Order Flows, Delta Hedging and Exchange Rate Dynamics

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ABSTRACT

This paper proposes a microstructure model of the FX options and spot markets. On both market segments, dealers receive customer order flows and use this private information strategically to speculate during interdealer rounds. This non-payoff information is first impounded in private dealers' inventories before affecting prices. Derivative trading impacts the equilibrium exchange rate via feedback effects of delta hedging strategies followed by option dealers to cover the FX risk embedded in their options portfolio. It is shown that depending on the correlation between spot and option order flows, the volatility of the exchange rate can either be amplified or reduced.

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1. Introduction

The FX microstructure literature focuses exclusively on spot order flows to explain the changes in the exchange rate (Lyons, 1997; Lyons, 2001; Evans and Lyons, 2002). It excludes *a priori* the possible influence of the derivative assets trading on the dynamics of the primary asset price. Such an approach would be warranted if markets were complete. The option price would therefore be given by the replication cost of the synthetic option' payoff with a portfolio composed of the risky asset and the risk-free asset. In this framework, options are indeed redundant assets so that neither their price, nor their order flows could provide relevant information about the dynamics of the underlying asset price. Such an idealised environment is described by a set of assumptions surrounding the Black and Scholes (1973) (thereafter, B&S) model or Garman and Kohlhagen (1983) (G&K) for currency options[1]. However, empirical evidence suggests that markets are incomplete notably due to stochastic volatility. Thus, option prices as well as option order flows are likely to convey specific information not only about the expected return of the underlying asset but also about its future volatility. The information may be all the more relevant that the share of trading in currency options markets in the total FX trading is growing. The BIS triennial survey of foreign exchange activity (2002) shows that the market average turnover in the FX options segment accounted for US$ 60 billions in April 2001, i.e. 15.5% of the turnover in the spot segment, which is in constant decline (US$ 387 billions). The notional amounts outstanding of options also provides an indication on the potential impact of dynamic hedging strategies on the spot market. Indeed option dealers use such strategies to cover the FX risk embedded in options portfolios, which involves taking positions in the spot market. At the end of 2002, this amount was equal to US$ 3 238 billions (BRI, 2003).[2]

Although the FX microstructure literature has ignored the interaction between option trading and the spot dynamics, research focusing on the stock markets has widely explored this area (Detemple and Selden, 1991; Back, 1993; John, Koticha and Subramanyam, 1994; Easley, O'Hara and Srinivas, 1998). The reason rests on the fact the stock microstructure models are grounded on the assumption that stock traders have private information about the future payoff. Traders with private information may prefer to trade on option markets due to lower costs and leverage effects (Black, 1976; Mayhew, Sarin and Shastri, 1995). If the private information is related

[1] Three key hypotheses about the underlying asset markets are postulated in standard derivative pricing theory, based on arbitrage arguments. Markets are assumed to be complete, frictionless and perfectly elastic.

2 However, the impact on the spot market via dynamic hedging of option dealers is more directly related to the difference between options bought and sold. The notional amount outstanding would then be reduced to 22.2 billions of US dollars.

to future volatility, traders may only intervene on the derivative market (Back, 1993; Cherian and Jarrow, 1998). However, this theoretical literature does not allow a consensus regarding the impact of option trading on the underlying asset market to be reached.[3] Whereas the existence of private information is a widely accepted assumption when the stock markets are considered, it is generally viewed as irrelevant for the foreign exchange market, perceived as the most informationally efficient market.

Other approaches unrelated to private information analyse the influence of derivative trading on the dynamics of the underlying asset price. The *feedback effect* literature highlights the impact of dynamic hedging behavior of option dealers on the underlying asset's equilibrium price and volatility. These strategies prove generally destabilising as option dealers are supposed to be net option writers so that they are buying in bullish spot market and selling in bearish spot market. In a partial equilibrium analysis, Platen and Schweizer (1998) and Frey and Stremme (1997), starting from a Black and Scholes economy, show that the parameters of the diffusion process for the price of the underlying asset become both time and price dependent. However, this kind of feedback effect is rather indirect because only the coefficients of the stochastic process followed by the underlying price react to the dynamic hedging strategies, but not the price itself. Garber and Spencer (1996) assess the extent to which dynamic hedging strategies may impinge the efficacy of interest rate defence of fixed exchange rate regimes. Krueger (1999) show that during currency crisis, the volatility can reach sufficient high levels so that the impact of interest rate changes on dynamic hedgers is likely to be small. Genotte and Leland (1990) evaluate the impact of such hedging strategies, assuming differences in information between market participants that induce relative illiquidity in the stock market. Due to its pro-cyclical impact, the feedback effect literature leads

3 In a general equilibrium framework with incomplete financial markets, Detemple and Selden (1991) show that the introduction of an option may increase the equilibrium stock price and decrease its volatility. Back (1993) extends Kyle's (1985) model of informed trading by introducing the options market. The volatility becomes stochastic, but its expected average level does not change. The intuition followed is similar to that of Grossman (1988). Option trading transmits information that would not be available if replaced with options synthesised by dynamic trading strategies. Easley, O'Hara and Srinivas (1998) investigate the informational role of transactions volume in options markets in a model with asymmetric information, where traders can either trade in the equity or in the option markets. They find that option volumes contain information about future stock prices. Other approaches consider a market that is incomplete without the options but which becomes complete when the options are introduced. Brennan and Cao (1996) show that including options does not change the price of the underlying asset. Cao's model (1999) endogenizes the acquisition of information so that the increasing incentive to collect information leads to a higher stock price and a reduction in its volatility.

to the prediction of an increase in the volatility of the underlying asset price. But this conclusion does not necessarily fit the empirical evidence (Mayhew, 2000). Furthermore, assuming that option dealers are always net option writers is not confirmed by the data reported by the BIS (2003): there are almost as many options bought as options sold by dealers.

The aim of this paper is to analyse the interaction between currency option trading and the intra-daily exchange rate dynamics. The FX interdealer microstructure model of Evans and Lyons (2002) is thus extended to include a derivative market segment. Their model is based on customer order flows, which dealers claim to be their most important source of information (Lyons, 1995; Yao, 1998). In this framework, customer order flow is the source of asymmetry among dealers. Both spot and option dealers are thus supposed to use their private information strategically in that their speculative demand will depend on the impact their trade will have on subsequent prices. Hence, the private information is impounded in dealers' inventories before being reflected in quoted prices. The interaction between the two markets is captured by the effect of delta hedging behaviour on the dynamics of the underlying asset price. In a partial equilibrium framework, it is shown that the volatility of the exchange rate could increase or decrease, depending on the correlation between the spot and the currency option order flows.

The paper is organised as follows. Section 2 describes the main features of the interdealer trading model of the spot and derivatives market segments. Section 3 displays the optimal quoting and trading strategies of spot and option dealers. Section 4 presents simulation results considering different correlation coefficients between the spot and the currency option order flows and different maturities for options. The last section concludes.

2. The interdealer trading model of the FX options and spot markets

2.1. Environment

The decentralised FX market is supposed to be composed of the spot and option market segments and to operate in discrete time. Trading is simultaneous on the two segments. There are N spot dealers, indexed by *i*, M options dealers indexed by *q*, a continuum of customers trading in the spot market, a continuum of customers trading in the options market and delta hedgers. All agents on the spot and currency options markets are rational and have the same risk aversion parameter γ. They are supposed to maximise a negative exponential utility function:

$$U_t = E_t\left[-\exp\left(-\gamma W_{t+1}\right)\right] \tag{1}$$

where E_t is the expectations operator conditional on agents' information and W_{t+1} is the wealth at the end of day *t+1*. There are three assets in the model, one riskless, one risky, and a call option. The daily interest rate on the risk free asset is denoted r_t . The daily payoff on the risky foreign asset at time *t*, denoted R_t , is realised and observed publicly at the beginning of the trading day. It is composed of a series of increments ΔR_t so that:

$$R_t = \sum_{s=1}^{t} \Delta R_s \tag{2}$$

The payoff increments ΔR_t are *i.i.d.* $\mathrm{N}\left(0, \sigma_R^2\right)$ and represent the flow of macroeconomic information publicly available, such as the change in interest rates. Each day is characterised by three trading rounds (Table 1).

Table 1 - Three trading rounds in the currency option and spot markets

Spot market	*Currency options market*
Round 1 • The payoff R_t is realised • Spot dealers quote • Customers trade with spot dealers	• Option dealers quote • Customers trade with option dealers
Round 2 • Spot dealers quote • Spot dealers trade with spot dealer • Spot and options interdealer order flows revealed	• Option dealers quote • Option dealers trade with option dealers • Option and spot interdealer order flows revealed
Round 3 • Spot dealers quote • Spot dealers trade with the public	• Option dealers trade with delta hedgers • Delta hedger trade with the public on the spot market

2.2. Quotes on the spot and derivative markets

The quoting rules described in Lyons (1997) prevail. Quoting is simultaneous, independent and required. Hence a dealer's quote on the spot or on the option market cannot be conditioned on the quotes of other spot and option dealers within a given round. Quotes are observable and available to all participants and each quote is a single price at which each dealer agrees to buy and sell any amount. Finally, no arbitrage requires that quotes are common to all dealers in all rounds, so that they can only be conditioned on public information.

Spot dealers quote simultaneously and independently the price of the foreign exchange rate, denoted $P_{i,k,t}$, where *k* corresponds the *k*-round price and *k=1,2,3*. On the derivatives segment, simultaneously and independently option dealers quote the Black and Scholes (1973) at-the-money-forward (ATMF) implied volatility. In the FX over-the-counter market, currency options are *de facto* quoted in B&S volatility rather than in option prices and ATMF options are also the most traded. The average expected volatility of the underlying asset price for a given maturity τ is denoted $V_{q,k,t}$. It is thus a single point on the volatility term structure. This volatility is updated during a trading day with the arrival of new public information. Implied volatility therefore fluctuates in an unpredictable way. Boolen and Whaley (2003) show that an important factor driving the change in implied volatility quotes is the net buying pressure from public order flow. Hence, in our microstructure framework, the implied volatility will vary with the stochastic net order flow from option customers.

In this context of time-varying volatility, fair option value and perfect hedge cannot be determined with certainty. Indeed, markets are incomplete due to the volatility risk in option trading. Engle and Rosenberg (2000) propose the following approximate valuation formula to price at-the-money-forward calls, which is referred to as the "Black-Scholes-plug-in formula" (thereafter BSP):

$$C_{k,t} \cong BSP\left(E_t\left[V_{k,t}\left(P_{k,t}\right)\right], P_{k,t}, \tau\right) \tag{3}$$

where $C_{k,t}$ stands for the price of ATMF calls with matuirty τ in round *k*. As in Engle and Rosenberg (2002), dependence on the strike price and on domestic and foreign interest rates are ignored. Implicitly, it is supposed that the most important variables of the call price are the price of the foreign currency and the implied volatility for a given maturity. As options are ATMF, the BSP price is relatively accurate because the BSP model relies on the approximate linearity of the B&S formula in the volatility parameter for ATMF options.[4] Hence, updating expected volatility based on dealers' expanded information set during the day will result in rational changes in the call price. Furthermore, as volatility quotes are constant and common to all option dealers within each round to avoid arbitrage opportunities, option dealers have no incentive to deviate from the BSP model within each round, once they believe it is the correct pricing model.[5] Indeed, as the option price is equivalent to the replication cost of a synthesised option, using another model would necessary give rise to a different option price and thus to arbitrage opportunities.

