The notes on the following pages provide additional detail for the appendix to "A Simultaneous Trade Model of the Foreign Exchange Hot Potato," by Richard K. Lyons, *Journal of International Economics*, 42 (1997) 275-298. These notes are referenced in that article as "available on request."

# Not For Publication: Available on Request

## A.2.4. Gaps in Appendix Section 2

Getting From Eq. (A4) to Eq. (A5)

$$\text{(A4)} \qquad \underset{D_{i1}}{\text{Max}}\; E_{P_2,\mu_{iF},F}\left[-\exp\left[-\theta D_{i1}(P_2-P_1) - (\mu_{iF}-P_2)\Sigma_2^{-1}(F-P_2)\right] \middle| \Omega_{Ti1}\right].$$

From the law of iterated projections the expectation can be written as:

$$E_{P_2,\mu_{iF}}\left[E_F\left[-\exp\left[-\theta D_{i1}(P_2-P_1)-(\mu_{iF}-P_2)\Sigma_2^{-1}(F-P_2)\right] \middle| \Omega_{Ti2}\right] \middle| \Omega_{Ti1}\right].$$

Using the fact that (i) $E[F-P_2 | \Omega_{Ti2}]=\mu_{iF}-P_2$ and (ii) the moment generating function for a normally–distributed random variable $x$ implies $E[\exp(kx)] = \exp(k\mu_x+k^2\sigma_x^2/2)$, we can rewrite the the inside expectation over F as:

$$\begin{aligned} &E_F\left[-\exp\left[-\theta D_{i1}(P_2-P_1) - (\mu_{iF}-P_2)\Sigma_2^{-1}(F-P_2)\right] \middle| \Omega_{Ti2}\right] \\ &\quad = -\exp\left[-\theta D_{i1}(P_2-P_1) - (\mu_{iF}-P_2)\Sigma_2^{-1}(\mu_{iF}-P_2) + [-(\mu_{iF}-P_2)\Sigma_2^{-1}]^2\Sigma_2/2\right] \\ &\quad = -\exp\left[-\theta D_{i1}(P_2-P_1) - \tfrac{1}{2}(\mu_{iF}-P_2)^2\Sigma_2^{-1}\right] \end{aligned}$$

where $[-(\mu_{iF}-P_2)\Sigma_2^{-1}]$ corresponds to k in (ii) above. This leaves the objective function with two remaining random variables:

$$\text{(A5)} \qquad \underset{D_{i1}}{\text{Max}}\; E_{P_2,\mu_{iF}}\left[-\exp\left[-\theta D_{i1}(P_2-P_1) - \tfrac{1}{2}(\mu_{iF}-P_2)^2\Sigma_2^{-1}\right] \middle| \Omega_{Ti1}\right].$$

Getting From Eq. (A11) to the First Order Condition

$$\text{(A11)} \qquad \underset{D_{i1}}{\text{Max}} - \int_{-\infty}^{\infty} \exp\Big[-\theta(D_{i1})(\Lambda_V D_{i1}+\gamma_{ci}c_i+\gamma_{si}s_i+\gamma_s s+\Lambda_V V'-P_1)$$
$$- \tfrac{1}{2}\Sigma_2^{-1}[\pi_D D_{i1}+\pi_{ci}c_i+\pi_{si}s_i+\pi_s s+\pi_{V'} V']^2 - \tfrac{1}{2}\Sigma_{V'}^{-1}(V'-\mu_{V'})^2\Big]\, dV'$$

Collecting terms involving the random variable V′ yields:

$$\underset{D_{i1}}{\text{Max}} -\int_{-\infty}^{\infty} \exp\left[-\tfrac{1}{2}\left[2\theta(D_{i1})(\Lambda_V D_{i1}+\gamma_{ci}c_i+\gamma_{si}s_i+\gamma_s s-P_1) + \Sigma_2^{-1}[\pi_D D_{i1}+\pi_{ci}c_i+\pi_{si}s_i+\pi_s s]^2 \right.\right.$$

$$+ 2\theta(D_{i1})\Lambda_V V' + \Sigma_2^{-1}[\pi_{V'}^2 V'^2+2\pi_{V'}V'(\pi_D D_{i1}+\pi_{ci}c_i+\pi_{si}s_i+\pi_s s)]$$

$$\left.\left. + \Sigma_{V'}^{-1}(V'^2-2\mu_{V'}V'+\mu_{V'}^2)\right]\right] dV'.$$

Now, factoring those terms involving V′ yields:

$$\underset{D_{i1}}{\text{Max}} -\int_{-\infty}^{\infty} \exp\left[-\tfrac{1}{2}\left[2\theta(D_{i1})(\Lambda_V D_{i1}+\gamma_{ci}c_i+\gamma_{si}s_i+\gamma_s s-P_1) + \Sigma_2^{-1}[\pi_D D_{i1}+\pi_{ci}c_i+\pi_{si}s_i+\pi_s s]^2 \right.\right.$$

$$\left.\left. + \Sigma_{V'}^{-1}\mu_{V'}^2 - A\left[\frac{B}{A}\right]^2 + A\left[V' + \frac{B}{A}\right]^2\right]\right] dV'$$

