Inventory Information

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Abstract

This paper introduces a new class of private information: inventory information. Inventory information lies in the gap between the main approaches to microstructure theory, the inventory approach and the information approach. In the inventory approach, agents do not speculate on the basis of private information. In the information approach, agents do speculate on the basis of private information, but that information is limited to a specific type—information about payoffs (numerators in a valuation model). Inventory information lies in the gap because it provides a basis for speculation, but is unrelated to payoffs. Speculation in this case is based on forecasting valuation denominators. This new class of private information brings clarity to markets like foreign exchange, where private information about payoffs is unlikely, yet there is empirical support for private information of some kind. We also assess empirical methodologies and conclude that results are often over-interpreted: traditional specifications cannot distinguish these classes of private information. We specify and estimate an empirical model with more resolving power. The results are consistent with the theory.

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Inventory Information

The fields of finance and information economics enjoy a strong, symbiotic relationship. Early finance work on pricing in common-information environments draws heavily from information economics, particularly from the economics of uncertainty (Debreu 1959, Arrow 1964). Later work shows strong interaction in the analysis of private-information environments (Akerlof 1970, Grossman 1976, Hellwig 1980, Diamond and Verrecchia 1981). Interaction between these fields is also evident from the deep connection between information and financial market efficiency. Financial markets in which prices reflect all common information are said to be semi-strong-form efficient. Financial markets in which prices reflect all common and all private information are said to be strong-form efficient (Roberts 1967).

Despite the importance of private information within both fields, work within finance—in particular microstructure finance—has employed a much narrower definition of private information. Microstructure theory specifies private information as information about a security’s payoffs, in the simplest case a terminal payoff. But from the perspective of information economics, private information is any information that (1) is not common knowledge and (2) produces a better price forecast than common information alone. This definition makes room for types of private information that microstructure theory has not yet addressed.

This paper introduces these new types of private information to microstructure theory, and examines their empirical implications. As a first cut, we distinguish two broad classes of private information: private payoff information and private non-payoff information. Private payoff information is superior information about “dividends,” a terminal payoff, or both. Private non-payoff information is superior information that is unrelated to payoffs, but is related to interim prices. Interim

1 Dividends is used here as an umbrella term that includes coupons in the case of a bond and short-term interest rates in the case of foreign exchange. Within the literature, payoff information typically takes the direct form of signals distributed about those payoffs. Less common but also within this class is information about payoffs that is indirect, for example, information about the trading environment that allows one to infer the private signals of others, and thereby forecast payoffs more accurately than the market at large. Madrigal (1996) models this type of private payoff information.
prices can depend on many variables beyond payoff expectations—as has long been known (e.g., preferences, endowments, trading constraints, and other features of the trading environment). What we add here is superior knowledge about these variables. Insofar as these variables affect interim prices without altering payoff expectations, superior knowledge of them is within the non-payoff class.

Though our model accommodates several types of private information within the non-payoff class, we highlight one type in particular—inventory information. Inventory information lies in the gap between the main approaches to microstructure theory, the inventory approach and the information approach. In the inventory approach, agents do not speculate on the basis of private information. In the information approach, agents do speculate on the basis of private information, but that information is limited to a specific type—information about payoffs (numerators in a cash-flow discount model). Inventory information lies in the gap because it provides a basis for speculation, but is unrelated to payoffs. Speculation in this case is based on forecasting valuation denominators.

The model we develop below isolates this phenomenon of forecasting valuation denominators. The model introduces dealers who have superior information about inventories in a multiple-dealer market. These inventories are—by construction—orthogonal to the risky-asset payoff. Also, to isolate the non-payoff information class, the model does not include any superior information about payoffs. Indeed, payoff expectations never change in the model. Though payoff expectations never change, prices do change, due to a changing risk premium. (Dealers are risk averse and are compensated for bearing inventory risk.) Superior knowledge of inventories, then, helps dealers forecast price because it helps forecast the marketwide compensation for inventory risk.²

This paper’s contribution is fourfold. First, by introducing private non-payoff information to microstructure theory, we help close a persistent gap between microstructure finance and information economics. Second, our results provide new perspective on the traditional dichotomy between inventory models and information models. This new perspective is distinct from the well-known fact that the world is

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² For models of inventory effects on price see Ho and Stoll (1983) and O’Hara and Oldfield (1986) among many others. Empirical evidence of these effects on price is provided in Lyons (1995) for the FX market and in Snell and Tonks (1995) for an equity market.
not dichotomous—it includes aspects from both the inventory and information approaches. Our model does not simply graft an inventory model onto an information model to account for this fact. Rather, it introduces a new informational concept, one that had fallen through the cracks between these two modeling approaches. Third, we show that non-payoff information can generate speculative interdealer trading, and can do so in ways that are not symmetric, which is ruled out in inventory models with complete risk sharing. Finally, we provide concrete guidance for specifying empirical models that discriminate between private information sub-classes.

The related literature includes three strands, two from the microstructure literature and one outside of microstructure. The two strands from the microstructure literature are the above-mentioned information models (e.g., Kyle 1985, Glosten and Milgrom 1985) and inventory models (e.g., Ho and Stoll 1981, Ho and Stoll 1983, O'Hara and Oldfield 1986). The key difference between our model and canonical information models is that we do not assume dealers are risk neutral. The risk neutrality assumed in canonical models rules out the direct price effects from non-payoff information that we highlight here. (Put differently, if canonical models did not include private payoff information, then liquidity trades could not affect price; this point also holds for more recent models such as that of Gennotte and Leland 1990, which includes "supply informed" traders.) The key difference between our model and microstructure's inventory models is that our model combines multiple dealers with a trading mechanism that allows them to trade on superior inventory information before it is reflected in price.\(^3\) The third related strand of literature addresses payoff-unrelated price effects, though it does so outside of a microstructure setting. Kraus and Smith (1989), for example, develop a multi-period rational expectations model to show that—for certain equilibria—prices can move between periods even though it is common knowledge that no new payoff information has arrived. They do not address the informational implications of private non-payoff information. Vayanos (1999) develops a Walrasian auction among agents whose only private information is endowment information. His focus is the welfare

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\(^3\) Though there is no information asymmetry across dealers in Vayanos (1998), that model does include a large non-dealer participant who trades strategically based on his endowment (which is payoff unrelated).
costs of the resulting strategic behavior. He does not consider strategic trading on private information about intertemporal price change, as we do here. In his model, the equilibrium with private information is the same as that when all private information is public.

The rest of the paper is in six sections. Section I motivates our analysis by framing inventory information in the context of past empirical work. Section II presents a trading model designed to isolate inventory information. Sections III and IV describe the model’s equilibrium and present our results on how inventory information motivates speculation. Section V returns to the empirical side—we estimate a simple model that takes account of inventory information; our results that are broadly consistent with our model’s predictions. Section VI concludes.

