

An Information Approach to International Currencies

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Abstract

Models of currency competition focus on the 5 percent of trading attributable to balance-of-payments flows. We introduce an information approach that focuses on the other 95 percent. Important departures from traditional models arise when transactions convey information. First, prices reveal different information depending on whether trades are direct or through vehicle currencies. Second, missing markets arise due to insufficiently symmetric information, rather than insufficient transactions scale. Third, the indeterminacy of equilibrium that arises in traditional models is resolved: currency trade patterns no longer concentrate arbitrarily on market size. Empirically, we provide a first analysis of transactions across a full market triangle: the euro, yen and US dollar. The estimated transaction effects on prices support the information approach.

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An Information Approach to International Currencies

Competition between national currencies remains a deep issue in both monetary and international economics. Why, for example, are the international roles of the euro and yen disproportionately small? Like much existing work, this paper addresses currency competition by focusing on currency exchange. Our approach, however, contrasts sharply with existing approaches. We admit the possibility that transactions convey incremental information, at least to those who observe them. When transactions can convey information, they play an added role in determining prices, providing a natural integration of money as a medium-of-exchange (transactions) and as a store-of-value (prices).

Information affects currency exchange by affecting the cost of transacting. Specifically, the cost of transacting is increasing in the amount of information that trades convey, which protects counterparties from informational disadvantage (Copeland and Galai 1983, Kyle 1985, Glosten and Milgrom 1985). In our model, this produces a percentage cost of individual trades that increases with trade size. It is important to emphasise that we are not referring to second order costs of transacting, such as bid-ask spreads, but to a first order effect: the price impact of trades. This contrasts with the rest of this literature, which assumes that the percentage cost of individual trades is unaffected by trade size. This difference has significant effect: in our setting people can manage the amount of information that their trades convey—and thereby their transaction costs—by choosing to trade currencies in different ways, such as through a vehicle currency. The question of equilibrium currency exchange is reframed.

The two traditional approaches to currency exchange are driven not by information, but by market size (Krugman 1980) or risk aversion (Black 1991, Hartmann 1998). A third line of research – more distant from ours – explains how international currencies arise more primitively, without exogenously attaching monetary values through preferences and technologies (Matsuyama et al. 1993 and Zhou 1997). For the market-size approach of Krugman, the basic idea is that transaction costs are a decreasing function of trading volume marketwide. Krugman is agnostic about the mechanism underlying this cost effect. His focus is instead on the increasing returns (and multiple equilibria) that result from market-size economies: the choice to trade particular currencies has effects on transaction costs that promote further trade in those currencies, and vice versa. The market-size approach was much enriched by Rey (2001), who uses a search-theoretic basis for why transaction costs should decrease with market size. The

search-theoretic basis for transaction costs that decrease with market size is not the only possible basis. For example, fixed costs to marketmaking also produces declining costs as market size grows. However, the work of Rey is particularly relevant in that she specifically analyzes the existence and use of vehicle currencies. That vehicle currencies can, and indeed generally must, arise is central to understanding equilibrium patterns of currency exchange (a point stressed by Krugman). Devereux and Shi (2005) advance the market size argument further by developing a dynamic general equilibrium model in which vehicle currencies economize on the number of currency trading posts.

Under the risk-aversion approach to currency exchange, price volatility becomes an additional determinant of transaction costs. When exchange rates are more volatile, the risk faced by liquidity-providing marketmakers increases, inducing them to increase the transaction spread they receive as compensation. Models in this second approach highlight the importance of exchange rate second moments in determining currency exchange patterns, a point that was absent in the earlier literature.

We choose an information approach here on both empirical and theoretical grounds. Empirically, a series of papers at both the micro (i.e., individual marketmaker) and market-wide levels has established that foreign exchange transactions do indeed convey information to those who observe them, with attendant price effects (e.g., Lyons 1995, Covrig and Melvin 2002, Evans and Lyons 2002a,b, Payne 2003, Bjornnes and Rime 2005, Killeen et al. 2006). Moreover, these price effects are significant: they can account for 40-80 percent of the daily variation in major rates (Evans and Lyons 2002b). This empirical perspective was not present at the time the two earlier approaches to international currencies were developed.

Theoretical motivation for our information approach has two main elements. The first relates to the type of information conveyed by transactions and why, theoretically, this role for transactions makes sense. For information models to be relevant to FX, it is not necessary that private information be of the “concentrated” type, e.g., inside information in the hands of one or a few individuals (regarding some macro variable). Rather, our information model is consistent with an economic environment where information is of the “dispersed” type, i.e., in which there are myriad bits of information relevant to exchange rates that are not observable to all agents and need to be aggregated via market processes. Think, for example, about heterogeneous shifts in money demands or risk preferences, both of which will produce currency transactions that, in turn, inform the market about those shifts.

The second theoretical motivation for our approach is linked to cross-sectional (versus time-series) variation in information environments. Specifically, our model helps identify

information differences across bilateral markets that can drive currency competition outcomes. Hartmann (1998) recognizes the possibility of modeling currency exchange using an information approach, but argues that “information-cost effects are transitory and therefore less relevant for the emergence of vehicle currencies” (page 43, footnote 3). The Hartmann argument against information-based analysis is a time-series argument: the information-cost effects he is considering are due to shocks to trading volume over time. Our focus here, in contrast, is on differences among markets cross-sectionally. Cross-sectional differences are by their nature persistent (see Easley, Hvidkjaer, and O’Hara (2002)). They include any variables that affect the per-dollar information intensity of flows in a given market. Examples include differences in the share of flows any one marketmaker sees, the relative importance of currency transactions tied to real trade, and the relative slopes of the markets’ net excess supply curves (i.e., the relative elasticity of economy-wide liquidity provision in the two markets).¹ These variables—as well as many others—affect the signal-to-noise ratio in flows, which translates into cross-market differences in transaction costs.

Analytical results from our information model include several important departures from traditional results. First, adding the information dimension resolves the traditional indeterminacy of currency trade patterns that arises in traditional models. The reason is that the convex trading costs faced by individual agents mitigate the concentrating force of market-size economies (the latter being the focus of Krugman 1980). Second, the pattern of currency trade is not driven by the pattern of real trade, as it is in traditional models (e.g., Krugman 1980 and Rey 2001), but instead by the pattern of asset trade. It is well known that currency trade resulting from underlying real trade accounts for less than 6 percent of trading volume in the major currency markets (though this number was much higher in the pre-1950 period when sterling was the world’s dominant currency). Of course, the Rey (2001) and Krugman (1980) models can also serve as metaphors for countries’ balance-of-payments accounts more broadly (i.e., not just real trade, but also the capital account). Still, the sum of capital-account transactions and those from real trade is too low to account for FX volume. Third, exchange rate levels are actually affected by whether transactions are executed directly or indirectly². This is because these trading methods do not generally reveal the same information (i.e., the equilibrium price impact from the two indirect trades does not in general equal the price impact from the equivalent direct trade). Fourth, our model provides new intuitions for why some currency pairs never trade directly: in

¹ See Hau and Rey (2002) for analysis of how supply curve slopes affect exchange rate determination.

² ‘Direct’ and ‘indirect’ are used here, in Krugman’s sense, as to whether or not a transaction is vehicled through a third currency: the terms do not allude to the method of execution, e.g. through a broker or directly.

information models of the type we analyze here, if the information available to various market participants is not sufficiently symmetric, then the market breaks down—there is insufficient incentive to participate (see, e.g., Bhattacharya and Spiegel 1991 and the earlier literature on “no-trade theorems”, e.g., Milgrom and Stokey 1982). The circumstances in our model that lead to the absence of direct trade—what Krugman calls total indirect exchange—are qualitatively different than those outlined by Krugman.

By modeling currency exchange patterns using a pure information model, we do not mean to imply that market size and volatility—the driving forces in the two earlier approaches—are irrelevant. Size and volatility are relevant. Rather, our objective is to learn what we can from a pure information model, given that existing literature has abstracted from the information dimension completely, and given evidence of this approach’s empirical relevance. Continuing to abstract completely from the information dimension is increasingly untenable.

Our empirical analysis is the first integrated analysis of transactions across a full market triangle: ¥/\$, \$/€ and ¥/€. Danielsson, Payne, and Luo (2002) analyze data for a full currency-market triangle (sterling-dollar-euro) , but treat each of the three markets separately: there is no integration of triangular arbitrage trading into their model. Data on the full triangle permits comparison of direct and indirect exchange for each (e.g., $¥/\$ \times \$/\text{€} = ¥/\text{€}$). We also formalize the incentives to engage in triangular arbitrage. Unlike the traditional market-size approach, our information model predicts that transactions should affect prices across markets (e.g., flow in the \$/€ market should convey information relevant to ¥/€ and ¥/\$ prices). This prediction is borne out.

The remainder of the paper is organized as follows. Section I presents our model of how information transmission affects currency exchange patterns. Section II extends the model to address vehicle currency determination. Section III presents supporting empirical work. Section IV concludes.

I. An Information Model of Currency Exchange Patterns

This section develops a model for analyzing direct versus indirect currency exchange. The basic structure is simple, drawing on the Kyle (1985) model, extended to allow for trading through vehicle currencies and for trading against any departures from triangular parity. We are not attempting to suggest that the foreign exchange market, in general, nor its brokered inter-dealer segment (EBS and Reuters), from which our data is derived, looks like a Kyle batch auction. EBS is, in fact, an order driven market with no market maker. What attracts us to the

Kyle model is that it offers us a metaphor for how an informed trader would choose to transact to maximize the value of private information. This in turn provides a way to characterize how information is incorporated into prices across markets given the strategic use of information by an informed trader.