where $$A \equiv \Sigma_{V'}^{-1}+\pi_{V'}^2\Sigma_2^{-1}$$

and $$B \equiv (\pi_{V'}/\Sigma_2)(\pi_D D_{i1}+\pi_{ci}c_i+\pi_{si}s_i+\pi_s s) - (\mu_{V'}/\Sigma_{V'}) + \theta\Lambda_V D_{i1}$$

which is equivalent to:

$$\underset{D_{i1}}{\text{Max}} -\exp\left[-\tfrac{1}{2}\left[2\theta(D_{i1})(\Lambda_V D_{i1}+\gamma_{ci}c_i+\gamma_{si}s_i+\gamma_s s-P_1) + \Sigma_2^{-1}[\pi_D D_{i1}+\pi_{ci}c_i+\pi_{si}s_i+\pi_s s]^2 \right.\right.$$

$$\left.\left. + \Sigma_{V'}^{-1}\mu_{V'}^2 - A\left[\frac{B}{A}\right]^2\right]\right] \left[\int_{-\infty}^{\infty} \exp\left[(-\tfrac{1}{2})A\left[V' + \frac{B}{A}\right]^2\right] dV'\right]$$

The integral within large square brackets is proportional to a cumulative normal density with a mean of $-B/A$ and a variance equal to $1/A$. Since the factor of proportionality is a function of the variance only, which does not include $D_{i1}$, maximizing this whole expression is equivalent to maximizing the expression in large curved brackets:

$$\underset{D_{i1}}{\text{Max}} \left\{2\theta(D_{i1})(\Lambda_V D_{i1}+\gamma_{ci}c_i+\gamma_{si}s_i+\gamma_s s-P_1) + \Sigma_2^{-1}[\pi_D D_{i1}+\pi_{ci}c_i+\pi_{si}s_i+\pi_s s]^2 \right.$$

$$\left. + \Sigma_{V'}^{-1}\mu_{V'}^2 - A\left[\frac{B}{A}\right]^2\right\}$$

The first order condition is:

$$\left[X_1^2A^{-1}-\pi_D^2\Sigma_2^{-1}-2\theta\Lambda_V\right]D_{i1} - \left[\theta\gamma_{ci}+\pi_D\pi_{ci}\Sigma_2^{-1}-X_1X_2A^{-1}\right]c_i$$

$$- \left[\theta\gamma_{si}+\pi_D\pi_{si}\Sigma_2^{-1}-X_1X_3A^{-1}\right]s_i - \left[\theta\gamma_s+\pi_D\pi_s\Sigma_2^{-1}-X_1X_4A^{-1}\right]s + \theta P_1 = 0$$

where the coefficients $X_1,\ldots,X_4$ are calculated in the next section.

### A.2.5. Coefficient Algebra (Available on Request)

Below we use repeatedly the following result. Define $x = \bar{x} + z_0$ and $y_i = x + z_i$, i=1,...,n. Let each $z_i$, i=0,...,n, be distributed independently and normally, $N(0,\Sigma_{z_i})$. Then:

$$E[x\,|\,y_1,\ldots,y_n] = \frac{\bar{x}\Sigma_{z_0}^{-1} + y_1\Sigma_{z_1}^{-1} + \ldots + y_n\Sigma_{z_n}^{-1}}{\Sigma_{z_0}^{-1} + \Sigma_{z_1}^{-1} + \ldots + \Sigma_{z_n}^{-1}}$$

where the denominator $= Var[x\,|\,y_1,\ldots,y_n]^{-1}$.

Calculating $\Lambda_{s1}$ in the period-one pricing rule

We want to determine $\Lambda_{s1}$ such that $E[D_{i1}\,|\,\Omega_{T1}]+E[c_i\,|\,\Omega_{T1}]=0$. From the first page of the appendix, we know that $E[c_i\,|\,\Omega_{T1}]=\lambda_{cs}s$. The $E[D_{i1}\,|\,\Omega_{T1}]$ term comes from the demand function in Eq. (A12). Specifically, recognizing that the coefficients on $c_i$, $s_i$, and s equal $(\beta_{11}-1-\delta_{ci})$, $(\beta_{21}-\delta_{si})$, and $(\beta_{31}-\delta_s)$, respectively, we can write:

$$(\beta_{11}-1-\delta_{ci})E[c_i\,|\,\Omega_{T1}] + (\beta_{21}-\delta_{si})E[s_i\,|\,\Omega_{T1}] + (\beta_{31}-\delta_s)s - (\theta/G)P_1 + E[c_i\,|\,\Omega_{T1}] = 0$$

collecting terms, and recognizing that $E[s_i\,|\,\Omega_{T1}]=\lambda_{cs}s$ we have:

$$P_1 = (G/\theta)[\lambda_{cs}(\beta_{11}+\beta_{21}-\delta_{ci}-\delta_{si})+\beta_{31}-\delta_s]s$$
$$\equiv \Lambda_{s1}s$$

Calculating $\lambda_{s2}$ and $\lambda_V$ in the period-two pricing rule

We want to determine $\{\lambda_{s2},\lambda_V\}$ such that $E[c\,|\,s,V] = \lambda_{s2}s + \lambda_V V$. Since nothing in the model provides information on the individual components of c (i.e. F and $\nu$), we can write:

$$E[F\,|\,s,V] = \lambda_{Fc}E[c\,|\,s,V].$$

Now, note that: $E[c\,|\,s] = E[c\,|\,c+\eta]$ and $\eta \sim N(0,\Sigma_\eta)$.

