\right) = 2\Omega \tag{8}$$

(7) therefore has the appealing property that the variance of the error (return or flow) at the twenty second frequency is twice that of the error at the ten second frequency.[8] In order to solve for the structural parameters in (7), note that if we let $x_\tau = P^{+}y_\tau$, then x_τ is the solution to

$$\underbrace{\begin{bmatrix} g_{11} & g_{12} & 1 & \beta \\ g_{21} & g_{22} & 0 & 1 \end{bmatrix}}_{[G\ Q^{-1}]=P} \underbrace{\begin{bmatrix} x_{1\tau} \\ x_{2\tau} \\ x_{3\tau} \\ x_{4\tau} \end{bmatrix}}_{x_\tau} = \underbrace{\begin{bmatrix} R_\tau \\ F_\tau \end{bmatrix}}_{y_\tau} \tag{9}$$

There are clearly an infinite number of solutions for x_τ and this is to be expected when considering the literature on simultaneous equation models; premultiply any structural form by a non-singular matrix and the reduced form, (4), will be unaffected. However, $[I_2\ \ I_2]\,P^{+}y_\tau$, the right hand side of the structural equation, (7), will have the general form

$$\begin{aligned} &\begin{bmatrix} \left(\frac{1}{g_{11}} + \frac{g_{12}g_{21}}{g_{11}|G|}\right) & -\left(\frac{g_{12}}{|G|}\right) \\ -\left(\frac{g_{21}}{|G|}\right) & \left(\frac{g_{11}}{|G|}\right) \end{bmatrix} \begin{bmatrix} R_\tau \\ F_\tau \end{bmatrix} \\ &+ \begin{bmatrix} \left(1 - \frac{1}{g_{11}} + \frac{g_{21}}{|G|}\right) & -\left(\frac{\beta}{g_{11}} + \frac{(g_{11}-g_{21}\beta)}{|G|}\right) \\ \left(\frac{g_{21}}{|G|}\right) & \left(1 - \frac{(g_{11}-g_{21}\beta)}{|G|}\right) \end{bmatrix} \begin{bmatrix} m_\tau \\ n_\tau \end{bmatrix} \end{aligned} \tag{10}$$

for any real values of m_τ and n_τ, $\tau = 1, \ldots, T$. For a proof, see Appendix A.2. Therefore there are an infinite number of structural VARs at the twenty second

[8] Note that the errors in the return and flow equations at the ten second frequency are serially uncorrelated and independent, as too are those at the twenty second frequency.

frequency that are consistent with the recursively ordered structural ten second VAR in (2). The first row of (10) can be interpreted as the left hand side of the structural return equation and the bottom row can be interpreted as the left hand side of the structural flow equation. However, a common choice of structural form in (7) is the Choleski solution. This essentially chooses the arbitrary values of m_τ and n_τ so that the R_τ term in the structural flow equation drops out. The system then becomes a recursively ordered structural VAR so that flows only depend on lagged returns and flows, and standard econometric methods can be used to estimate the model. For all other solutions though, the coefficient on contemporaneous returns in the flow equation will depend on g_{21}. Only if g_{21} equals zero will there be no contemporaneous feedback trading. In Appendix A.1, g_{21} is shown to be equal to ϕ_{21}, the coefficient on lagged returns in the flow equation in the ten second VAR. This is perfectly intuitive. If the order flow from seconds 11 to 20 depends on the return from seconds 1 to 10, then part of the order flow from seconds 1 to 20 will depend on part of the return from seconds 1 to 20, i.e. contemporaneous feedback trading exists! The question then becomes, on what grounds should the Choleski solution be chosen? The justification for choosing the Choleski solution is always on the basis that it makes life easier; the model becomes just identified and estimation can take place simply using OLS. In this paper, we suggest that one should use the data to tell us what the coefficient on returns in the flow equation should be, rather than assuming it to be zero. The data to be used are those from the GBP/EUR and USD/GBP markets, done through using instrumental variables. In Section 3.1 we show that using data to calculate the effect of contemporaneous returns on flows, rather than assuming the coefficient to be zero, can have serious implications when calculating the price impact of trades and hence on estimates of the information content of trades.

2 The VAR Model with Feedback Trading

The VAR model, originally introduced by Sims (1980) and implemented in the microstructure literature by Hasbrouck (1991), is a simple statistical framework that allows us to examine the relationships between asset returns and trading activity; more specifically, order flows. By interpreting order flow shocks as private information, one can examine the price impact of such shocks and therefore give a quantitative estimate of the information content of trades. The greater the price impact, the more information trades are argued to carry.

2.1 Model design

The standard VAR model allows asset returns to depend on contemporaneous order flows but not the converse. From Section 1 we saw that when data are aggregated, even at still very high frequencies, this recursive ordering may not be valid. Here we allow both returns and order flows to depend on each other contemporaneously. The model we wish to estimate can be written as:

$$y_t + By_t = c + \sum_{j=1}^{p} \phi_j y_{t-j} + \epsilon_t \qquad t = 1, \ldots, T \tag{11}$$

In our example, y_t is simply the 2×1 vector of endogenous variables at date t, i.e. returns and flows; $y_t = [R_t \ F_t]'$. In the appendix we generalise the analysis from the 2 to the n variable case.[9] c is a 2×1 vector of constants, the summation term contains the lags of the VAR and ϵ_t is a 2×1 vector of residuals with zero mean and variance matrix Ω, assumed to be diagonal. B is the 2×2 matrix of structural parameters

[9] In the model of Engle and Patton (2004), returns of ask prices and returns of bid prices are considered separately, so y_t need not be restricted to be a 2×1 vector. In the original Hasbrouck (1991) setting, y_t also contained a number of trade related variables including trade sign and the interactions between trade sign and volume and spread.

with zeros along the main diagonal.[10]

$$B = \begin{bmatrix} 0 & -b_{12} \\ -b_{21} & 0 \end{bmatrix} \tag{12}$$

b_{12} represents the contemporaneous effect of flows on returns and b_{21} represents the contemporaneous feedback trading parameter. For each equation, $i = 1,\ 2$, we can stack the k exogenous and $n-1(=1)$ endogenous regressors into a $(1+k)\times 1$ vector, z_{it}. Stacking these vectors across the T observations allows us to write

$$y_i = z_i \Pi_i + \epsilon_i \qquad i = 1,\ 2 \tag{13}$$

Π_i is a $(1+k)\times 1$ vector of parameters. y_i is the $T \times 1$ vector of the scalar y_{it}s ($y_{it} \in \{R_t,\ F_t\}$) and similarly for ϵ_i. z_i is the $T \times (1+k)$ matrix formed by stacking the T, $(1+k)\times 1$, z_{it} vectors. In matrix form, (13) can be written:

$$\underbrace{\begin{bmatrix} R \\ F \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} z_1 & 0_{T\times(1+k)} \\ 0_{T\times(1+k)} & z_2 \end{bmatrix}}_{Z} \underbrace{\begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}}_{\pi} + \underbrace{\begin{bmatrix} \epsilon^R \\ \epsilon^F \end{bmatrix}}_{\epsilon} \tag{14}$$

Y is $2T\times 1$, Z is $2T \times 2(1+k)$, π is $2(1+k)\times 1$ and ϵ is $2T\times 1$. Writing the system in this form will help us calculate the distribution of the impulse response functions in Section 2.3, since they will be functions of the distribution of the $2(1+k)\times 1$ vector, π.

2.2 Instrumental variables

Since each of the equations in (13) contain endogenous variables on the right hand side, we estimate using instrumental variables. For the $1+k$ variables in z_{it} we use

[10] We separate y_t and By_t in (11) as this simplifies the notation in the appendix when we calculate the distribution of the impulse response functions.

the $g+k$ instruments w_{it}, $i = 1,\ 2$ and $g \geq 1$. For a greater explanation of the use of these variables as instruments, see Section 2.5. Using two stage least squares, the IV estimator for Π_i, $i = 1,\ 2$, is denoted $\hat{\Pi}_i$ and calculated as:

$$\hat{\Pi}_i = \underbrace{\left[z_i' w_i \left(w_i' w_i\right)^{-1} w_i' z_i\right]^{-1} z_i' w_i \left(w_i' w_i\right)^{-1}}_{Q^{ii}} w_i' y_i \tag{15}$$

where w_i is simply the $T \times (g+k)$ matrix formed by stacking the T w_{it} vectors of instruments. Using standard instrumental variables methods, the distribution of $\hat{\pi}$, the $2(1+k) \times 1$ vector of parameters in (14) is given by:

$$\sqrt{T}\left(\hat{\pi} - \pi\right) \xrightarrow{d} N\left(0\ ,\ \underbrace{Q^P S^P Q^{P\prime}}_{\Sigma_\pi}\right) \tag{16}$$

where Q^P is the population analogue of Q, given below.

$$Q = \begin{bmatrix} Q^{11} & 0 \\ 0 & Q^{22} \end{bmatrix} \tag{17}$$

and $S^P = E\left[\zeta_t \zeta_t'\right]$ where

$$\zeta_t = \begin{bmatrix} w_{1t}\epsilon_{1t} \\ w_{2t}\epsilon_{2t} \end{bmatrix} \tag{18}$$

For more on the derivation of the distribution of $\hat{\pi}$, see Appendix A.3, which uses the results of Hamilton (1994).