Noise traders in the forex market are analogous to part of the outer ring of market participants in Lyons (2001). Deals that passively execute the instructions of commercial customers fall into this category. Commercial customers participate in the forex market for non-strategic purposes, such as exogenous portfolio rebalancing and foreign trade purposes. Any inventory created by such a trade will be quickly offset by the dealer and such countervailing trades are also part of the noise trade element. The distinction between the dealers and informed traders is more problematic. Though the current model does not specify the distinction between limit and market orders, it is straightforward to associate those submitting market orders as informed traders while those submitting limit orders as analogous to market makers who earn normal profits only. Since the latter are both participating dealers, these are indeed agents that are changing roles over time.

Though simple, our model is rich enough to embed the cross-market information links that are particular to FX. Information links across markets arise in the model because order flow in one market conveys information relevant for the pricing of the other exchange rates. This information relates to fundamental cash flows and is conveyed by the trades of agents who have amassed superior information of this type (e.g., through observation of market transaction flows correlated with future cash flows, but not yet fully impounded in price). An example is trades of the non-financial corporate sector that, when aggregated, convey information about the current state of the macroeconomy. Though there is an existing multi-market version of the Kyle model in the literature (Chowdhry and Nanda 1991), it does not include the triangular parity relationships across markets that are our focus here. Indeed, to our knowledge our model is the first optimizing model to address these issues. Our theoretical approach could, in principle, be applied to any situation where one security can be synthesised from a set of others, so long as all of the constituent securities are independently traded. Another obvious application is foreign exchange futures contracts along with the associated home and foreign interest rate contracts.

There is another feature of the forex market that motivates our use of the Kyle (1985) framework. Aggregation of dispersed information takes time. During that time some participants (e.g., commercial banks with large forex customer bases) are observing informative transaction flows before this information is fully impounded in price, which results in information advantages becoming increasingly concentrated. This squares with the strategic informed trading that is the

hallmark of Kyle-type models. Do the largest players in this market (e.g., large commercial banks, hedge funds) in fact trade strategically in order to minimize their price impact? For anyone who has observed these markets, the answer is a clear yes. This is precisely what the Kyle model is designed to capture. The five commercial banks with the largest market share in the major forex markets now account for over 60 percent of turnover (Euromoney, 2008).

Payoffs, information, and triangular parity

There are three markets in the model: market 1 (for concreteness, \$/€), market 2 (¥/\$), and market 3 (¥/€). Let p^1 , p^2 , and p^3 denote the change in the log of prices in the three markets, measured as dollars per euro, yen per dollar, and yen per euro, respectively. If triangular parity were to hold (i.e., no triangular arbitrage opportunities), the following would also be an identity:

$$(1) \quad p^3 = p^1 + p^2$$

As will be clear below, this relationship is not as trivial in the micro context as it is assumed to be in international macroeconomics. In particular, the issue of how triangular parity is actually achieved needs to be addressed. For part of the market, this is an irrelevant point because most minor currencies have inter-dealer markets with only one other currency, usually the dollar and in some cases the euro. Cross-rates to currencies other than the dollar are then arithmetically calculated using indirect rates. However, the yen, sterling and Swiss franc all have independent inter-dealer markets with *both* the dollar and the euro. The seven currency pairs in these triangles (USD/EUR, USD/JPY, USD/GBP, USD/CHF, EUR/JPY, EUR/GBP and EUR/CHF) constitute 63% of the total world market (BIS, 2007, Table 4).

The first point to note is that as an empirical matter, exact triangular parity in these pairs does not hold at every instant. To see this, we show the deviation from parity in Figure 2 which plots the euro/yen exchange rate calculated indirectly (i.e., via a transaction through the dollar) as a ratio of the direct euro/yen rate, logged and multiplied by 10,000 i.e. expressed in basis points. The data cover all of 1999, sampled daily at 9pm GMT (when the trading day in New York winds down). The source is Olsen & Associates, Zurich; we are grateful to Richard Olsen for his generosity. Importantly, these data were prepared with full knowledge of how they were going to be used, i.e., with an eye toward eliminating any non-synchronization problems. Though the mean is nearly zero (no deviation), the range is from +34 basis points to -30 basis points, with a standard deviation of nearly 7 basis points. For comparison, average bid-ask spreads for brokered inter-dealer trades around that time were roughly 1, 1, and 3 basis points for €/\$, \$/¥ and €/¥,

respectively.³ Ito and Hashimoto (2006) show that spreads on EBS are at their peak at 9 pm and are elevated above the average by between a third and a around a half. This suggests that deviations from triangular parity are important even at this particular time of day. That triangular parity does not hold at every instant has already been observed by De Jong, Mahieu, and Schotman (1998) using high-frequency data from the Reuters FFX system. They model adjustment to triangular parity in the \$, ¥, DM triangle as an error correction mechanism, but do not explore behavioral aspects of the disparity. The work of Evans (2002) is perhaps even more compelling in this regard: looking within a single market (DM/\$), he finds that prices exhibit significant dispersion that is purely cross-sectional (using four months of ultra-high frequency transactions data). Akram, Rime, and Sarno (2006) also find evidence of pure arbitrage opportunities in high-precision transactions data.

Let v^1 denote the random increment in the log fundamental value of the bilateral \$/€ pair (i.e., terminal payoff, measured as dollars per euro). Analogously, v^2 is the random increment in the log fundamental value for the bilateral ¥/\$ pair (yen per dollar) and v^3 is the random increment in the log fundamental value for the bilateral ¥/€ pair (yen per euro). These value innovations are i.i.d. Normal, with mean zero and scalar variance Σ_v^j . We assume that prior to the value innovations the three fundamental values are common knowledge.

The assumption that the variance-covariance matrix of the private signals is diagonal is a simplification required to make the model tractable. There are substantial benefits from this simplification. Without this, there would be a full set of off diagonal covariances as additional fundamentals. We are agnostic about the nature of these fundamentals. What matters is that they are *not* common knowledge. This completely rules out innovations in traditional monetary fundamentals such output and money. In relatively high frequency data such as the daily data that we consider in Section IIIa, this is a reasonable approximation but in lower frequency data, the model would have to take these into account. However, in developing the model, we simply thought of the v^j as tomorrow's exchange rate innovation, p_{t+1}^j , expected by informed trader j . Her expectation about the other two exchange rates in the triangle is zero, though she knows that the variances of the private signals of the other informed traders are non-zero. In general, the private nature of the signal imposes no structure on the off-diagonal elements of the variance covariance matrix of the vector $\begin{pmatrix} p_{t+1}^3 & p_{t+1}^2 & p_{t+1}^1 \end{pmatrix}$. The variance of tomorrow's expected deviation from triangular deviation must simply be non-zero. So long as the variance covariance

³ See Hau, Killeen and Moore (2002b), Table 3. Data are from August 1999.

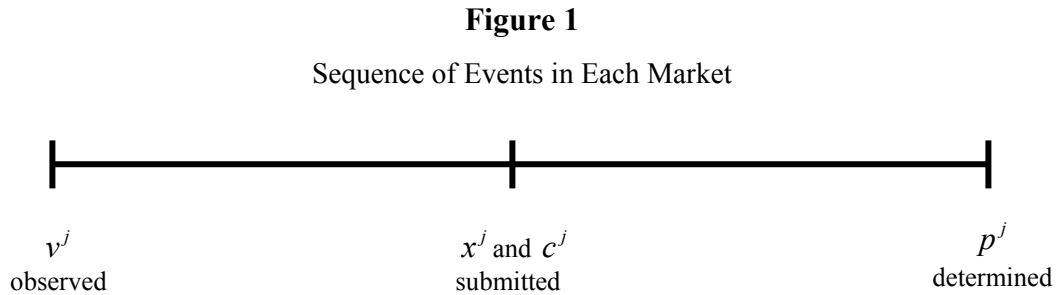
matrix is non-singular, this will be so. Our additional restriction that the variance covariance matrix is diagonal is conceptual unnecessary but analytically convenient.

Timing

The trading day is divided into three events, as illustrated in Figure 1 (more on notation and specifics below). Our empirical analysis is at the daily frequency, so it is not important for our model to capture intraday variation in liquidity (e.g., by including multiple intraday trading rounds for each trader type). First, the informed traders observe the value innovation v^j in their respective market. Second, orders are submitted by both informed traders (x^j) and liquidity-motivated traders (c^j), where a positive order denotes a purchase and a negative order denotes a sale. Third, marketmakers set prices to clear markets based on the orders submitted and based on any information conveyed by those orders. We now describe each of these three agent types—marketmakers, liquidity traders, and informed traders.

Marketmakers

The three marketmakers are competitive (meaning they expect to earn zero profit) and risk-neutral. Marketmakers cannot distinguish between trader types (i.e., they observe the sum of all



orders submitted, not the components separately). Because competition ensures that each marketmaker sets price such that expected profit is zero, marketmakers will necessarily set price at their expectation of the fundamental value conditional on order flows observed. Though the three marketmakers may see each others' order flows, the assumption that innovations are orthogonal means that only own order flow is relevant for price setting; order flow in another

market conveys no additional information. This is the inference problem marketmakers must solve.

Liquidity traders

There are three aggregate liquidity trades, one in each of the three markets, which are random and exogenous. (These can represent, for example, aggregations of large numbers of underlying liquidity demands in each market.) We have denoted the three aggregate liquidity trades as c^1 , c^2 , and c^3 respectively. Each of the liquidity trades is distributed Normal $(0, \Sigma_c^j)$, where Σ_c^j is a scalar variance. The three liquidity trades and the three fundamental values are mutually independent. Note that these relative liquidity-trade variances are one way to calibrate market size (i.e., to provide links to the market-size approach to international currencies in Krugman 1980 and Rey 2001).