The linear trading rule in Eq. (A15) and the definition of V in Eq. (6) imply:

$$V = \sum_{i=1}^{n} T_{i1} = \beta_{11}\sum_{i=1}^{n} c_i + \beta_{21}\sum_{i=1}^{n} s_i + n(\beta_{31}-\beta_{41}\Lambda_{s1})s.$$

To make the signal in V orthogonal to the noise in s, we construct the statistic $Z_1$ where:

$$Z_1 \equiv \frac{V-n(\beta_{31}-\beta_{41}\Lambda_{s1})s}{n(\beta_{11}+\beta_{21})} = c + \frac{\beta_{11}}{n(\beta_{11}+\beta_{21})}\sum_{i=1}^{n} x_i + \frac{\beta_{21}}{n(\beta_{11}+\beta_{21})}\sum_{i=1}^{n} \xi_i.$$

Note that $(Z_1-c) \sim N(0,\Sigma_{Z1})$, where $\Sigma_{Z1} \equiv (1/n)[\beta_{11}/(\beta_{11}+\beta_{21})]^2\Sigma_x + (1/n)[\beta_{21}/(\beta_{11}+\beta_{21})]^2\Sigma_\xi$. Note also that the last two terms of $Z_1$ are distributed independently of s. Hence, we can write:

$$E[c\,|\,s,V] = \frac{s\Sigma_\eta^{-1}+Z_1\Sigma_{Z1}^{-1}}{\Sigma_c^{-1}+\Sigma_\eta^{-1}+\Sigma_{Z1}^{-1}}$$

Finally, note that $Z_1\Sigma_{Z1}^{-1} = [\Sigma_{Z1}n(\beta_{11}+\beta_{21})]^{-1}V - [\Sigma_{Z1}^{-1}(\beta_{31}-\beta_{41}\Lambda_{s1})/(\beta_{11}+\beta_{21})]s$. Hence:

$$E[c\,|\,s,V] = \Big[[\Sigma_\eta^{-1}-\Sigma_{Z1}^{-1}(\beta_{31}-\beta_{41}\Lambda_{s1})/(\beta_{11}+\beta_{21})] \,/\, (\Sigma_c^{-1}+\Sigma_\eta^{-1}+\Sigma_{Z1}^{-1})\Big]s + \Big[\Sigma_{Z1}n(\beta_{11}+\beta_{21})(\Sigma_c^{-1}+\Sigma_\eta^{-1}+\Sigma_{Z1}^{-1})\Big]^{-1}V$$

$$\equiv \lambda_{s2}s + \lambda_V V$$

<u>Calculating the $\phi$'s</u>

We want to determine $\{\phi_{ci},\phi_{si},\phi_s,\phi_V\}$ such that $E[F\,|\,c_i,s_i,s,P_1,T_{i1},T'_{i1},V,P_2] = \phi_{ci}c_i + \phi_{si}s_i+\phi_s s+\phi_V V$. (Note that equilibrium $P_1$, $P_2$, $T_{i1}$, and $T'_{i1}$ are informationally redundant here given the other conditioning variables.) Again, because nothing in the model provides information on the individual components of c, we can write:

$$E[F\,|\,c_i,s_i,s,V] = \lambda_{Fc}E[c\,|\,c_i,s_i,s,V]$$

Now, $E[c\,|\,c_i] = E[c\,|\,c+x_i]$ and $x_i \sim N(0,\Sigma_x)$

$$E[c \mid s_i] = E[c \mid c+\xi_i] \text{ and } \xi_i \sim N(0,\Sigma_\xi)$$

$$E[c \mid s] = E[c \mid c+\eta] \text{ and } \eta \sim N(0,\Sigma_\eta)$$

$$E[c \mid V] = E[c \mid \Sigma_i T_{i1}]$$

To make the signal in V orthogonal to the noise in $c_i$, $s_i$, and s, construct the statistic $Z_2$ where:

$$Z_2 \equiv [V-T_{i1}-(n-1)(\beta_{31}-\beta_{41}\Lambda_{s1})s][(n-1)(\beta_{11}+\beta_{21})]^{-1}$$
$$= c + \beta_{11}[(n-1)(\beta_{11}+\beta_{21})]^{-1}\sum_{k\neq i}^{n} x_k + \beta_{21}[(n-1)(\beta_{11}+\beta_{21})]^{-1}\sum_{k\neq i}^{n}\xi_k.$$

Note that $(Z_2-c) \sim N(0,\Sigma_{Z2})$, where:

$$\Sigma_{Z2} \equiv (n-1)^{-1}[\beta_{11}/(\beta_{11}+\beta_{21})]^2\Sigma_x + (n-1)^{-1}[\beta_{21}/(\beta_{11}+\beta_{21})]^2\Sigma_\xi.$$

