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IMPATIENCE VERSUS INCENTIVES

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IMPATIENCE VERSUS INCENTIVES

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This paper studies the dynamics of long-term contracts in repeated principal–agent relationships with an impatient agent. Despite the absence of exogenous uncertainty, Pareto-optimal dynamic contracts generically oscillate between favoring the principal and favoring the agent.

KEYWORDS: Dynamic contracts, endogenous cycles, principal–agent models, relational contracts, differential discounting, repeated games.

1. INTRODUCTION

WE STUDY OPTIMAL CONTRACTING in a repeated principal–agent framework where the agent is more impatient than the principal. Differential discounting creates gains from trade across time and tends to push optimal contracts toward favoring the more patient principal in the long run. This *impatience* force conflicts with the *incentives* force that tends to push optimal contracts toward favoring the agent in the long run. We show that Pareto-optimal contracts generically resolve this impatience versus incentives conflict by oscillating between favoring the principal and favoring the agent over time.

In our model, a contract stipulates, for each period, a transfer to the agent and an action. Our model admits a broad interpretation of "action." It can be effort by the agent or investment by the principal or a collaborative venture by both. Each action generates a surplus, a portion of which is transferred to the agent according to the contract. If the agent deviates from the contract, he receives a deviation payoff that is a function of the action that was supposed to be taken, but loses a fraction of the stipulated transfer. We study contracts where the agent never wants to deviate and the principal's participation constraint is satisfied in each period.

In this setting, we first prove the existence of a unique stationary Pareto-optimal contract—the steady state. We find that the associated steady-state action does not maximize static surplus, but, instead, also accounts for dynamic trading gains resulting from differential discounting. We then show that all nonstationary Pareto-optimal contracts oscillate around this focal steady state. These contracts may feature oscillating transfers with constant action or co-moving oscillating transfers and actions. Oscillation can be damped, converg-

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ing to the steady state, or can persist in the long run. In the latter case, the amplitude of oscillation grows over time, causing even arbitrarily low participation constraints to bind in the long run, distorting contract dynamics.

There are two features of the model that drive the oscillation phenomenon. First, the agent is more impatient than the principal. Second, the agent loses a fraction of the stipulated transfer when he deviates. The first feature ensures that the agent's incentive-compatibility (IC) constraint binds in Pareto-optimal contracts. Otherwise, it would be incentive compatible to further front-load transfers to the impatient agent and reap dynamic trading gains. Binding IC constraints plus the second feature imply that any above-steady-state transfer to the agent must be followed by a below-steady-state transfer, and any below-steady-state transfer must be followed by an above-steady-state transfer. Oscillation emerges.[2]

Our setting is broadly applicable, in the spirit of Ray (2002), and nests environments studied in many influential papers, such as Thomas and Worrall (1988), Thomas and Worrall (1994), and Albuquerque and Hopenhayn (2004). Yet, we show that by adding even an infinitesimal amount of relative impatience on the agent side, virtually all Pareto-optimal contracts oscillate around a focal Pareto-optimal steady state. This contrasts with the standard result under equal discounting (e.g., Becker and Stigler (1974), Harris and Holmstrom (1982), and Ray (2002)), in which the timing of pay does not create any value, and any Pareto-optimal payoff can be sustained by a contract that favors the agent in the long run.

The rest of the paper is organized as follows: Section 2 describes our general model and its applicability to various agency problems, and provides a basic intuition for the oscillation principle. Section 3 presents all formal results.

2. MODEL

The model is an infinitely repeated principal–agent relationship with perfect public information and transferable utility. It consists of the following ingredients: discount factors $\delta_P \geq \delta_A$ for the principal and agent, respectively; a principal outside option O_P; an abstract action set $\mathcal{A}$ where each action $a \in \mathcal{A}$ produces surplus $\pi(a)$; and a *best possible* deviation payoff function $D(u_A, a)$ for the agent when the equilibrium path currently calls for a transfer $u_A \in \mathbb{R}$ to the agent and action a.[3] Following Ray (2002), one can think of D as the result of a maximization over a potentially large set of available deviations. We assume that $D(u_A, a)$ takes the quasilinear form $(1-\theta)u_A + d(a)$ with $\theta \in [0, 1]$. Here

[2]Thus, cycles require neither exogenous shocks/uncertainty (see Aguiar, Amador, and Gopinath (2009)) nor self-fulfilling changes in beliefs as in Zhu, Wright, and He (2015), Gu, Mattesini, Monnet, and Wright (2013), and Rocheteau and Wright (2013).

[3]In some applications, it may be natural to impose a constraint on u_A so that it must be nonnegative or nonpositive. For technical reasons, we do not assume such constraints. In many cases, however, the optimal contracts we derive satisfy these additional constraints automatically.

θ is the fraction of the current-period transfer lost under deviation, and $d(a)$ is the residual deviation payoff that depends on a and includes all subsequent agent payoffs derived from an outside option or under a punishment equilibrium within the game. We make no assumptions on π or d except that the map $(\pi, d): \mathcal{A} \to \mathbb{R}^2$ has compact image.