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[4] As noted by Engle and Rosenberg (2000), the effect of the volatility risk premium should be small, because average volatility is used rather than average volatility under a risk-neutral measure.

5 Cherian and Jarrow (1998) show that when dealers share the same beliefs that the B&S formula is the correct option pricing model, this results in a self-fulfilling rational equilibrium option price.

2.3. *Trading rounds*

2.3.1 Round 1

After the realisation of the payoff of holding foreign exchange, each spot dealer *i* quotes $P_{i,1,t}$, at which he agrees to sell or buy any amount, whereas simultaneously and independently each options dealer *q* quotes at-the-money-forward implied volatility $V_{q,1,t}$. Each spot dealer *i* receives a net order flow denoted $x_{i,1,t}$, which is normally distributed with zero mean and known standard deviation σ_x. Each option dealer *q* receives a net order flow denoted $y_{q,1,t}$, which is also normally distributed with mean 0 and known standard deviation σ_y. Whereas the spot order flow refers to the difference between the buying and selling position in foreign exchange received by the dealer, the option order flow denotes the difference between the number of calls bought and sold by option customers. Spot and derivative flows are supposed to be independent of the payoff increment ΔR_t. Each spot order flow $x_{i,1,t}$ is executed at the quoted price $P_{i,1,t}$, where positive (negative) $x_{i,1,t}$ denotes a buy (sell) order from customers. Each option order flow $T_{i,2,t}$ is executed at price $C_{q,1,t}$ according to equation 3, where positive (negative) $D_{i,2,t}$ means that customers are nets buyers (sellers) of calls. Both order flows on the spot and derivative market segment are liquidity demand shocks from the public, which are private information for the dealer who receives it.

2.3.2 Round 2

Round 2 is an interdealer trading round. Simultaneously and independently, each spot dealer quotes $P_{i,2,t}$ and each options dealer quotes $V_{q,2,t}$. These quotes are observable and available to all dealers on both markets. During this round, dealers attempt to reduce their risk exposure stemming from their trade with customers, but they also use their private information to speculate. This private information will be first impounded in each dealer' round 2 inventory before affecting the round 3 quotes. Let $D_{i,2,t}$ be the speculative demand from the spot dealer *i* and $E\left[T'_{i,2,t} \middle| \Omega_{i,2,t}\right]$, the expectation of dealer *i* incoming flows from other dealers, then the net outgoing order for dealer *i*, $T_{i,2,t}$ will be:

$$T_{i,2,t} = D_{i,2,t} + x_{i,1,t} + E\left[T'_{i,2,t} \middle| \Omega_{i,2,t}\right] \tag{4}$$

Let $D^{O}_{q,2,t}$ be the speculative demand from the option dealer *q* and $E\left[O^{'}_{q,2,t}\middle|\Omega_{q,2,t}\right]$ the expectation of dealer *q* incoming flows from other option dealers, then the net outgoing order for dealer *q*, $O_{q,2,t}$, will be:

$$O_{q,2,t} = D^{O}_{q,2,t} + y_{q,1,t} + E\left[O^{'}_{q,2,t}\middle|\Omega_{q,2,t}\right] \tag{5}$$

At the end of round 2, spot and options interdealer order flows are revealed to all market participants and thus become public information. A net positive spot interdealer order flow indicates that buy orders are greater than sell orders, whereas a positive options order flow denotes that dealers are net buyers of call options.

$$X_t \equiv \sum_{i=1}^{n} T_{i,2,t} \text{ and } Y_t \equiv \sum_{q=1}^{m} O_{q,2,t} \tag{6}$$

2.3.3. Round 3

In round 3, option dealers trade with delta hedgers so as to hold no net position in options at the end of the trading day. Distinguishing two classes of agents in the derivative market is quite artificial because option dealers use *de facto* delta hedging strategies to cover their exchange rate risk embedded in their options portfolio. But this allows the logic of delta hedging to be presented in a simplified manner. Option trading is indeed quite different from underlying asset trading, in that there is no need to keep real inventories of options, because in equilibrium, the point is to decide which optimal hedging strategy to use. Furthermore, due to transaction costs, delta hedging cannot operate in continuous time and rebalancing the position in the underlying currency can only take place in discrete time. Hence, it is supposed that delta hedgers will have to determine and implement the best hedging strategy on a daily frequency in round 3.

The purpose of delta hedging is to eliminate the first order sensivity of the options portfolio value with respect to the price of the underlying asset (the foreign currency). To do so, delta hedgers replicate the payoff of the options sold or bought by a self-financing portfolio composed of bonds and of the foreign asset. Let consider the hedge of a short position in one call that gives the right to its holder to buy an amount *z* of foreign currency at maturity if the option is exercised. Delta hedging this position in round 3 involves to buy $\Delta^{P}_{3,t}$ units of the foreign currency at the price $P_{3,t}$ and to borrow $e^{-rt}\left(C_{3,t} - \Delta^{P}_{3,t} P_{3,t}\right)$ on the domestic market. If the foreign currency appreciates, the increase in the value of the spot position will be exactly compensated by the decrease in the value of the portfolio composed of one call and conversely if the exchange rate depreciates. The value of

$\Delta^{P}_{3,t}$ is equal to *z*, the quantity of foreign currency likely to be bought at maturity specified in the option contract, times the delta of the call option.

The delta of a call option is given by the first derivative of the BSP formula with respect to P_t at the current level of volatility:

$$\Delta_{3,t} = \Delta^{BS}_{3,t} + \upsilon_{3,t}\partial V_{3,t} / \partial P_{3,t} \tag{7}$$

where $\Delta^{BS}_{3,t}$ is the delta of the B&S formula and $\upsilon_{3,t}$ is the vega of the call, i.e. the first derivative of the call price with respect to volatility. Equation 7 indicates that a change in the underlying price affects the call price not only through the delta under constant volatility but also through the vega of the call option and a shift in the average expected volatility. However, in order to simplify calculations, it is supposed that delta hedgers, when covering their exchange rate risk on the spot market, use a delta deriving from the standard B&S model.[6] That amounts to neglecting the change in average expected volatility due to a first order change in the price of the foreign exchange. It is supposed that the delta formula is known by all market participants.

In round 3, delta hedgers will trade with the public to be immunised against the exchange rate risk at the end of the day. However, delta hedging does not allow all the risk to be removed. The change in value of a delta hedged portfolio due to interest rates change, time decay and to the gamma are ignored. Their effect especially on long-term options can be considered of second order. However, the volatility risk for delta hedgers remains significant. When quoting the round 3 implied volatility, option dealers have therefore to know not only the size of the total options flow that will be absorbed by delta hedgers, but also their volatility risk bearing capacity. Simultaneously and independently they quote a volatility so that delta hedgers accept to absorb their options inventory imbalances.

On the spot market, dealers trade with the public to have a zero net position at the end of the day. The public encompasses non-dealer agents whose risk bearing capacity is greater than that of spot dealers. The number of customers is indeed supposed to be large relative to the *N* dealers (in a convergence sense). Their capacity is however limited, so that the aggregate demand of liquidity suppliers is not perfectly elastic. They will therefore

[6] The delta of ATMF options is typically equal to 0.50. If the price of the foreign currency increases in round 3 with respect to the round 2, the call becomes in-the-money and the value of the delta is then greater than 0.50. Conversely, if the exchange rate depreciates, the call becomes out-of-the-money and its delta becomes inferior to 0.50.

ask for a lower price to accept to hold larger positions in foreign currency. Their motive to trade in round 3 is non-stochastic but purely speculative. The rational round 3 exchange rate quote should be set so that liquidity suppliers accept to absorb the inventories imbalance on the spot market and the flow arising from delta hedging behaviour on the derivative market. Spot dealers are thus supposed to know the aggregate demand from the spot segment and the risk bearing capacity of the public. But they also are supposed to perfectly infer the net demand from delta hedgers. To do so, spot dealers must know the delta formula and be able to deduce the round 3 implied volatility from the interdealer option order flow. This implies that the optimal trading rule followed by option dealers is known by spot dealers as well as the volatility risk bearing capacity of delta hedgers.

2.4. Objective function and demands

2.4.1 Objective function of spot dealers

Each spot dealer determines exchange rate quotes and his speculative demand by maximising a negative, exponential utility function. Spot dealers are supposed to have a closed position at the end of each day.

$$\underset{P_{i,1},P_{i,2},P_{i,3},D_{i,2}}{Max} \quad E\left[-\exp\left(-\gamma W_{i,3}\right)\middle|\Omega_i\right] \tag{8}$$

$$\text{s.t. } W_{i,3,t} = W_{i,0,t} + x_{i,1,t}P_{i,1,t} + T^{'}_{i,2,t}P_{i,2,t} - T_{i,2,t}P^{'}_{i,2,t} - \left(x_{i,1,t} + T^{'}_{i,2,t} - T_{i,2,t}\right)P^{'}_{i,3,t}$$

where $W_{i,0,t}$ is the wealth of dealer *i* at the beginning of the first trading round, and a ' denotes a trade or a quote received by dealer *i* from other dealers.

2.4.2. Objective function of option dealers

The optimisation problem for each options dealer is also defined over four variables, the three implied volatility quotes $V_{q,1,t}$, $V_{q,2,t}$, $V_{q,3,t}$ and its speculative demand in round 2 $D^{O}_{q,2,t}$. Options dealers are supposed to hold zero inventory at the end of the day.

$$\underset{V_{q,1},V_{q,2},V_{q,3},D^{O}_{q,2},}{Max} \quad E\left[-\exp\left(-\gamma W^{O}_{q,3}\right)\middle|\Omega_q\right] \tag{9}$$

$$\text{s.t. } W^{O}_{q,3,t} = W^{O}_{q,0,t} + y_{q,1,t}C_{q,1,t} + O^{'}_{q,2,t}C_{q,2,t} - O_{q,2,t}C^{'}_{q,2,t} - \left(y_{q,1,t} + O^{'}_{q,2,t} - O_{q,2,t}\right)C^{'}_{3,t}$$

where $W_{q,0,t}$ is the wealth of dealer *q* at the beginning of the first trading round, and a ' denotes a trade or a quote received by dealer *q* from other option dealers. The information sets of both classes of dealers at each round are summarised in Appendix A.