Note that the last two terms of $Z_2$ are distributed independently of $c_i$, $s_i$, and s. Hence, we can write:

$$\text{(A15)} \qquad E[c \mid c_i,s_i,s,V] = \frac{c_i\Sigma_x^{-1}+s_i\Sigma_\xi^{-1}+s\Sigma_\eta^{-1}+Z_2\Sigma_{Z2}^{-1}}{\Sigma_c^{-1}+\Sigma_x^{-1}+\Sigma_\xi^{-1}+\Sigma_\eta^{-1}+\Sigma_{Z2}^{-1}}.$$

Finally, note that:

$$Z_2\Sigma_{Z2}^{-1} = \Sigma_{Z2}^{-1}[(n-1)(\beta_{11}+\beta_{21})]^{-1}V - \Sigma_{Z2}^{-1}[(n-1)(\beta_{11}+\beta_{21})]^{-1}[(\beta_{11}c_i+\beta_{21}s_i+(\beta_{31}-\beta_{41}\Lambda_{s1})s]$$
$$- \Sigma_{Z2}^{-1}[(\beta_{11}+\beta_{21})^{-1}(\beta_{31}-\beta_{41}\Lambda_{s1})]s$$

Taking account of the $c_i$, $s_i$, and s terms in $Z_2$, we have:

$$\phi_{ci} \equiv \lambda_{Fc}\Sigma_{Ec}\Sigma_x^{-1} - \lambda_{Fc}\Sigma_{Ec}\Sigma_{Z2}^{-1}[(n-1)(\beta_{11}+\beta_{21})]^{-1}\beta_{11},$$

$$\phi_{si} \equiv \lambda_{Fc}\Sigma_{Ec}\Sigma_\xi^{-1} - \lambda_{Fc}\Sigma_{Ec}\Sigma_{Z2}^{-1}[(n-1)(\beta_{11}+\beta_{21})]^{-1}\beta_{21}, \qquad \text{and}$$

$$\phi_s \equiv \lambda_{Fc}\Sigma_{Ec}\Sigma_\eta^{-1} - \lambda_{Fc}\Sigma_{Ec}\Sigma_{Z2}^{-1}n[(n-1)(\beta_{11}+\beta_{21})]^{-1}(\beta_{31}-\beta_{41}\Lambda_{s1})$$

where $\lambda_{Fc}$ comes from $E[F \mid \Omega_i]=\lambda_{Fc}E[c \mid \Omega_i]$, and $\Sigma_{Ec} \equiv (\Sigma_c^{-1}+\Sigma_x^{-1}+\Sigma_\xi^{-1}+\Sigma_\eta^{-1}+\Sigma_{Z2}^{-1})^{-1}$. (Note from the outset of appendix section A.2.5. that $\Sigma_{Ec}$ — the inverse of the denominator in Eq. (A15) — is equal to the conditional variance of c with respect to $\Omega_{Ti2}$.) Finally, from $Z_2$ we have:

$$\phi_V \equiv \lambda_{Fc}\Sigma_{Ec}\Sigma_{Z2}^{-1}[(n-1)(\beta_{11}+\beta_{21})]^{-1}.$$

Calculating the $\rho$'s

We want to determine $\{\rho_{ci},\rho_{si},\rho_s\}$ such that $E[c_j|c_i,s_i,s]= \rho_{ci}c_i+\rho_{si}s_i+\rho_s s$. Note:
$E[c_j|c_i,s_i,s] = E[c+x_j|c_i,s_i,s] = E[c|c_i,s_i,s]$

Since $c_i$, $s_i$, and s are signals of c with precisions $\Sigma_x^{-1}$, $\Sigma_\xi^{-1}$, and $\Sigma_\eta^{-1}$ respectively, we can write:

$$E[c|c_i,s_i,s] = (\Sigma_x^{-1}/\Sigma_c^{-1}+\Sigma_x^{-1}+\Sigma_\xi^{-1}+\Sigma_\eta^{-1})c_i + (\Sigma_\xi^{-1}/\Sigma_c^{-1}+\Sigma_x^{-1}+\Sigma_\xi^{-1}+\Sigma_\eta^{-1})s_i$$
$$+ (\Sigma_\eta^{-1}/\Sigma_c^{-1}+\Sigma_x^{-1}+\Sigma_\xi^{-1}+\Sigma_\eta^{-1})s \equiv \rho_{ci}c_i+\rho_{si}s_i+\rho_s s$$

Calculating the Period–One $\delta$'s

We want to determine $\{\delta_{ci},\delta_{si},\delta_s\}$ such that $E[T'_{i1}|c_i,s_i,s] = \delta_{ci}c_i+\delta_{si}s_i+\delta_s$.

$$\begin{aligned}
E[T'_{i1}|c_i,s_i,s] &= E[T_{j1}|c_i,s_i,s] = E[\beta_{11}c_j+\beta_{21}s_j+(\beta_{31}-\beta_{41}\Lambda_{s1})s|c_i,s_i,s] \\
&= \beta_{11}E[c_j|c_i,s_i,s] + \beta_{21}E[s_j|c_i,s_i,s] + (\beta_{31}-\beta_{41}\Lambda_{s1})s \\
&= \beta_{11}E[c|c_i,s_i,s] + \beta_{21}E[c|c_i,s_i,s] + (\beta_{31}-\beta_{41}\Lambda_{s1})s \\
&= (\beta_{11}+\beta_{21})(\rho_{ci}c_i+\rho_{si}s_i+\rho_s) + (\beta_{31}-\beta_{41}\Lambda_{s1})s \\
&= [(\beta_{11}+\beta_{21})\rho_{ci}]c_i + [(\beta_{11}+\beta_{21})\rho_{si}]s_i + [(\beta_{11}+\beta_{21})\rho_s+(\beta_{31}-\beta_{41}\Lambda_{s1})]s \\
&\equiv \delta_{ci}c_i+\delta_{si}s_i+\delta_s
\end{aligned}$$