Informed traders

There are three risk-neutral informed traders. Each observes the realization of fundamental value in one currency pair. This specification with the informed trader observing the fundamental value without noise follows Kyle, but need not: adding noise is straightforward but adds nothing to the economics of the problem. The informed traders and marketmakers are risk neutral here so it is simply the expectation of fundamental value that matters for determining their decision rules. We have denoted the three informed trades as x^1 , x^2 , and x^3 respectively. The task of the informed trader is to select the size of his trade to maximise profit (which is affected by his trading costs, in the form of his trades' price impact).

To fix ideas, let us first solve for a simple version of the model in which each informed trader trades only in his own market. Since the innovations are orthogonal, a trader has no incentive to trade directly in a market in which he has no information advantage. The only type of trading that we are ruling out is arbitrage trading: this is introduced in the next section. Using the informed trader in market 1 as an example, that problem can be expressed as:

$$(2) \quad \underset{x^1}{Max} \ E \left[x^1 (v^1 - p^1) \right]$$

MM Rule:
$$p^1 = E \left[v^1 \mid c^1 + x^1 \right]$$

As in the original Kyle (1985) model, we solve for the linear equilibrium. Given our assumption that liquidity trades and value innovations are Normally distributed, this gives linear pricing rules for each of the three marketmakers of the form:

$$(3) \quad p^j = \lambda_j (c^j + x^j) \quad \text{for } j=1,2,3$$

or, in matrix notation:

$$(4) \quad \begin{bmatrix} p^1 \\ p^2 \\ p^3 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} c^1 + x^1 \\ c^2 + x^2 \\ c^3 + x^3 \end{bmatrix}$$

The solution (from Kyle) is:

$$(5) \quad x^j = \beta_j v^j \quad \text{for } j=1,2,3$$

where:
$$\beta_j = \left(\Sigma_c^j / \Sigma_v^j \right)^{1/2} \quad \text{for } j=1,2,3$$

and:
$$\lambda_j = \left(2\beta_j \right)^{-1} \quad \text{for } j=1,2,3$$

We know from this equilibrium that:

$$Var[v^j] = 2Var[v^j | c^j + x^j]$$

And in our case we can write departures from triangular parity as:

$$Var[p^1 + p^2 - p^3] = \lambda_1^2 Var[c^1 + \beta_1 v^1] + \lambda_2^2 Var[c^2 + \beta_2 v^2] + \lambda_3^2 Var[c^3 + \beta_3 v^3]$$

Since the innovations are independent, the Kyle result goes through straightforwardly: strategic trading by the informed traders cuts the variance of deviations from triangular parity in observed exchange rates to half of the variance of deviations from triangular parity in the innovations. The

important feature here is that it is not optimal for informed traders to eliminate *all* of the deviation from triangular parity.

II. Adding Vehicle Currencies

II.a Analytical Framework

We need to extend the Kyle model to allow informed agents to engage in two additional trade types: one-way arbitrage and two-way arbitrage. Though we do not add them here, one could also introduce strategic liquidity trading, for example, allowing the liquidity traders to trade either directly or indirectly. This extension to arbitrage trading is non-trivial. One-way arbitrage is building a position less expensively by doing the trade indirectly through the other two markets. Two-way arbitrage is a round-trip trade to profit from departures from triangular parity. The distinction between one-way and two-way arbitrage is well established in international economics. It was first introduced by Deardorff (1979): it was developed by Callier (1981), Bahmani-Oskooee et al. (1985) and Clinton (1988). We will see that adding these trade types means that order flow in each market is no longer irrelevant for price determination in the other markets (i.e., the matrix of λ 's in the equivalent of equation 4 is no longer diagonal).

Let the subscripts d , i , and a refer to direct, indirect, and arbitrage (two-way) trades respectively, so that x_i^1 , for example, denotes the indirect trade of the trader who is informed about the market 1 value innovation v^1 . As an indirect trade, x_i^1 is part of the order flow in both market 2 and market 3, but not part of order flow in market 1. Including these trade types implies a fundamental re-specification of the informed traders' problem. Beyond this re-specification, the model has the same structure as that above (save, of course, that the flows that each marketmaker now sees include six components rather than two: the liquidity trade c^1 and direct informed trade x_d^1 as before, plus an indirect and an arbitrage trade from each of the two other-market informed traders).

Though the notation above seems simple enough, it embeds a subtlety that warrants further attention. Specifically, getting the sign of the trade direction right in Kyle-type models is crucial. So, take for example the indirect trade of the informed trader in market 3, x_i^3 . When positive, this is indirect buying of euros, and paying of yen, which we refer to as "buying ¥/€". Buying ¥/€ indirectly therefore involves buying ¥/\$ and also buying \$/€, i.e., when $x_i^3 > 0$, then

this constitutes positive order flow in both markets 1 and 2. This is not the case for indirect trading by the other two insiders, however. Consider the indirect trade of the informed trader in market 2, x_i^2 : when positive, this is indirect buying of dollars, and paying of yen. Buying ¥/\$ indirectly therefore involves buying ¥/€ and *selling* \$/€, i.e., when $x_i^2 > 0$, this constitutes positive order flow in market 3 but negative order flow in market 1. Similarly, buying \$/€ indirectly involves buying ¥/€ and *selling* ¥/\$, i.e., when $x_i^1 > 0$, this constitutes positive order flow in market 3 but negative order flow in market 2. These same sign issues affect the two-way arbitrage trades. An arbitrage trade x_a^3 will denote a sale in market 3, and therefore purchases in both markets 1 and 2. An arbitrage trade x_a^1 will denote a sale in market 1, a purchase in market 3, and a *sale* in market 2. An arbitrage trade x_a^2 will denote a sale in market 2, a purchase in market 3, and a *sale* in market 1.

The task of the informed trader is to select the size and location of his trades so as to maximise profit. This is of course affected by the extent to which the informed trader's actions move price. Each informed trader still observes the fundamental innovation in one of the three markets. When he trades indirectly, the trades he places in the other two markets eliminate any net position in the vehicle currency. (For example, the trader in the ¥/€ market now has the option of taking positions in ¥/€ indirectly via equal dollar-sized trades in the ¥/\$ and \$/€ markets—one a dollar purchase and the other a sale, to eliminate any net dollar position.). When he trades for arbitrage, the trades he places in the three markets eliminate any net position in any of the three currencies, save for any dollar profit generated from the arbitrage. Buying a given quantity indirectly will not, in general, involve the same trade quantity in both indirect markets. The approximation involved in specifying the problem as in equation (6) is second-order, however (see the second part of Appendix 1).

First, marketmakers set regret-free prices based on the full matrix of order flows:

$$(6) \quad \begin{bmatrix} p^1 \\ p^2 \\ p^3 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} \begin{bmatrix} c^1 + x_d^1 - x_i^2 + x_i^3 - x_a^2 + x_a^3 \\ c^2 + x_d^2 - x_i^1 + x_i^3 - x_a^1 + x_a^3 \\ c^3 + x_d^3 + x_i^1 + x_i^2 + x_a^1 + x_a^2 \end{bmatrix}$$

Naturally, the informed traders integrate this pricing rule into their strategy. Consider, for example, the problem for the informed trader in market 3, the ¥/€ trader. He has to account for the impact of his order flow in all markets because he is trading both directly and indirectly.

$$(7) \quad \underset{x_d^3, x_i^3, x_a^3}{Max} E \left[x_d^3 (v^3 - p^3) + x_i^3 (v^3 - \tilde{p}^3) + x_a^3 (p^3 - \tilde{v}^3) \right]$$

where the indirect price for currency 3 is denoted as $\tilde{p}^3 \equiv p^1 + p^2$ and the arbitrage buying price for currency 3 purchased in the other two markets is denoted as $\tilde{v}^3 \equiv p^1 + p^2$, the two being equal. Equation (7) presumes that triangular arbitrage does not hold instantaneously, otherwise each trader would only care about the direct trades in her own market. Equation (7) is maximized subject to equation (6) with respect to x_d^3, x_i^3 and x_a^3 . Note that the two-way arbitrage flows x_a^1, x_a^2 and x_a^3 do not appear in the own-market order flows in equation (6). These are effectively components of the direct trades x_d^1, x_d^2 and x_d^3 respectively. This is because, from the point of view of the optimizing trader, it is the *total* number of direct trades that exploits her information monopoly in the own-market. As far as direct trades are concerned, the trader is indifferent as how they are labeled. The market maker only sees the total flow in any event.