I. Relation to Past Empirical Work

Empirical work on information effects divides into two methodologies—structural models and time-series models.\(^4\)

Structural Models

Structural models are variations on the following specification:

\[
\Delta P_t = \beta_1 Q_t + \beta_2 (I_t - I_{t-1}) + \beta_3 (D_t - D_{t-1}) + \epsilon_t
\]

were \(\Delta P_t\) is the change in transaction price from \(t-1\) to \(t\), \(Q_t\) is the signed order size, \(I_t\) is the dealer’s inventory, and \(D_t\) is an indicator variable for capturing bid-ask bounce: value=1 if \(Q_t\) is a purchase and value=-1 if \(Q_t\) is a sale. (Depending on the model, the error term \(\epsilon_t\) may have structure, and the restrictions that single coefficients \(\beta_2\) and \(\beta_3\) summarize inventory effects and bid-ask bounce may not apply.)

The key coefficient here is \(\beta_1\). Asymmetric information models predict a positive \(\beta_1\): dealers protect against adverse selection by increasing the customer’s

\(^4\) Work using structural models includes Glosten and Harris (1988), Madhavan and Smidt (1991), and Foster and Viswanathan (1993), all of which address the NYSE; structural models in a multi-
purchase price $P_t$ for larger purchases and reducing the customer’s selling price for larger sales ($Q_t < 0$ if sale). Estimates of $\beta_1$ in the literature are uniformly positive and significant, in both specialist and multiple-dealer settings.

Because these empirical models are derived assuming private information about payoffs, a significant $\beta_1$ is interpreted as evidence of private information about payoffs. But as our model clarifies, private non-payoff information produces the same finding. This does not, however, imply observational equivalence; indeed, one contribution of our model is to provide a set of payoff-unrelated factors that empirical specifications can integrate to disentangle these information classes.

**Time Series Models**

Time series models take the form:

$$P_t = F_t + E_t$$

where $P_t$ is the transaction price at time $t$, $F_t$ is a fundamental—or information—component that follows a random-walk, and $E_t$ is a stationary pricing error. The identifying assumption under this approach is that information, public or private, has only permanent effects on price. This departs from our information-economic definition above, which states that private information need only (1) not be common knowledge and (2) produce a superior price forecast than public information alone. Indeed, in our model below the price component forecastable with inventory information is temporary. In our judgment, then, the time-series definition of information is too narrow, and hinders articulation of a richer taxonomy of information types.

**II. A Model of Interdealer Trading**

Our multiple-dealer model corresponds most closely to trading in the largest multiple-dealer market—the market for spot foreign exchange (FX).\(^5\) This market is

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\(^5\) The model should not be viewed however as applying to this market only. Other important markets with a similar multiple-dealer structure include the London Stock Exchange and most bond markets.

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an interesting target because superior information about payoffs is unlikely, yet empirical work finds evidence of information asymmetries (Lyons 1995, Yao 1998, Ito et al. 1998, Covrig and Melvin 1998, Cheung and Wong 1998, Evans 1997, and Payne 1999). The model includes n dealers who behave strategically and a large number of competitive customers. All dealers and customers have identical negative exponential utility defined over terminal wealth. Though the model opens with a round of customer-dealer trading, all the action in is the two following rounds of interdealer trading. These two rounds of interdealer trading are the basis for grouping events into two periods—periods one and two.

A key feature of the model is that trading within a period occurs simultaneously. This simultaneous-trade approach is in the spirit of simultaneous-move games (cf, sequential-move games). Simultaneous trading has the effect of constraining dealers’ conditioning information: within any period dealers cannot condition on that period’s realization of others’ trades. We consider this level of conditioning information more realistic than that implicit in rational-expectations models (see Hellwig 1982 for another method of relaxing the strong assumption about conditioning information in rational-expectations models). Realism aside, though, the essential implication is that constraining conditioning information in this way allows dealers to trade on inventory information before it is reflected in price. Thus, unlike traditional trading models, in our model dealers actually have room to exploit inventory information. The lack of room to exploit it in traditional models helps explain why it has been overlooked in the literature for so long.

There are two assets, one riskless and one risky. The payoff on the risky asset is realized after the second round of interdealer trading, with the gross return on the riskless asset normalized to one. The risky asset is in zero supply initially, with a payoff of $F$, where $F$ is normally distributed about 0 with known variance $\Sigma_F$:

\begin{equation}
F \sim N(0, \Sigma_F)
\end{equation}

The seven events of the model occur in the following sequence (see Figure 1):

<table>
<thead>
<tr>
<th>Period 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(E1)</td>
<td>Dealers quote</td>
</tr>
<tr>
<td>(E2)</td>
<td>Customers trade with dealers</td>
</tr>
</tbody>
</table>
(E3) Dealers trade with dealers\(^6\)
(E4) Interdealer order flow is observed

**Period 2**
(E5) Dealers quote
(E6) Dealers trade with dealers
(E7) Payoff F realized

**II.1. Quoting Rules**

In both periods the first event is dealer quoting. Let \(P_{it}\) denote the quote of dealer \(i\) in period \(t\). The rules governing dealer quotes are:

(R1) Quoting is simultaneous, independent, and required
(R2) Quotes are observable and available to all participants
(R3) Each quote is a single price at which the dealer agrees buy and sell any amount\(^7\)

Rule 1 places this model in the simultaneous-trade approach to dealer markets. Though the sequential-trade approach (Glosten and Milgrom 1985) is popular for single-dealer modeling, it becomes unwieldy in multiple-dealer settings. Simultaneous moves in the foreign exchange market, for example, occur through electronic dealing products that allow simultaneous quotes and simultaneous trades. The key implication of rule R1 is that \(P_{it}\) is cannot be conditioned on \(P_{jt}\). That quotes are required squares with the fact that in actual multiple-dealer markets, refusing to quote violates an implicit contract of reciprocal immediacy and can be punished (e.g., by reciprocating with refusals).

Rule 2 defines the first of the model’s three dimensions of transparency: quotes are fully transparent (customer trades and interdealer trades, the other two dimensions, are not, per below). That quotes are observable is tantamount to assuming costless search. The last rule prevents a dealer from exiting the game at times of informational disadvantage.