2.3 Impulse response functions

To calculate the impulse response functions, we convert the VAR of (11) into its MA(∞) representation. It is simple to show that (11) can be written as[11]

$$y_t = \mu + \Psi_0 \left(I_2 + B\right)^{-1} \epsilon_t + \Psi_1 \left(I_2 + B\right)^{-1} \epsilon_{t-1} + \Psi_2 \left(I_2 + B\right)^{-1} \epsilon_{t-2} + \ldots \tag{19}$$

where μ is the unconditional mean of the vector y_t. B is given in (12) and Ψ_S, $S = 0, 1, 2, \ldots$, is given by

$$\Psi_S = \left(I_2 + B\right)^{-1} \phi_1 \Psi_{S-1} + \left(I_2 + B\right)^{-1} \phi_2 \Psi_{S-2} + \ldots + \left(I_2 + B\right)^{-1} \phi_P \Psi_{S-P} \tag{20}$$

where ϕ_j, $j = 1, \ldots, P$, are the coefficients on the lags in the VAR in (11) and $\Psi_0 = I_2$, the 2×2 identity matrix. $\Psi_k = 0_{2\times 2}\ \forall\ k < 0$. The impacts of ϵ_t on y_{t+S} are shown by the impulse response functions, H_S, where H_S is the 2×2 coefficient matrix on ϵ_{t-S} in (19).

$$H_S = \Psi_S \left(I_2 + B\right)^{-1} \tag{21}$$

We therefore introduce a one unit order flow shock to each of the VARs and examine the effect of the feedback trading parameter by comparing the IRFs with and without this feedback trading. In order to determine whether the non-feedback IRF is significantly different from the IRF of the unrestricted VAR, we have to calculate the distribution of this feedback impulse response and we do so analytically using the delta method. From (21), H_S depends only on the parameters in the structural VAR, the distribution of which is shown in (16). If $\hat{H}_S = \hat{\Psi}_S \left(I_2 + \hat{B}\right)^{-1}$, where the

[11] See Hamilton (1994) for example.

caret denotes parameter estimate, and if $\hat{h}_S = vec\left(\hat{H}'_S\right)$ then the distribution of the IRF parameters is given, using (16), as

$$\sqrt{T}\left(\hat{h}_S - h_S\right) \xrightarrow{d} N\left(0\ ,\ G_S \Sigma_\pi G'_S\right) \tag{22}$$

where G_S is $4 \times 2(1+k)$ and equals $\frac{\partial h_S}{\partial \pi'}$. However, to calculate G_S, one cannot use the results of Hamilton (1994). The structural VARs considered in Hamilton (1994) are estimated from the reduced form and the structural parameters are backed out from the variance/covariance matrix of the residuals. In that way, the distribution of the IRFs depends, not only on the distribution of the reduced form parameters, but also on the distribution of the variance/covariance matrix of the errors. Since we use instrumental variables to estimate the structural parameters directly, the distribution of the IRFs in (22) will depend only on the distribution of the π parameters and *not* on the distribution of the variance/covariance matrix of residuals. In Appendix A.4 we show, using methods similar to those of Hamilton (1994) and Lütkepohl (1990), that G_S can be written as

$$G_S = \frac{\partial h_S}{\partial \pi'} = \left[I_4 + (I_2 \otimes B')\right]^{-1}\left[\frac{\partial \psi_S}{\partial \pi'} - (H_S \otimes I_2)\, S_{B'} \frac{\partial \Theta_{B'}}{\partial \pi'}\right] \tag{23}$$

where $S_{B'}$ and $\frac{\partial \Theta_{B'}}{\partial \pi'}$ are shown to be matrices of zeros and ones, and $\frac{\partial \psi_S}{\partial \pi'}$ is given by

$$\begin{aligned}\frac{\partial \psi_S}{\partial \pi'} = &- \left(I_2 \otimes \left[\phi_1 \Psi_{S-1} + \ldots + \phi_P \Psi_{S-P}\right]'\right)\left((I_2 + B)^{-1} \otimes \left((I_2 + B)^{-1}\right)'\right) S_{B'} \frac{\partial \Theta_{B'}}{\partial \pi'} \\ &+ \sum_{j=1}^{P}\left\{\left((I_2 + B)^{-1} \otimes \Psi'_{S-j}\right)\frac{\partial \Theta_{\phi'_j}}{\partial \pi'} + \left((I_2 + B)^{-1}\phi_j \otimes I_2\right)\frac{\partial \psi_{S-j}}{\partial \pi'}\right\}\end{aligned} \tag{24}$$

where, again, $\frac{\partial \Theta_{\phi'_j}}{\partial \pi'}$, $j = 1, \ldots, P$, are matrices of zeros and ones. Using (22), (23) and (24) we can then calculate the distribution of the impulse response functions and therefore see whether the restricted/non-feedback IRF is significantly different from the unrestricted impulse response.[12]

2.4 Data

The market we consider is that of the spot USD/EUR (US dollar per euro) inter-dealer foreign exchange market, taken from the Reuters D2000-2 electronic trading system. The data we consider as instruments are those from the USD/GBP (US dollar per pound sterling) and GBP/EUR (pound sterling per euro). When sampling the data we record the last transaction price in each period (one minute or five minutes) and the order flow, defined as the number of buyer initiated trades minus the number of seller initiated trades. Unfortunately we have no information on traded quantities. However, to the extent that earlier work has shown little size variation in trades on this dealing system (Payne 2003) and that in other applications it is the number rather than aggregate size of trades that has been shown to matter for prices and volatility (Jones, Kaul, and Lipson 1994), we expect that this limitation will not distort our results. Furthermore, even when both the number and size of trades have been available, research has often focussed on the former measure of trading activity (Hasbrouck 1991). We also decide to remove certain sparse trading periods from our sample. These include weekends, the overnight period, defined as 1800 to 0600 GMT (BST in the summer months) where trading activity was found to be very thin and some public holidays including Christmas, New Year, Easter (Good Friday and Easter Monday) and the May Day bank holiday. Periods where the D2000-2 data feed broke down were also excluded. These periods are defined as those where no transactions (and hence no price changes) occurred for at least thirty minutes during the day in any of the three FX markets, i.e. if the data feed broke down on GBP/EUR or USD/GBP but not USD/EUR then those data are still excluded, purely because the GBP/EUR and USD/GBP data are needed in the construction of the instruments.

[12] The distribution of the cumulative IRFs can be calculated quite easily from (22). See Lütkepohl (1990).

This filtering process reduced the total number of observations to 90949 at the one minute frequency and 18401 at the five minute frequency. Table 1 contains statistical information on exchange rate returns, defined as 100 times the logarithmic difference in prices, transaction frequencies and order flows for our filtered data sample.

2.5 Instrumenting the endogenous variables

The IV estimator and its distribution, reported in Section 2.2 are standard results. The main question at this point concerns what instruments one can use and how good they are at instrumenting the endogenous regressors. Since the data available to us include not only USD/EUR returns and transactions but also those from the USD/GBP and GBP/EUR markets, the statistics from these other two markets seem prime candidates for use as instruments. Previous research has documented the cross effects of order flow on exchange rates. Evans and Lyons (2002a) document the role that order flow in one currency has in determining exchange rates in other markets. In particular DEM/USD (Deutsche mark-dollar) and CHF/USD (Swiss franc-dollar) order flows have significant effects on a number of other dollar exchange rates. These cross market effects are also documented in Daníelsson, Luo, and Payne (2002) and Love and Payne (2003), both of which consider USD/EUR, GBP/EUR and USD/GBP and the former also examining the JPY/USD market. Theoretical explanations as to why cross effects of order flow exist are also presented in Lyons and Moore (2003), which examines the triangle of rates between the US dollar, euro and yen.

Since the triangle of rates between the dollar, sterling and euro form a strict cointegrating system, using contemporaneous USD/GBP and GBP/EUR returns as instruments for USD/EUR returns is likely to be problematic. Following a shock to USD/EUR returns at date t for example, this will affect not only the USD/EUR rate but also one or both of the sterling rates, otherwise clear arbitrage opportunities would result. In which case date t USD/GBP and GBP/EUR returns will be correlated with the date t error in the USD/EUR return equation. The use of contemporaneous USD/GBP and GBP/EUR returns as instruments for the endogenous USD/EUR variables will then result in biased parameter estimates just as OLS estimates would. Instead, we consider lags of sterling returns which should not be correlated with the errors and hence stand a good chance of being valid instruments.

Also, under conditions of no arbitrage, it is clear that the USD/EUR return at time t will equal the sum of the returns in the USD/GBP and GBP/EUR markets. Therefore, using lags of *both* sterling returns as instruments will be problematic since, unless the coefficients on the sterling returns are different in the first stage regression in the 2SLS procedure, we will essentially be using a 'synthetic' lagged USD/EUR return to instrument for contemporaneous returns. However, the lagged USD/EUR return is effectively being used as an instrument for itself, since it too is included in the VAR. For this reason we only use one of either USD/GBP or GBP/EUR returns as instruments. Which return series we use will depend on how good they are at instrumenting for the endogenous regressors. Since the no arbitrage problem does not hold for our order flow series; USD/EUR order flow in period t does not have to equal the sum of the USD/GBP and GBP/EUR flows, we consider both USD/GBP and GBP/EUR order flows as candidate instruments for the contemporaneous USD/EUR flow regressor.

Researchers have often pointed to the pitfalls of using weak instruments and the bias that such instruments introduce. See for example Buse (1992), Bound, Jaeger, and Baker (1995), Wang and Zivot (1998) and Staiger and Stock (1997). It is therefore vital that we examine the quality of our instruments. In our two equation case, in returns and flows, we only have one endogenous regressor and so test the quality of our instruments using the procedure discussed in Pagan and Robertson (1998). For cases with multiple endogenous regressors, see Shea (1996) and Hall, Rudebusch, and Wilcox (1996). To test the quality of our instruments we run a regression of the endogenous regressor in each of the structural equations in (11) on all the exogenous variables (lagged USD/EUR flows and returns) as well as the candidate instruments. A Wald test is then performed on the coefficients of those instruments.