The first-order condition with respect to direct trading x_d^3 is:

$$\begin{aligned} E_{v^3} \left[v^3 - \left\{ \lambda_{33} (x_a^1 + x_i^1 + x_a^2 + x_i^2 + x_d^3) + \lambda_{32} (-x_a^1 - x_i^1 + x_d^2 + x_a^3 + x_i^3) \right. \right. \\ \left. \left. + \lambda_{31} (x_d^1 - x_a^2 - x_i^2 + x_a^3 + x_i^3) \right\} \right] \\ = \lambda_{33} x_d^3 + (\lambda_{13} + \lambda_{23}) x_i^3 + (\lambda_{13} + \lambda_{23} - \lambda_{33}) x_a^3 \end{aligned}$$

and the first-order condition with respect to indirect trading (one-way arbitrage) x_i^3 is:

$$\begin{aligned} E_{v^3} \left[v^3 - \left\{ \lambda_{13} (x_a^1 + x_i^1 + x_a^2 + x_i^2 + x_d^3) + \lambda_{23} (x_a^1 + x_i^1 + x_a^2 + x_i^2 + x_d^3) \right. \right. \\ \left. \left. + \lambda_{12} (-x_a^1 - x_i^1 + x_d^2 + x_a^3 + x_i^3) + \lambda_{22} (-x_a^1 - x_i^1 + x_d^2 + x_a^3 + x_i^3) \right\} \right] \\ = (\lambda_{31} + \lambda_{32}) x_d^3 + (\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}) x_i^3 + (\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22} - \lambda_{31} - \lambda_{32}) x_a^3 \end{aligned}$$

Finally, the first order condition with respect to two-way arbitrage x_a^3 is:

$$\begin{aligned}
& E \left[\begin{aligned} & -\lambda_{13} (x_a^1 + x_i^1 + x_a^2 + x_i^2 + x_d^3) - \lambda_{23} (x_a^1 + x_i^1 + x_a^2 + x_i^2 + x_d^3) + \\ & \lambda_{33} (x_a^1 + x_i^1 + x_a^2 + x_i^2 + x_d^3) - \lambda_{12} (-x_a^1 - x_i^1 + x_d^2 + x_a^3 + x_i^3) - \\ & \lambda_{22} (-x_a^1 - x_i^1 + x_d^2 + x_a^3 + x_i^3) + \lambda_{32} (-x_a^1 - x_i^1 + x_d^2 + x_a^3 + x_i^3) - \\ & \lambda_{11} (x_d^1 - x_a^2 - x_i^2 + x_a^3 + x_i^3) - \lambda_{21} (x_d^1 - x_a^2 - x_i^2 + x_a^3 + x_i^3) + \\ & \lambda_{31} (x_d^1 - x_a^2 - x_i^2 + x_a^3 + x_i^3) \end{aligned} \right] \\
& = (\lambda_{31} + \lambda_{32}) x_d^3 + (\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}) x_i^3 + (\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22} - \lambda_{31} - \lambda_{32}) x_a^3
\end{aligned}$$

These conditions are interpretable as follows: the informed trader stops building his position at the point where the expected gap between price and fundamental value is equal to the full impact on price from the informed trader's trades (the right-hand side in both cases representing that full impact on price). To trade more aggressively would reduce expected profit due to heightened price impact. For the first two equations, the “fundamental value” is the superior information the trader has regarding the value of the ¥/€ pair. For the last equation, it is particularly noteworthy that the informed trader does not drive two-way arbitrage opportunities to zero: two-way arbitrage trading has a systems effect on profits through its price impact, and in general the solution does not imply driving these opportunities to zero.

The signal v^3 provides no information about the other two traders' demands because of the orthogonality of the innovations. Consequently their expected value is zero. Using the above first order conditions, we can then solve for the optimal trading pattern of the ¥/€ investor⁴:

$$(8) \quad x_d^3 = \left(\frac{(\lambda_{31} + \lambda_{32})(\lambda_{13} + \lambda_{23} + \lambda_{33}) - 2\lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})}{2\lambda_{33}\{(\lambda_{13} + \lambda_{23})(\lambda_{31} + \lambda_{32}) - \lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})\}} \right) v^3$$

$$(9) \quad x_i^3 = \left(\frac{-\left\{ \begin{aligned} & \lambda_{13}^2 + \lambda_{23}^2 + \lambda_{23}(\lambda_{31} + \lambda_{32} - 2\lambda_{33}) + \lambda_{13}(2\lambda_{23} + \lambda_{31} + \lambda_{32} - 2\lambda_{33}) \\ & - 2\lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}) + \lambda_{33}(\lambda_{31} + \lambda_{32} + \lambda_{33}) \end{aligned} \right\}}{2\lambda_{33}\{(\lambda_{13} + \lambda_{23})(\lambda_{31} + \lambda_{32}) - \lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})\}} \right) v^3$$

⁴To assist the reader, the derivation of equations (8)-(12) is available in the form of a Mathematica notebook. The solutions for the other two traders are available in the same format.

$$(10) \quad x_a^3 = \left(\frac{(\lambda_{13} + \lambda_{23})(\lambda_{13} + \lambda_{23} + \lambda_{31} + \lambda_{32}) - \lambda_{33}(2\lambda_{11} + 2\lambda_{12} + 2\lambda_{21} + 2\lambda_{22} + \lambda_{13} + \lambda_{23} - \lambda_{31} - \lambda_{32})}{2\lambda_{33}\{(\lambda_{13} + \lambda_{23})(\lambda_{31} + \lambda_{32}) - \lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})\}} \right) v^3$$

Analogous expressions for the \$/€ and ¥/\$ investors can also be derived. To avoid clutter, they are relegated to Appendix2.

The solution to the model involves finding a set of price impact coefficients which satisfy equation (6) and the requirement that the market maker earn zero profits in expectation. This is spelt out in some detail in Appendix 2. As can be seen in that appendix, this involves finding a solution to the matrix equation:

$$(11) \quad \Lambda - \Sigma_v \mathbf{A}' [\mathbf{A} \Sigma_v \mathbf{A}' + \Sigma_c]^{-1} = \mathbf{0}$$

where the 3×3 matrices $\Lambda, \mathbf{A}, \Sigma_v, \Sigma_c$ and $\mathbf{0}$ are defined in equations(28), (22), (26) and $\mathbf{0}$ is the zero matrix. This gives us nine equations in the nine unknown price impact coefficients in equation (6) to solved in terms of the six variances in Σ_c^j and Σ_v^j $j = 1, 2, 3$ and. A number of points should be obvious. Firstly, we should be astonished if a solution exists for all values of the variances Σ_c^j and Σ_v^j . This is because we know that most currency triangles do not exist at all. The second point is that equation (11) is highly non-linear unlike the simple one-asset Kyle case. In Section IIIb, we examine an approximate solution.

IIb Some results on Vehicle Currencies

Even without a full solution, we are already in a position to use the model to address a number of fundamental questions regarding vehicle currency use. Specifically, ratios of the expressions (8)-(10) pin down the relative intensity of direct, indirect and arbitrage trading within and across currencies.

(i) What determines the choice of direct versus vehicle trade?

Consider the decision whether to trade directly in market 3, the ¥/€ market, or to use the \$ as a vehicle for arbitrage trading, both one and two-way. The total of arbitrage trading is the sum of x_i^3 in equation (9) and x_a^3 in equation (10). This is:

$$(12) \quad x_i^3 + x_a^3 = \left(\frac{\lambda_{13} + \lambda_{23} - \lambda_{33}}{2\{(\lambda_{13} + \lambda_{23})(\lambda_{31} + \lambda_{32}) - \lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})\}} \right) v^3$$

The ratio of arbitrage to direct ¥/€ trading is equation (12) divided by (8):

$$(13) \quad \frac{x_i^3 + x_a^3}{x_d^3} = \frac{\lambda_{33}(\lambda_{13} + \lambda_{23} - \lambda_{33})}{-2\lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}) + (\lambda_{31} + \lambda_{32})(\lambda_{13} + \lambda_{23} + \lambda_{33})}$$

This can be summarized in the following proposition:

Proposition 1: The ratio of vehicle to direct trade depends on their relative price impact and is given by equation (13)

An important feature of equation (13) is the expression $\lambda_{13} + \lambda_{23} - \lambda_{33}$. If $\lambda_{13} + \lambda_{23} = \lambda_{33}$, it means that the immediate price impact of a ¥/€ trade is exactly the same in its own market, λ_{33} , as its combined immediate effects on the other two markets in the triangle, \$/€ and ¥/€, $\lambda_{13} + \lambda_{23}$. If this condition is met, arbitrage trading is zero as can be seen from equations (12) and (13). For vehicle trading to occur at all, the choice between direct and arbitrage trading, both one-way and two-way, has to have different implications for the cost of trading. However, λ_{33} and $\lambda_{13} + \lambda_{23}$ are not the only determinants of relative price impact and it would be simplistic to focus on them alone. Trades affect all markets and just those in which they are immediately executed. That is why the measure in equation (13) contains all of the nine price impact coefficients in the matrix of equation (6).

(ii) What determines the relative importance of a currency as a vehicle?

To answer this we need to compare use of the dollar as a vehicle (i.e., arbitrage ¥/€ trading via the dollar) to use of the euro as a vehicle (i.e., arbitrage ¥/\$ trading via the euro). Equation (12) provides an expression for the former. The analogous expression for the euro is from equations (18) and (19) in Appendix 2:

$$(14) \quad x_i^2 + x_a^2 = \left(\frac{\lambda_{32} - \lambda_{12} - \lambda_{22}}{2 \{ (\lambda_{12} - \lambda_{32})(\lambda_{21} + \lambda_{23}) - \lambda_{22}(\lambda_{11} - \lambda_{13} - \lambda_{31} + \lambda_{33}) \}} \right) v^2$$

The ratio of arbitrage trading through the dollar relative to the euro is equation (12) divided by (14):

$$(15) \quad \frac{\{ (\lambda_{21} - \lambda_{23})(\lambda_{12} - \lambda_{32}) - \lambda_{22}(\lambda_{11} - \lambda_{13} - \lambda_{31} + \lambda_{33}) \} (\lambda_{13} + \lambda_{23} - \lambda_{33}) \left(\frac{v^3}{v^2} \right)}{\{ (\lambda_{13} + \lambda_{23})(\lambda_{31} + \lambda_{32}) - \lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}) \} (\lambda_{32} - \lambda_{12} - \lambda_{22}) \left(\frac{v^3}{v^2} \right)}$$

This leads to the following proposition:

Proposition 2: The relative importance of two currencies as vehicles is determined by the relative difference in the relative cost of direct and one and two-way arbitrage trades.