\(^6\) At this stage, it may seem strange that we do not allow dealers to change their prices between events E2 and E3 in response to the customer orders received at E2. In equilibrium, however, dealers would not alter their prices even if they were given the opportunity. Though this will be clearer later, the intuition comes from that fact that no-arbitrage implies that dealer quotes for trading at E3 must be conditioned on common information, and the customer flows at E2 are not common information.
II.2. Customer Trades

Customer market-orders are independent of the payoff $F$. They occur in period-one only and are cleared at the receiving dealer’s period-one quote $P_{i1}$. Each customer trade is assigned—or preferred—to a single dealer, resulting from a bilateral customer relationship for example. The net customer order received by a particular dealer is distributed normally about 0, with known variance $\Sigma_c$:

$$c_i \sim N(0, \Sigma_c)$$

We use the convention that $c_i$ is positive for net customer purchases and negative for net sales. Though the assumptions of preferencing and exogeneity of $c_i$ appear strong at this stage, we show below that equilibrium dealer quotes in period one are all the same, and conditional on public information this price is also unbiased.\(^8\)

Customer trades embody the second of the model’s three dimensions of transparency: $c_i$ is not observed by other dealers. This is important—these customer trades are the private non-payoff information in the model. In foreign exchange, dealers have no direct information about other banks’ customer trades.\(^9\)

II.3. Interdealer Trading Rules

The model’s two-period structure is designed around the interdealer trading that occurs in each period. Let $T_{it}$ denote the net outgoing interdealer order placed by dealer $i$ in period $t$; let $T'_{it}$ denote the net incoming interdealer order received by

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\(^7\) The sizes tradable at quoted prices in the FX market, for example, are very large relative to other markets—currently they are good for up to $10$ million.

\(^8\) Recall from the quoting rules that these trades face no spread. If our focus here were endogenizing the number of dealers, we could certainly include a size-dependent spread for customer trades. We focus instead on the information dimension of the model.

\(^9\) Our specification accords well with practitioner survey responses that “better information” and “large customer base” are the two main sources of competitive advantage for large players in the FX market (Cheung and Wong 1998).
dealer i in period t, placed by other dealers. The rules governing interdealer trading are as follows:

(R4) Trading is simultaneous and independent
(R5) Trading with multiple partners is feasible
(R6) Trades are divided equally among dealers with the same quote if this is a quote at which a transaction is desired.

Rule R4 generates an interesting role for $T_{iT_i}^t$ in the model: because interdealer trading is simultaneous and independent, $T_{iT_i}^t$ is not conditioned on $T_{iT_i}^t$, so $T_{iT_i}^t$ is an unavoidable disturbance to dealer i’s position in period t that must be carried into the following period. The restriction in rule R6—that trades are split equally if quotes are common—can be relaxed. For example, allowing a split into $m < n$ equal fractions is straightforward as long as $m$ is known. (An unknown $m$ generates a non-normal position disturbance.) This relaxation would provide less risk sharing, but would not affect the path of price, nor would it affect dealers’ incentive to speculate on the basis of private non-payoff information (as we will see below).

Consider now the determination of dealer i’s outgoing interdealer orders in each period. We sign all orders according to the party initiating the trade. Thus, $T_{iT_i}^t$ is positive for dealer i purchases, and $T_{iT_i}^t$ is positive for purchases by other dealers from dealer i. Consequently, a positive $c_i$ or $T_{iT_i}^t$ corresponds to a dealer i sale. Letting $D_{iT_i}$ denote dealer i’s speculative demand, we have by definition:

\[
T_{iT_i}^t = D_{iT_i}^t + c_i + E[T_{iT_i}^t | \Omega_{T_{iT_i}^t}]
\]

\[
T_{iT_i}^t = D_{iT_i}^t - D_{iT_i}^t + T_{iT_i}^t - E[T_{iT_i}^t | \Omega_{T_{iT_i}^t}] + E[T_{iT_i}^t | \Omega_{T_{iT_i}^t}]
\]

where $\Omega_{T_{iT_i}^t}$ and $\Omega_{T_{iT_i}^t}$ denote dealer i’s information sets at the time of trading in periods one and two, respectively.

Eq. (3) clarifies that dealer orders include both an information-driven component $D_{iT_i}^t$ and inventory control components $c_i$ and $E[T_{iT_i}^t | \Omega_{T_{iT_i}^t}]$. Trades with customers must be offset in interdealer trading to establish the desired position $D_{iT_i}^t$. Dealers also do their best to offset the incoming dealer order $T_{iT_i}^t$ (which they cannot know ex-ante due to simultaneous trading). In period two inventory control has four components, three from the realized period-one position and one from the offset of the
incoming $T_{i1}$ (the plus sign preceding $T_{i1}$ in Eq. (4) reflects that $T_{i1}>0$ is a dealer i sale in period one).

II.4. The Last Period-One Event: Interdealer Order Flow Observed

The last event in period one defines the third of the model’s three dimensions of transparency – that applying to interdealer trades. At the close of period one all dealers observe period-one interdealer order flow:

\[
V \equiv \sum_{i=1}^{n} T_{ii}
\]

This sum over $T_{i1}$ is net demand – the difference in buy and sell orders. In foreign exchange $V$ is the information on interdealer order-flow provided by interdealer brokers (see Lyons 1999 for details). According to actual dealers, this is an essential source of real-time information.

Note that we specify this as an exact measure, which maximizes the transparency difference across trade types (customer-dealer with zero transparency and interdealer with complete transparency). As noted above, FX trades between customers and dealers do indeed have zero transparency. It is not the case that the actual transparency of interdealer trades is complete. It will be clear, however, from our results that adding noise to Eq. (5) has no qualitative impact, so we stick to this simpler specification.

II.5. Dealer Objectives and Information Sets

Each dealer determines quotes and speculative demand by maximizing a negative exponential utility function defined over terminal wealth. Letting $W_{it}$ denote the end-of-period $t$ wealth of dealer $i$, we have:
\[
\begin{align*}
\text{MAX} & \quad E[-\exp(-\theta W_{i2} | \Omega_i)] \\
\text{s.t.} & \quad W_{i2} = W_{i0} + c_i(P_{i1} - P'_{i1}) + \left(D_{i1} + E[T_{i1}^0 | \Omega_{i1}]ight)(P'_{i2} - P'_{i0}) + \left(D_{i2} + E[T_{i2}^2 | \Omega_{i2}]ight)(F - P'_{i2}) \\
& \quad - T_{i1}^0(P'_{i2} - P'_{i1}) - T_{i2}^0(F - P'_{i2})
\end{align*}
\]

where \(P_{i1}\) is dealer \(i\)'s period-one quote, \(P'\) denotes a quote or trade received by dealer \(i\), and \(F\) is the terminal payoff on the risky asset. The last two terms in final wealth capture the position disturbances from incoming dealer trades. The conditioning information \(\Omega_i\) at each decision node (2 quotes and 2 trades) is summarized in the Appendix.

### III. Equilibrium

The equilibrium concept we use is that of Perfect Bayesian Equilibrium, or PBE. Under PBE, Bayes rule is used to update beliefs and strategies are sequentially rational given those beliefs.

#### III.1. Equilibrium Quoting Strategies

Solving for the symmetric PBE, first we consider properties of optimal quoting strategies. The following proposition addresses period-one quotes.