2.4.3 The public

The public's total demand for the risky asset is given by maximising the expected utility in equation (1) subject to the following budget constraint:

$$W_{t+1} = W_t + h_{3,t}\left(P_{3,t+1} + R_{t+1} - P_{3,t}\right) \quad (10)$$

where W_t is the wealth at the end of time *t*. The demand for the foreign currency in round 3, denoted $h_{3,t}$ is thus a linear function of its expected return conditional on public information.

$$h_{3,t}\left(P_{3,t}\right) = \theta\left(E\left[P_{3,t+1} + R_{t+1} \middle| \Omega_{3,t}\right] - P_{3,t}\right) \quad (11)$$

where the positive coefficient θ captures the risk bearing capacity of the public, with $\theta = \left(\gamma\sigma^2_{P+R}\right)^{-1}$ and σ^2_{P+R} denotes the conditional variance of $P_{3,t+1} + R_{t+1}$.

2.4.4 The delta hedgers

On the assumption that the change in the value of a delta hedged portfolio only depends on the variation of the volatility between *t* and *t+1*, the budgetary constraint of delta hedgers simplifies to: [7]

$$W_{t+1} = W_t + h^O_{3,t}\upsilon_{3,t}\left(V_{3,t+1} - V_{3,t}\right) \quad (12)$$

where $\upsilon_{3,t}$ is the vega of the call option, that is the first derivative of the call price with respect to a small change in the volatility: $\upsilon_{3,t} = P_{3,t}\sqrt{\tau}e^{-r^*_t}N'\left(d_{3,t}\right)$. Maximising expected utility in equation (1) subject to this budgetary constraint will determine the demand of delta hedgers for call options in round 3, $h^O_{3,t}$.

$$h^O_{3,t}\left(V_{3,t}\right) = \theta^O\upsilon_{3,t}\left(E\left[V_{3,t+1} \middle| \Omega_{3,t}\right] - V_{3,t}\right) \quad (13)$$

where the positive parameter θ^O captures the volatility risk bearing capacity of delta hedgers, with $\theta^O = (\gamma\sigma^2_C)^{-1}$ and σ^2_C denoting the conditional variance of the call price conditioned on public information in round 3 at time *t*. Delta hedgers will then accept to buy or to sell options from option dealers at a price that depends on the expected change of implied volatility between round 3 of period *t* and *t+1* and on the sensitivity

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[7] The fact that the oustanding amount of options hold by delta hedgers is no more delta hedged due to the change in the price of the foreign exchange in round 1 is not taken into account in this specification. This amounts to neglecting the impact of rebalancing options portfolio on the round 3 implied volatility and to ignoring the effect of dynamic hedging the stock of options on the equilibrium exchange rate.

of their options portfolio with respect to a small change in the implied volatility. The higher the quantity delta hedgers have to sell (buy), the higher (lower) the implied volatility that option dealers have to quote in round 3.

2.5. Market clearing

On the derivative market, the round 3 implied volatility quote should be such that the delta hedgers' demand matches the quantity of options liquidated by option dealers. As there is no leakage during interdealer trading rounds, the amount of options to be sold or bought by option dealers corresponds exactly to the net selling or buying position of customers in round 1.

$$h_{3,t}^{O}\left(V_{3,t}\right) = -\sum_{q=1}^{m} y_{q,1,t} \tag{14}$$

On the spot market in round 3, the liquidity suppliers will have to absorb not only the demand of spot dealers, which is exactly the net aggregate customers order flows, but also the flow arising from delta hedgers who cover their exchange rate risk on the spot market, according to the delta of their portfolio $\sum_{q=1}^{m} y_{q,1,t}\Delta_{3,t}^{P}$. The round 3 market-clearing price of the foreign currency should therefore satisfy:

$$h_{3}\left(P_{3,t}\right) = -\left(\sum_{i=1}^{n} x_{i,1,t} + \sum_{q=1}^{m}\left[y_{q,1,t}\Delta_{3,t}^{P}\left(P_{3,t}, V_{3,t}\right)\right]\right) \tag{15}$$

These two market clearing conditions on the derivative and spot markets lead to the following equilibrium system (see Appendixes B and C for details):

$$\begin{cases} P_{3,t} = E_t\left[P_{3,t+1} + R_{t+1} \middle| \Omega_{3,t}\right] + \lambda X_t + \lambda_t^{O'} Y_t \Delta_{3,t}^{P}\left(P_{3,t}, V_{3,t}\right) \\ V_{3,t} = E_t\left[V_{3,t+1} \middle| \Omega_{3,t}\right] + \lambda_t^{O} Y_t / \upsilon_{3,t}\left(P_{3,t}, V_{3,t}\right) \end{cases}$$

where $\lambda = (\alpha\theta)^{-1}$, $\lambda_t^{O} = \left(\beta_t \theta^{O}\right)^{-1}$ and $\lambda_t^{O'} = \left(\beta_t \theta\right)^{-1}$, with α and β_t entering the optimal trading rule of respectively the spot and options dealers. As $\Delta_{3,t}^{P}\left(P_{3,t}, V_{3,t}\right)$ and $\upsilon_{3,t}\left(P_{3,t}, V_{3,t}\right)$ are non-linear functions in the round 3 price of the foreign currency and in implied volatility, there is no closed-form solution for the simultaneous equilibrium on the two markets. However, as the change in delta and in vega between round 2 and round 3 only depends on the change of $P_{2,t}$ and $V_{2,t}$, using the Taylor first order linearisation allows an approximate analytical solution to be provided.

3. Equilibrium and implications

As in Evans and Lyons (2002), the equilibrium concept used refers to the Bayesian-Nash Equilibrium (BNE). Under this equilibrium concept, agents update beliefs according to Bayes rule and given these beliefs, quoting and trading strategies followed by dealers are sequentially rational. Details on the proofs are provided in the Appendixes B and C.

3.1. Equilibrium quoting strategies

The assumption of no arbitrage within each round implies that all dealers quote a common price (the exchange rate on the spot market and the implied volatility on the derivative segment). As in a given market the quoted price is common to all dealers, it is therefore conditioned on public information only. The common information shared by all dealers at the beginning of round 1 includes the payoff increments, the round 3 quote of the foreign exchange and of the implied volatility at time *t-1*.

3.1.1 The option market

In the derivative market, it is supposed that the innovation in payoff does not modify the average expected volatility of option dealers. Hence, the only relevant information for option dealers at the beginning of round 1 is the round 3 volatility quote of the previous day. Interdealer FX option order flows are observed at the end of round 2. This information will be reflected in the round 3 implied volatility quote.

Proposition 1: A quoting strategy on the option market is consistent with symmetric BNE only if the implied volatility quotes in rounds 1 and 2 are common across dealers and equal to:

$$V_{1,t} = V_{2,t} = V_{3,t-1} \tag{16}$$

where $V_{3,t-1}$ is the round 3 implied volatility quote from the previous day.

Proposition 2: A quoting strategy is consistent with symmetric BNE only if the common round 3 implied volatility quote is equal to:

$$V_{3,t} = V_{2,t} + \frac{\lambda_t^O Y_t \sqrt{2\pi}}{\sqrt{\tau} P_{3,t}} \tag{17}$$

***Proposition 3*:** If delta hedgers hold rational expectations and option dealers quote according to propositions 1 and 2, the change in the volatility quote from one day to the other is given by:

$$\Delta V_{3,t} = \frac{\lambda_t^O Y_t \sqrt{2\pi}}{\sqrt{\tau} P_{3,t}} \tag{18}$$

where λ_t^O is positive. Propositions 2 and 3 show how the change in implied volatility quote is related to the interdealer FX option order flows and to the sensitivity of the call price relative to a small change in volatility. This sensitivity is captured by $\sqrt{\tau}P_{3,t} / \sqrt{2\pi}$, which stems from the first order linearisation of the vega. The higher this sensitivity the lower the change in implied volatility needed so that delta hedgers accept to absorb the aggregate customers portfolio shift, where $\lambda_t^O Y_t = \lambda_t^O \sum_q O_{q,2} = \lambda_t^O \beta_t \sum_q y_{q,1,t}$.

3.1.2 The spot market

Proposition 4: A quoting strategy on the spot market is consistent with symmetric BNE only if the quoted prices in rounds 1 and 2 are common across dealers and set to:

$$P_{1,t} = P_{2,t} = P_{3,t-1} + \Delta R_t \tag{19}$$

where $P_{3,t-1}$ is the round 3 price of the previous day and ΔR_t is the payoff innovation observed at the beginning of round 1.

Proposition 5: A quoting strategy on the spot market is consistent with symmetric BNE only if the common round 3 price is set to:

$$P_{3,t} = P_{2,t} + \frac{\lambda X_t + \lambda_t^{O'} Y_t z\left(\Delta_{2,t} + \left(V_{3,t} - V_{2,t}\right)\sqrt{\tau} / \sqrt{2\pi}\right)}{\left(1 - \lambda_t^{O'} Y_t z \Gamma_{2,t}\right)} \tag{20}$$

where $\Delta_{2,t}$ and $\Gamma_{2,t}$ are respectively the delta and the gamma of each call option held by option dealers in round 2 and *z* is the amount of foreign exchange that the holder of one call is allowed to buy if the option is exercised at maturity.

***Proposition 6*:** In a BNE where the public holds rational expectations, the change in the round 3 price of the foreign exchange rate between period *t-1* and *t* is given by:

$$\Delta P_t = \Delta R_t + \frac{\lambda X_t + \lambda_t^{O'} Y_t z\left(\Delta_{2,t} + \left(V_{3,t} - V_{2,t}\right)\sqrt{\tau} / \sqrt{2\pi}\right)}{\left(1 - \lambda_t^{O'} Y_t z \Gamma_{2,t}\right)} \tag{21}$$

The change in the price of the foreign exchange is a function of the payoff increment, of the spot and option interdealer order flows and of the intra-daily change in the implied volatility. The variation from period *t-1* to *t* is therefore related to market conditions prevailing in round 2 on the spot and derivative segments. Finally equilibrium quotes in round 3 on both market segments are interdependent and simultaneous. Indeed, the equilibrium on the option market depends on the round 3 price quoted on the spot segment and the equilibrium round 3 exchange rate is a function of the change in volatility quote during the day.

3.2. Equilibrium trading strategies

The appendix C shows that the optimal trading rule on the spot and derivative markets can be expressed as a linear function of the order flows received by dealers on their own market in round 1. Although dealers receive customers' orders that are different in magnitude, the relation between the size of these orders and the dealer' net outgoing orders is the same across all dealers in a given market. However, the optimal trading rule is not identical in the spot and in the derivative market segments.[8]

Proposition 7: An optimal trading strategy on the option market that conforms to the BNE is given by:

$$O_{q,2,t} = \beta_t y_{q,1,t} \text{ for } \forall \text{q.} \tag{22}$$

where $\beta_t > 1$.