Calculating the $\gamma$'s

$$\begin{aligned}
P_2 &= \Lambda_{s2}s + \Lambda_V(T_{i1}+V') \\
&= \Lambda_{s2}s + \Lambda_V(D_{i1}+c_i+E[T'_{i1}|c_i,s_i,s]) + \Lambda_V V' \\
&= \Lambda_V D_{i1} + \Lambda_V(1+\delta_{ci})c_i + \Lambda_V\delta_{si}s_i + (\Lambda_V\delta_s+\Lambda_{s2})s + \Lambda_V V' \\
&\equiv \Lambda_V D_{i1} + \gamma_{ci}c_i + \gamma_{si}s_i + \gamma_s s + \Lambda_V V'
\end{aligned}$$

Calculating the $\pi$'s

We want to determine $\{\pi_D,\pi_{ci},\pi_{si},\pi_s,\pi_{V'}\}$ such that $\mu_{iF}-P_2= \pi_D D_{i1}+\pi_{ci}c_i+\pi_{si}s_i+\pi_s s+ \pi_{V'}V'$, where $\mu_{iF}\equiv E[F|c_i,s_i,s,P_1,T_{i1},T'_{i1},V,P_2]$. From Eq. (A6) we have:

$$\mu_{iF}-P_2 = \phi_{ci}c_i + \phi_{si}s_i + \phi_s s + \phi_V V - P_2.$$

But: (i) $V = V' + T_{i1}$

(ii) $P_2 = \Lambda_{s2}s + \Lambda_V V = \Lambda_{s2}s + \Lambda_V V' + \Lambda_V T_{i1}$

(iii) $T_{i1} = D_{i1} + c_i + \delta_{ci}c_i + \delta_{si}s_i + \delta_s s.$

Making these substitutions yields:

$$\begin{aligned}
\mu_{iF} - P_2 &= \phi_{ci}c_i + \phi_{si}s_i + \phi_s s + \phi_V(V'+T_{i1}) - (\Lambda_{s2}s + \Lambda_V V' + \Lambda_V T_{i1}) \\
&= \phi_{ci}c_i + \phi_{si}s_i + (\phi_s - \Lambda_{s2})s + (\phi_V - \Lambda_V)V' + (\phi_V - \Lambda_V)T_{i1} \\
&= (\phi_V - \Lambda_V)D_{i1} + [\phi_{ci} + (\phi_V - \Lambda_V)(1+\delta_{ci})]c_i + [\phi_{si} + (\phi_V - \Lambda_V)\delta_{si}]s_i \\
&\qquad + [(\phi_s - \Lambda_{s2}) + (\phi_V - \Lambda_V)\delta_s]s + (\phi_V - \Lambda_V)V' \\
&\equiv \pi_D D_{i1} + \pi_{ci}c_i + \pi_{si}s_i + \pi_s s + \pi_{V'}V'.
\end{aligned}$$

<u>Calculating the X's</u>

From the optimization problem in Eq. (A11) we get the first order condition:

$$\begin{aligned}
&4\theta\Lambda_V D_{i1} + 2\theta(\gamma_{ci}c_i + \gamma_{si}s_i + \gamma_s s - P_1) + 2\pi_D \Sigma_2^{-1}(\pi_D D_{i1} + \pi_{ci}c_i + \pi_{si}s_i + \pi_s s) \\
&\qquad - 2A^{-1}B(\pi_D \pi_{V'} \Sigma_2^{-1} + \theta\Lambda_V) = 0
\end{aligned}$$

where B is defined above in appendix section A.2.4.. Dividing by $-2$, substituting for B, and recognizing that $\mu_{V'} = (n-1)E[T'_{i1} | c_i, s_i, s, P_1] = (n-1)(\delta_{ci}c_i + \delta_{si}s_i + \delta_s s)$ yields:

$$\begin{aligned}
&\left[(\pi_D \pi_{V'} \Sigma_2^{-1} + \theta\Lambda_V)^2 A^{-1} - \pi_D^2 \Sigma_2^{-1} - 2\theta\Lambda_V\right] D_{i1} \\
&- \left[\theta\gamma_{ci} + \pi_D \pi_{ci} \Sigma_2^{-1} - (\pi_D \pi_{V'} \Sigma_2^{-1} + \theta\Lambda_V)[\pi_{ci}\pi_{V'}\Sigma_2^{-1} - (n-1)\delta_{ci}\Sigma_{V'}^{-1}]A^{-1}\right] c_i \\
&- \left[\theta\gamma_{si} + \pi_D \pi_{si} \Sigma_2^{-1} - (\pi_D \pi_{V'} \Sigma_2^{-1} + \theta\Lambda_V)[\pi_{si}\pi_{V'}\Sigma_2^{-1} - (n-1)\delta_{si}\Sigma_{V'}^{-1}]A^{-1}\right] s_i \\
&- \left[\theta\gamma_s + \pi_D \pi_s \Sigma_2^{-1} - (\pi_D \pi_{V'} \Sigma_2^{-1} + \theta\Lambda_V)[\pi_s \pi_{V'}\Sigma_2^{-1} - (n-1)\delta_s \Sigma_{V'}^{-1}]A^{-1}\right] s + \theta P_1 = 0
\end{aligned}$$