This follows from equations (12), (14) and (15). We know, from proposition 1, that the dollar will only be used as a vehicle if $\lambda_{13} + \lambda_{23} \neq \lambda_{33}$ and that it is the extent to which this is violated, *in any direction*, that determines the magnitude of its vehicle role. The same point applies to the vehicle role of the euro, in respect of which the analogous condition is $\lambda_{12} + \lambda_{22} \neq \lambda_{32}$. It is the relative deviation from these two relative conditions that determines the relative usefulness of the two currencies as vehicles. A natural definition of the relative vehicleness of the dollar vis-a vis the euro is:

$$(16) \quad \frac{|\lambda_{13} + \lambda_{23} - \lambda_{33}|}{|\lambda_{12} + \lambda_{22} - \lambda_{32}|}$$

Again, the result is intuitive and appealing: the vehicle role of a currency is determined by the relative price impact of its indirect trades. The lower that price impact, the more the currency will be used as a vehicle. It is worth noting that our analysis allows for indirect trades to take place *through all currencies*. This is different from most existing theory. There is no one vehicle currency, just different degrees of “vehicleness”. Exceptions include Krugman (1984) and Hartmann (1998), both of whom consider more than one vehicle currency in a many-currency

world (e.g., the legacy DM as a regional European vehicle). Nevertheless, the concentrating force of market-size economies in their models is quite different from the dispersing force—avoidance of price impact—identified here.

(iii) Does the choice between direct and indirect trade affect the informativeness of prices?

This is an important question, one that does not arise within the two traditional approaches to currency competition. The answer boils down to whether direct and indirect trades reveal the same information.

Corollary 1: Whether transactions are executed directly or indirectly affects the equilibrium levels of the exchange rates.

In general, a marketmaker facing order flow from indirect trading cannot recover the solution that would have resulted if order flow contained only direct trade. The logic underlying this is clear: the different markets involve different noise realizations, so there is no way to arrive at the same inferences from direct and indirect trade, despite known ex-ante noise variances.

This follows from our model in a very simple sense. We are not making a special claim that there are multiple equilibria, though it is well known that there may be more than one solution to Kyle-type problems. The choice between direct and indirect trading is the outcome of optimal behavior by traders. What we are saying is that deviations from the optimal mix between the two types of trade execution affect the value of all endogenous variables including spot exchange rate returns. This is a novel feature of this model which has not been claimed in the existing exchange rate determination literature.

(iv) What parameters determine whether one of the markets shuts down altogether?

In the traditional market-size and marketmaker-risk approaches to international currencies, the conditions under which one of the three markets in a currency triangle shuts down are well known. (Krugman 1980 calls this total indirect exchange, i.e., no direct exchange between a currency pair takes place.) For example, total indirect exchange is more likely in Krugman's analysis when: (1) the balance of real trade between the two missing-market countries is closer to zero, (2) the balance of real trade between each of the two missing-market countries and the third country is further from zero, and (3) the rate at which transaction costs fall with market size is high (see, e.g., Krugman 1980, equations 9 and 10).

The conditions under which the whole market shuts down in information models are quite different. The essential parameters in information models, including ours, are those governing the degree of information asymmetry (see Glosten and Milgrom 1985, Glosten 1989, and Bhattacharya and Spiegel 1991). If information asymmetry in a given market becomes too high, then it is no longer possible for the marketmaker to find a price at which the market clears and he expects to earn zero profit. When the marketmaker exits, no liquidity is provided, and the market shuts down.

To take this any further in our model, we are required to specify an exact solution to equation (11). What we would need to identify is the domain of the exogenous variances - Σ_c^j and Σ_v^j $j = 1, 2, 3$ - for which a solution does not exist. In terms of the microstructure literature, this means the shutting down of a market. In the vehicle currency tradition, it corresponds to the emergence of total indirect exchange.

As a matter of policy, this framework presents a radically different view on what is required to jump-start a missing currency market. This ability to fundamentally shift perspective is one of the approach's strengths.

III. Model Validation

IIIa Empirical Analysis

Data

The order flow data we use to estimate the matrix of price impact coefficients λ in equation (6) is from Electronic Broking Services (EBS), the leading foreign-exchange broker.⁵ These data span the twelve-month period, 1999, and cover the world's largest currency triangle consisting of the \$/€, ¥/\$, and ¥/€ markets. Our daily exchange rate data are closing New York mid-prices and are provided by Olsen and Associates. This matches the EBS 'day' precisely, which runs for 24 hours from 9pm GMT the previous evening. Saturdays and Sundays were excluded.

⁵ We measure order flow here as the *cash value* of buyer- minus seller-initiated trades. An alternative would be to use the net number of buys minus sells. The latter choice of order flow measure has a long tradition in empirical microstructure finance (e.g., Hasbrouck 1991). The results do not differ substantially when the numbers measure is used.

On EBS, currency ratios are defined so that nominal rates are typically greater than unity. Hence dollar/yen is measured in yen per dollar and the volume of trades is measured in (millions of) dollars. Similarly euro/yen is measured in yen per euro with volume measured in (millions of) euro. Euro/dollar defied this convention for much of its early life as its value dropped to less \$1 for a sustained period. However at its launch date, it was valued conventionally in dollars per euro and its volumes in (millions of) euro.

Table 1 provides descriptive statistics for the variables used in the estimation. These are the order flows, \$/€ Flow, ¥/\$ Flow, ¥/€ Flow and the three spot returns, $\Delta p^{$/€}$, $\Delta p^{¥/$}$, and $\Delta p^{¥/€}$.

Results

A number of writers, notably Bjornes and Rime (2005) and Killeen, Lyons and Moore (2006) provide evidence that exchange rate levels and cumulative order flow are cointegrated in high frequency data. However, Berger, Chaboud, Chernenko, Howorka, and Wright (2006) have cast doubt on this in longer horizon data. More recently Chinn and Moore (2008) has restored a long-term role for cumulative order flow but only in the context of a hybrid version of the traditional monetary model which is only applicable in low frequency data.. The Kyle framework that is outlined in the theoretical part of the paper does not entertain the possibility of such cointegration. It seems natural therefore to proceed to estimating equation (6).

The first three rows of Table 2 present the results. The coefficients are measured in basis points, i.e. the price impact in basis points per billion dollars of currency units. Note that all the coefficients are highly significant. The \overline{R}^2 statistics are all remarkably high relative to macro empirical models of log exchange rate changes (consistent with Evans-Lyons 2002b). There is some evidence that the residuals in the dollar-euro equation are negatively auto-correlated (consistent with evidence on that pair over the same period in Hau, Killeen and Moore 2002). Hausman specification tests for endogeneity of order flow (using lagged order flow as instruments) show that simultaneity is overwhelmingly rejected (not reported).

It is interesting that the matrix is obviously not symmetric. The last column measures the price impact of euro-yen trades across all three markets: the coefficients are noticeably large. The implication is that traders choosing to trade in this market are providing particularly informative signals.

From these estimates we can assess the relative cost of vehicle to direct trading in each of the markets as suggested by equation (13) in Proposition 1. The results are presented in Table 3. To understand the Table, consider the last row first as this explicitly quantifies equation (13). In

market 3, the ¥/€ market, the estimate of .85 is an index of the relative importance of using the dollar as a vehicle or trading directly. The last column is a test that the index is significantly different from zero. The analogous estimates for the \$/€ and ¥/\$ markets are given in rows 1 and 2 respectively. They are very close and are both an order of magnitude lower than the figure in the last row, though they are both statistically significant. This means that direct trading is more predominant in the \$/€ and ¥/\$ markets when compared to the ¥/€ market. It also confirms that the dollar is used more as a vehicle currency than either the euro or the yen.

Interestingly, though, our model does predict that some vehicle trading will still occur in equilibrium, even though intuition (and past modes of analyzing vehicle trading) would rule it out. The reason is that price impact needs to be appreciated as a “system”: traders compare the marginal costs of trading across a number of trading alternatives, and the full-system price impact of continuing to trade directly can become relatively costly.

Proposition 2 suggests that the relative vehicle role of a currency is determined by the relative price impact of its one and two way arbitrage trades. The measure of the relative vehicleness of the dollar relative to the euro is given in equation (16). Table 4 provides the results of estimating this. By this index, the dollar is significantly (at 10%) more important as a vehicle currency than either the euro or the yen (rows 1 and 2). By contrast, row 3 suggests that it is difficult to tell the vehicle roles of the euro and yen apart. This is a surprising feature as one might have mistakenly guessed from currency shares that the euro had a relatively more important vehicle role. It appears that both are overshadowed by the dollar. According to BIS 2007, Table 3, the dollar’s April 2007 share is 86.3%, the euro’s 37% and the yen’s is 16.5%. However, our data come from the first year of the euro in 1999. In both the 1998 and 2001 BIS surveys, the yen’s share was over 20%.

One final empirical point: the last row of Table 2 presents the result of regressing the deviation from triangular arbitrage on the three order flows. They are individually and jointly insignificant. Deviations from triangular arbitrage are consistent with ‘no-arbitrage’ because the price impact of the order flow required to close out an apparent arbitrage opportunity would eliminate the arbitrage opportunity altogether. If the deviation from triangular arbitrage were affected by the order flow, this would imply that the ‘optimum’ deviation had not been arrived at. Therefore this result is reassuringly consistent with the model.