Proposition 8: An optimal trading strategy on the spot market that conforms to the BNE is given by:

$$T_{i,2,t} = \alpha x_{i,1,t} \quad \text{for } \forall \text{ i.} \tag{23}$$

where $\alpha > 1$. In both market segments, dealers are thus able to infer the aggregate portfolio shift in round 1 from the interdealer order flows revealed at the end of round 2. They also know that the public in the spot market and the delta hedgers on the derivative market will have to absorb these portfolio shifts and that this will induce prices to change accordingly.

In this framework, options are not redundant assets as derivative trading will impact on the option price through the change in implied volatility and finally on the dynamics of the underlying asset through delta hedging behaviour. Speculative demand on the options market will indeed influence the total interdealer order flow at the end of round 2 and thus the round 3 implied volatility quote. The larger the positive FX option order flow, the higher the implied volatility in round 3, the greater the flows from delta hedging behaviour of option dealers and the higher the price of the foreign exchange. But the exchange rate shift between round 2 and round 3 may be dampened if currency options order flows are negatively correlated with spot flows.

4. Simulation results

[8] Whereas the optimal trading rule in the spot market involves a constant parameter α, the optimal trading rule in the derivative segment is related to market conditions prevailing in round 2 and especially to the round 2 price of the foreign currency (see Appendix C).

The aim of this section is twofold. First it is devoted at assessing how FX options trading may impact the equilibrium exchange rate. As the equilibrium is simultaneous on the spot and on the derivative segment, the implied volatility quote that clears the options market also depends on the equilibrium price of the foreign exchange. In order to analyse how the two markets interact, a sensitivity analysis is conducted according to the amount of FX options order flows, its positive or negative correlation with spot flows and with respect to the maturity of traded options. Predicted equilibrium exchange rate and implied volatility, however, stem from an approximate model that rests on three simplifications necessary to get a closed-form solution. Hence, this section is also directed at measuring the distance between the approximated equilibrium and the numerical equilibrium solution that does not rely on any simplification. The scope of validity of the approximate equilibrium equations presented in section 3 is then evaluated.

4.1. Equilibrium and FX option order flows

The sensitivity analysis of the change in equilibrium exchange rate and implied volatility between round 2 and round 3 is based on the following benchmark scenario. The round 2 price of the foreign exchange is supposed to be equal to 1, the round 2 implied volatility to 10%, the domestic interest rate to 2%, the foreign interest rate is postulated to be null. The risk aversion γ is equal to 3, the parameters α and β catching the speculative behaviour of dealers are set to 1.2.[9] The daily volatility of the exchange rate σ is 1% and that of the call option σ_C is 1.2%. Given the postulated value for the parameters, the spot order flow is calibrated to yield a 1% change in the exchange rate when no derivative trading is accounted for. X_t is thus supposed to equal 40. This provides a benchmark to study how the introduction of FX options order flows will impact on the equilibrium. It is furthermore supposed that the amount of foreign currency that the holder of a call is allowed to buy, z, is equal to 1. FX options turnover at present accounts for around 15% of spot trading. Given the growing development of the derivative segment, simulations will consider a range for options order flows up to half that prevailing on the spot market. Furthermore, the impact on the equilibrium exchange rate does not only depend on the quantity of traded options, but also on the net aggregate buying or selling position of customers. Hence, simulations should consider positive and negative correlations between the spot and option order flows. Finally as the value of the

[9] Dealers on the options market expecting, for example, an increase in round 3 implied volatility because they have received net positive call order flows from their customers will try to be net buyers of volatility at the end of round 2 (β>1). In round 3, if implied volatility increases they will make a profit from the sale of their options portfolio.

delta, used to cover foreign exchange risk, depends on the maturity of options, the sensitivity of the equilibrium to this parameter has also to be evaluated.

Figure 1 displays the equilibrium exchange rate relative to the amount of FX option order flows with respect to spot order flows, when the latter are positive. Positive spot order flows indicate that customers have a net aggregate buying position of the foreign exchange. With no derivative trading, the benchmark scenario predicts a 1% increase in the price of the foreign exchange. When the currency options order flow is positive, meaning that customers are net buyers of calls, the exchange rate response is amplified. Indeed for derivative order flows equal to half the spot flows, the change in the exchange rate ranges from 1.26% for the five-years maturity to 1.30% for the three-months' time to expiration. The exchange rate variation is then all the higher that the maturity is short. The delta is indeed a decreasing function of the time to maturity when options are in-the-money, i.e. with a delta superior to 0.5. As the price of the foreign exchange rises due to positive spot order flows, options that were previously at-the-money-forward become in-the-money. Hence the delta of the options portfolio decreases with longer maturity, so that the additional flows from delta hedging behaviour are limited and the exchange rate response is lower for the five years maturity than for the three-month maturity. The difference in exchange rate response with respect to the maturity of traded options appears however rather slight. Besides, when FX option order flows are negative, delta hedging behaviour will tend to reduce the equilibrium value of the exchange rate relative to its initial value of 1.01, because selling flows from delta hedgers will have to be absorbed in a buying spot market. Both the relative decline in the price of the foreign exchange and the decrease in equilibrium implied volatility will feedback into a reduction of the delta of the options portfolio. The downturn in the equilibrium price is thus slightly limited by the decrease of the delta. Moreover, this fall is all the more pronounced, the shorter the time to expiration of options. Indeed, the shorter the maturity, the higher the delta and the greater the selling flows from delta hedging on the spot market. The change in the exchange rate is then dampened to 0.72% relative to the 1% increase with no derivative trading (-0.27% compared with the initial 1.01 exchange rate) when the three months maturity is concerned and added up to 0.75% for the five years maturity (-0.25%). Hence, the exchange rate response is not linear with respect to the amount of FX option order flows. For a given maturity and a given amount of options order flows, the magnitude of the exchange rate change is stronger for positive than for negative option flows, because the value of the delta is also lower for negative than for positive FX option order flows.

Figure 2 displays the equilibrium exchange rate path with respect to FX options order flows when spot order flows are negative. With no derivative trading, the price of the foreign exchange decreases by 1%. Compared with the situation where spot flows are positive, the change in the exchange rate is not symmetrical and is of smaller size. Indeed, options that were initially at-the-money-forward then become out-of-the-money, i.e. with a delta inferior to 0.5. So for a given amount of FX option order flows, the sensitivity of the exchange rate response is weaker. Besides, The delta of out-of-the-money options is an increasing function of the time to expiration. Thus when FX options order flows are negative, the exchange rate response is all the more magnified that the maturity is far away. In fact, the change in the equilibrium price is –1.20% when the three-months' maturity is considered, whereas it turns to be –1.24% for the five-years' maturity. By contrast, positive FX option order flows dampen the initial 1% decrease of the exchange rate to 0.78% for the three-months' maturity to 0.75% for the five-years' maturity. So, the exchange rate response to delta hedging strategies is asymmetric regarding the positive or negative spot order flows.

Hence, the model predicts a destabilising effect of derivative trading on the spot dynamics when the FX options order flow is of the same sign as that of the spot order flow. Indeed, delta hedging behaviours exacerbate the exchange rate change. This magnifying effect is more important when short-term options are considered and when spot order flows are positive. By contrast, when spot order flows are negative, the amplified response of the price of the foreign exchange is observed for long-term options. But when derivative and spot order flows are negatively correlated, the impact on the exchange rate proves to be stabilising as it reduces the daily volatility. This stabilising effect is all the more significant, the longer the options' term, whatever the net position on the spot market.

Figures 3 and 4 present the equilibrium implied volatility relative to FX option order flows, when spot order flows are respectively positive and negative. Figure 3 shows that implied volatility increases when FX options order flows are positive from an initial level of 10% up to levels ranging from 10.80% for the five-years' maturity to 13.56% for the three-months' maturity. That result directly stems from the equilibrium implied volatility equation. It is a decreasing function of the options' maturity, so that the volatility response is all the more dampened that options have a long maturity. Conversely implied volatility decreases when options order flows are negative turning to levels ranging from 6.42% for the three-months' maturity to 9.20% for the five-years' maturity. The decline in volatility is then all the higher that the maturity is short. Figure 4 shows that equilibrium implied volatility is not sharply different if spot order flows are negative, especially for long maturities. Indeed, equilibrium quotes range from 10.81% for the five-years maturity to 13.64% for the three-months' time to expiration when FX option order flows are positive. These figures respectively equal 6.35% and 9.18% when derivative flows indicate a net aggregate selling position. The rather limited difference in implied volatility whether spot order flows are positive or negative can be explained as follows. First, its influence is only indirect via the equilibrium exchange rate. When spot order flows are positive, the price of the foreign exchange increases. This tends to limit the increase (decrease) in implied volatility when positive (negative) FX option order flows are accounted for. By contrast, negative spot order flows lead to a decline in the exchange rate, which tends to amplify the reaction of the implied volatility: it increases (decreases) more when FX options order flows are positive (negative). This phenomenon is more marked for short-term maturity because the volatility equilibrium is a decreasing function of the time to expiration. Hence, the impact of exchange rate on the implied volatility equilibrium is higher when traded options have a short-term maturity. But for long-term options, the impact of the spot market on the equilibrium in the derivative market is almost insignificant. In this case, the implied volatility response to FX option order flows is indeed almost linear, whether spot order flows are positive or negative.

The equilibrium exchange rate is very sensitive to the sign of spot order flows and to a lesser extent to the sign of the FX option order flows. The influence of the equilibrium implied volatility on the equilibrium exchange rate is only indirect via the value of the delta. Insofar as this value depends prominently on the change in the exchange rate rather than on a change in implied volatility, its impact on the spot market is really limited. The equilibrium implied volatility is sensitive to the FX option order flows and to a far lesser extent to the equilibrium exchange rate. Although the price of the foreign exchange enters the equilibrium implied volatility equation, its effect is quite limited. The influence of spot order flows is only indirect through the equilibrium price of the foreign exchange.

4.2. Accuracy of the approximate equilibrium

These predictions regarding the behaviour of the equilibrium exchange rate and implied volatility come from a model that resorts to three approximations. The equilibrium equations displayed in section 3 are based on the B&S delta formula rather than the BSP delta that is obtained by differentiating equation 3 with respect to the underlying asset price. The change in the average expected volatility due to a first order change in the exchange rate is thus ignored. Second, they depend on the first-order linearisation of the vega formula that enters the equation of options demand from delta hedgers. This linearisation involves that the round 3 approximate vega is equal to a vega of at-the-money-forward options portfolio. But as the round 3 price of the foreign exchange and implied volatility change relative to their round 2 value, the options in round 3 are of course no more at-the-money-forward. Third, a first-order linearisation of the B&S delta formula is used to provide a closed-form equilibrium exchange rate. Hence, the accuracy of the approximate equilibrium has to be evaluated and compared with simulations resting on a model without any simplification to provide the scope of validity of the projected paths.