For the endogenous USD/EUR return regressor we begin by using the first lag of either USD/GBP or GBP/EUR returns and continue increasing the lag length of the instruments until no more explanatory power is added by their inclusion.[13] For the USD/EUR flow regressor, we start by considering contemporaneous USD/GBP and GBP/EUR flows and increase the lag length in a similar fashion. The results of these Wald tests are reported in Table 2 for the one and five minute frequency VARs.

For the one minute frequency VAR, three lags of USD/GBP and GBP/EUR flows were chosen to instrument for USD/EUR flows in the return equation, while two lags of USD/GBP returns were chosen as instruments for USD/EUR returns in the flow equation. Testing how good the instruments were for the USD/EUR flows in the return equation produced a Wald test of 28.09. For returns in the flow equation, the instrument Wald test was 236.13. The 1% critical values for the corresponding χ^2 distributions are 16.81 and 9.21 respectively, suggesting that the chosen variables are good instruments for the endogenous regressors. When considering the five minute frequency VAR, contemporaneous and one lag of both USD/GBP and GBP/EUR flows were chosen to act as instruments for USD/EUR flows in the return equation, while in the flow equation, two lags of GBP/EUR returns were chosen to instrument for USD/EUR returns. The regression of USD/EUR flows on all exogenous variables and instruments produced a Wald test of 7109.01, while testing how good the GBP/EUR returns are as instruments for returns in the flow equation produced a Wald test of 62.26. Again both of these are significant at the 1% level, suggesting that these variables make good instruments.[14] We also considered sampling the data at lower frequencies. However, at anything lower than the five

[13]The choice as to which sterling series to use, is made based on the overall fit of this first stage regression.

[14]It may appear strange that different variables were chosen to act as instruments at the one and five minute frequencies. This may be reconciled when one considers the different FX market dynamics at the different frequencies. With different dynamics and cross correlations at one and five minute frequencies, it may be unsurprising to find different choices of instruments. The question of which variables to include as instruments is, after all, an empirical one. The huge Wald statistic on the instruments for USD/EUR flows (7109.01) comes primarily from the use of contemporaneous USD/GBP and GBP/EUR flows as instruments. Surprisingly, contemporaneous flows were not of any use for the one-minute VAR, suggesting possibly delayed information spill-overs from one market to another.

minute frequency, the instrument Wald test became insignificant, even at the 5% level, suggesting that neither USD/GBP or GBP/EUR variables would be good at instrumenting for the endogenous USD/EUR returns and flows. Since only lagged returns are suggested as instruments, due to the problems of using contemporaneous USD/GBP and GBP/EUR returns explained above, as soon as one considers lower frequency data, the ability of these lagged variables to instrument for USD/EUR returns is likely to fall. At the hourly or daily frequency for example, returns and flows will no longer be serially correlated. If USD/EUR returns are not correlated with its own lag, it is highly unlikely that they will be correlated with the lags of USD/GBP or GBP/EUR returns. At the lower frequencies, even the fifteen minute level, the candidate instruments became very weak. Therefore the estimations were only performed for the one and five minute VARs.

3 Estimation Results

Estimation of the feedback VAR necessitates the use of instrumental variables. However, this raises the question of whether one needs instruments for order flows in the return equation when estimating the restricted/recursively ordered VAR. Since the aim of this paper is to compare the two VARs (with and without feedback trading) we decide to estimate the non-feedback VAR equation by equation using OLS since this is the method used to estimate the VAR in the existing literature. In this way, we will compare the 'true', unrestricted VAR to that which would have been estimated using current best practice.

The estimation results of the one minute VAR with and without feedback trading are shown in Table 3. The lag lengths of the VAR were chosen using the Schwartz Information Criterion and this resulted in seven lags of returns and three for order flows. Each VAR was estimated equation by equation and heteroscedasticity and autocorrelation consistent standard errors were calculated using the Newey-West method. The left panel shows the estimation results of the feedback VAR. There are a number of important findings. Firstly, as one would expect, returns depend positively on contemporaneous order flow. One way buying (selling) pressure causes positive (negative) returns intra minute. Returns also display negative serial correlation, as have been found in Payne (2003) and Evans (2002), both of whom consider the Deutsche mark-dollar market. Returns also depend negatively on lagged own order flow, although the explanatory power of these variables in the determination of returns is surprisingly low compared to previous studies; the R^2 is only 11.2%.[15] Of more interest in this paper are the results for the order flow equation. Order flow appears to depend positively on contemporaneous returns, with a coefficient that is significant at the 1% level. This suggests that following a positive return in one minute, traders 'buy into' the currency in that same period, possibly because they expect further price changes in the same direction. However, this is not consistent with the negative serial correlation observed in one minute returns. On the other hand, positive intra minute feedback trading is consistent with the positive effect of lagged returns on order flows, seen in both versions of the VAR. This lagged feedback trading phenomenon is considered in more detail by Cohen and Shin (2003) in the US treasury market. Without feedback trading, the R^2 in the flow equation is only 8.2%. However, when contemporaneous feedback trading is allowed, the R^2 increases to 26.5%, suggesting that contemporaneous price changes are an important determinant of order flows. Even so, the purpose of this paper is not to explain from where order flows originate, but to document the importance of contemporaneous feedback trading that has been overlooked in previous studies. The importance and interpretation of contemporaneous feedback trading will be considered in Section 3.1.

Table 4 gives the VAR results when the data are sampled at the five minute frequency. Again the VARs were estimated equation by equation and the Newey-West method was used to correct for heteroscedasticity and serially correlated errors. The

[15] This raises the question of how good lagged sterling flows are at instrumenting for contemporaneous USD/EUR flows at the one minute frequency. The dramatic reduction in R^2 from non-feedback to feedback VAR, along with the dramatic reduction in the t-stat on contemporaneous flows, suggests either a huge mis-specification in the non-feedback VAR, or the use of instruments which are not as strong as the Wald test suggests. However, this problem is not apparent in the 5 minute frequency VAR, in which the feedback trading parameter is larger and makes more of a difference.

Schwartz Information Criterion suggested using three lags of returns and one of flows. The results are similar, but more pronounced, than those from the higher/one minute frequency VAR. Again, order flows have a positive and significant effect on contemporaneous returns, as one would expect, and returns display negative serial correlation and depend negatively on lagged flows. The explanatory power of these variables for five minute returns is quite high, with an R^2 of 31.1%. When examining the flow equation, we again find evidence of feedback trading. Flows depend positively, not only on lagged returns, but also on contemporaneous five minute returns. The coefficient on contemporaneous returns in the flow equation is significant at the 5% level and quantitatively very large; the size of the contemporaneous feedback trading parameter is more than two and a half times larger than that on the first lag of returns (56.27 versus 21.23).[16] Indeed, if there is positive feedback trading (lagged and contemporaneous) at the one minute frequency, this, by definition, will be shown as contemporaneous feedback trading at the five minute frequency, as demonstrated in Section 1. The results therefore suggest that positive feedback trading is present in the spot USD/EUR market and significant at high frequencies. Intra minute feedback trading is significant but not large, possibly because of the time it takes for traders to react to the price movements. At the five minute frequency, however, intra period feedback trading becomes much larger as any lagged feedback trading at higher frequencies gets incorporated into the contemporaneous feedback effect. We can interpret the effects of this feedback trading using standard VAR analysis, namely impulse response functions. This is the focus of the next section.

3.1 Implications of contemporaneous feedback trading: Impulse response functions

In order to evaluate the informativeness of trades, a common approach is to use impulse response functions (IRFs). Shocking the system with an order flow shock,

[16]For the one minute VAR, contemporaneous feedback trading had a coefficient of 27.02, compared with 24.12 for the first lag of returns.

ϵ_{it}, where i corresponds to the order flow equation in (11), can be interpreted as examining the effect of private information. The larger the impact such a shock has on returns, the more informative order flows are argued to be. By comparing the impulse response functions following an order flow shock in the two VARs (feedback trading versus non-feedback trading) we can examine how important contemporaneous feedback trading is. Intuitively, by ignoring the positive feedback trading (in the recursively ordered structural VAR) it is likely that any order flow shock will have a smaller impact on returns. The existence of positive feedback trading will cause the price impact of order flow shocks/private information to be larger than when feedback trading is ignored, i.e. trades carry *more* information than previous estimates suggest.

The impulse response functions following a one unit order flow shock are shown in the top panel of Figure 1 for the one minute frequency VAR. A number of features can be noted.

- The impact of the order flow shock is almost immediate. Following the one unit shock in the feedback VAR, this causes a 1.09 basis point return and after ten minutes the cumulative return is 1.06 basis points.
- When contemporaneous feedback trading is allowed, the effect of the order flow shock is larger than when contemporaneous feedback trading is prohibited; the feedback impulse response is more than double that in the non-feedback VAR. However, the non-feedback VAR impulse response function is not significantly different from the feedback IRF, i.e. the non-feedback IRF lies within the 95% confidence bound of that from the unrestricted VAR.[17]

This suggests that at the one minute frequency, the difference between the two impulse response functions is *economically* significant, if not statistically so (at the 5%

[17] Impulse responses were also done using a one standard deviation order flow shock but this had no effect on our results. This is because the standard deviations of order flow shocks were very similar in both VAR specifications; see Table 3.

level). Evaluating the informativeness of order flows by considering their price impact will result in a bias if a recursively ordered VAR is considered. However, the statistical significance of this bias is questionable.

The results from the five minute frequency VAR are more pronounced and suggest a much more important role for feedback trading in the interpretation of IRFs. The impulse response functions following a one unit order flow shock are shown in the bottom panel of Figure 1. The notable features are given below.