**PROPOSITION 1:** A quoting strategy is consistent with symmetric PBE only if the period-one quote is common across dealers with:

\[P_i = E[F] = 0.\]
Proofs of all propositions appear the appendix. This result is rather intuitive however. First, rational quotes must be common to avoid arbitrage since quotes are single prices, available to all dealers, and good for any size. That the common price is 0, i.e., unbiased conditional on public information, is necessary for market clearing. Specifically, market clearing requires that dealer demand in period one offsets customer demand.

\[ \sum_i T_{ii} - T_{ii}' = \sum_i D_{ii} + c_i + E[T_{ii}' | \Omega_{T2}] - T_{ii}' = 0 \]  

(7)

At the time of quoting, the expectation of (7) is:

\[ E[c_i | \Omega_{T1}] + E[D_{ii}(P_1) | \Omega_{T1}] = 0 \]  

(8)

where \( \Omega_{T1} \) is public information available for quoting. (Since \( P_1 \) is common it is necessarily conditioned on public information only.) At the time of quoting in period one there is nothing in \( \Omega_{T1} \) that helps estimate \( c_i \) so \( E[c_i | \Omega_{T1}]=0 \). The only value of \( P_1 \) for which \( E[D_{ii}(P_1) | \Omega_{T1}]=0 \) is \( P_1=0 \) since \( D_{ii}(0)=0 \) and \( D_{ii}'<0 \).

An implication of common quotes is that each dealer receives exactly one order in period one from the dealer on his right (per trading rule R6). This order corresponds to the position disturbance \( T_{ii}' \) in the dealer's problem in Eq. (6). The next proposition addresses period-two quotes:

PROPOSITION 2: A quoting strategy is consistent with symmetric PBE only if the common period-two quote is:

\[ P_2 = \lambda V, \quad \lambda > 0. \]

The value of the constant \( \lambda \) is presented in the Appendix. The no-arbitrage argument that establishes common quotes is the same as that for proposition 1. Like \( P_1 \), \( P_2 \) necessarily depends only on public information. Here, the additional public information is the interdealer order-flow \( V \).

Intuition for \( \lambda>0 \) is important because \( P_2 \) is what motivates speculation in this model. The market clearing condition is similar to that in period one:
Taking the expectation using public information in period 2, we get:

\[(10) \sum \mathbb{E}[c_i | \Omega_{T2}] + \mathbb{E}[D_i\prime(P_2) | \Omega_{T2}] = 0\]

In this case, however, \(\mathbb{E}[c_i | \Omega_{T2}]\neq 0\) since interdealer order flow \(V\) is contained in \(\Omega_{T2}\) and provides information about \(c_i\). A negative \(V\), for example, means that average \(T_{i1}\) is negative – dealers are selling in interdealer trading. This implies that, prior to interdealer trading, customers sold on average; the negative \(V\) reflects dealers’ laying off long positions from buying from customers – the \(c_i\) term in Eq. (3). If customers sold in period one then in period two dealers are long (interdealer trading in period one does not affect the average dealer position). To clear the market, expected F-P\(_2\) must be positive to induce dealers to hold this long position. \(P_2\) must therefore fall below \(\mathbb{E}[F | \Omega_{T2}] = 0\) to provide the positive return needed to compensate dealers. The end result is that the negative \(V\) drives a reduction in price, that is, \(\lambda > 0\), as we set out to show. As in proposition 1, any price other than \(P_2 = \lambda V\) is incompatible with equilibrium since \(D_i\prime < 0\).

### III.2. Equilibrium Trading Strategies

Given the quoting strategy described in propositions 1 and 2, the following optimal trading strategy corresponds to symmetric linear equilibrium:

**PROPOSITION 3:** The trading strategy profile:

\[
T_{i1} = \beta_1 c_i \\
T_{i2} = \beta_2 c_i + \beta_3 T_{i1}' - \beta_4 P_2
\]

\(\forall i \in \{1,\ldots,n\}\) is a perfect Bayesian equilibrium.

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10 Notice that adding customer trading in period two would not alter this property that \(P_2\) is proportional to \(V\) unless the induced second-round customer demand is perfectly elastic.
The values of the $\beta$ coefficients are presented in the Appendix. Recall that the quoting rules for $P_1$ and $P_2$ are linear in $E[F]$ and $V$, respectively. The trading rules have a corresponding linear structure deriving from exponential utility and normality, which generate linear speculative demand. The derivations in the appendix insure that the quoting and trading strategies are mutually consistent and sequentially rational. These strategies impound dealer recognition that individual actions affect price.

**IV. Speculation Based on Inventory Information**

The trading strategies in proposition 3 have implications for the role of private non-payoff information, and more specifically, for the role of inventory information. For example, the coefficient in the period-one trading rule implies that non-payoff information motivates dealer speculation.

**PROPOSITION 4:** Even though the model contains no private payoff information, dealers still speculate based on private non-payoff information.

This proposition follows directly from the expression for $\beta_1$ in proposition 3. Specifically, the appendix shows that $\beta_1 = 1 + \delta$ and that $\delta > 0$. In the case of no speculation, $\beta_1 = 1$ since a non-speculative dealer would simply offset his customer trade with an interdealer trade, one-for-one. Instead, the dealer chooses to open a risky position, using his private information as the basis.

Here is the intuition behind the result. Recall from propositions 1 and 2 that $P_1 = 0$ and that $P_2 = \lambda V$. The dealer can therefore open a risky position at a price of zero and can close it at $\lambda V$. At the time of trading in period one the dealer does not know the realization of $V$. But since $V$ is simply the sum over all the dealers’ period-one trades $T_{i1}$, each dealer knows one component of $V$, namely his own trade. This creates a risk-return tradeoff that would not otherwise exist (the risk, of course, being the realization of the other dealers’ trades in $V$). To see why $\delta$ is positive, i.e., why the dealer rides herd on his customer order, notice that the positive $\lambda$ driving $P_2$
means that price will move in the same direction as the dealer’s order $T_{1i}$. After receiving a positive customer trade $c_i$, the dealer is short. A non-speculative dealer would buy just enough in the interdealer market to cover that short. A speculative dealer buys more than this because the long position will profit from the price increase his trade induces.

The next proposition generalizes our simple example.

PROPOSITION 5: Superior information about other determinants of $P_2$ also qualifies as private non-payoff information, in particular superior information about $n$, $\theta$, $\Sigma_c$, and $\Sigma_2$.

This proposition follows directly from the proof of proposition 2, which shows that $P_2$ is a function of all these parameters, none of which provides information about payoffs. Though our model focuses on the inventory information partition of the larger non-payoff class, proposition 5 clarifies that many other partitions exist (indeed, there are many partitions beyond those specified in the proposition).