Simulations with respect to the amount of FX option order flows, to their correlation with spot flows and according the options maturity have been conducted to gauge the forecast errors. Figures 5 and 6 present the difference in equilibrium exchange rate path with respect to currency options order flows when the spot flow is respectively positive and negative. The curves represent the difference between the numerical and the approximate solutions. In most cases, the approximate model leads to overestimate the exchange rate response when FX options trading is introduced whether spot order flows are positive or negative. The approximate delta tends indeed to overestimate the numerical delta when FX option order flows are positive, whereas it underestimates it when options flows are negative.[10] This distance is decreasing with maturity. However, in all the cases, the percentage error is less than 1%. The highest error is equal to 0.008% for the three-months' maturity, when spot and FX option order flows are positive. When they are both negative, the highest forecast errors grow to 0.024% for the greatest amount of option flows. The accuracy of the approximate model for this range of derivative trading appears to be quite good. However, for shorter maturity, the errors could potentially grow and become significant.

Figures 7 and 8 display the difference between the numerical equilibrium implied volatility and that coming from the approximate closed form solution. When spot order flows are positive, the percentage deviation is almost always less than 1%. The difference is all the lower that the maturity of options is long. The numerical equilibrium volatility is higher when FX option order flows are positive and lower when derivative flows are negative. This is due to the first order linearisation of the vega that amounts to postulating that the value of the normal density function in round 3 is at its highest level whereas any variation of the exchange rate or of the implied volatility in round 3 reduces this value as its argument is no more equal to zero. However, errors slightly

[10] For the three-months' maturity however, when spot flows are positive and when FX option flows range between one quarter to one half of spot flows, the approximate delta is higher than the numerical delta. So the equilibrium exchange rate coming from the approximate model is lower than the numerical equilibrium solution.

superior to 1% are observed when negative FX options order flows amount to around one half of spot flows for the three-months' maturity. When both spot and FX option order flows are negative, forecast errors turn to be significant for three-months options flows ranging from one quarter to one half of spot flows. The error is equal to 6.2% for the highest amount of FX options flows at a three-months' maturity and 1.2% for the six-months' maturity when FX option flows are negative.

These simulations have shown that the most problematic approximation is related to the linearisation of the vega. Indeed, it introduces significant differences in the equilibrium implied volatility at least for short-term options when FX option order flows amount to half that of spot flows. However, to the extent that implied volatility does not significantly alter the equilibrium on the spot market, this valuation error does not reflect in the equilibrium exchange rate. Hence, the approximate model provides acceptable equilibrium paths of the price of the foreign exchange at least for the maturities considered and the amount of FX option order flows. It seems however that for shorter maturity (less than three months) and important amounts of derivative trading, errors could become significant. But if currently observed FX option order flows relative to spot flows are considered, even for shorter maturity, the approximate model provides accurate predictions regarding the behaviour of the equilibrium exchange rate.

5. Conclusion

This paper proposes a FX microstructure model of interdealer trading on the spot and options markets that explicitly accounts for interactions between the two market segments. Both prices are directly related to interdealer order flows in their own market. But the equilibrium on the derivative market is also influenced by the price of the foreign exchange and the equilibrium exchange rate depends on the FX option order flows and indirectly on the equilibrium implied volatility through its impact on the value of the delta. The equilibrium on the two market segments is thus simultaneous. It is shown that depending on the correlation between spot and option order flows, the daily volatility of the exchange rate can either be amplified or reduced.

When the order flows on the two market segments are positively correlated, delta hedging strategies amplify the exchange rate response. For positive spot order flows indicating that customers are net buyers of the foreign currency, the increase in the exchange rate is all the higher that the maturity of options is short. By contrast, when spot order flows are negative, the response is all the more pronounced that the time to expiration of options is far away. Furthermore, when order flows on the two markets are negatively correlated, delta hedging has a stabilising impact on the exchange rate to the extent that it limits its variation. This effect grows when the maturity of options is long. Finally, simulations show that depending on the sign of spot order flows, the exchange rate response is asymmetric to FX option order flows. Its impact is indeed more pronounced when spot flows are positive, this influence being all the higher that traded options have a short maturity. As far as the equilibrium implied volatility is concerned, its change is mainly driven by FX option order flows and the effect of the equilibrium exchange rate is very limited. Volatility increases (decreases) with positive (negative) option flows, the rise (fall) being all the higher than the maturity of options is short.

Moreover the equilibrium equations for the spot and derivative markets depend on some first-order approximations. Compared with numerical solutions that do not rely on any simplification, these approximations proved to be accurate regarding the equilibrium exchange rate, whereas differences may be significant for the equilibrium volatility when the amount FX option order flows are important relative to spot flows.

Finally, the proposed model does not account for the potential additional effect on the equilibrium exchange rate of dynamic hedging of the risk embedded in older outstanding amount of options portfolio. Indeed, only the order flows of the current period are considered. A possible extension could then include the impact of hedging the previous stock that are no more perfectly delta hedged with the change in the equilibrium exchange rate at the beginning of the day.

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Appendix A. Information sets of dealers

The first three information sets for each market segment are available to individual dealers at the time of quoting in each round, whereas the last three are public information for each round.

Spot dealers information sets

$$\Omega_{i,1,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, {}_{i=1}^{n}\{P_{i,1,t}\}, x_{i,1,t}, V_{1,t} \right\}$$

$$\Omega_{i,2,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, {}_{i=1}^{n}\{P_{i,1,t}, P_{i,2,t}\}, x_{i,1,t}, V_{1,t}, V_{2,t} \right\}$$

$$\Omega_{i,3,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, {}_{i=1}^{n}\{P_{i,1,t}, P_{i,2,t}, P_{i,3,t}\}, x_{i,1,t}, V_{1,t}, V_{2,t}, V_{3,t}, T_{i,2,t}, T'_{i,2,t}, X_t, Y_t \right\}$$

$$\Omega_{1,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, {}_{i=1}^{n}\{P_{i,1,t}\}, V_{1,t} \right\}$$

$$\Omega_{2,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, {}_{i=1}^{n}\{P_{i,1,t}, P_{i,2,t}\}, V_{1,t}, V_{2,t} \right\}$$

$$\Omega_{3,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, {}_{i=1}^{n}\{P_{i,1,t}, P_{i,2,t}, P_{i,3,t}\}, V_{1,t}, V_{2,t}, V_{3,t}, X_t, Y_t \right\}$$

Option dealers information sets

$$\Omega^O_{q,1,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, P_{1,t}, y_{q,1,t}, {}_{q=1}^{m}\{V_{q,1,t}\} \right\}$$

$$\Omega^O_{q,2,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, P_{1,t}, P_{2,t}, y_{q,1,t}, {}_{q=1}^{m}\{V_{q,1,t}, V_{q,2,t}\} \right\}$$

$$\Omega^O_{q,3,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, P_{1,t}, P_{2,t}, P_{3,t}, y_{q,1,t}, {}_{q=1}^{m}\{V_{q,1,t}, V_{q,2,t}, V_{q,3,t}\}, O_{q,2,t}, O'_{q,2,t}, Y_t, X_t \right\}$$

$$\Omega^O_{1,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, P_{1,t}, {}_{q=1}^{m}\{V_{q,1,t}\} \right\}$$

$$\Omega^O_{2,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, P_{1,t}, P_{2,t}, {}_{q=1}^{m}\{V_{q,1,t}, V_{q,2,t}\} \right\}$$

$$\Omega^O_{3,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, P_{1,t}, P_{2,t}, P_{3,t}, {}_{q=1}^{m}\{V_{q,1,t}, V_{q,2,t}, V_{q,3,t}\}, Y_t, X_t \right\}$$

Appendix B - Proofs of the optimal quoting strategies

Returns are independent across each trading round, the stochastic environment dealers face being unchanged. Thus the maximisation problem for each dealer reduces to determine an independent optimal quoting and trading strategy for each round. Spot and option dealers maximise a negative exponential utility function, $U_t = E_t[-\exp(-\gamma W_{t+1})]$ and the terminal wealth is distributed $N(\mu, \sigma^2)$. Hence, maximising $E[U(W)] = -\exp[-\gamma(\mu - \theta\sigma^2/2)]$ is equivalent to maximise $(\mu - \gamma\sigma^2/2)$.

Proof of propositions 1, 2 and 3: price determination on the options market

The assumption of no arbitrage opportunity requires that implied volatility quotes must be observable to all dealers and also common across dealers, the quote being a single price at which each dealer engaged to absorb any quantity. Hence, common quotes imply that prices can only be conditioned on public information. In rounds 1 and 2, the public information includes the round 3 implied volatility and exchange rate quotes of the previous day as well as the value of the payoff increment ΔR_t. The equilibrium level of implied volatility results from the optimal dealers and delta hedgers trading rules.

$$E\left[y_{q,1,t} \middle| \Omega_{1,t}\right] + E\left[D^{O}_{q,2,t}\left(V_{1,t}\right) \middle| \Omega_{1,t}\right] = 0 \tag{24}$$

$$E\left[y_{q,1,t} \middle| \Omega_{1,t}\right] + E\left[D^{O}_{q,2,t}\left(V_{2,t}\right) \middle| \Omega_{1,t}\right] = 0 \tag{25}$$

$$E\left[\sum_{q=1}^{m} y_{q,1,t} \middle| \Omega_{3,t}\right] + E\left[h^{O}_{3,t}\left(V_{3,t}\right) \middle| \Omega_{3,t}\right] = 0 \tag{26}$$

Equations 24 and 25 state that the demand of each options dealer is expected to absorb the demand from customers at the quoted prices that clear the market in round 1 and 2. There is nothing in the information set $\Omega_{1,t}$ that allows the customers order flows to be predicted, so that $E\left[y_{q,1,t} \middle| \Omega_{1,t}\right] = 0$. The only round 1 quote that is consistent with $E\left[D^{O}_{q,2,t}\left(V_{1,t}\right) \middle| \Omega_{1,t}\right] = 0$ is therefore: $V_{2,t} = E\left[V_{3,t} \middle| \Omega_{2,t}\right] = V_{1,t}$. Hence, the rational round 1 implied volatility quote, conditioned on public information is $V_{1,t} = V_{3,t-1}$. (It is supposed that the payoff information revealed at the beginning of round 1 does not impact on the implied volatility quote.) With the same logic, at the beginning of round 2, the inter-dealer order flows do not enter the information set, so $E\left[D^{O}_{q,2,t}\left(V_{2,t}\right) \middle| \Omega_{2,t}\right] = 0$ and $E\left[D^{O}_{q,2,t}\left(V_{2,t}\right) \middle| \Omega_{2,t}\right] = 0$. The only volatility quote that clears the market during the interdealer trading round is $V_{2,t} = E\left[V_{3,t} \middle| \Omega_{2,t}\right] = V_{1,t}$ as there is no new public information revealed between rounds 1 and 2. $V_{1,t}$ and $V_{2,t}$ are therefore unbiased predictors of the future round' volatility quote, conditional on public information.