- On impact of the order flow shock in the feedback VAR, this causes a return of 1.12 basis points. The cumulative return is 1.02 basis points after thirty minutes.

- Again, as in the one minute VAR results, the effect of a one unit order flow shock is larger when feedback trading is allowed. On impact of the order flow shock in the non-feedback VAR, the return is only 0.43 basis points, i.e. the feedback IRF is over two and a half times that of the non-feedback VAR. However, for the five minute frequency case, the non-feedback IRF is significantly different from the unrestricted impulse response, i.e. it lies outside the 95% confidence bound.

Therefore, at the five minute frequency, feedback trading appears to have important consequences when trying to calculate the price impact/informativeness of order flow. The IRF that is commonly computed (that does not allow contemporaneous feedback trading) is significantly below the 'true' IRF which does allow such trading strategies. The price impact of order flow, and hence proxies for the informativeness of such trades, is therefore larger than is commonly believed, implying that trades carry more information than previous studies suggest. The feedback trading that occurs both contemporaneously and also with lags at the one minute frequency, has significant repercussions when modelling five minute data without feedback trading, as is commonly done.

4 Discussion and Interpretation

We have shown that feedback trading in the USD/EUR spot FX market does exist even at high frequencies, specifically the one and five minute sampling frequencies. At the one minute frequency, even though the non-feedback impulse response is not different from the unrestricted IRF in a statistical sense, it is different in an economic sense; the feedback impulse response is more than twice that of the non-feedback IRF, implying trades carry over twice as much information than current estimates suggest. At the five minute frequency, the contemporaneous effect of returns on order flows (feedback trading) is significant and causes the IRFs with and without feedback trading to differ significantly, economically and statistically. Indeed, if feedback trading occurs at the one minute frequency in the lags and also contemporaneously, then by definition, such trading strategies will appear contemporaneous at the five minute frequency. This positive feedback trading causes the price impact of unanticipated order flow shock (representing private information) to be larger compared to when contemporaneous feedback trading is ruled out. The price impact of private information/order flow shocks that is commonly calculated will then be biased downwards compared with the true impact.

The existence and profitability of feedback trading strategies has been considered in a number of papers. De Long, Shleifer, Summers, and Waldmann (1990) build a model of feedback trading with rational speculators who will buy (sell) when the price rises (falls). The profitability of a number of feedback trading strategies in stock markets is considered in Jegadeesh and Titman (1993) and the existence of high frequency positive feedback trading in the US treasury market is documented in Cohen and Shin (2003). In this paper we have labelled the effects of contemporaneous asset returns on order flows as feedback trading effects. However, there may be other reasons why date t order flows depend on contemporaneous asset returns. Firstly, traders wishing to trade large quantities may break up their trade into a number of smaller sized transactions. By walking up and down the limit order book and splitting a large buy order into a number of smaller trades, to be executed within a short time interval, this will be shown up in the VAR as order flows depending on contemporaneous asset returns. Even though traders are not ultimately wishing to trade based on previous price changes; the decision to trade was made some time earlier, this will still manifest itself as date t order flow depending (statistically) on date t asset returns. Imposing a recursively ordered structural VAR will still be a mis-specification. Another reason why order flow may appear dependent on contemporaneous returns in aggregated data is because of the existence of stop-loss orders (Osler 2002). If the price of an asset falls to a certain level, traders may initiate sell orders in order to *stop losses* from getting any larger. In which case, negative returns induce negative order flow immediately. In any case, what we have demonstrated in this paper is that order flow at date t depends positively on date t asset returns in the spot FX market. Whether this is due to the splitting of dealers' trades, stop loss orders or due to 'pure' feedback trading based on extrapolative expectations of future price changes, is irrelevant. In all cases, the assumption of a recursively ordered structural VAR will result in a misspecified model and in a bias in any estimate of the price impact/informativeness of trades.

This paper does not try to explain why feedback trading may occur in the foreign exchange market, or indeed how profitable such strategies may or may not be. The purpose of the paper is to analyse the effects of contemporaneous feedback trading on estimates of the price impact of order flows. Such measures are commonly used as proxies for the informativeness of trades (how much information trades carry). As shown in this study, these measures are underestimated if feedback trading is ignored and the data are sampled at anything other than at the highest frequencies. Following a positive order flow shock, representing private information, this causes a positive return due to the asymmetric information (Kyle 1985, Glosten and Milgrom 1985) or inventory control channels (Lyons 1995). If such price changes induce further trades, which in turn cause price changes, etc., then the total effect of the order flow shock will be greater than when contemporaneous feedback trading is prohibited.

To our knowledge, only one other paper has tried to examine contemporaneous feedback trading in the foreign exchange market. Evans and Lyons (2003) use a VAR model, as we do, in returns and order flows but is not as general as the procedure outlined above. Evans and Lyons split order flows into two types, both having different roles. They assume returns depend on contemporaneous 'informational' trades while 'feedback' trades depend on contemporaneous returns. The order flow measure available from the data is simply the sum of these two components and the model is estimated by GMM, with expected future price changes being used to instrument for the second structural parameter. This is more restrictive than our approach since we do not, in any way, split trades into different motives. By using variables obtained from other FX markets as instruments we are able to estimate an otherwise unidentified model.[18] At the five minute frequency we find that after a one unit order flow shock, this causes a return of approximately 1.12 basis points whereas when feedback trading is ignored, the return is only 0.43 basis point. The non-feedback IRF is significantly different from that of the unrestricted VAR, implying that feedback trading makes a difference when calculating the price informativeness of trades.

5 Conclusions

Microstructure theory suggests that trades carry information and hence have permanent effects on prices. The information content of these trades is normally quantified by examining their price impact (Hasbrouck 1991) after fitting the data to a vector autoregression. However, common practise is to allow returns to depend on contemporaneous order flows but not the converse. The recursively ordered structural VAR that results can then be estimated quite easily. Although intuitive at ultra-high frequencies, tick by tick, as soon as one starts aggregating the data, any feedback trading (that by definition can only occur in the lags of tick by tick data) will appear contemporaneous. The recursively ordered VAR then becomes misspecified and can have important repercussions when examining the price impact of order flow shocks.

[18] At the daily frequency Evans and Lyons (2003) find evidence of negative feedback trading!

In this paper we use standard instrumental variables techniques in order to estimate a VAR model that allows contemporaneous feedback trading. Feedback trading is found to be significant and positive at the one and five minute frequencies, with the implication that the price impact of order flows is underestimated when such trading strategies are not allowed. Trades, in the form of order flow shocks, therefore carry more information/have a larger impact on asset prices than previously believed.

References

Bank for International Settlements, 2001, 71st annual report, section 5, 'Foreign exchange markets'. B.I.S., Basle.

Blanchard, O., and D. Quah, 1989, The dynamic effects of aggregate demand and supply disturbances, American Economic Review, 79, 655–673.

Bound, J., D. Jaeger, and R. Baker, 1995, Problems with instrumental variables estimation when the correlation between the instruments and the endogenous explanatory variable is weak, Journal of the American Statistical Association, 90, 443–450.

Buse, A., 1992, The bias of instrumental variable estimators, Econometrica, 60, 173–180.

Cohen, B., and H. Shin, 2003, Positive feedback trading under stress: Evidence from the US Treasury securities market, Working Paper, London School of Economics.

Daníelsson, J., J. Luo, and R. Payne, 2002, Exchange rate determination and inter-market order flow effects, mimeo, LSE.

De Long, J., A. Shleifer, L. Summers, and R. Waldmann, 1990, Positive feedback investment strategies and destabilizing rational speculation, Journal of Finance, 45, 379–395.

Dufour, A., and R. Engle, 2000, Time and the Price Impact of a Trade, Journal of Finance, 55, 2467–2498.

Easley, D., and M. O'Hara, 1987, Price, Trade Size and Information in Securities Markets, Journal of Financial Economics, 19, 69–90.

Engle, R., and A. Patton, 2004, Impacts of trades in an error-correction model of quote prices, Journal of Financial Markets, 7, 1–25.

Evans, M., 2002, FX trading and exchange rate dynamics, Journal of Finance, 57, 2405–2447.

Evans, M., and R. Lyons, 2002a, Informational integration and FX trading, Journal of International Money and Finance, 21, 807–831.

Evans, M., and R. Lyons, 2002b, Order flow and Exchange Rate Dynamics, Journal of Political Economy, 110, 170–180.

Evans, M., and R. Lyons, 2003, How is Macro News Transmitted to Exchange Rates?, Working Paper.

Glosten, L., and P. Milgrom, 1985, Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders, Journal of Financial Economics, 14, 71–100.

Green, C., 2003, Economic news and the impact of trading on bond prices, Journal of Finance, forthcoming.

Hall, A., G. Rudebusch, and D. Wilcox, 1996, Judging instrument relevance in instrumental variables estimation, International Economic Review, 37, 283–298.

Hamilton, J., 1994, Time Series Analysis. Princeton University Press, Princeton, New Jersey.

Harvey, A., 1989, Forecasting, Structural Time-Series Models and the Kalman Filter. Cambridge University Press, Cambridge.

Hasbrouck, J., 1991, Measuring the Information Content of Stock Trades, Journal of Finance, 46(1), 179–206.

Jegadeesh, N., and S. Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, Journal of Finance, 48, 65–91.

Jones, C., G. Kaul, and M. Lipson, 1994, Transactions, Volume and Volatility, Review of Financial Studies, 7, 631–651.

Kyle, A., 1985, Continuous Auctions and Insider Trading, Econometrica, 53, 1315–1335.

Love, R., and R. Payne, 2003, Macroeconomic news, order flows and exchange rates, Working Paper, London School of Economics.