The reason we specify the model without payoff information is to establish that this is not a case of payoff-irrelevant trades being mistaken for payoff-relevant trades, as occurs in other models in the literature. Given this modeling choice, it is helpful to link our result to more familiar contexts, in particular those with private payoff information. The next proposition makes this link by addressing the robustness of private non-payoff information as a motive for speculation.

PROPOSITION 6: Introducing private payoff information does not preclude dealers from speculating based on private non-payoff information.

This proposition also follows directly from the earlier propositions. Specifically, introducing private information about $F$ does not alter the risk-premium effect on $P_2$ necessary to clear the interdealer market (as described following proposition 2). Clearly, introducing private payoff information will alter the demand function described in appendix Eq. (A12), but one component of that demand function will still represent non-payoff information.
V. Some Empirical Guidance

Recall that traditional time series models take the form:

\[ P_t = F_t + E_t \]

where \( P_t \) is the transaction price at time \( t \), \( F_t \) is a fundamental—or information—component that follows a random-walk, and \( E_t \) is a stationary pricing error. The key identifying assumption under this approach is that information, public or private, has only permanent effects. In our model, however, the price component corresponding to inventory information is temporary. The traditional time-series approach therefore has no power to detect inventory information. Or more precisely, it has no power to disentangle inventory information from pricing errors. To identify inventory information using this approach, one would have to impose additional structure on the components of \( E_t \).

In the case of structural models, we offer the following strategy for identification. Recall that the key coefficient in structural models is that on incoming signed order flow, \( \beta_1 \):

\[ \Delta P_t = \beta_1 Q_t + \beta_2 (I_t - I_{t-1}) + \beta_3 (D_t - D_{t-1}) + \epsilon_t \]

Asymmetric information models predict a positive \( \beta_1 \): dealers protect against adverse selection by increasing the customer’s purchase price \( P_t \) for larger purchases and reducing the customer’s selling price for larger sales (\( Q_t < 0 \) if sale).

Estimates of \( \beta_1 \) in the literature are uniformly positive and significant, in both specialist and multiple-dealer settings. Note, however, that these regressions include only contemporaneous incoming order flow. Because these structural models are derived assuming private payoff information, a significant \( \beta_1 \) is interpreted as evidence of permanent price effects. Our model clarifies, though, that private non-payoff information produces the same finding, namely a positive contemporaneous effect on price.
For structural models, we offer an estimation strategy for discriminating private payoff information from private non-payoff information. The inventory information introduced in this paper is such that the positive contemporaneous effect reflected in $\beta_1$ dissipates over time as the expected return differential is realized. Lagged order flow should therefore correlate with current price change, but with a negative sign (see also Spiegel and Subrahmanyam 1995). This provides a means of decomposing the information effect—measured by $\beta_1$—into permanent and transitory parts, the former reflecting payoff information and the latter reflecting inventory information or other forms of non-payoff information.

To provide guidance for empiricists, we implement this strategy by applying the structural model of Madhavan and Smidt (1991) to the spot FX market. We choose the FX market because, as noted in section II above, the structure of our multiple-dealer model in section III corresponds most closely to that market. In addition, the FX market is particularly interesting because superior information about payoffs is unlikely. Nevertheless, empirical work provides evidence of private information. For example, evidence of private information is provided in Lyons (1995). That paper applies a variant of the Madhavan-Smidt model and finds that FX spreads have an adverse-selection component. Here we revisit these findings in light of our section III and IV results.\(^{11}\)

The baseline Madhavan-Smidt model takes the form:

\[
\Delta P_t = \beta_0 + \beta_1 Q_{j,t} + \beta_2 I_{t} + \beta_3 I_{t-1} + \beta_4 D_{t} + \beta_5 D_{t-1} + \epsilon_{t}
\]

where, as described above, a significant positive $\beta_1$ is interpreted as evidence of private payoff information. (The subscript $i$ denotes the dealer receiving the signorder $Q$ from dealer $j$.) Here we estimate an alternative that includes past order flow:

\[
\Delta P_{it} = \beta_0 + \beta_1 Q_{j,t} + \sum_{k=1}^{T} \gamma_k Q_{j,t-k} + \beta_2 I_{it} + \beta_3 I_{i,t-1} + \beta_4 D_{i,t} + \beta_5 D_{i,t-1} + \epsilon_{it}
\]

\(^{11}\) Our data are the same as those in Lyons (1995). To conserve space, we refer readers to the data description in that paper.
There are two features of the data we want to test. First, we want to determine whether the coefficients on past order flow, the $\gamma_k$, are negative. If negative, this implies that the lagged effects of order flow offset the impact effect (measured by $\beta_1$).

Second, we want to test the null that:

$$\sum_{k=1}^{T} \gamma_k = 0$$

Under this null, the information effect of order flow on price is, in the end, transitory, even if the impact effect measured by $\beta_1$ is significantly positive.

To estimate equation (12) we need to take a position on values for $T$ in the summation, and on the shape of the coefficients $\gamma_k$. Though the ideas in equations (12) and (13) are inspired by our Section III model, that model is not designed to pin down a specific value for $T$, nor the shape of the coefficients $\gamma_k$. With data on five days of FX trading—a total of 837 observations (i.e., incoming transactions)—we do not have the statistical power to extend the lag length $T$ very far. We estimate three different lag-length specifications: no lagged flow whatsoever (i.e., a baseline Madhavan and Smidt model), lagged flow over the previous 25 incoming transactions $Q_{lt}$, and lagged flow over the previous 50 incoming transactions $Q_{lt}$. For the shape of the coefficients $\gamma_k$, we adopt a simple polynomial distributed lag specification, constrained to equal zero at the near and far ends (i.e., at lag 0 and at lag $T+1$), but free to take any values between lags 1 and $T$.

Table 1 presents our results. The first row is the baseline Madhavan-Smidt model, i.e., it does not include any lagged order flow $Q_{lt-k}$. All of the coefficients are signed as the model predicts and all are significant. When lagged order flow is included under the $T=25$ and $T=50$ specifications, we find that the coefficients in the

---

12 One might presume that extending the lag length $T$ would increase the correlation between lagged flow and inventory $I_t$, leading to a multicollinearity problem. This is not the case here because the lagged flow includes only incoming transactions, whereas inventory includes all transactions, including outgoing trades at other dealers’ prices and trades through interdealer brokers. (For more on these specific transaction types, and their relation to our data, see Lyons 1995.)