III.b Approximate Solution

The solution to the model involves finding a set of price impact coefficients which satisfy equation (6) and the requirement that the market maker earn zero profits in expectation. This is spelt out in some detail in Appendix 2. As can be seen in that appendix, this involves finding a solution to the matrix equation (11) which is repeated here for convenience:

$$\Lambda - \Sigma_v A' [A \Sigma_v A' + \Sigma_c]^{-1} = \mathbf{0}$$

where the 3×3 matrices $\Lambda, A, \Sigma_v, \Sigma_c$ and $\mathbf{0}$ are defined in equations (28), (22), (26) and $\mathbf{0}$ is the zero matrix. This gives us nine equations in the nine unknown price impact coefficients in equation (6) to be solved in terms of the six variances in Σ_c^j and Σ_v^j $j = 1, 2, 3$. Since the full currency triangle *does* exist for $\$/\text{€}/\text{¥}$, we assume that the estimated values of the price impact coefficients that are reported in the first three rows of Table 5 is a solution for *some* set of six fundamental variances Σ_c^j and Σ_v^j . We take a first order Taylor's approximation of equation (11) around the estimated price impact coefficients. What follows is based on that approximation⁶.

To evaluate the model, we need some idea of the possible range of the values of variances Σ_c^j and Σ_v^j . Obvious candidates for the variances of the price signals are the empirical variances of the spot returns that are reported (expressed as standard deviations and measured in basis points) in the penultimate column of Table 1. The variances of uninformed trades have no empirical analogue but they can be 'guesstimated' as follows. The univariate Evans-Lyons price impact parameters can be given the simple Kyle interpretation in equation (5). From that estimated values for the standard deviations of uninformed trades are

$$(17) \quad \sqrt{\Sigma_c^j} = \frac{\sqrt{\Sigma_v^j}}{2\lambda_j} \quad j=1,2,3$$

The three estimated Evans-Lyons parameters are 37, 47 and 201 basis points per billion currency units for $\$/\text{€}$, $\text{¥}/\text{\$}$ and $\text{¥}/\text{€}$ respectively. The implied values for the standard deviation of uninformed trades are (from equation (17)) 0.792, 0.881 and 0.227 billion currency units respectively. Note that these are approximately 2/3 of the values for the standard deviations of the total order flows in Table 1.

These values are used as the benchmark for a model simulation. The implied price impact parameters are reported in Table 5. Comparing Tables 2 and 5, there are a number of qualitatively striking features. Firstly, the pattern of signs is the same in both cases. In particular,

the negative impact of dollar/yen flow on the dollar/euro spot return and vice versa is successfully replicated. As in Table 2, both coefficients are similar in magnitude. However, the absolute value of the simulated coefficients is approximately four times larger than the estimated coefficients. Secondly, the ranking of own price coefficients on the diagonal is similar with the own price effect of $\$/\epsilon$ being the smallest and that of \yen/ϵ being very much the largest. In fact, both the $\$/\epsilon$ and \yen/ϵ simulated coefficients are close in magnitude both economically and statistically to the estimated coefficients. The simulated $\yen/\$$ coefficient is somewhat larger than the corresponding estimated coefficient. The third point is that the matrix of simulated, like the estimated coefficients, are strikingly asymmetric. To see this, consider the last column in each case. The price impact of \yen/ϵ flows is noticeably higher in every market. The simulated coefficients in the $\$/\epsilon$ and $\yen/\$$ markets are, however, approximately five times higher than the estimated coefficients, though they are, once again, the correct sign.

In summary, the simulated coefficients are of the correct sign, order of magnitude and relative magnitude as the estimated coefficients. The model does not, of course fit perfectly. For example, a test of the null that the simulated coefficients of Table 5 are not significantly different from the estimated coefficients of table 2 is easily rejected⁷.

⁶ A Mathematica notebook is available in which equation (11), the results of appendix 2 as well as the Taylor's approximation are derived.

⁷ Details on request.

IV. Conclusions

It would be wrong to contend that transaction costs arise wholly due to adverse selection, as they do in our information model. Our objective here is to provide a first analysis of how the presence of dispersed information can affect international currency competition and the pattern of cross-currency exchange. To isolate the new forces at work, our analysis does not include pure market-size effects nor marketmaker-risk effects on transaction costs. To the extent our information approach resonates, integration with these two earlier approaches is a natural next stage. The notion that vehicleness is due to the ability to hide informed trades is new, but in ways closely related to the traditional view of market size. In both models vehicleness is determined by the “ease” of trade execution. The difference is that our model focuses on the ease of execution of informed trades whereas in the traditional model that refers to any trade (as there is no heterogeneity across trades). Therefore, both models predict that vehicleness is greater in currencies with deeper markets.

Our approach brings two fresh elements to existing work focusing on market size and marketmaker risk. First, we recognize that transaction flows play an important role beyond market size in that they also convey price-relevant information. Given that empirically these flows are a first-order driver of exchange rate adjustment, this element is surely important for analyzing international currency competition. Second, our analysis shows that the operative flow concept for understanding the information dimension is order flow, as opposed to the flow concept stressed by the traditional analysis, namely real trade (or balance of payments) flows. Real trade flows are a small component of currency transaction flows. Put another way, the analysis shows that it is not the gross amount of trading that matters, but rather its composition. This point is absent from the existing literature.

Consider the following example of how neglecting this information role of transactions can affect international currency competition. When the euro was launched, most people felt the new currency would enjoy reduced transaction costs (relative to those for the DM), thereby increasing the international role of the euro relative to that of the dollar. But Hau, Killeen, and Moore (2002a,b) show that relative to those for the DM, euro transaction costs *increased*. The explanation they offer turns on a structural shift relating to the information role of transactions. Specifically, they argue that introduction of the euro caused higher spreads because it increased the information content of order flow by eliminating a particular source of uninformative trading (the inventory management trades of marketmakers in the now-defunct Euro-zone cross-rate markets like FF/DM). Thus, their model, too, stresses the transaction mix (versus total volume).

Our analysis of vehicle currency determination leads to two powerful results. First, exchange rate levels are actually affected by whether transactions are executed directly versus indirectly; these trading methods do not reveal the same information. This possibility, even if obvious given the information framework, was not considered in earlier work. Second, our model provides a new understanding for why there are so few currency triangles for which a significant market exists in all three legs (the dollar-euro-yen triangle we study here being a notable exception). The conditions leading to the absence of direct trade in our model are of a completely different nature than those outlined by Krugman. What are the welfare consequences of having only two tradable legs to a currency triangle? And what role might policy play to increase this likelihood of full triangle trading? These are promising directions for further analytical work.

Though our model's predictions are rather strong, they are nevertheless borne out in the data. Specifically, we find that transactions affect prices across markets, as the information approach predicts. An approximate solution to the model around the estimated coefficients has aspects of consistency with plausible values of the fundamental variances.

One policy implication is that what matters is not the total amount of trading (as is the focus of the existing international-currency literature), but the composition of trading. For example, one might ask whether the US dollar's current dominance as an international currency is structural. The answer from the model is that dominance depends on two key parameters: the uninformative-trade variances Σ_c and the fundamental variances Σ_v . Knocking the dollar from its dominant role would need to involve changes in one or more of these parameters. Policy choices certainly affect the first two of these. (For example, sterilized intervention and other officially directed trading will affect the uninformative-trade variances Σ_c .) Finally, to those who are concerned that the Kyle framework is inappropriate for the foreign exchange market, we pose the following question: do you believe that the largest players in this market (e.g., large commercial banks, hedge funds) trade strategically in order to minimize their price impact? For anyone who has observed these markets, the answer is a resounding yes. This is precisely what the Kyle model is designed to capture. Of course, the model should be viewed as metaphoric in that the better-informed players do not actually see fundamentals without noise. But adding noise to the informational advantage of the better-informed players would not change the model's solution qualitatively. The model is, in fact, much more broadly applicable than is suggested by the term "insider trading" from the title of Kyle (1985).

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Table 1
Summary Statistics

Variable	Unit		Mean		Standard Deviation		AR(1) Coefficient
\$/€ Flow	€ billions		0.515		1.1		0.264
¥/\$ Flow	\$ billions		0.904		1.351		0.127
¥/€ Flow	€ billions		0.153		0.322		0.133
$\Delta p^{$/€}$	Basis points		-6.2		58.6		-0.124
$\Delta p^{¥/$}$	Basis points		-3.5		82.8		-0.043
$\Delta p^{¥/€}$	Basis points		-9.6		91.3		0.013

For the period 5th January 1999 to 31st December 1999, we report for the order flows, \$/€ Flow, ¥/\$ Flow, ¥/€ Flow and the three spot returns, $\Delta p^{$/€}$, $\Delta p^{¥/$}$ and $\Delta p^{¥/€}$, the mean, standard deviation and the first order autocorrelation coefficient (AR(1) coefficient). There are 259 observations corresponding to all Mondays to Fridays inclusive. The first observation is lost to differencing.

Table 2
The matrix of price impact coefficients

	Regressor			Diagnostics	
<u>Dep. Variable</u>	\$/€ Flow	¥/\$ Flow	¥/€ Flow	DW	$\overline{R^2}$
$\Delta p^{\$/\epsilon}$	26.37 (9.4)	-16.00 (6.7)	79.20 (7.3)	2.48	56%
$\Delta p^{\yen/\$}$	-16.02 (4.5)	36.94 (10.8)	55.90 (3.2)	1.89	61%
$\Delta p^{\yen/\epsilon}$	11.21 (3.2)	21.81 (5.3)	131.36 (6.0)	1.96	56%
$\Delta p^{\$/\epsilon} - \Delta p^{\yen/\$} - \Delta p^{\yen/\epsilon}$	0.87 (1.15)	0.86 (1.2)	-3.76 (1.16)	2.85	-0.4%

Notes: (i) The coefficients have the dimension “basis points per billions of currency units”. Newey-West heteroscedasticity and serial correlation adjusted t-statistics in parentheses (lag window is 5). The t-stats are tests of the hypothesis that the associated coefficient is zero. Estimated using Generalized Method of Moments. Sample: daily data for the year 1999. Flow data are from EBS (Electronic Broking Systems). Exchange rate data are from Olsen & Associates.