A contract is a sequence of transfers and actions $\{(\tilde{u}_{A,t}, \tilde{a}_t)\}_{t=0}^{\infty}$. There exist public randomization devices, so each $\tilde{u}_{A,t}$ and $\tilde{a}_t$ can be random. For every agent promised value U_A, the Principal's Problem is the maximization

$$\max_{\{(\tilde{u}_{A,t}, \tilde{a}_t)\}_{t=0}^{\infty}} \mathbb{E}_0\left[\sum_{t=0}^{\infty} \delta_P^t\big(\pi(\tilde{a}_t) - \tilde{u}_{A,t}\big)\right]$$

s.t.

$$\text{(1)} \qquad \tilde{u}_{A,t} + \delta_A U_{A,t+1} \geq D(\tilde{u}_{A,t}, \tilde{a}_t) := (1-\theta)\tilde{u}_{A,t} + d(\tilde{a}_t)$$

$$\forall t \;\forall \text{ realizations of } (\tilde{a}_t, \tilde{u}_{A,t}),$$

$$\text{(2)} \qquad U_{P,t} \geq O_P \quad \forall t,$$

$$\text{(3)} \qquad U_{A,0} \geq U_A,$$

where $U_{A,t} := \mathbb{E}_t[\sum_{s=t}^{\infty} \delta_A^{s-t} \tilde{u}_{A,s}]$ and $U_{P,t} := \mathbb{E}_t[\sum_{s=t}^{\infty} \delta_P^{s-t}(\pi(\tilde{a}_s) - \tilde{u}_{A,s})]$ denote the date t continuation payoffs of the agent and the principal.

Expression (1) is the agent's incentive-compatibility constraint, which requires, for every random realization, that the agent's date t continuation payoff $\tilde{u}_{A,t} + \delta_A U_{A,t+1}$ be weakly larger than his deviation payoff $D(\tilde{u}_{A,t}, \tilde{a}_t)$.

Expression (2) is the principal's interim participation constraint.[4] From now on, all contracts are assumed to satisfy (1) and (2). Our goal is to show that when $\delta_P > \delta_A$, the following theorem is generically true.

THEOREM 1: *There exists a unique steady state: a Pareto-optimal contract with a constant continuation payoff process* $\{(U_{A,t}, U_{P,t})\}_{t=0}^{\infty}$. *The steady-state action* a^s *does not maximize static surplus* $\pi(a)$:

$$\text{(4)} \qquad a^s = \arg\max_{a \in \mathcal{A}} \pi(a) - \frac{\delta_P - \delta_A}{\theta\delta_P + (1-\theta)\delta_A} d(a).$$

If $\theta = 0$ *or* 1, *every non-steady-state Pareto-optimal contract has a nonrandom continuation payoff process that converges monotonically to the steady state. If* $\theta \in (0, 1)$, *every non-steady-state Pareto-optimal contract has a nonrandom continuation payoff process that oscillates around the steady state. This oscillation persists in the long run if and only if* $\theta \in [\frac{\delta_A}{1+\delta_A}, \frac{\delta_A}{\delta_P+\delta_A}]$.

[4] Alternatively, one can impose an agent interim participation constraint, or both principal and agent interim participation constraints. The results do not change. Further discussion is provided at the end of the main text; see Remark 2.

Informally, “generically true” means true when corner conditions do not get in the way. For example, an action solving (4) generically does not maximize surplus, but there are clearly exceptions for certain discrete or sufficiently kinked π and d.[5] On the other hand, if one only wants to prove that there are Pareto-optimal payoffs that can only be supported by oscillating contracts, then no further assumptions need to be made except for O_P being sufficiently low.

Our flexible model can speak to a wide spectrum of agency problems.[6]

EXAMPLE 1: A government (A) allows a multinational firm (P) to invest I in the country. Investment generates output $Y(I)$. The contract stipulates a transfer of taxes τ to the government. The government can expropriate output up to $Y(I)$, but then forfeits tax income forever.

EXAMPLE 2: An entrepreneur (A) seeks a lender (P) to help finance a product. There are a number of different ways to develop the product. Each option o_i requires some outlay I_{o_i} from the lender and generates some return Y_{o_i} for the entrepreneur. The contract stipulates a loan repayment R. The entrepreneur can keep Y_{o_i} and strategically default on R, in which case the lender can take the entrepreneur to court. With probability $1-\theta$, the lender prevails and recoups R; otherwise, he receives nothing.

EXAMPLE 3: An owner (P) has access to a set of projects and can choose to collaborate with a worker (A) to implement a subset of them. Each subset $\{p_i\}$ requires effort cost $c_{\{p_i\}}$ from the worker and $C_{\{p_i\}}$ from the owner, and produces $\sum_{\{p_i\}} e_{p_i}$ for the owner. In return, the worker receives an up-front wage w. The worker can shirk and keep the wage, in which case the worker is fired but does capture unemployment benefits valued at $\delta_A O_A$.