13 The first-order moving average in the error term is derived in Madhavan and Smidt (1991). We refer readers to the source for details about why this error structure arises under the null. The results for this first-row specification are identical to those reported for one of the specifications in Lyons (1995).
baseline model retain their sign and significance, including, most notably, the coefficient on contemporaneous order flow, \( \beta_1 \) (despite the fact that including these lags reduces the number of available observations substantially). The sum of the coefficients on lagged order flow, \( \Sigma \gamma_k \), is negative, as predicted by our model, and the size is roughly equal to the coefficient \( \beta_1 \). This sum is not significant at conventional levels, however. Figure 2 plots the values of the estimates of the individual \( \gamma_k \). For both lag lengths, all of the individual estimates are negative. Though these negative individual estimates, too, are consistent with our model, none of them is significant at conventional levels.

Figure 2
Individual \( \gamma_k \) estimates

```
Lag length
```

Finally, we address the null hypothesis expressed in equation (13). This is a test of whether the sum of the coefficients on \( Q_{t-k} \) from \( k=0 \) to \( k=T \) is equal to zero. This would be the case if the information effect of order flow on price is transitory. The point estimates and standard errors for the sum of \( Q_{t-k} \) from \( k=0 \) to \( k=T \) are:

<table>
<thead>
<tr>
<th>( T = 25 )</th>
<th>( T = 50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>–0.02</td>
<td>–1.04</td>
</tr>
<tr>
<td>(1.48)</td>
<td>(3.03)</td>
</tr>
</tbody>
</table>
Thus, though the initial effect of $Q_{jt}$—i.e., $\beta_1$—remains positive and significant when lagged order flow is included, we cannot reject the null that this price effect dissipates over time, consistent with private non-payoff information. Of course, our estimates of $\Sigma \gamma_k$ are not very precise, so this test has low power. Nevertheless, we believe our application serves its purpose, namely to help point the way for further examination of private non-payoff information and its implications.

VI. Conclusion

From the perspective of information economics, microstructure theory has evolved under a rather narrow specification of private information. This paper broadens that specification. In particular, it introduces a class of private information that is unrelated to payoffs, yet still provides a motive for speculation. The information is a motive for speculation because it correlates with future price. There are many factors that affect future price beyond expectations of payoffs. Examples include features of the trading environment like traders’ risk aversion, trading constraints, endowments, and many others. Insofar as these affect price without altering payoff expectations, superior knowledge of them qualifies as non-payoff information.

The model we present here is designed as a specific example of private non-payoff information, not as a general treatment. Specifically, our example is based on superior information about dealer inventories, which we call inventory information. Superior knowledge of inventories helps a dealer to forecast price because it helps forecast the marketwide compensation for inventory risk. This information is the sole driver of dealers’ speculative demands. Note that our model does not rely on liquidity trades that masquerade as informative trades; indeed, we specified the model without any private payoff information to make this distinction clear.

This class of information has implications for both theoretical and empirical microstructure. Theoretically, our model lies in the gap between asymmetric information models (which highlight payoff information) and inventory models (which highlight price effects that are payoff-unrelated, but do not motivate speculation). Our model is thus in the spirit of recent work that spans the two previously
distinct approaches. Our spanning is of a very different nature, however, because it is not simply a grafting of one approach onto another.

Empirically, our model suggests that earlier work finding private information effects on price might very well have found effects from the non-payoff class, as opposed to the payoff class which was used to interpret the findings. Past empirical models are simply not designed to discriminate. The empirical results we provide here are a first step in addressing these issues more fully. These results are broadly consistent with predictions from the model we develop, but should not be considered conclusive. Other applications within the literature are readily apparent, and we consider this an interesting direction for further work.
Appendix

This appendix repeatedly uses certain information sets and conditional expectations. To simplify notation, we present these at the outset for reference.

Information Sets

\[ \Omega_{T1} \equiv \{ c_i, i^n \{ P_1 \} \} \]
\[ \Omega_{T2} \equiv \{ c_i, i^n \{ P_1 \}, T_1, T_2, V, i^n \{ P_2 \} \} \]
\[ \Omega_{T1} \equiv \{ i^n \{ P_1 \} \} \]
\[ \Omega_{T2} \equiv \{ i^n \{ P_2 \}, V, i^n \{ P_2 \} \} \]

The first two are the private information sets available to each dealer \( i \) at the time of trading in each of the two periods. The second two are the public information sets available at the time of trading in each period.

Conditional Variances

This appendix also repeatedly uses several conditional variances. These variances do not depend on conditioning variables’ realizations. For example, \( \Sigma_2 \equiv \text{Var}(F_{-P_2} | \Omega_{T2}) \) does not depend on dealer \( i \)’s realization of \( c_i \). \( \Sigma_2 \) is thus common to all dealers and known in period one. (It is a convenient property of the normal distribution that realizations of conditioning variables affect the conditional mean but not the precision of the condition mean.) This predetermination of period-two conditional variances is essential to the period-one quoting and trading rules.

A.1. Proof of Propositions 1 and 2: Price Determination

Rational quotes must be common to avoid arbitrage under quoting rules (R1)-(R3), trading rules (R4)-(R6), and risk-aversion. With common prices, the level necessarily depends only on commonly observed information. (Prices are thus never relevant as conditioning variables since they depend deterministically on commonly observed variables already in the information sets.) Accordingly, the equations that pin down the equilibrium price in each period are necessarily conditioned on public information.

The equations that pin down equilibrium price are Eqs. (3) and (4), the dealer trading rules in each period. Per above, when conditioned on public information they must be consistent with equilibrium price. This implies the following key relations:

\[ \text{(A1)} \quad E[c_i | \Omega_{T1}] + E[D_{1i}(P_1) | \Omega_{T1}] = 0 \]
\[ \text{(A2)} \quad E[c_i | \Omega_{T2}] + E[D_{2i}(P_2) | \Omega_{T2}] = 0 \]

where the additional terms in Eqs. (3) and (4) cancel when projected on public
information due to symmetry and the law of iterated projections. Eqs. (A1) and (A2) simply state that in expectation, net dealer demand must absorb the demand from customers. The reason they pin down equilibrium price is that all prices except the one that satisfies each will generate net excess demand in interdealer trading, which cannot be reconciled since dealers trade among themselves.

That $P_1=0$ follows directly from Eq. (2), which pins down $E[c_i | \Omega T_1]$, and Eq. (A12), which provides the expression for $E[D_{i1} | \Omega T_1]$ from the demand function. At the public information unbiased price $P_1=E[F | \Omega T_1]=0$, $E[D_{i1} | \Omega T_1]=0$. This is the only price at which the relation in Eq. (A1) is satisfied. Note from the demand function in Eq. (A12) that the coefficient on $c_i$ equals $\beta_1^{-1}$. In effect, the above analysis postulates that this is indeed the form of the demand function; we show below that the optimal rule does take this form.