Table 3
Measure of Absolute Vehicleness (Proposition 1)

Market	Vehicle Currency	Ratio of arbitrage to direct trading	t statistic
\$/€	¥	0.07	2.48
¥/\$	€	0.08	2.33
¥/€	\$	0.85	3.10

Notes: (i) Estimates of absolute vehicleness measure in Equation (13) of Proposition 1. (ii) The t-stats are tests of the hypothesis that the associated coefficient is zero.

Table 4
Measure of Relative Vehicleness (Proposition 2)

Comparison	Ratio	t-statistic
Dollar to Yen	4.33	1.86
Dollar to Euro	4.35	1.92
Euro to Yen	0.99	1.53

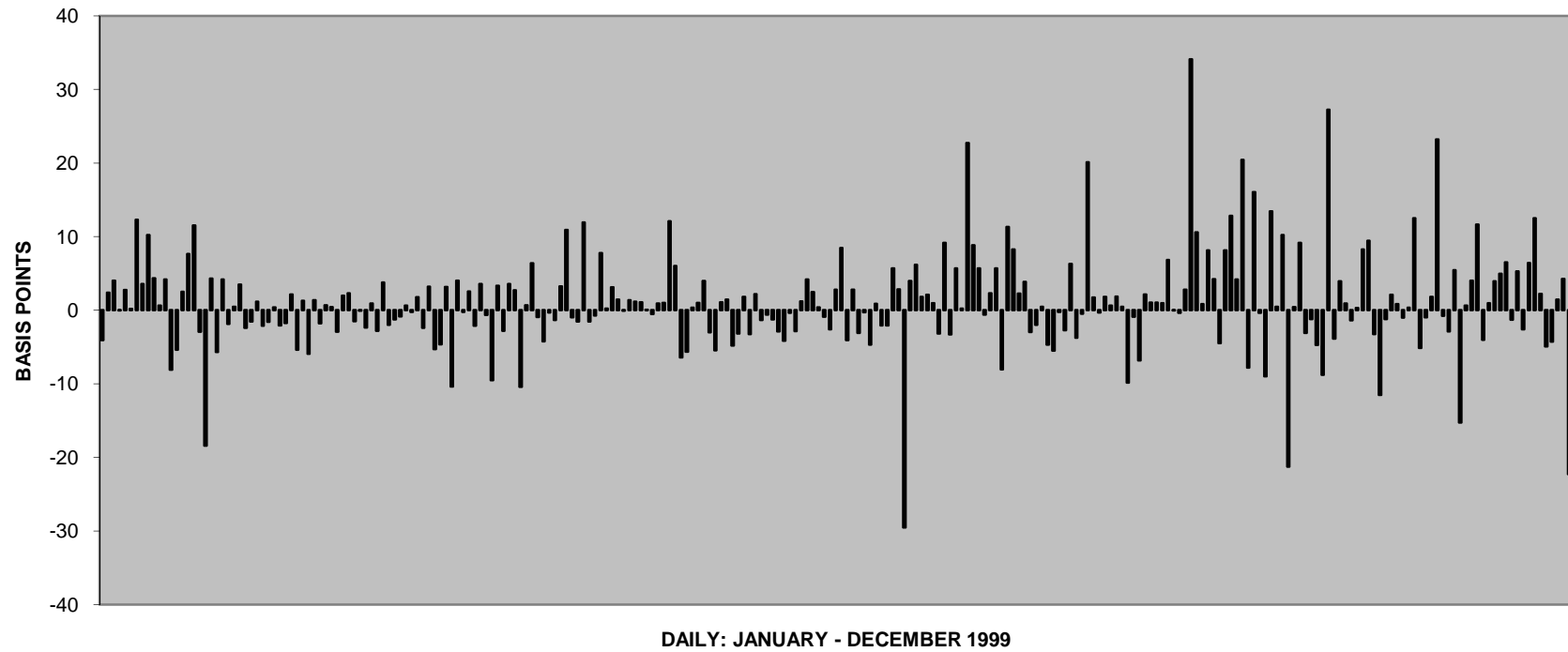
Note: (i) Estimates of relative vehicleness measure in Equation (16) of Proposition 2. (ii) The t-stats are tests of the hypothesis that the associated coefficient is zero.

Table 5
The matrix of implied price impact coefficients

	Regressor		
<u>Dep. Variable</u>	\$/€ Flow	¥/\$ Flow	¥/€ Flow
$\Delta p^{S/\text{€}}$	26.83	-68.56	356.17
$\Delta p^{\text{¥}/\$}$	-70.70	61.73	282.84
$\Delta p^{\text{¥}/\text{€}}$	71.11	109.14	176.13

Note: The coefficients have the dimension “basis points per billions of currency units”.

Figure 2
DEVIATION FROM $\$/\text{€}/\text{¥}$ TRIANGULAR PARITY



Appendix 1

(1) Two-way arbitrage trading:

Consider the case of a two-way (i.e., round trip) arbitrage trade in the euro-yen market. Without loss of generality, suppose $P^{\text{€/¥}} > P^{\text{$/€}} P^{\text{$/¥}}$. To effect two-way arbitrage, one would want to:

- (1) Buy x_a^3 euro at price $P^{\text{$/€}}$ in the euro-dollar market. This costs $x_a^3 P^{\text{$/€}}$ dollars.
- (2) Fund this by selling yen for dollars in the dollar-yen market. To yield $x_a^3 P^{\text{$/€}}$ dollars, one needs to exchange $x_a^3 P^{\text{$/€}} P^{\text{$/¥}}$ yen.
- (3) Sell x_a^3 euro at price $P^{\text{€/¥}}$ in the euro-yen market. This yields $x_a^3 P^{\text{€/¥}}$ yen.

The profit is $x_a^3 (P^{\text{€/¥}} - P^{\text{$/€}} P^{\text{$/¥}})$ yen. Now assume that in the previous period, triangular parity obtained:

$$\bar{P}^{\text{€/¥}} = \bar{P}^{\text{$/€}} \bar{P}^{\text{$/¥}}$$

Let us now define $(1 + p^i) \equiv \frac{P^i}{\bar{P}^i}$. The profit, measured as % change of yen per euro, is:

$$\begin{aligned} x_a^3 \left(\frac{P^{\text{€/¥}}}{\bar{P}^{\text{€/¥}}} - \frac{P^{\text{$/€}} P^{\text{$/¥}}}{\bar{P}^{\text{$/€}} \bar{P}^{\text{$/¥}}} \right) &= x_a^3 \left[(1 + p^{\text{€/¥}}) - (1 + p^{\text{$/€}})(1 + p^{\text{$/¥}}) \right] \\ &= x_a^3 \left[p^{\text{€/¥}} - (p^{\text{$/€}} + p^{\text{$/¥}} + p^{\text{$/€}} p^{\text{$/¥}}) \right] \\ &\approx x_a^3 \left[p^{\text{€/¥}} - (p^{\text{$/€}} + p^{\text{$/¥}}) \right] \end{aligned}$$

This is the implied expression in equation (7). In the opposite case with $P^{\text{€/¥}} < P^{\text{$/€}} P^{\text{$/¥}}$, both the direction of trade and the unit % profit have the opposite sign. Their product remains the definition of profit.

(2) Scaling order flows in equation (6)

Consider the following example where an incremental euro is purchased using yen, but the trade is executed indirectly (through the dollar). Without loss of generality, assume that the

matrix in equation (6) is diagonal, i.e., that there are no cross-price effects from order flow. To effect this indirect trade, one would:

- (1) Buy *one* euro, in the euro-dollar market, when previous dollar price of euro is $P^{\$/\epsilon}$. This purchase causes price appreciation according to $\lambda^{\$/\epsilon}$. The cost is $P^{\$/\epsilon} (1 + \lambda^{\$/\epsilon})$ dollars.
- (2) Fund this by buying dollars in the dollar-yen market, when the previous yen price of dollars is $P^{\$/\yen}$. Again, trading causes price appreciation according to $\lambda^{\$/\yen}$. A purchase of just *one* \$ in that market costs $P^{\$/\yen} (1 + \lambda^{\$/\yen})$ yen. Consequently, to purchase the necessary $P^{\$/\epsilon} (1 + \lambda^{\$/\epsilon})$ dollars, it will cost $P^{\$/\epsilon} P^{\$/\yen} (1 + \lambda^{\$/\epsilon}) (1 + \lambda^{\$/\yen})$ yen.

The gross cost measured as a % change of yen per euro is this $(1 + \lambda^{\$/\epsilon}) (1 + \lambda^{\$/\yen})$. Taking logs, the approximate net cost is $\lambda^{\$/\epsilon} + \lambda^{\$/\yen}$. This is the approximation implied by equation (6). Given that the λ 's are of the same order as returns, the approximation has negligible economic significance.