Lütkepohl, H., 1990, Asymptotic distributions of impulse response functions and forecast error variance decompositions of vector autoregressive models, The Review of Economics and Statistics, 72, 116–125.

Lyons, R., 1995, Tests of Microstructural Hypotheses in the Foreign Exchange Market, Journal of Financial Economics, 39, 321–351.

——— , 2001, The Microstructure Approach to Exchange Rates. MIT Press, Boston, U.S.A.

Lyons, R., and M. Moore, 2003, An information approach to international currencies, Working Paper.

Madhavan, A., and S. Smidt, 1991, A Bayesian Model of Intraday Specialist Pricing, Journal of Financial Economics, 30, 99–134.

Magnus, J., and H. Neudecker, 1988, Matrix Differential Calculus with Applications in Statistics and Econometrics. New York: Wiley.

O'Hara, M., 1995, Market Microstructure Theory. Blackwell Publishers, Oxford, U.K.

Osler, C., 2002, Stop-loss orders and price cascades in currency markets, working paper, Federal Reserve Bank of New York.

Pagan, A., and J. Robertson, 1998, Structural models of the liquidity effect, The Review of Economics and Statistics, 80, 202–217.

Payne, R., 2003, Informed trade in spot foreign exchange markets: an empirical investigation, Journal of International Economics, 61, 307–329.

Shea, J., 1996, Instrument relevance in multivariate linear models: a simple measure, unpublished manuscript, University of Wisconsin.

Sims, C., 1980, Macroeconomics and reality, Econometrica, 48, 1–48.

Staiger, D., and J. Stock, 1997, Instrumental variables regression with weak instruments, Econometrica, 65, 557–586.

Wang, J., and E. Zivot, 1998, Inference on structural parameters in instrumental variables regression with weak instruments, Econometrica, 66, 1389–1404.

Table 1: Summary Statistics on USD/EUR Exchange Rate Returns and Flows

Variable	Mean	Variance	Skewness	Kurtosis	Autocorrelation (lags)			
					1	5	10	20
1 minute frequency								
returns	-0.000160	0.000815	0.201	28.1	-0.0259*	-0.0157*	-0.0128*	-0.00177
absolute returns	0.0166	0.000540	4.35	48.2	0.224*	0.132*	0.108*	0.0875*
flows	0.0380	12.1	-0.379	20.3	0.232*	0.0299*	-0.00214	-0.000590
trades	4.02	19.1	2.41	16.2	0.567*	0.412*	0.357*	0.294*
5 minute frequency								
returns	-0.000820	0.00374	-0.00431	7.84	-0.0448*	0.00631	-0.000700	0.00559
absolute returns	0.0422	0.00196	2.42	13.0	0.195*	0.101*	0.0738*	0.0553*
flows	0.190	94.2	-0.106	9.91	0.124*	0.0171*	-0.00752	0.00198
trades	19.9	295.1	1.99	11.1	0.685*	0.451*	0.312*	0.108*

Notes: The USD/EUR exchange rate is defined as the number of dollars (numerator currency) per euro (denominator currency). Returns are defined as 100 times the first difference of the logarithm of the exchange rate. Positive order flow in the USD/EUR market implies net purchases of euro. * denotes significance at the 5% level or less.

Table 2
Instrumenting USD/EUR returns and flows

Return equation

$$R_t^{USD/EUR} = c + b\,F_t^{USD/EUR} + \sum_{i=1}^{m} u_i R_{t-i}^{USD/EUR} + \sum_{j=1}^{n} v_j F_{t-j}^{USD/EUR} + \epsilon_t^R$$

	1 min frequency	5 min frequency
Instruments for $F_t^{USD/EUR}$ (flows)	$\underbrace{\begin{matrix} F_{t-1}^{USD/GBP} & F_{t-1}^{GBP/EUR} \\ F_{t-2}^{USD/GBP} & F_{t-2}^{GBP/EUR} \\ F_{t-3}^{USD/GBP} & F_{t-3}^{GBP/EUR} \end{matrix}}_{w_t}$	$\underbrace{\begin{matrix} F_t^{USD/GBP} \\ F_{t-1}^{USD/GBP} \\ F_t^{GBP/EUR} \\ F_{t-1}^{GBP/EUR} \end{matrix}}_{w_t}$

Frequency	Wald test	1% critical value	Degrees of freedom
1 min	28.09	16.81	6
5 min	7109.01	13.28	4

Flow equation

$$F_t^{USD/EUR} = c + b\,R_t^{USD/EUR} + \sum_{i=1}^{m} u_i R_{t-i}^{USD/EUR} + \sum_{j=1}^{n} v_j F_{t-j}^{USD/EUR} + \epsilon_t^F$$

	1 min frequency	5 min frequency
Instruments for $R_t^{USD/EUR}$ (returns)	$\underbrace{\begin{matrix} R_{t-1}^{USD/GBP} \\ R_{t-2}^{USD/GBP} \end{matrix}}_{w_t}$	$\underbrace{\begin{matrix} R_{t-1}^{GBP/EUR} \\ R_{t-2}^{GBP/EUR} \end{matrix}}_{w_t}$

Frequency	Wald test	1% critical value	Degrees of freedom
1 min	236.13	9.21	2
5 min	62.26	9.21	2

Notes: R_t^x is 100 × the log first difference of exchange rate x at date t. F_t^x is the order flow for exchange rate x, defined as the number of buyer less the number of seller initiated transactions in period t.

Table 3: USD/EUR VAR results (1 minute frequency)

	Feedback VAR				Non-feedback VAR			
	R_t equation		F_t equation		R_t equation		F_t equation	
$constant$	-0.000519^a	(-4.21)	0.0435^a	(3.88)	-0.000376^a	(-4.75)	0.0381^a	(3.44)
$flow_t$	0.00841^a	(4.17)			0.00468^a	(57.61)		
$return_t$			27.02^a	(3.13)				
$return_{t-1}$	-0.249^a	(-5.28)	24.12^a	(26.55)	-0.165^a	(-15.89)	22.52^a	(28.86)
$return_{t-2}$	-0.0684^a	(-4.37)	7.31^a	(12.83)	-0.0420^a	(-6.01)	7.07^a	(12.47)
$return_{t-3}$	-0.0258^a	(-2.99)	2.91^a	(4.09)	-0.0152^b	(-2.19)	2.86^a	(4.02)
$return_{t-4}$	-0.0346^a	(-4.58)	2.83^a	(5.05)	-0.0255^a	(-4.85)	2.45^a	(4.44)
$return_{t-5}$	-0.0366^a	(-5.20)	2.65^a	(5.47)	-0.0286^a	(-5.41)	2.16^a	(4.67)
$return_{t-6}$	-0.0267^a	(-4.47)	2.16^a	(4.64)	-0.0197^a	(-4.82)	1.87^a	(4.09)
$return_{t-7}$	-0.0171^a	(-3.64)	0.994^b	(2.04)	-0.0146^a	(-3.74)	0.688	(1.44)
$flow_{t-1}$	-0.000399^c	(-1.76)	0.0954^a	(12.64)	0.00000878	(0.16)	0.109^a	(17.94)
$flow_{t-2}$	-0.000191^a	(-2.88)	0.0213^a	(3.60)	-0.000113^a	(-2.74)	0.0209^a	(3.53)
$flow_{t-3}$	-0.000268^a	(-3.37)	0.0313^a	(3.33)	-0.000152^a	(-2.88)	0.0311^a	(3.31)
R^2	0.112		0.265		0.302		0.082	
$\hat{\sigma}^2$	0.0285		3.33		0.0239		3.33	

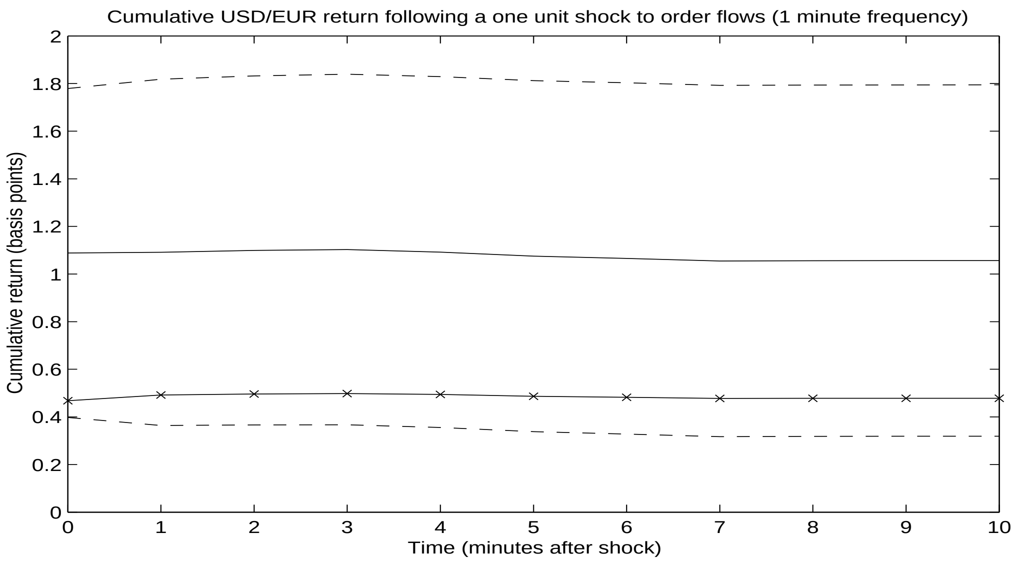
Notes: The data cover the eight month period from 1st December 1999 to 24th July 2000. The USD/EUR exchange rate is defined as the number of dollars (numerator currency) per euro (denominator currency). Returns are defined as 100 times the log first difference of the exchange rate. Positive order flow in the USD/EUR market implies net purchases of euro. a, b, c denote significance at the 1%, 5% and 10% levels respectively. Newey-West corrected T-stats in parentheses.