$$\begin{aligned}
\text{or} \quad &\left[X_1^2 A^{-1} - \pi_D^2 \Sigma_2^{-1} - 2\theta\Lambda_V\right] D_{i1} - \left[\theta\gamma_{ci} + \pi_D \pi_{ci} \Sigma_2^{-1} - X_1 X_2 A^{-1}\right] c_i \\
&- \left[\theta\gamma_{si} + \pi_D \pi_{si} \Sigma_2^{-1} - X_1 X_3 A^{-1}\right] s_i - \left[\theta\gamma_s + \pi_D \pi_s \Sigma_2^{-1} - X_1 X_4 A^{-1}\right] s + \theta P_1 = 0
\end{aligned}$$

<u>Calculating $\lambda_{cis}$ and $\lambda_{ciV}$ in the period–two pricing rule</u>

We want to determine $\{\lambda_{cis}, \lambda_{ciV}\}$ such that $E[c_i | s, V] = \lambda_{cis}s + \lambda_{ciV}V$. Note:

$$E[c_i|s,V] = E[c|s,V] + E[x_i|s,V] = \lambda_{s2}s + \lambda_V V + E[x_i|s,V]$$

Now, to determine $E[x_i|s,V]$ we construct a statistic $Z_3$:

$$Z_3 \equiv (1/n\beta_{11})[V - n(\beta_{31} - \beta_{41}\Lambda_{s1})s] = n^{-1}\sum_i^n x_i + (\beta_{11}+\beta_{21})\beta_{11}^{-1}c + (\beta_{21}/n\beta_{11})\sum_i^n \xi_i.$$

From the last two terms, we define:

$$\Sigma_{Z3} \equiv (\beta_{11}+\beta_{21})^2\beta_{11}^{-2}\Sigma_c + (\beta_{21}/\beta_{11})^2 n^{-1}\Sigma_\xi$$

with which we get:

$$E[x_i|s,V] = [\Sigma_x/(\Sigma_x+\Sigma_{Z3})]Z_3 = [\Sigma_x/(\Sigma_x+\Sigma_{Z3})](1/n\beta_{11})[V - n(\beta_{31}-\beta_{41}\Lambda_{s1})s]$$

implying:

$$E[c_i|s,V] = \Big[\lambda_{s2} - [\Sigma_x/(\Sigma_x+\Sigma_{Z3})](1/\beta_{11})(\beta_{31}-\beta_{41}\Lambda_{s1})\Big]s + \Big[\lambda_V + [\Sigma_x/(\Sigma_x+\Sigma_{Z3})](1/n\beta_{11})\Big]V$$
$$\equiv \lambda_{cis}s + \lambda_{ciV}V$$

<u>Calculating the Period–Two $\delta$'s</u>

We want to determine $\{\delta_{ci2}, \delta_{si2}, \delta_{s2}, \delta_{T'2}, \delta_{V2}\}$ such that $E[T'_{i2}|\Omega_{Ti2}] = \delta_{ci2}c_i + \delta_{si2}s_i + \delta_{s2}s + \delta_{T'2}T'_{i1} + \delta_{V2}V$. This is a rather tedious calculation. We will use the notation $T_{k1} \equiv T'_{i1}$, i.e., dealer i receives dealer k's orders (dealer i is to dealer k's left). We begin with:

$$\begin{aligned} E[T'_{i2}|\Omega_{Ti2}] &= E[\beta_{12}c_k + \beta_{22}s_k + \beta_{32}s + \beta_{42}T'_{k1} + \beta_{52}V - \beta_{62}P_2|\Omega_{Ti2}] \\ &= \beta_{12}E[c_k|\Omega_{Ti2}] + \beta_{22}E[s_k|\Omega_{Ti2}] + \beta_{42}E[T'_{k1}|\Omega_{Ti2}] \\ &\quad + \beta_{32}s + \beta_{52}V - \beta_{62}(\Lambda_{s2}s + \Lambda_V V) \end{aligned}$$

The three remaining expectations need to be solved. The first two can be collapsed into one. To see this, note that the conditioning set together with the trading rule in Eq. (A13) allows us to write:

$$E[s_k|\Omega_{Ti2}] = -(\beta_{11}/\beta_{21})E[c_k|\Omega_{Ti2}] + (1/\beta_{21})T_{k1} - (\beta_{31}-\beta_{41}\Lambda_{s1})(1/\beta_{21})s$$

which implies:

$$\text{(A16)} \quad \begin{aligned} E[T'_{i2}|\Omega_{Ti2}] &= [\beta_{12} - (\beta_{22}\beta_{11}/\beta_{21})]E[c_k|\Omega_{Ti2}] + \beta_{42}E[T'_{k1}|\Omega_{Ti2}] \\ &\quad + [\beta_{32} - (\beta_{31}-\beta_{41}\Lambda_{s1})(\beta_{22}/\beta_{21}) - \beta_{62}\Lambda_{s2}]s + (\beta_{22}/\beta_{21})T_{k1} + (\beta_{52}-\beta_{62}\Lambda_V)V \end{aligned}$$