The third equation (26) establishes that the round 3 implied volatility quote should be set so that delta hedgers accept to absorb the total initial order flows from option customers. The volatility quote should adjust to ensure that: $h_{3,t}^{O}(V_{3,t})+\sum_{q=1}^{m} y_{q,1,t}=0$. Given the postulated optimal trading rule on the option market, (which is proved in appendix C) and since inter-dealer option order flows are included in $\Omega_{3,t}$, we have $E\left[\sum_{q=1}^{m} y_{q,1,t} \middle| \Omega_{3,t}\right]=\frac{Y_t}{\beta_t}$. Furthermore, the maximisation problem of delta hedgers can be written as follows:

$$\underset{V_{3,t}}{Max} \quad h_{3,t}^{O}\left(E\left[C_{3,t+1} \middle| \Omega_{3,t}\right]-C_{3,t}\right)-\left(h_{3,t}^{O}\right)^2 \frac{\gamma}{2}\sigma_C^2 \tag{27}$$

where σ_C is the standard deviation of the call price. The first order condition yields: $h_{3,t}^{O}=\frac{E\left[C_{3,t+1} \middle| \Omega_{3,t}\right]-C_{3,t}}{\gamma\sigma_C^2}$. Ignoring the impact of time decay, of interest rates changes and of second-order factors on the change in the call price, the change in the value of a delta hedged portfolio only depends on the variation of volatility: $dC=\upsilon dV$, where υ is the vega of the call option. Hence, the demand from delta hedgers can be rewritten as $h_{3,t}^{O}=\frac{\upsilon_{3,t}\left(E\left[V_{3,t+1} \middle| \Omega_{3,t}\right]-V_{3,t}\right)}{\gamma\sigma_C^2}$. Let $\theta^{O}=\left(\gamma\sigma_C^2\right)^{-1}$, this leads to the equation 13 in the text. The round 3 market-clearing quote is therefore given by:

$$V_{3,t}=E\left[V_{3,t+1} \middle| \Omega_{3,t}\right]+\frac{\lambda_t^{O} Y_t}{\upsilon_{3,t}} \tag{28}$$

with $\lambda_t^{O}=\left(\beta_t\theta^{O}\right)^{-1}$ and $\upsilon_{3,t}=P_{3,t}\sqrt{\tau}N'\left(d_{3,t}\right)$ where $N'\left(d_{3,t}\right)=\frac{1}{\sqrt{2\pi}}\exp\left\{-\left(d_{3,t}\right)^2\right\}$. $\upsilon_{3,t}$ is a non-linear function of $V_{3,t}$. In order to have a closed-form solution, $\upsilon_{3,t}$ is linearised around its round 2 value.

$$\upsilon_{3,t}=\upsilon_{2,t}+\frac{\partial\upsilon_{2,t}}{\partial P_{2,t}}\left(P_{3,t}-P_{2,t}\right)+\frac{\partial\upsilon_{2,t}}{\partial V_{2,t}}\left(V_{3,t}-V_{2,t}\right) \tag{29}$$

As options in round 1 and 2 are at-the-money-forward, their delta $\Delta=e^{-r^*\tau}N(d)$ is equal to 0.5. Neglecting the discounting factor $e^{-r^*\tau}$, this entails that $d_{2,t}$ entering the cumulative normal distribution is 0. Hence, $N'\left(d_{2,t}\right)=1/\sqrt{2\pi}$ so that $\upsilon_{2,t}=P_{2,t}\frac{\sqrt{\tau}}{\sqrt{2\pi}}$, $\frac{\partial\upsilon_{2,t}}{\partial V_{2,t}}=0$ and $\frac{\partial\upsilon_{2,t}}{\partial P_{2,t}}=\frac{\sqrt{\tau}}{\sqrt{2\pi}}$.

So the round 3 approximate implied volatility can be rewritten as: $V_{3,t}=E\left[V_{3,t+1} \middle| \Omega_{3,t}\right]+\frac{\lambda_t^{O} Y_t\sqrt{2\pi}}{\sqrt{\tau}P_{3,t}}$. As in equilibrium $E\left[V_{3,t+1} \middle| \Omega_{3,t}\right]=V_{3,t-1}$, the rational expectations solution of this equation leads to equation 18 in the text and together with proposition 1 to equation 17. The round 3 volatility is the sum of the risk premium determined by cumulative portfolio shifts divided by the round 3 price of the foreign exchange rate.

Proof of propositions 4, 5 and 6: price determination on the spot market

$$E\left[x_{i,1,t} \middle| \Omega_{1,t}\right] + E\left[D_{i,2,t}\left(P_{1,t}\right) \middle| \Omega_{1,t}\right] = 0 \tag{30}$$

$$E\left[x_{i,1,t} \middle| \Omega_{1,t}\right] + E\left[D_{i,2,t}\left(P_{2,t}\right) \middle| \Omega_{1,t}\right] = 0 \tag{31}$$

$$E\left[\sum_{i=1}^{n} x_{i,1,t} \middle| \Omega_{3,t}\right] + E\left[\sum_{q=1}^{m} y_{q,1,t} \Delta_{3,t}^{P}\left(P_{3,t}, V_{3,t}\right) \middle| \Omega_{3,t}\right] + E\left[h_{3,t}\left(P_{3,t}\right) \middle| \Omega_{3,t}\right] = 0 \tag{32}$$

Equation 30 and 31 state that the demand of spot dealers is expected to absorb the customers' demand at prices that clear the market in rounds 1 and 2. There is nothing in $\Omega_{1,t}$ and $\Omega_{2,t}$ that can help to predict the price of the foreign exchange in the subsequent round. Hence, $E\left[D_{i,2,t}\left(P_{1,t}\right) \middle| \Omega_{1,t}\right] = 0$ and $E\left[D_{i,2,t}\left(P_{2,t}\right) \middle| \Omega_{1,t}\right] = 0$. The round 1 equilibrium quote on the spot market is therefore $P_{1,t} = E\left[P_{2,t} \middle| \Omega_{1,t}\right]$. As the round 3 quote of the previous period and the payoff increment are included in the information set $\Omega_{1,t}$, $P_{1,t} = E\left[P_{2,t} \middle| \Omega_{1,t}\right] = P_{3,t-1} + \Delta R_t$. As no new public information is revealed between rounds 1 and 2, the rational quote in round 2 is $P_{2,t} = E\left[P_{3,t} \middle| \Omega_{2,t}\right] = P_{1,t}$. Equation 32 states that in expectation, the liquidity suppliers will have to absorb not only the demand from customers on the spot market, but also the flow arising from delta hedgers who cover their exchange rate risk.

Since the specification of the liquidity suppliers is: $h_3\left(P_{3,t}\right) = \theta\left(E\left[P_{3,t+1} + R_{t+1} \middle| \Omega_{3,t}\right] - P_{3,t}\right)$ and given the optimal trading rules on the two FX market segments, $E\left[\sum_{i=1}^{n} x_{i,1,t} \middle| \Omega_{3,t}\right] = \frac{X_t}{\alpha}$ and $E\left[\sum_{q=1}^{m} y_{q,1,t} \middle| \Omega_{3,t}\right] = \frac{Y_t}{\beta_t}$, the round 3 equilibrium price that clears the market must satisfy:

$$P_{3,t} = E_t\left[P_{3,t+1} + R_{t+1} \middle| \Omega_{3,t}\right] + \frac{X_t}{\alpha\theta} + \frac{Y_t}{\beta_t \theta} \Delta_{3,t}^{P}\left(P_{3,t}, V_{3,t}\right) \tag{33}$$

where $\Delta_{3,t}^{P} = z\Delta_{3,t}$ and z being the amount of foreign exchange that the holder of one call is allowed to buy if the option is exercised at maturity. The delta formula is non-linear in the exchange rate and in the implied volatility. In order to have a closed form solution, the first order linear Taylor approximation for the delta value is used. Ignoring the influence of time decay and of interest rates, the round 3 delta value is linearised around its round 2 value, which is known.

$$\Delta_{3,t}\left(P_{3,t},V_{3,t}\right)=\Delta_{2,t}\left(P_{2,t},V_{2,t}\right)+\left(P_{3,t}-P_{2,t}\right)\frac{\partial\Delta_{2,t}\left(P_{2,t},V_{2,t}\right)}{\partial P_{2,t}}+\left(V_{3,t}-V_{2,t}\right)\frac{\partial\Delta_{2,t}\left(P_{2,t},V_{2,t}\right)}{\partial V_{2,t}}$$

Hence the round 3 delta will only depend on the change in the exchange rate and implied volatility quotes between rounds 2 and 3. Ignoring the arguments of the delta function to simplify notation and given that $\frac{\partial\Delta_{2,t}}{\partial V_{2,t}}=\frac{\sqrt{\tau}}{\sqrt{2\pi}}$ for at-the-money forward calls, the round 3 equilibrium quote is given by:

$$P_{3,t}=\frac{1}{\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)}\left[E\left[P_{3,t+1}+R_{t+1}\middle|\Omega_{3,t}\right]+\lambda X_t+\lambda_t^{O'}Y_t z\left(\Delta_{2,t}-P_{2,t}\Gamma_{2,t}+\left(V_{3,t}-V_{2,t}\right)\frac{\sqrt{\tau}}{\sqrt{2\pi}}\right)\right]$$

where $\lambda=(\alpha\theta)^{-1}$, $\lambda_t^{O'}=(\beta_t\theta)^{-1}$ and $\frac{\partial\Delta_{2,t}}{\partial P_{2,t}}=\Gamma_{2,t}$. Furthermore, as in equilibrium $E\left[P_{3,t+1}\middle|\Omega_{3,t}\right]=P_{3,t-1}+\Delta R_t$, this yield eq. 21 in the text and together with proposition 4 to eq. 20.

Market clearing on the spot and option market

At the end of round 2, both spot and option interdealer order flows become public information. As spot dealers are supposed to know the optimal trading rule on the derivative market, they are able to perfectly forecast the subsequent implied volatility quote. Furthermore, option dealers are able to perfectly forecast the impact of delta hedging strategies on the round 3 exchange rate quote, because they know the risk bearing capacity of the public and the optimal trading rule prevailing on the spot market. The equilibrium on the spot and on the derivative markets in round 3 is thus simultaneous and is given by the solution of the following system.

$$\begin{cases} P_{3,t}=P_{2,t}+\dfrac{\lambda X_t+\lambda_t^{O'}Y_t z\left(\Delta_{2,t}+\left(V_{3,t}-V_{2,t}\right)\sqrt{\tau}/\sqrt{2\pi}\right)}{\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)} \\ V_{3,t}=V_{2,t}+\dfrac{\lambda_t^{O}Y_t\sqrt{2\pi}}{P_{3,t}\sqrt{\tau}} \end{cases}$$

The round 3 market clearing price of the foreign exchange rate and that of the implied volatility are given by:

$$\begin{cases} P_{3,t}^{*}=\dfrac{P_{2,t}\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)+\lambda X_t+\lambda_t^{O'}Y_t z\Delta_{2,t}+\sqrt{\left[P_{2,t}\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)+\lambda X_t+\lambda_t^{O'}Y_t z\Delta_{2,t}\right]^2+4\lambda_t^{O'}\lambda_t^{O}z\left(Y_t\right)^2\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)}}{2\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)} \\ V_{3,t}^{*}=V_{2,t}+\dfrac{2\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)\lambda^{O}Y_t\sqrt{2\pi}}{\sqrt{\tau}\left\{P_{2,t}\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)+\lambda X_t+\lambda_t^{O'}Y_t z\Delta_{2,t}+\sqrt{\left[P_{2,t}\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)+\lambda X_t+\lambda_t^{O'}Y_t z\Delta_{2,t}\right]^2+4z\lambda_t^{O'}\lambda_t^{O}\left(Y_t\right)^2\left(1-\lambda_t^{O'}Y_t z\Gamma_{2,t}\right)}\right\}} \end{cases} \quad (34)$$

Appendix C. Proofs of the optimal trading strategies

On the spot market, during interdealer trading, each dealer will receive a share 1/(N-1) of every other interdealer trade, whereas on the derivative market, each dealer will receive 1/(M-1) of interdealer options trade. This is directly related to the assumption of common interdealer quote and corresponds respectively to the disturbance term $T^{'}_{i,2,t}$ and $O^{'}_{q,2,t}$ in the maximisation problem of dealers. During this round 2, each dealer will speculate on the subsequent price change on the basis of his private information.