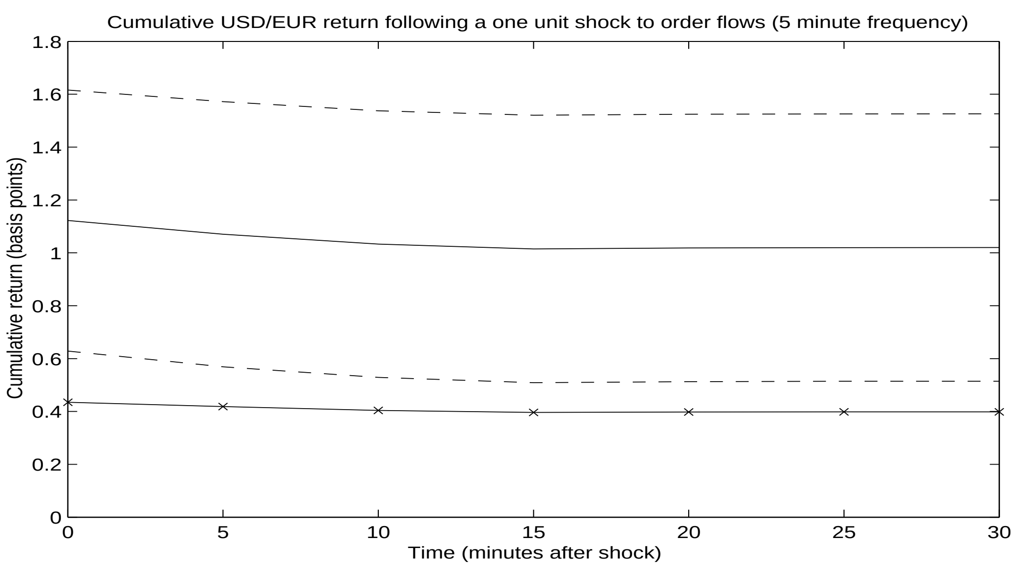
Table 4: USD/EUR VAR results (5 minute frequency)

	Feedback VAR				Non-feedback VAR			
	R_t equation		F_t equation		R_t equation		F_t equation	
$constant$	-0.00232^a	(-6.30)	0.254^a	(3.48)	-0.00181^a	(-5.48)	0.201^a	(2.85)
$flow_t$	0.00688^a	(63.47)			0.00435^a	(45.95)		
$return_t$			56.27^b	(2.44)				
$return_{t-1}$	-0.183^a	(-12.28)	21.23^a	(9.41)	-0.138^a	(-9.82)	17.83^a	(10.33)
$return_{t-2}$	-0.0660^a	(-7.72)	6.15^a	(3.72)	-0.0559^a	(-6.79)	3.98^a	(2.90)
$return_{t-3}$	-0.0337^a	(-4.53)	3.08^b	(2.29)	-0.0288^a	(-4.41)	1.94	(1.54)
$flow_{t-1}$	-0.000213^a	(-2.78)	0.0398^b	(2.57)	-0.0000979	(-1.20)	0.045^a	(2.98)
R^2	0.311		0.375		0.469		0.022	
$\hat{\sigma}^2$	0.0501		9.60		0.0446		9.60	

Notes: The data cover the eight month period from 1st December 1999 to 24th July 2000. The USD/EUR exchange rate is defined as the number of dollars (numerator currency) per euro (denominator currency). Returns are defined as 100 times the log first difference of the exchange rate. Positive order flow in the USD/EUR market implies net purchases of euro. a, b, c denote significance at the 1%, 5% and 10% levels respectively. Newey-West corrected T-stats in parentheses.

Figure 1. Impulse Response Functions for Feedback and Non-feedback VARs





Notes: The figures plot the impulse response functions following a one unit order flow shock. The shock was introduced into the estimated VAR of (11) and the cumulative return calculated. In both plots, the solid line gives the impulse response function from the feedback VAR and the dashed lines trace out a 95% confidence interval for the IRF derived from (22). The crossed line gives the impulse response from the non-feedback VAR.

A Appendix

A.1 Aggregation of the ultra-high frequency VAR in model (1)

The methods used here are taken from Harvey (1989). The reduced form of the ten second frequency VAR is given in (3). For convenience, this is given in (A.1) below.

$$y_t = Q^{-1}\alpha + Q^{-1}\phi y_{t-1} + Q^{-1}\epsilon_t \qquad t = 1, \ldots, 2T \tag{A.1}$$

Let μ denote the unconditional mean of the stationary vector process, y_t. In which case we can write

$$y_t - \mu = Q^{-1}\phi\left(y_{t-1} - \mu\right) + \underbrace{Q^{-1}\epsilon_t}_{u_t} \tag{A.2}$$

Putting (A.2) into state space form, gives the state and observation equations as (A.3) and (A.4) respectively.

$$\begin{gathered}\underbrace{\begin{bmatrix} y_{t+1} - \mu \\ y_t - \mu \end{bmatrix}}_{\xi_{t+1}} = \underbrace{\begin{bmatrix} Q^{-1}\phi & 0 \\ I_2 & 0 \end{bmatrix}}_{F}\underbrace{\begin{bmatrix} y_t - \mu \\ y_{t-1} - \mu \end{bmatrix}}_{\xi_t} + \underbrace{\begin{bmatrix} u_{t+1} \\ 0 \end{bmatrix}}_{v_{t+1}} \\ Var\left(v_{t+1}\right) = \Sigma = \begin{bmatrix} Q^{-1}\Omega\left(Q^{-1}\right)' & 0 \\ 0 & 0 \end{bmatrix}\end{gathered} \tag{A.3}$$

$$y_t = \mu + \underbrace{[I_2 \ 0]}_{H'}\begin{bmatrix} y_t - \mu \\ y_{t-1} - \mu \end{bmatrix} \tag{A.4}$$

Let τ denote timing at the twenty second frequency, $\tau = 1, \ldots, T$, and t denote timing at the ten second frequency, $t = 1, \ldots, 2T$. If y_t^f denotes the cumulator variable, i.e. $y_t^f = y_t$ for the first ten seconds of a twenty second period and $y_t^f = y_t + y_{t-1}$ for the second ten second period, then:

$$\begin{aligned} y_{2(\tau-1)+1}^f &= y_{2(\tau-1)+1} \\ y_{2(\tau-1)+2}^f &= y_{2(\tau-1)+1} + y_{2(\tau-1)+2} \end{aligned} \tag{A.5}$$

The cumulator variable at the ten second frequency, but at times $t = 2,\ 4,\ 6, \ldots,$ can therefore be given, using the observation equation, (A.4), as

$$y_t^f = y_{t-1}^f + \mu + H' F \xi_{t-1} + H' v_t \tag{A.6}$$

At times $t = 1,\ 3,\ 5, \ldots,$ the cumulator variable is given by

$$y_{t-1}^f = \mu + H' \xi_{t-1} \tag{A.7}$$

Substituting y_{t-1}^f into (A.6) and using $\xi_{t-1} = F \xi_{t-2} + v_{t-1}$ gives

$$y_t^f = 2\mu + H' \left(F + F^2 \right) \xi_{t-2} + \underbrace{H' \left(I_2 + F \right) v_{t-1} + H' v_t}_{H' \bar{v}_t^f} \tag{A.8}$$

Since the cumulator function, y_t^f, at times $t = 2,\ 4,\ 6, \ldots,$ is the same as y_τ, the data sampled at the twenty second frequency, then we can write

$$y_\tau = 2\mu + H' \left(F + F^2 \right) \begin{bmatrix} y_{\tau-1} - 2\mu \\ y_{\tau-2} - 2\mu \end{bmatrix} + H' \bar{v}_t^f \tag{A.9}$$

Noting that $H' \left(F + F^2 \right) = \left[Q^{-1} \phi \left(I_2 + Q^{-1} \phi \right) \quad 0 \right]$ then the reduced form model at the twenty second frequency can be written as

$$y_\tau = \underbrace{\left(I_2 - Q^{-1}\phi\left(I_2 + Q^{-1}\phi\right)\right)2\mu}_{\mu^+} + \underbrace{Q^{-1}\phi\left(I_2 + Q^{-1}\phi\right)}_{A} y_{\tau-1} + e_\tau \tag{A.10}$$

where $e_\tau = [I_2\ 0]\,\bar{v}_t^f$. $Var\left(e_\tau\right)$ is therefore given by

$$\begin{aligned} Var\left(e_\tau\right) &= [I_2\ 0]\,Var\left(\bar{v}_t^f\right)\begin{bmatrix} I_2 \\ 0 \end{bmatrix} = [I_2\ 0]\left[\left(I_2 + F\right)\Sigma\left(I_2 + F\right)' + \Sigma\right]\begin{bmatrix} I_2 \\ 0 \end{bmatrix} \\ &= \left(I_2 + Q^{-1}\phi\right)Q^{-1}\Omega\left(Q^{-1}\right)'\left(I_2 + Q^{-1}\phi\right)' + Q^{-1}\Omega\left(Q^{-1}\right)' \\ &= G\Omega G' + Q^{-1}\Omega\left(Q^{-1}\right)' \end{aligned} \tag{A.11}$$

where $G = \left(I_2 + Q^{-1}\phi\right)Q^{-1}$. Expanding this gives us the elements of G.