In the second period a bias in $P_2$ is necessary for (A2) to hold. First consider the term $E[c_i | \Omega T_2]$. Given $P_1$ and $P_2$ are common across dealers and conditioned only public information, the only variable in $\Omega T_2$ relevant for determining $P_2$ is $V$, interdealer order flow from period one. We have:

$$E[c_i | \Omega T_2] = E[c_i | V] = E[c_i | \Sigma \beta_1 c_i] = (1/n\beta_1)V$$

To determine $E[D_{i2} | \Omega T_2]$ in Eq. (A2), we use normality and exponential utility to write:

$$E[D_{i2} | \Omega T_2] = E[F-P_2 | \Omega T_2]/\theta \Sigma = -P_2/\theta \Sigma.$$  

These expressions in Eq. (A2) imply:

$$P_2 = (\theta \Sigma /n\beta_1)V \equiv \lambda V$$

with $\lambda>0$ unambiguously.

**A.2. Proof of Proposition 3: Optimal Trading Strategies**

The derivation of trading strategies has 3 steps. First we establish the dealer’s problem as a maximization over realizations of the order flow $V$. Next, because each dealer accounts for his own impact on $V$, we redefine the problem over a random variable that is independent of a dealer’s own actions. Finally, we solve this maximization problem. Our approach to this problem relates closely to that in Lyons (1997), though we present less detail here.

**A.2.1. Step One: Maximization Over the Random Variable $V$**

To solve the dynamic programming problem we first determine the period-two desired position for use in the period-one first order condition. Under normality

---

14 This is clear enough for Eq. (3). For Eq. (4), it is clear once one recognizes that $E[-D_{i1} + T'_{i1} - E[T'_{i1} | \Omega T_1] | \Omega T_2] = E[c_i | \Omega T_2]$, which follows under symmetry from the fact that $E[T'_{i1} | \Omega T_2] = E[D_{i1} + c_i + E[T'_{i1} | \Omega T_1] | \Omega T_2]$. 

---
and negative exponential utility it is well known that the period-two desired position is:

$$D_{i2} = (\mu_{iF} - P_2)(\theta \Sigma_2)^{-1}$$

where $$\mu_{iF} = E[F|\Omega_{T_2}]$$, $$\theta$$ is the coefficient of absolute risk aversion, and $$\Sigma_2$$ denotes $$\text{Var}[F|\Omega_{T_2}] = \Sigma_F$$. Omitting terms unrelated to $$D_{i2}$$, we can write the dealers’ problem as:

$$\text{Max}_{D_{i2}} E_{\lambda_{i2}, \theta_{i2}, \Sigma_2} \left[ -\exp(-\theta(D_{i2} - T_{i2}))(P_2 - P_1) - \frac{1}{2}(\mu_{i2} - P_2)^{\Sigma_2^{-1}}(F - P_2) | \Omega_{T_2} \right]$$

We can use the moment generating function for the normally distributed variable $$F$$ to express the problem as:

$$\text{Max}_{D_{i2}} E_{\lambda_{i2}, \theta_{i2}, \Sigma_2} \left[ -\exp(-\theta(D_{i2} - T_{i2}))(P_2 - P_1) - \frac{1}{2}(\mu_{i2} - \Sigma_2^{-1}) | \Omega_{T_2} \right]$$

which leaves the objective function with three remaining random variables.

Now, an expression for $$\mu_{iF}$$ is central to the model since this summarizes each dealer’s end-of-period-one beliefs as a function of each contingency. Since our specification lacks private payoff information this is easy:

$$\mu_{iF} = E[F|\Omega_{T_2}] = 0$$

Using the expressions for $$P_1$$ and $$P_2$$ from propositions 1 and 2, the dealer’s problem is now a function of a single random variable $$V$$:

$$\text{Max}_{D_{i2}} E_{V} \left[ -\exp(-\theta(D_{i2} - \mu_{i2})^2 V - \frac{1}{2}(\lambda V)^2 \hat{\Sigma}_2^{-1} | \Omega_{T_1} \right]$$

where

$$\hat{\Sigma}_2^{-1} = \Sigma_2^{-1} - \theta^2 \text{Var}[T_{i1}^1 | \Omega_{T_1}, P]$$

$$\mu_{i1} = E[T_{i1}^1 | \Omega_{T_1}, P] = (V - T_{i1}) / (n - 1)$$

### A.2.2. Step Two: Accounting for Dealer i’s Impact on V

Given dealer $$i$$ considers the effect of her own order on $$V$$, it would not be appropriate to take the expectation over $$V$$. We define a new variable on which dealer $$i$$ has no effect, $$\hat{V}$$, which is the interdealer order flow netted of dealer $$i$$’s trade:

$$\hat{V} = V - T_{i1}$$

From Eq. (3), we know that:
\[ T_{i1} = D_{i1} + c_i + E[T_{i1} \mid \Omega_{T_{i1}}]. \]

The last term in this expression is 0 under our specification since (i) customer trades are mean-zero and independent across dealers and (ii) there is no information in the model other than customer trades to motivate speculative demand.

(A9) \[ E[T_{i1} \mid \Omega_{T_{i1}}] = 0 \]

Making the substitution for \( T_{i1} \), the objective function in Eq. (A7) becomes a maximization over \( V \) — a random variable independent of own actions:

(A10) \[
\max_{D_{i1}} E_V \left[ -\exp \left( -\theta \left( D_{i1} - \frac{V'}{n-1} \right) \left( \lambda D_{i1} + \lambda c_i + \lambda V' \right) - \frac{1}{2} \left( \lambda D_{i1} + \lambda c_i + \lambda V' \right)^2 \hat{\Sigma}_2^{-1} \right) \mid \Omega_{T_{i1}} \right].
\]

A.2.3. Step Three: Solving the Maximization Problem

To express the expectation in Eq. (A10) in terms of an integral, we use the fact that \( V \) is normally distributed about a conditional mean of zero. Let \( \Sigma_V \) denote \( \text{Var}[V \mid \Omega_{T_{i1}}] \). The following objective function is proportional to that in Eq. (A10):

(A11) \[
\max_{\theta, \lambda} -\int_{-\infty}^{\infty} \exp \left[ -\theta D_{i1} \left( \lambda D_{i1} + \lambda c_i + \lambda V' \right) - \frac{1}{2} \left( \lambda D_{i1} + \lambda c_i + \lambda V' \right)^2 \hat{\Sigma}_2^{-1} \right] dV'
\]

Here, we draw on a solution technique used by Kim and Verrecchia (1991, pages 317-319). Within the square brackets, collecting terms involving \( V \) yields:

\[
-\frac{1}{2} \left[ 2 \theta \lambda D_{i1}^2 + 2 \theta \lambda c_i D_{i1} + \hat{\Sigma}_2^{-1} \lambda^2 D_{i1}^2 + \hat{\Sigma}_2^{-1} \lambda c_i^2 + 2 \hat{\Sigma}_2^{-1} \lambda^2 c_i D_{i1} \right]
\]

\[
- \frac{1}{2} \left[ 2 \left( \theta \lambda D_{i1} + \hat{\Sigma}_2^{-1} \lambda^2 - \frac{\theta \lambda}{n-1} (D_{i1} + c_i) \right) V' + \left( \hat{\Sigma}_2^{-1} \lambda^2 + \Sigma_V^{-1} - \frac{2 \theta \lambda}{n-1} \right) V'^2 \right]
\]