Now only two expectations remain to be solved, $E[c_k|\Omega_{Ti2}]$ and $E[T'_{k1}|\Omega_{Ti2}]$. For the first:

$$E[c_k|\Omega_{Ti2}] = E[c|\Omega_{Ti2}] + E[x_k|\Omega_{Ti2}]$$

The first of these terms appears in Eq. (A15). For the second, the key conditioning variable is $T_{k1}$. To extract the signal, define the statistic $Z_4$:

$$\begin{aligned} Z_4 &\equiv [T_{k1}-(\beta_{31}-\beta_{41}\Lambda_{s1})s-(\beta_{11}+\beta_{21})E[c|\Omega_{Ti2}]]\beta_{11}^{-1} \\ &= x_k + (\beta_{11}+\beta_{21})\beta_{11}^{-1}(c-E[c|\Omega_{Ti2}]) + \beta_{21}\beta_{11}^{-1}\xi_k \end{aligned}$$

Since $x_k$, $(c-E[c|\Omega_{Ti2}])$, and $\xi_k$ are mutually independent, we can write:

$$E[x_k|\Omega_{Ti2}] = \lambda_{Z4}Z_4 \quad \text{where } \lambda_{Z4} \equiv \Sigma_x \,/\, [\Sigma_x+(\beta_{11}+\beta_{21})^2\beta_{11}^{-2}\Sigma_{Ec}+(\beta_{21}/\beta_{11})^2\Sigma_\xi].$$

Thus, from this expression and Eq. (A15) we can express the expectation $E[c_k|\Omega_{Ti2}]$ as:

$$\begin{aligned} E[c_k|\Omega_{Ti2}] &= E[c|\Omega_{Ti2}] + E[x_k|\Omega_{Ti2}] \\ &= E[c|\Omega_{Ti2}] + \lambda_{Z4}\beta_{11}^{-1}[T_{k1}-(\beta_{31}-\beta_{41}\Lambda_{s1})s-(\beta_{11}+\beta_{21})E[c|\Omega_{Ti2}]] \\ &= [1-\lambda_{Z4}\beta_{11}^{-1}(\beta_{11}+\beta_{21})]E[c|\Omega_{Ti2}] + \lambda_{Z4}\beta_{11}^{-1}[T_{k1}-(\beta_{31}-\beta_{41}\Lambda_{s1})s] \\ &= [1-\lambda_{Z4}\beta_{11}^{-1}(\beta_{11}+\beta_{21})]\Sigma_{Ec}(c_i\Sigma_x^{-1}+s_i\Sigma_\xi^{-1}+s\Sigma_\eta^{-1}+Z_2\Sigma_{Z2}^{-1}) + \lambda_{Z4}\beta_{11}^{-1}[T_{k1}-(\beta_{31}-\beta_{41}\Lambda_{s1})s] \end{aligned}$$

Letting K denote $[1-\lambda_{Z4}\beta_{11}^{-1}(\beta_{11}+\beta_{21})]\Sigma_{Ec}$ and collecting terms we have:

$$= K\Sigma_x^{-1}c_i + K\Sigma_\xi^{-1}s_i + [K\Sigma_\eta^{-1}-\lambda_{Z4}\beta_{11}^{-1}(\beta_{31}-\beta_{41}\Lambda_{s1})]s + \lambda_{Z4}\beta_{11}^{-1}T_{k1} + K\Sigma_{Z2}^{-1}Z_2.$$

Recall from the calculations for Eq. (A15) that:

$$Z_2 \equiv [V-T_{i1}-(n-1)(\beta_{31}-\beta_{41}\Lambda_{s1})s][(n-1)(\beta_{11}+\beta_{21})]^{-1}.$$

Letting R denote $[(n-1)(\beta_{11}+\beta_{21})]^{-1}$, we have a condensed expression for $E[c_k|\Omega_{Ti2}]$:

$$\begin{aligned} E[c_k|\Omega_{Ti2}] = {} & (K\Sigma_x^{-1}-KR\Sigma_{Z2}^{-1}\beta_{11})c_i + (K\Sigma_\xi^{-1}-KR\Sigma_{Z2}^{-1}\beta_{21})s_i \\ & + [K\Sigma_\eta^{-1}-[\lambda_{Z4}\beta_{11}^{-1}+nKR\Sigma_{Z2}^{-1}](\beta_{31}-\beta_{41}\Lambda_{s1})]s + \lambda_{Z4}\beta_{11}^{-1}T_{k1} + KR\Sigma_{Z2}^{-1}V \end{aligned}$$