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} 1 + \phi_{11} + \beta\phi_{21} & \left(1 + \phi_{11} + \beta\phi_{21}\right)\beta + \phi_{12} + \beta\phi_{22} \\ \phi_{21} & \phi_{12} + \beta\phi_{22} \end{bmatrix} \tag{A.12}$$

A.2 Derivation of the structural form of the twenty second VAR

The structural VAR at the twenty second frequency is given in (7) and x_τ, $= P^+ y_\tau$, is the solution to the equation in (9). The general solution for $x_{i\tau}$, $i = 1, \ldots, 4$, can be found using Gaussian elimination, resulting in

$$
\begin{aligned}
x_{1\tau} &= \left(\frac{1}{g_{11}} + \frac{g_{12}g_{21}}{|G|}\right) R_\tau - \left(\frac{g_{12}}{|G|}\right) F_\tau - \left(\frac{1}{g_{11}} - \frac{g_{21}}{|G|}\right) m_\tau - \left(\frac{\beta}{g_{11}} + \frac{(g_{11} - g_{21}\beta)}{|G|}\right) n_\tau \\
x_{2\tau} &= -\left(\frac{g_{21}}{|G|}\right) R_\tau + \left(\frac{g_{11}}{|G|}\right) F_\tau + \left(\frac{g_{21}}{|G|}\right) m_\tau - \left(\frac{g_{11} - g_{21}\beta}{|G|}\right) n_\tau \\
x_{3\tau} &= m_\tau \\
x_{4\tau} &= n_\tau
\end{aligned}
\tag{A.13}
$$

for any real values of m_τ and n_τ, $\tau = 1, \ldots, T$, implying an infinite number of solutions. The right hand side of (7) can be written as $[I_2 \; I_2]\, x_\tau$, and the coefficients on R_τ and F_τ are the structural parameters of interest. Noting that $[I_2 \; I_2]\, x_\tau = [(x_{1\tau} + x_{3\tau}) \quad (x_{2\tau} + x_{4\tau})]'$ gives us (10).

A.3 Distribution of $\hat{\pi}$

The notation and methods used here are similar to those of Hamilton (1994) but we allow IV estimation rather than simple OLS. In this appendix we also generalise the 2 variable case, in returns, R_t, and flows, F_t, to the n variable case, since one may wish to model a number of returns and trade characteristics. From (15)

$$
\begin{aligned}
\sqrt{T}\left(\hat{\Pi}_i - \Pi_i\right) = &\left[\left(\frac{1}{T}\sum_{t=1}^{T} z_{it}w'_{it}\right)\left(\frac{1}{T}\sum_{t=1}^{T} w_{it}w'_{it}\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T} w_{it}z'_{it}\right)\right]^{-1} \\
&\left(\frac{1}{T}\sum_{t=1}^{T} z_{it}w'_{it}\right)\left(\frac{1}{T}\sum_{t=1}^{T} w_{it}w'_{it}\right)^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{it}\epsilon_{it}
\end{aligned}
\tag{A.14}
$$

$$\text{Let} \qquad \underset{(g+k)\times(g+k)}{Q_{ww}^{ii}} = \frac{1}{T}\sum_{t=1}^{T} w_{it}w_{it}' \qquad \text{and also} \qquad Q_{ww}^{ii} \longrightarrow E\left[w_{it}w_{it}'\right] \quad \text{(A.15)}$$

$$\underset{(n-1+k)\times(g+k)}{Q_{zw}^{ii}} = \frac{1}{T}\sum_{t=1}^{T} z_{it}w_{it}' \qquad Q_{zw}^{ii} \longrightarrow E\left[z_{it}w_{it}'\right] \quad \text{(A.16)}$$

$$\underset{(g+k)\times(n-1+k)}{Q_{wz}^{ii}} = \frac{1}{T}\sum_{t=1}^{T} w_{it}z_{it}' \qquad Q_{wz}^{ii} \longrightarrow E\left[w_{it}z_{it}'\right] \quad \text{(A.17)}$$

Also assume that these expectations exist and are finite. This basically states that the instruments are correlated with the right hand side regressors in (13). The instruments, w_{it}, have the property that

$$plim\frac{1}{T}\sum_{t=1}^{T} w_{it}\epsilon_{it} = 0 \qquad i = 1, \ldots, n \tag{A.18}$$

so that the instruments are uncorrelated with the errors in the original VAR. Let $Q^{ii} = \left[Q_{zw}^{ii}Q_{ww}^{ii^{-1}}Q_{wz}^{ii}\right]^{-1} Q_{zw}^{ii}Q_{ww}^{ii^{-1}}$, then from (A.14),

$$\sqrt{T}\left(\hat{\Pi}_i - \Pi_i\right) = Q^{ii}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{it}\epsilon_{it} \qquad i = 1, \ldots, n \tag{A.19}$$

Stacking these up for $i = 1, \ldots, n$, noting that $\pi = [\Pi_1' \ \Pi_2' \ \ldots \ \Pi_n']'$, i.e. $\pi = vec\,(\Pi')$ where $\Pi' = [\Pi_1 \ \Pi_2 \ \ldots \ \Pi_n]$, then we obtain

$$\sqrt{T}\,(\hat{\pi} - \pi) = \begin{bmatrix} Q^{11}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{1t}\epsilon_{1t} \\ Q^{22}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{2t}\epsilon_{2t} \\ \vdots \\ Q^{nn}\frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{nt}\epsilon_{nt} \end{bmatrix} \tag{A.20}$$

which can be written

$$\sqrt{T}\,(\hat{\pi}-\pi) = \underbrace{\begin{bmatrix} Q^{11} & 0 & & 0 \\ 0 & Q^{22} & & \\ & & \ddots & \\ 0 & & & Q^{nn} \end{bmatrix}}_{Q} \begin{bmatrix} \frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{1t}\epsilon_{1t} \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{2t}\epsilon_{2t} \\ \vdots \\ \frac{1}{\sqrt{T}}\sum_{t=1}^{T} w_{nt}\epsilon_{nt} \end{bmatrix} \tag{A.21}$$

$$= Q\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\zeta_t \tag{A.22}$$

where

$$\zeta_t = \begin{bmatrix} w_{1t}\epsilon_{1t} \\ w_{2t}\epsilon_{2t} \\ \vdots \\ w_{nt}\epsilon_{nt} \end{bmatrix} \tag{A.23}$$

Let $S = \frac{1}{T}\sum_{t=1}^{T}\zeta_t\zeta_t'$ and let $S \longrightarrow E\left[\zeta_t\zeta_t'\right] = S^P$ which we assume exists and is finite. If the population analogues of Q and S are denoted with superscript Ps then

$$\sqrt{T}\,(\hat{\pi}-\pi) \xrightarrow{d} N\left(0\ ,\ Q^P S^P Q^{P\prime}\right) \tag{A.24}$$

In the empirical application of the paper, S^P is estimated using the Newey-West method, i.e.

$$\hat{\pi} \approx N\left(\pi\ ,\ \frac{Q\hat{S}Q'}{T}\right) \tag{A.25}$$

where Q is as in (A.21) and $\hat{S}$ is given by

$$\hat{S} = \hat{S}_0 + \sum_{v=1}^{q}\left[1-\frac{v}{q+1}\right]\left(\hat{S}_v + \hat{S}_v'\right) \tag{A.26}$$

where

$$\hat{S}_v = \begin{bmatrix} \frac{1}{T}\sum_{t=v+1}^{T} w_{1t}\epsilon_{1t}\epsilon_{1t-v}w'_{1t-v} & 0 & \cdots & 0 \\ 0 & \frac{1}{T}\sum_{t=v+1}^{T} w_{2t}\epsilon_{2t}\epsilon_{2t-v}w'_{2t-v} & & \\ \vdots & & \ddots & \\ 0 & & & \frac{1}{T}\sum_{t=v+1}^{T} w_{nt}\epsilon_{nt}\epsilon_{nt-v}w'_{nt-v} \end{bmatrix} \tag{A.27}$$

This is the same as estimating the system using instrumental variables equation by equation and making the Newey-West correction on each equation in turn.

A.4 Distribution of the impulse response functions

The results outlined here are similar to those given in Lütkepohl (1990) and Hamilton (1994). However, the structural forms of the VAR that they consider are estimated from reduced forms, with the structural parameters backed out of the variance/covariance matrix of the residuals. In which case the distribution of the IRFs will be functions of the distribution of the parameters of the VAR but also of the distribution of the variance/covariance matrix of the errors. Since our structural VAR is estimated directly, using instrumental variables, the distribution of our IRFs will depend only on the distribution of the VAR parameters. The differences between our results and those in Lütkepohl (1990) and Hamilton (1994) are non-trivial.