Appendix 2

(1) The market maker's problem

To sketch the solution to the model, we start by investing in an economical notation for the full list of demands as in equations (8), (9) and (10). For each trader, her direct demand is written as:

$$\begin{aligned}
 (18) \quad x_d^1 &= \left(\frac{(\lambda_{13} - \lambda_{12})(\lambda_{11} - \lambda_{21} + \lambda_{31}) - 2\lambda_{11}(\lambda_{22} - \lambda_{23} - \lambda_{32} + \lambda_{33})}{2\lambda_{11}\{(\lambda_{13} - \lambda_{12})(\lambda_{31} - \lambda_{21}) - \lambda_{11}(\lambda_{22} - \lambda_{23} - \lambda_{32} + \lambda_{33})\}} \right) v^1 = \beta^1 v^1 \\
 x_d^2 &= \frac{(\lambda_{23} - \lambda_{21})(\lambda_{22} + \lambda_{32} - \lambda_{12}) - 2\lambda_{22}(\lambda_{11} - \lambda_{13} - \lambda_{31} + \lambda_{33})}{2\lambda_{22}\{(\lambda_{23} - \lambda_{21})(\lambda_{32} - \lambda_{12}) - \lambda_{22}(\lambda_{11} - \lambda_{13} - \lambda_{31} + \lambda_{33})\}} v^2 = \beta^2 v^2 \\
 x_d^3 &= \left(\frac{(\lambda_{31} + \lambda_{32})(\lambda_{13} + \lambda_{23} + \lambda_{33}) - 2\lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})}{2\lambda_{33}\{(\lambda_{13} + \lambda_{23})(\lambda_{31} + \lambda_{32}) - \lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})\}} \right) v^3 = \beta^3 v^3
 \end{aligned}$$

The indirect demands are:

$$\begin{aligned}
 (19) \quad x_i^1 &= \frac{-\left\{ \lambda_{11}^2 + (\lambda_{21} - \lambda_{31})(\lambda_{12} - \lambda_{13} + \lambda_{21} - \lambda_{31}) + \lambda_{11}(-\lambda_{12} + \lambda_{13} + 2(\lambda_{21} - \lambda_{22} + \lambda_{23} - \lambda_{31} + \lambda_{32} + \lambda_{33})) \right\}}{2\lambda_{11}\{(\lambda_{13} - \lambda_{12})(\lambda_{31} - \lambda_{21}) - \lambda_{11}(\lambda_{22} - \lambda_{23} - \lambda_{32} + \lambda_{33})\}} v^1 = \gamma^1 v^1 \\
 x_i^2 &= \frac{-\left\{ \lambda_{12}^2 + \lambda_{32}^2 + \lambda_{32}(-\lambda_{21} + \lambda_{23} - 2\lambda_{22}) + \lambda_{12}(-2\lambda_{32} + \lambda_{21} - \lambda_{23} + 2\lambda_{22}) \right\} - 2\lambda_{22}(\lambda_{11} - \lambda_{13} - \lambda_{31} + \lambda_{33}) + \lambda_{22}(\lambda_{23} - \lambda_{21} + \lambda_{22})}{2\lambda_{22}\{(\lambda_{23} - \lambda_{21})(\lambda_{32} - \lambda_{12}) - \lambda_{22}(\lambda_{11} - \lambda_{13} - \lambda_{31} + \lambda_{33})\}} \gamma^2 v^2 \\
 x_i^3 &= \left(\frac{-\left\{ \lambda_{13}^2 + \lambda_{23}^2 + \lambda_{23}(\lambda_{31} + \lambda_{32} - 2\lambda_{33}) + \lambda_{13}(2\lambda_{23} + \lambda_{31} + \lambda_{32} - 2\lambda_{33}) \right\} - 2\lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22}) + \lambda_{33}(\lambda_{31} + \lambda_{32} + \lambda_{33})}{2\lambda_{33}\{(\lambda_{13} + \lambda_{23})(\lambda_{31} + \lambda_{32}) - \lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})\}} \right) v^3 = \gamma^3 v^3
 \end{aligned}$$

And the two-way arbitrage demands are:

$$\begin{aligned}
x_a^1 &= \left(\frac{(\lambda_{21} - \lambda_{31})(\lambda_{12} - \lambda_{13} + \lambda_{21} - \lambda_{31})}{2\lambda_{11} \{(\lambda_{13} - \lambda_{12})(\lambda_{31} - \lambda_{21}) - \lambda_{11}(\lambda_{22} - \lambda_{23} - \lambda_{32} + \lambda_{33})\}} \right) v^1 = \delta^1 v^1 \\
(20) \ x_a^2 &= \frac{\left(\left(\lambda_{12}^2 - 2\lambda_{11}\lambda_{22} + 2\lambda_{13}\lambda_{22} - \lambda_{21}\lambda_{22} + \lambda_{22}\lambda_{23} + 2\lambda_{22}\lambda_{31} \right. \right. \\
&\quad \left. \left. + \lambda_{12}(\lambda_{21} + \lambda_{22} - \lambda_{23} - 2\lambda_{32}) - \lambda_{21}\lambda_{32} - \lambda_{22}\lambda_{32} + \lambda_{23}\lambda_{32} + \lambda_{32}^2 - 2\lambda_{22}\lambda_{33} \right) \right)}{2\lambda_{22}((\lambda_{23} - \lambda_{21})(\lambda_{32} - \lambda_{12}) - \lambda_{22}(\lambda_{11} - \lambda_{13} - \lambda_{31} + \lambda_{33}))} = \delta^2 v^2 \\
x_a^3 &= \left(\frac{(\lambda_{13} + \lambda_{23})(\lambda_{13} + \lambda_{23} + \lambda_{31} + \lambda_{32})}{2\lambda_{33} \{(\lambda_{13} + \lambda_{23})(\lambda_{31} + \lambda_{32}) - \lambda_{33}(\lambda_{11} + \lambda_{12} + \lambda_{21} + \lambda_{22})\}} \right) v^3 = \delta^3 v^3
\end{aligned}$$

Defining y^j $j = 1, 2, 3$ as the order flow in each of the three markets, we have from equation (6):

$$(21) \quad \begin{bmatrix} y^1 \\ y^2 \\ y^3 \end{bmatrix} = \begin{bmatrix} c^1 + x_d^1 - x_i^2 + x_i^3 - x_a^2 + x_a^3 \\ c^2 + x_d^2 - x_i^1 + x_i^3 - x_a^1 + x_a^3 \\ c^3 + x_d^3 + x_i^1 + x_i^2 + x_a^1 + x_a^2 \end{bmatrix}$$

Define the matrix

$$(22) \quad \mathbf{A} = \begin{bmatrix} \beta^1 & -(\gamma^2 + \delta^2) & \gamma^3 + \delta^3 \\ -(\gamma^1 + \delta^1) & \beta^2 & \gamma^3 + \delta^3 \\ \gamma^1 + \delta^1 & \gamma^2 + \delta^2 & \beta^3 \end{bmatrix}$$

Then (21) and (22) imply:

$$(23) \quad \begin{bmatrix} y^1 \\ y^2 \\ y^3 \end{bmatrix} = \mathbf{A} \begin{bmatrix} v^1 \\ v^2 \\ v^3 \end{bmatrix} + \begin{bmatrix} c^1 \\ c^2 \\ c^3 \end{bmatrix}$$

Recalling that we have assumed:

$$(24) \quad \begin{bmatrix} c^1 \\ c^2 \\ c^3 \end{bmatrix} \sim N \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_c^1 & 0 & 0 \\ 0 & \Sigma_c^2 & 0 \\ 0 & 0 & \Sigma_c^3 \end{bmatrix} \right\}$$

And

$$(25) \quad \begin{bmatrix} v^1 \\ v^2 \\ v^3 \end{bmatrix} \sim N \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_v^1 & 0 & 0 \\ 0 & \Sigma_v^2 & 0 \\ 0 & 0 & \Sigma_v^3 \end{bmatrix} \right\}$$

We also label

$$(26) \quad \begin{aligned} & \begin{bmatrix} \Sigma_c^1 & 0 & 0 \\ 0 & \Sigma_c^2 & 0 \\ 0 & 0 & \Sigma_c^3 \end{bmatrix} = \Sigma_c \\ & \begin{bmatrix} \Sigma_v^1 & 0 & 0 \\ 0 & \Sigma_v^2 & 0 \\ 0 & 0 & \Sigma_v^3 \end{bmatrix} = \Sigma_v \\ & \begin{bmatrix} y^1 \\ y^2 \\ y^3 \end{bmatrix} = \mathbf{y} \\ & \begin{bmatrix} v^1 \\ v^2 \\ v^3 \end{bmatrix} = \mathbf{v} \\ & \begin{bmatrix} c^1 \\ c^2 \\ c^3 \end{bmatrix} = \mathbf{c} \end{aligned}$$

From the properties of the normal distribution, the conditional expectation of the signals given the order flows is:

$$(27) \quad E[\mathbf{v} | \mathbf{y}] = \Sigma_v \mathbf{A}' [\mathbf{A} \Sigma_v \mathbf{A}' + \Sigma_c]^{-1}$$

We write the matrix of price impact coefficients in equation (6) as:

$$(28) \quad \mathbf{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix}$$

And the vector of spot returns as:

$$(29) \quad \mathbf{p} = \begin{bmatrix} p^1 \\ p^2 \\ p^3 \end{bmatrix}$$

From equations (6), (28) and (29), the market maker sets prices so that

$$(30) \quad \mathbf{p} = \mathbf{\Lambda} \mathbf{y}$$

Since the market maker is risk neutral and earns zero profits, equations (27) and (30) imply:

$$(31) \quad \mathbf{\Lambda} = \mathbf{\Sigma}_v \mathbf{A}' [\mathbf{A} \mathbf{\Sigma}_v \mathbf{A}' + \mathbf{\Sigma}_\epsilon]^{-1}$$

Where $\mathbf{0}$ is the 3×3 zero matrix. This gives us nine equations in the nine unknown price impact coefficients in equation (28) to solved in terms of the six variances in $\mathbf{\Sigma}_\epsilon$ and $\mathbf{\Sigma}_v$.