This leaves $E[T'_{k1}|\Omega_{Ti2}]$ as the remaining expectation component of Eq. (A16) to be solved. The key conditioning variable here is V since, due to symmetry, V summarizes the information available regarding dealer k's position disturbance $T'_{k1}$ (from the dealer to dealer k's right):

$$\begin{aligned} E[T'_{k1}|\Omega_{Ti2}] &= (V-T_{i1}-T_{k1})/(n-2) \\ &= -(n-2)^{-1}[\beta_{11}c_i+\beta_{21}s_i+(\beta_{31}-\beta_{41}\Lambda_{s1})s+T_{k1}-V] \end{aligned}$$

We can now plug these expectation values back into Eq. (A16). Letting J denote

$[\beta_{12}-(\beta_{22}\beta_{11}/\beta_{21})]$, we have:

$$
\begin{aligned}
E[T'_{i2}|\Omega_{Ti2}] = \\
&+ \left[JK(\Sigma_x^{-1}-R\Sigma_{Z2}^{-1}\beta_{11})-\beta_{42}(n-2)^{-1}\beta_{11}\right] c_i \\
&+ \left[JK(\Sigma_\xi^{-1}-R\Sigma_{Z2}^{-1}\beta_{21})-\beta_{42}(n-2)^{-1}\beta_{21}\right] s_i \\
&+ \left[JK\Sigma_\eta^{-1}-[J\lambda_{Z4}\beta_{11}^{-1}+nJKR\Sigma_{Z2}^{-1}+\beta_{22}/\beta_{21}+\beta_{42}(n-2)^{-1}](\beta_{31}-\beta_{41}\Lambda_{s1})+\beta_{32}-\beta_{62}\Lambda_{s2}\right] s \\
&+ \left[J\lambda_{Z4}\beta_{11}^{-1}+(\beta_{22}/\beta_{21})-\beta_{42}(n-2)^{-1}\right] T_{k1} \\
&+ \left[JKR\Sigma_{Z2}^{-1}+\beta_{42}(n-2)^{-1}+(\beta_{52}-\beta_{62}\Lambda_V)\right] V \\
&\equiv \delta_{ci2}c_i + \delta_{si2}s_i + \delta_{s2}s + \delta_{T'2}T'_{i1} + \delta_{V2}V \qquad \text{(recall that } T_{k1}\equiv T'_{i1})
\end{aligned}
$$

Calculating $\Sigma_2$

$$
\begin{aligned}
\Sigma_2 &\equiv \mathrm{Var}[F-P_2|c_i,s_i,s,P_1,T_{i1},T'_{i1},V,P_2] = \mathrm{Var}[(c-\nu)-P_2|c_i,s_i,s,V] \\
&= \mathrm{Var}[c|c_i,s_i,s,V] + \mathrm{Var}[\nu|c_i,s_i,s,V] \\
&= 1/(\Sigma_c^{-1}+\Sigma_x^{-1}+\Sigma_\xi^{-1}+\Sigma_\eta^{-1}+\Sigma_{Z2}^{-1}) + \Sigma_\nu
\end{aligned}
$$

where the last line uses the conditional variance of c from the denominator of Eq. (A15).

Calculating $\Sigma_{V'}$

$$
\begin{aligned}
&\mathrm{Var}[V'|c_i,s_i,s,P_1] = E[(V'-\mu_{V'})^2|c_i,s_i,s,P_1] \\
&= E[(\sum_{j\neq i}^{n} T_{j1}-\mu_{V'})^2|c_i,s_i,s,P_1] \\
&= E[(\sum_{j\neq i}^{n}(\beta_{11}c_j+\beta_{21}s_j+\beta_{31}s-\beta_{41}P_1)-\mu_{V'})^2|c_i,s_i,s,P_1] \\
&= E[(\beta_{11}\sum_{j\neq i}^{n}c_j+\beta_{21}\sum_{j\neq i}^{n}s_j+(n-1)\beta_{31}s-(n-1)\beta_{41}P_1-\mu_{V'})^2|c_i,s_i,s,P_1] \\
&= E[(\beta_{11}\sum_{j\neq i}^{n}(c+x_j)+\beta_{21}\sum_{j\neq i}^{n}(c+\xi_j)+(n-1)\beta_{31}s-(n-1)\beta_{41}P_1-\mu_{V'})^2|c_i,s_i,s,P_1] \\
&= E\left[\left[(n-1)(\beta_{11}+\beta_{21})c+\beta_{11}\sum_{j\neq i}^{n}x_j+\beta_{21}\sum_{j\neq i}^{n}\xi_j+(n-1)\beta_{31}s-(n-1)\beta_{41}P_1-\mu_{V'}\right]^2|c_i,s_i,s,P_1\right] \\
&= (n-1)^2(\beta_{11}+\beta_{21})^2(\Sigma_c^{-1}+\Sigma_x^{-1}+\Sigma_\xi^{-1}+\Sigma_\eta^{-1})^{-1} + (n-1)\beta_{11}^2\Sigma_x + (n-1)\beta_{21}^2\Sigma_\xi \\
&\equiv \Sigma_{V'}
\end{aligned}
$$

The penultimate line uses the fact that $\mathrm{Var}[c|c_i,s_i,s] = (\Sigma_c^{-1}+\Sigma_x^{-1}+\Sigma_\xi^{-1}+\Sigma_\eta^{-1})^{-1}$.