Proof of the optimal spot trading strategy (proposition 8): $T_{i,2} = \alpha x_{i,1}$

The optimisation problem that solves the speculative demand for each spot dealer can be written as follows:

$$\underset{D_{i2}}{Max} \quad D_{i,2}\left(E\left[P_3 \middle| \Omega_{i,2}\right] - P_2\right) - \left(D_{i,2}\right)^2 \frac{\gamma}{2}\sigma^2 \tag{35}$$

where the information set is $\Omega_{i,2,t} \equiv \left\{ {}_{s=1}^{t}\{\Delta R_s\}, {}_{i=1}^{n}\{P_{i,1,t}, P_{i,2,t}\}, x_{i,t}, V_{1,t}, V_{2,t} \right\}$ and where σ^2 denotes the conditional variance of $E\left[P_3 \middle| \Omega_{i,2}\right] - P_2$. Spot dealers know that the round 3 exchange rate quote will clear the net demand stemming from the spot market and the net demand of delta hedgers who will cover their foreign exchange risk on the spot market (see Proposition 5). As the information set of spot dealers $\Omega_{i,2,t}$ contains no information about the option interdealer order flows, $E\left[\lambda^{O'} Y \Delta_3^P \middle| \Omega_{i,2}\right] = 0$. Furthermore, the only relevant information to rationally predict $E\left[\lambda X \middle| \Omega_{i,2}\right]$ is the outgoing order flow of the spot dealer. Hence, $E\left[\lambda X \middle| \Omega_{i,2}\right] = E\left[\lambda \sum_{i=1}^{n} T_{i,2} \middle| \Omega_{i,2}\right] = \lambda T_{i,2}$. Hence, each dealer expects a change in the price of the foreign exchange that is proportional to its own trade.

$$E\left[P_3 \middle| \Omega_{i,2}\right] - P_2 = E\left[\lambda X + \lambda^{O'} Y \Delta_3^P \middle| \Omega_{i,2}\right] = \lambda T_{i,2} \tag{36}$$

Trade arising from customers is distributed $N(0, \sigma_x)$ and independent across dealers, so that $E\left[T^{'}_{i,2} \middle| \Omega_{i,2}\right] = 0$ and $T_{i,2} = D_{i,2} + x_{i,1}$. Hence, the maximisation problem becomes:

$$\underset{D_{i2}}{Max} \quad D_{i,2}\lambda\left(D_{i,2} + x_{i,1}\right) - \left(D_{i,2}\right)^2 \frac{\gamma}{2}\sigma^2 \tag{37}$$

The first order condition yields:

$$2\lambda D_{i,2} + \lambda x_{i,1} - \gamma\sigma^2 D_{i,2} = 0 \tag{38}$$

The speculative demand of spot dealers is therefore:

$$D_{i,2} = \left[\frac{\lambda}{\gamma\sigma^2 - 2\lambda} \right] x_{i,1} \tag{39}$$

The outgoing order flow from dealer *i* will therefore equal to:

$$T_{i,2} = \left[\frac{\lambda}{\gamma\sigma^2 - 2\lambda} + 1 \right] x_{i,1} = \left[\frac{\gamma\sigma^2 - \lambda}{\gamma\sigma^2 - 2\lambda} \right] x_{i,1} \equiv \alpha x_{i,1} \tag{40}$$

The condition $\gamma\sigma^2 - 2\lambda > 0$ ensures that $\alpha > 1$.

Proof of the optimal options trading strategy (proposition 7): $O_{q,2} = \beta_t y_{q,1}$

The optimisation problem solving the speculative demand of each options dealer is given by:

$$\underset{D^O_{q,2}}{Max} \quad D^O_{q,2}\left(E\left[C_3\left(P_3, V_3\right)\middle|\Omega_{q,2}\right] - C_2\left(P_2, V_2\right)\right) - \left(D^O_{q,2}\right)^2 \frac{\gamma}{2}\sigma_C^2 \tag{41}$$

where σ_C^2 denotes the conditional variance of $E\left[C_3\middle|\Omega_2\right] - C_2$. As nothing in the information set $\Omega_{q,2}$ allows the direction of the round 3 exchange rate to be predicted, option dealers rationally expect it to be: $E\left[P_3\middle|\Omega_{q,2}\right] = P_2$. Hence, the only variable that is expected to influence the round 3 call price is the implied volatility change from round 2 to round 3 when interest rate changes, time decay and second order factors are neglected.

$$E\left[C_3\middle|\Omega_{q,2}\right] - C_2 = \upsilon_2\left(E\left[V_3\middle|\Omega_{q,2}\right] - V_2\right) \tag{42}$$

where υ_2 is the vega of the at-the-money-forward call option. Each options dealer uses strategically his private information related to the order flows he received from his customers to predict the change in implied volatility quote between round 2 and 3.

$$E\left[V_3\middle|\Omega_{q,2}\right] - V_2 = E\left[\lambda^O Y\middle|\Omega_{q,2}\right] = E\left[\lambda^O \sum_{q=1}^{m} O_{q,2}\middle|\Omega_{q,2}\right] = \lambda^O O_{q,2} \tag{43}$$

With the same logic as that on the spot market, $E\left[O^{'}_{q,2}\middle|\Omega_{q,2}\right] = 0$, so that the dealer *q* outgoing order flows are equal to $O_{q,2} = D^O_{q,2} + y_{q,1}$. The options dealer maximisation problem can then be rewritten as:

$$\underset{D^O_{q,2}}{Max} \quad D^O_{q,2}\upsilon_2\lambda^O\left(D^O_{q,2} + y_{q,1}\right) - \left(D^O_{q,2}\right)^2 \frac{\gamma}{2}\sigma_C^2 \tag{44}$$

The first order condition yields:

$$2\upsilon_2\lambda^O D_{q,2}^O + \upsilon_2\lambda^O y_{q,1} - \gamma\sigma_C^2 D_{q,2}^O = 0 \tag{45}$$

The speculative demand from the options dealer is thus equal to:

$$D_{q,2}^O = \left[\frac{\upsilon_2\lambda^O}{\gamma\sigma_C^2 - 2\upsilon_2\lambda^O}\right] y_{q,1} \tag{46}$$

As $O_{q,2} = D_{q,2}^O + y_{q,1}$, the dealer q optimal trading strategy will be:

$$O_{q,2} = \left[\frac{\upsilon_2\lambda^O}{\gamma\sigma_C^2 - 2\upsilon_2\lambda^O} + 1\right] y_{q,1} \equiv \beta y_{q,1} \tag{47}$$

The condition $\gamma\sigma_C^2 - 2\upsilon_2\lambda^O > 0$ ensures that $\beta_t > 1$.