Now, factoring terms involving \( V \) yields:

\[
-\frac{1}{2} \left[ 2 \theta \lambda D_{i1}^2 + 2 \theta \lambda c_i D_{i1} + \hat{\Sigma}_2^{-1} \lambda^2 D_{i1}^2 + \hat{\Sigma}_2^{-1} \lambda c_i^2 + 2 \hat{\Sigma}_2^{-1} \lambda^2 c_i D_{i1} \right]
\]

\[
- \frac{1}{2} \left[ A \left( V' + \frac{B}{A} \right)^2 - \left( \frac{B^2}{A} \right) \right]
\]

where:

\[
A \equiv \hat{\Sigma}_2^{-1} \lambda^2 + \Sigma_V^{-1} - \frac{2 \theta \lambda}{n-1} \quad \text{and} \quad B \equiv \theta \lambda D_{i1} + \left( \hat{\Sigma}_2^{-1} \lambda^2 - \frac{\theta \lambda}{n-1} \right) (D_{i1} + c_i)
\]

This allows us to write the dealer’s problem as:
\[
\text{Max}_{D_{i1}} \exp \left[ -\frac{1}{2} \left( \theta \lambda D_{i1}^2 + 2 \theta \lambda c_i D_{i1} + \hat{\Sigma}_2^{-1} \lambda^2 D_{i1}^2 + \hat{\Sigma}_2^{-1} \lambda^2 c_i^2 + 2 \hat{\Sigma}_2^{-1} \lambda^2 c_i D_{i1} - \frac{(B^2)}{A} \right) \right]
\]

\[
\times \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{2} \left( A \left( V' + \frac{B}{A} \right)^2 \right) \right]
\]

The integral in the second line is proportional to a cumulative normal density with mean \(-B/A\) and variance \(1/A\). Since this proportional relationship is not a function of \(D_{i1}\), maximizing this whole expression is equivalent to maximizing the expression in the first line within curly brackets. This produces a first order condition of:

\[
\left( \frac{n}{n-1} \theta \lambda + C - C^2 / A \right) D_{i1} + \left( C + \frac{\theta \lambda}{n-1} - \left( \hat{\Sigma}_2^{-1} \lambda^2 - \frac{\theta \lambda}{n-1} \right) C / A \right) c_i = 0
\]

where:

\[
C \equiv \frac{n-2}{n-1} \theta \lambda + \hat{\Sigma}_2^{-1} \lambda^2
\]

which implies a speculative demand of:

\[
(A12) \quad D_{i1} = \left( -\frac{C \left( A - \hat{\Sigma}_2^{-1} \lambda^2 \right) + \left( A + C \right) \theta \lambda / (n-1)}{A(n \theta \lambda / (n-1) + C) - C^2} \right) c_i \equiv \delta c_i
\]

For the maximization problem to have a solution, the second order condition implies that the denominator in \(A(12)\) must be negative and thus \(\delta > 0\). The fact that \(T_{i1}=D_{i1}+c_i\) imply:

\[
(A13) \quad T_{i1} = (1 + \delta)c_i \equiv \beta_i c_i
\]

Combine \(A(12)\) and \(A(13)\), we get:

\[
(A14) \quad \frac{\theta^2 \Sigma_2 (1/\Sigma_c - \theta^2 \Sigma_2 \beta)}{\beta^3 \left[ \left( A(n \theta \lambda / (n-1) + C) - C^2 \right) \right]} = \beta n(n-1)
\]

Equation \(A(14)\) simplifies to a quadratic equation:

\[
(A15) \quad \beta^2 - \frac{n-1}{n} \left( \frac{2}{\theta^2 \Sigma_2 \Sigma_c} + 1 \right) \beta + \frac{(n-1)^2}{n^2 \theta^2 \Sigma_2 \Sigma_c} = 0
\]

There are two positive roots:

\[
(A16) \quad \beta = \left( \frac{n-1}{n} \right) \left( \frac{2 + x \pm \sqrt{4 + x^2}}{2x} \right)
\]
where \[ x = \theta^2 \Sigma \Sigma_c \]

However, only the larger root satisfy the second order condition. Thus, there are at most one equilibrium. In addition, we need to impose the condition that \( A > 0 \), such that the integral in (A11) is well defined. The positivity condition of \( A \) reduces to the following constraint:

\[
\theta^2 \Sigma \Sigma_c < \frac{n^2(n-2)}{(n+2)(n-1)}.
\]

Clearly, there is a nonempty set of parameters that satisfy this constraint. Moreover, given this constraint, the equilibrium described in the model is unique.

The period-two trading rule follows directly from Eqs. (4), (A3), and (A6).
Notation

$P_{i1}$: dealer i’s quote in period one.
$c_i$: net customer order received by dealer i.
$T_{i1}$: dealer i’s net outgoing order to other dealers in period one.
$V$: net interdealer order flow in period one.
$P_{i2}$: dealer i’s quote in period two.
$T_{i2}$: dealer i’s net outgoing order to other dealers in period two.
$F$: payoff on the risky asset.
Table 1
Model estimates

\[ \Delta P_t = \beta_0 + \beta_1 Q_{jt} + \sum_{k=1}^{T} \gamma_k Q_{jt-k} + \beta_2 I_t + \beta_3 I_{t-1} + \beta_4 D_t + \beta_5 D_{t-1} + v_t + \beta_6 v_{t-1} \]

<table>
<thead>
<tr>
<th></th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \Sigma \gamma_k )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
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<th>( \beta_6 )</th>
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T-statistics in parentheses. \( \Delta P_t \) is the change in the incoming transaction price (DM/$) from t-1 to t. Q_{jt} is the incoming order transacted at dealer i’s quoted price, positive for purchases (i.e., executed at the offer) and negative for sales (at the bid). It is dealer i’s inventory at the end of period t. D_t is an indicator variable with value 1 if the incoming order is a purchase and value –1 if a sale. All quantity variables are in $ millions. All coefficients are multiplied by 10^5. Sample: August 3-7, 1992. We use the Hildreth-Lu procedure to estimate the moving average coefficient \( \beta_6 \) (see Madhavan and Smidt 1991 for the derivation of this first-order moving-average error term). The coefficients \( \gamma_k \) are estimated as a polynomial distributed lag with coefficients constrained at beginning and end to equal zero (i.e., at lag 0 and at lag T+1), but free to take any values between lags 1 and T.
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