The impulse response functions are given by H_S in (21). Using $\hat{h}_S = vec\left(\hat{H}'_S\right)$, the distribution of the IRFs can be calculated from (22). For convenience, this is given below in (A.28).

$$\sqrt{T}\left(\hat{h}_S - h_S\right) \xrightarrow{d} N\left(0\ ,\ G_S \Sigma_\pi G'_S\right) \tag{A.28}$$

G_S is the matrix formed from the derivatives of each of the elements in the vector h_S with respect to each of the elements in the vector π, i.e. $G_S = \frac{\partial h_S}{\partial \pi'}$. Below, we show how these are calculated using $H_S = \Psi_S \left(I_n + B\right)^{-1}$ and noting that Ψ_S and B

depend on the π parameters, the distribution of which is given in (16).

$$\begin{aligned} H_S\left(I_n+B\right) &= \Psi_S \\ \Rightarrow \left(I_n+B'\right)H_S' &= \Psi_S' \end{aligned} \tag{A.29}$$

Letting η denote an element of π, then differentiating (A.29) with respect to η gives

$$\left(I_n+B'\right)\frac{\partial H_s'}{\partial\eta}+\frac{\partial B'}{\partial\eta}H_S'=\frac{\partial\Psi_S'}{\partial\eta} \tag{A.30}$$

Using the result that $vec\left(ABC\right)=\left(C'\otimes A\right)vec(B)$ and letting $\psi_S=vec\left(\Psi_S'\right)$, then (A.30) can be written as

$$\frac{\partial h_S}{\partial\eta}=\left[I_{n^2}+\left(I_n\otimes B'\right)\right]^{-1}\left[\frac{\partial\psi_S}{\partial\eta}-\left(H_S\otimes I_n\right)\frac{\partial vec\left(B'\right)}{\partial\eta}\right] \tag{A.31}$$

We then need expressions for $\frac{\partial\psi_S}{\partial\eta}$ and $\frac{\partial vec(B')}{\partial\eta}$. Start by considering $\frac{\partial vec(B')}{\partial\eta}$. From (12), and using the n variable case,

$$B=\begin{bmatrix} 0 & -b_{12} & \cdots & -b_{1n} \\ -b_{21} & 0 & \cdots & -b_{2n} \\ \vdots & & & \vdots \\ -b_{n1} & -b_{n2} & \cdots & 0 \end{bmatrix}\Rightarrow B'=\begin{bmatrix} 0 & -b_{21} & \cdots & -b_{n1} \\ -b_{12} & 0 & \cdots & -b_{n2} \\ \vdots & & & \vdots \\ -b_{1n} & -b_{2n} & \cdots & 0 \end{bmatrix} \tag{A.32}$$

Write

$$vec\left(B'\right)=S_{B'}\Theta_{B'} \tag{A.33}$$

where $vec\left(B'\right)$ is the $n^2\times 1$ vector formed by stacking the n^2 elements of B'. Since B has zeros along the diagonal, there are only n^2-n structural parameters that are estimated. Hence $\Theta_{B'}$ is the $(n^2-n)\times 1$ vector of the $-b_{ij}$ parameters, $i,j=1,\ldots,n$, $i \neq j$. $S_{B'}$ is the $n^2 \times (n^2-n)$ matrix of zeros and ones that maps the elements of $\Theta_{B'}$ onto $vec\,(B')$. Therefore

$$\frac{\partial vec\,(B')}{\partial \eta} = S_{B'}\frac{\partial \Theta_{B'}}{\partial \eta} \tag{A.34}$$

Since $\Pi' = [\Pi_1\ \Pi_2\ \ldots\ \Pi_n]$, then Π' can be written as

$$\Pi' = \underbrace{\begin{bmatrix} b_{12} & b_{21} & \cdots & b_{n1} \\ b_{13} & b_{23} & & b_{n2} \\ \vdots & \vdots & & \vdots \\ b_{1n} & b_{2n} & \cdots & b_{nn-1} \\ \hline & c' & & \\ & \phi_1' & & \\ & \vdots & & \\ & \phi_P' & & \end{bmatrix}}_{n\ columns} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} n-1\ rows \\ \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} k\ rows \end{array} \tag{A.35}$$

Noting that $\pi = vec\,(\Pi')$, it then becomes clear that $\frac{\partial \Theta_{B'}}{\partial \pi'}$ is an $(n^2-n)\times n\,(n-1+k)$ matrix of zeros and ones, i.e.

$$\frac{\partial \Theta_{B'}}{\partial \pi'} = \begin{bmatrix} -I_{n-1} & & & & 0_{(n-1)\times(n-1)(n-1+k)+k} \\ 0_{(n-1)\times(n-1+k)} & -I_{n-1} & & & 0_{(n-1)\times(n-2)(n-1+k)+k} \\ 0_{(n-1)\times 2(n-1+k)} & & -I_{n-1} & & 0_{(n-1)\times(n-3)(n-1+k)+k} \\ \vdots & & & & \vdots \\ 0_{(n-1)\times(n-1)(n-1+k)} & & & -I_{n-1} & 0_{(n-1)\times k} \end{bmatrix} \tag{A.36}$$

Since $\frac{\partial \Theta_{B'}}{\partial \pi'}$ is formed by stacking the $n\,(n-1+k)$ $\frac{\partial \Theta_{B'}}{\partial \eta}$ vectors, each of which are $(n^2-n)\times 1$ and $\frac{\partial h_S}{\partial \pi'}$ is formed by stacking the $n\,(n-1+k)$ $\frac{\partial h_S}{\partial \eta}$ vectors, then we have our first expression, the matrix analogue of $\frac{\partial vec(B')}{\partial \eta}$. We now need an expression for $\frac{\partial \psi_S}{\partial \eta}$. From (20) we can write

$$\Psi'_S = \Psi'_{S-1}\phi'_1\left((I_n+B)^{-1}\right)' + \Psi'_{S-2}\phi'_2\left((I_n+B)^{-1}\right)' + \ldots + \Psi'_{S-P}\phi'_P\left((I_n+B)^{-1}\right)' \tag{A.37}$$

Differentiating this with respect to η, an element of π, and rearranging results in

$$\begin{aligned}\frac{\partial \Psi'_S}{\partial \eta} =& \left[\phi_1\Psi_{S-1} + \phi_2\Psi_{S-2} + \ldots + \phi_P\Psi_{S-P}\right]' \frac{\partial\left((I_n+B)^{-1}\right)'}{\partial \eta} \\ &+ \Psi'_{S-1}\frac{\partial \phi'_1}{\partial \eta}\left((I_n+B)^{-1}\right)' + \ldots + \Psi'_{S-P}\frac{\partial \phi'_P}{\partial \eta}\left((I_n+B)^{-1}\right)' \\ &+ \frac{\partial \Psi'_{S-1}}{\partial \eta}\left((I_n+B)^{-1}\phi_1\right)' + \ldots + \frac{\partial \Psi'_{S-P}}{\partial \eta}\left((I_n+B)^{-1}\phi_P\right)'\end{aligned} \tag{A.38}$$

and implementing the vec operator, where $\psi_S = vec\left(\Psi'_S\right)$, gives us

$$\begin{aligned}\frac{\partial \psi_S}{\partial \eta} =& \left(I_n \otimes \left[\phi_1\Psi_{S-1} + \phi_2\Psi_{S-2} + \ldots + \phi_P\Psi_{S-P}\right]'\right) \frac{\partial vec\left((I_n+B)^{-1}\right)'}{\partial \eta} \\ &+ \left((I_n+B)^{-1} \otimes \Psi'_{S-1}\right)\frac{\partial vec\left(\phi'_1\right)}{\partial \eta} + \ldots + \left((I_n+B)^{-1} \otimes \Psi'_{S-P}\right)\frac{\partial vec\left(\phi'_P\right)}{\partial \eta} \\ &+ \left((I_n+B)^{-1}\phi_1 \otimes I_n\right)\frac{\partial \psi_{S-1}}{\partial \eta} + \ldots + \left((I_n+B)^{-1}\phi_P \otimes I_n\right)\frac{\partial \psi_{S-P}}{\partial \eta}\end{aligned} \tag{A.39}$$

Stack the elements of the $n \times n$ matrix ϕ'_j in an $n^2 \times 1$ vector, $\Theta_{\phi'_j}$, i.e. $\Theta_{\phi'_j} = vec\left(\phi'_j\right)$

then from (A.35), it can be seen that $\frac{\partial\Theta_{\phi_j'}}{\partial\pi'}$ can be written as[19]

$$\frac{\partial\Theta_{\phi_j'}}{\partial\pi'} = \begin{bmatrix} 0_{n\times nj} & I_n & 0_{n\times(n-1)(n-1+k)+(P-j)n} \\ 0_{n\times(n-1+k)+nj} & I_n & 0_{n\times(n-2)(n-1+k)+(P-j)n} \\ \vdots & & \vdots \\ 0_{n\times(n-1)(n-1+k)+nj} & I_n & 0_{n\times(P-j)n} \end{bmatrix} \tag{A.40}$$

Again, since $\frac{\partial\psi_S}{\partial\pi'}$ is formed by stacking the $n(n-1+k)$ $\frac{\partial\psi_S}{\partial\eta}$ vectors, then (A.40) gives us the matrix analogue terms of $\frac{\partial vec\left(\phi_j'\right)}{\partial\eta}$. The last term we need is $\frac{\partial vec\left((I_n+B)^{-1}\right)'}{\partial\eta}$. Using the results of Magnus and Neudecker (1988), pages 96, 148 and 151, we can see that

$$\frac{\partial vec\left((I_n+B)^{-1}\right)'}{\partial\eta} = -\left[(I_n+B)^{-1}\otimes\left((I_n+B)^{-1}\right)'\right]S_{B'}\frac{\partial\Theta_{B'}}{\partial\pi'} \tag{A.41}$$

where $S_{B'}$ and $\frac{\partial\Theta_{B'}}{\partial\pi'}$ are defined as before. Substituting (A.41) into (A.39) and summing the elements in the second and third rows will give us our definition of $\frac{\partial\psi_S}{\partial\pi'}$ when we stack the $n(n-1+k)$ $\frac{\partial\psi_S}{\partial\eta}$ vectors. This is (24) in the text. Stacking the $n(n-1+k)$ $\frac{\partial h_S}{\partial\eta}$ vectors in (A.31) will give the result in (23), which we can then use to calculate the distribution of the IRFs.

[19] If we have different lag lengths of returns and flows, as in our empirical model, these $\frac{\partial\Theta_{\phi_j'}}{\partial\pi'}$ matrices need to be altered slightly. Also $vec\left(\phi_j'\right) = S_{\phi_j'}\Theta_{\phi_j'}$ and $S_{\phi_j'}$ will not in general equal the identity matrix.