FIGURES – EQUILIBRIUM EXCHANGE RATE AND IMPLIED VOLATILITY

Figure 1 - Equilibrium exchange rate: positive spot order flows

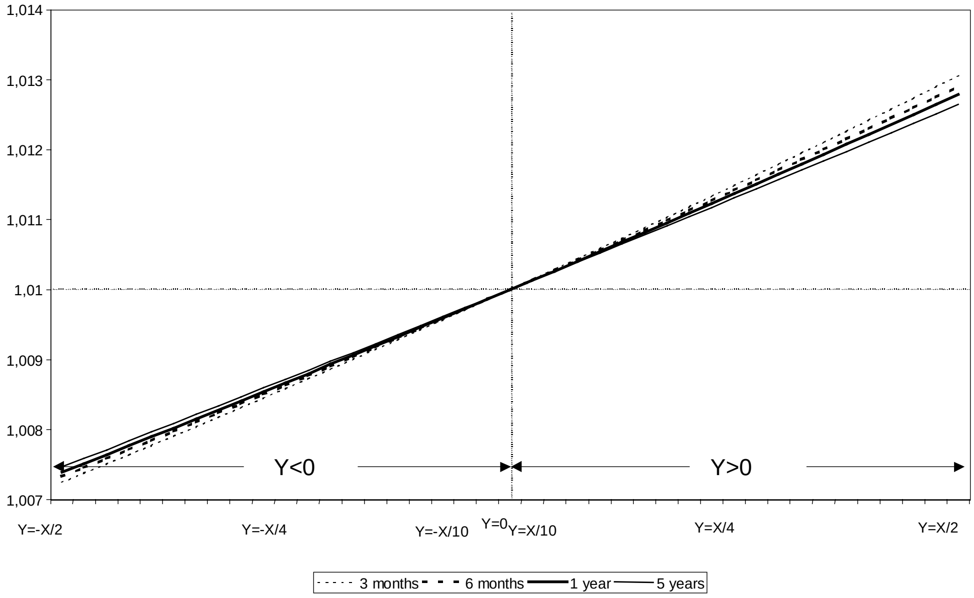

Figure 2 - Equilibrium exchange rate: negative spot order flows

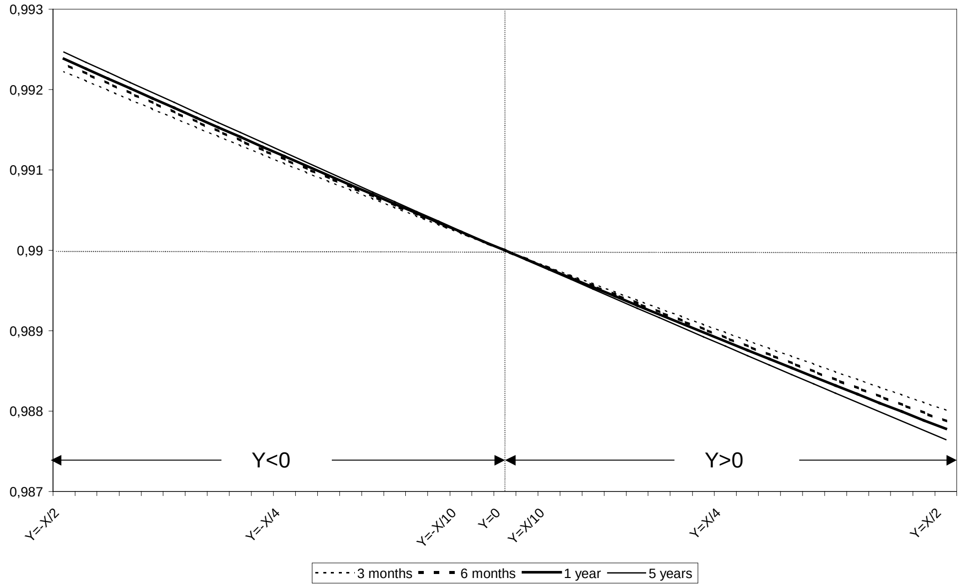

Figure 3 - Equilibrium implied volatility: positive spot order flows

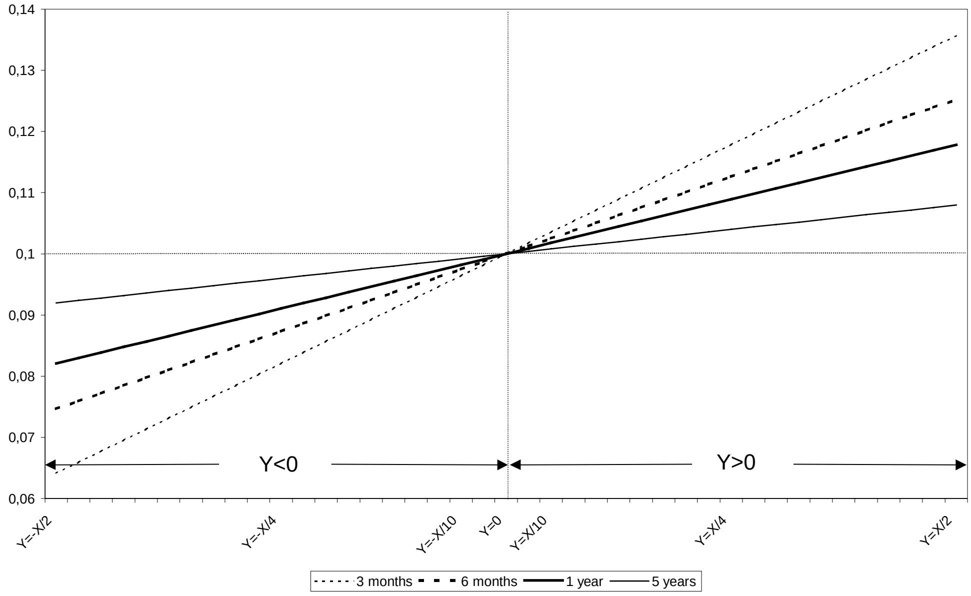

Figure 4 - Equilibrium implied volatility: negative spot order flows

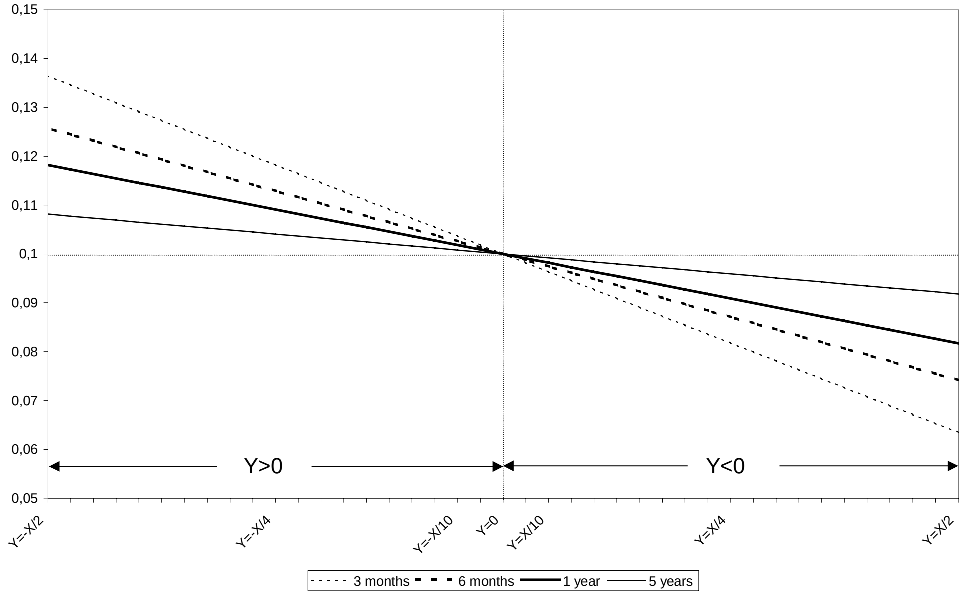

Figure 5 - Difference in equilibrium exchange rate: positive spot order flows

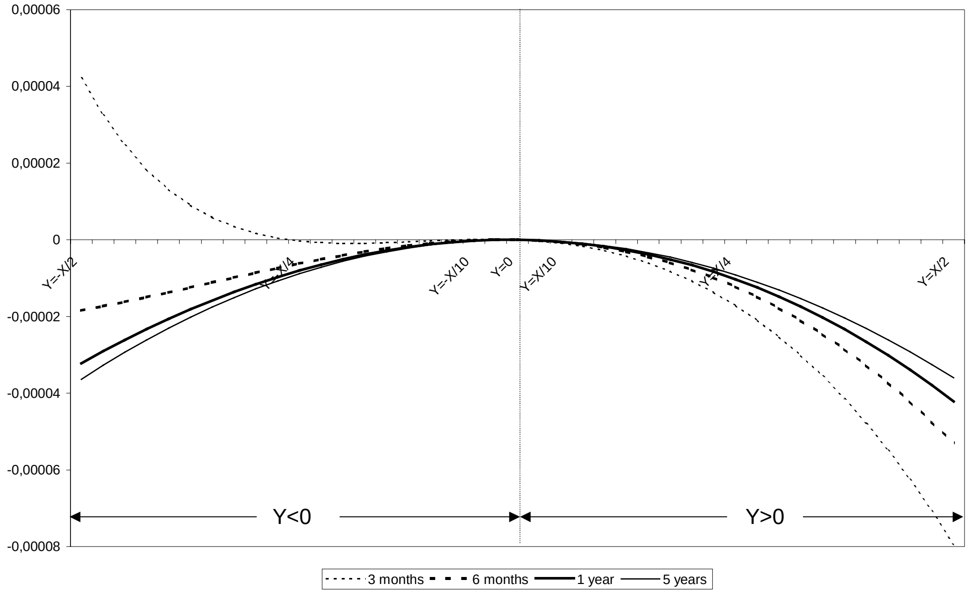

Figure 6 - Difference in equilibrium exchange rate: negative spot order flows

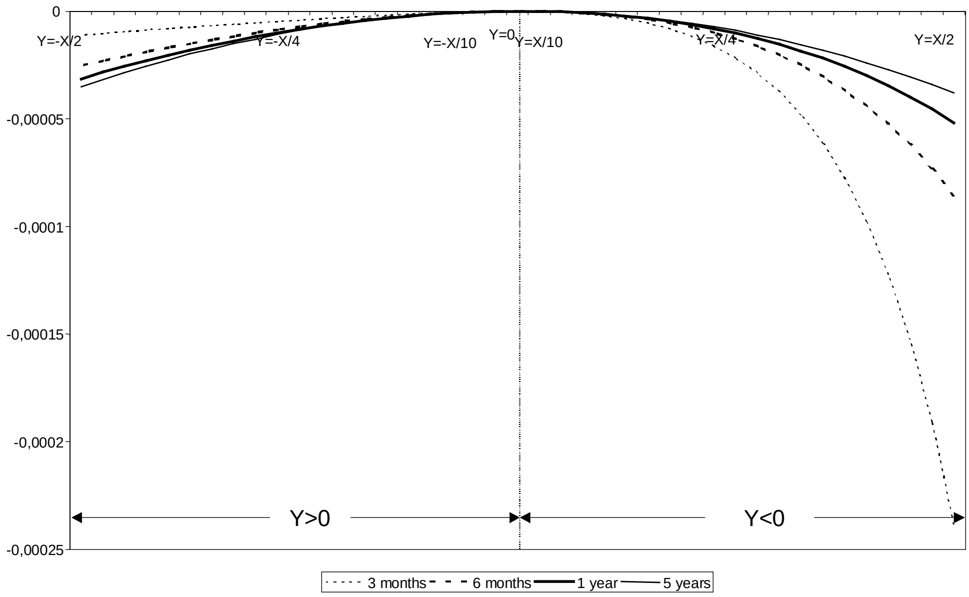

Figure 7 - Difference in equilibrium implied volatility: positive spot order flows

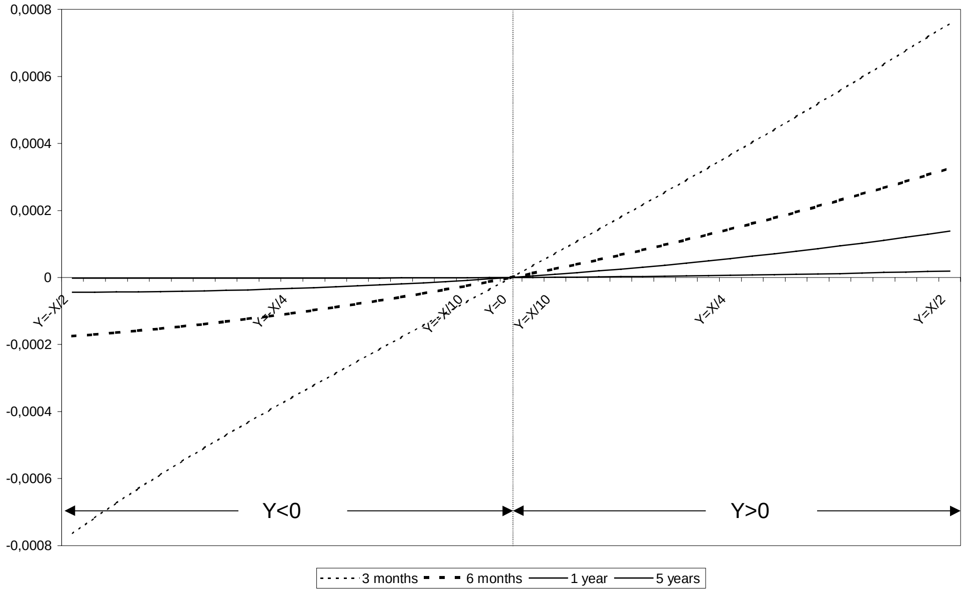

Figure 8 - Difference in equilibrium implied volatility: negative spot order flows