The three examples demonstrate how our model can encode various timing and pay conventions (see Table I). In Example 1, the firm pays taxes *after* observing whether the government has expropriated or not. So the government does not receive any tax revenue in the period it expropriates: $\theta = 1$ and $D = d$. If expropriation occurs, the strongest punishment equilibrium involves the firm withdrawing from the country and so the government receives no future tax revenue either. Therefore, if the government does expropriate, its best

[5]Here is a simple way to formalize the theorem: Replace the current model with a newer model whose $\mathrm{Im}(\mathcal{A})$ is the convex hull of the current one. This does not affect the dynamics of Pareto-optimal contracts (Lemma 1). Then slightly perturb the newer model so that the $\mathrm{Im}(\mathcal{A})$ is now a strictly convex approximation of the previous one with smooth boundary. Theorem 1 now holds for all sufficiently low O_P.

[6]Example 1 is drawn from Thomas and Worrall (1994) and Opp (2012); Example 2 is a modified version of Albuquerque and Hopenhayn (2004) or Clementi and Hopenhayn (2006); Example 3 is inspired by Ray (2002) and Thomas and Worrall (1988).

TABLE I

MAPPING THE EXAMPLES TO THE MODEL[a]

	$\mathcal{A}$	u_A	π	D	θ	d
Example 1	I	τ	$Y(I)-I$	$Y(I)$	1	$Y(I)$
Example 2	o_i	$Y_{o_i}-R$	$Y_{o_i}-I_{o_i}$	$Y_{o_i}-(1-\theta)R$	θ	θY_{o_i}
Example 3	$\{p_i\}$	$w-c_{\{p_i\}}$	$\sum_{\{p_i\}} e_{p_i}-c_{\{p_i\}}-C_{\{p_i\}}$	w	0	$c_{\{p_i\}}+\delta_A O_A$

[a]Notice that the action can be taken by the principal (Example 1), the agent (Example 2), or both (Example 3); it can be pecuniary or nonpecuniary, and it can be a single object or a set.

strategy is to steal the *entire* firm output: $D = Y(I)$. A contract $\{u_{A,t}, I_t\}_{t=0}^{\infty}$ is incentive-compatible if and only if $U_{A,t} \geq Y(I_t)$ for all t. In Example 2, the lender has recourse *after* a default and $(1-\theta)$ represents the probability of contractual enforcement. Alternatively, it could be that the borrower puts up collateral *before* any action, and that θ represents the collateral requirement as in Kiyotaki and Moore (1997).[7] In Example 3, the wage to the worker is prepaid and so $\theta = 0$ captures the fact that even if the worker shirks, he keeps the wage that has already been paid to him. Of course, if the worker can only abscond with a fraction of the wage due to some inefficiencies, such as the banker in Calomiris and Kahn (1991), then θ can represent the portion of the wage lost during deviation.

Examples 2 and 3 also highlight an important aspect of the model's flexibility. In many applications, it is more natural to think of the agent's utility u_A as a sum $m + h(a)$, where $h(a)$ is the component intrinsic to the action a stipulated by the contract and m is the monetary transfer stipulated by the contract. This is in contrast to the model setup where the entire u_A is thought of as the transfer.

The difference is important because when $u_A = m + h(a)$, it is more natural to assume that the θ parameter only applies to the m component of u_A, so that one should think of D as $D(m, a) = (1-\theta)m + \hat{d}(a)$ instead of $D(u_A, a) = (1-\theta)u_A + d(a)$. However, Example 2 shows how the model can easily accommodate this mismatch. Simply decompose $\hat{d}(a)$ into $(1-\theta)h(a) + (\hat{d}(a) - (1-\theta)h(a))$, define $d(a) := \hat{d}(a) - (1-\theta)h(a)$, and now $D(m, a)$ can be written in the correct form $D(u_A, a)$.[8]

[7]The settings of Geanakoplos (2009) and Brunnermeier and Pedersen (2009) show how the collateralization parameter θ can emerge endogenously.

[8]The quantity $d(a) := \hat{d}(a) - (1-\theta)h(a)$ lacks the economic significance of $\hat{d}(a)$, but that is of no concern since the model does not require d to satisfy anything beyond having a compact image.

REMARK 1: Any transferable utility model where agent utility has the more common form $u_A(m, a) := m + h(a)$ and $D(m, a) = (1 - \theta)m + \hat{d}(a)$ is quasi-linear in the monetary transfer with $\theta \in [0, 1]$ can be mapped into the model.

The Oscillation Principle

The basic intuition for oscillation around the steady state relies on two features of the model: $\theta > 0$ and relative impatience of the agent ($\delta_A < \delta_P$). In particular, a nontrivial action set is actually not necessary for oscillation to emerge. Thus, to highlight the basic mechanics of oscillation, we will, for now, consider the simplest version of the model where the action set is a singleton $\{a^s\}$. Let u^s_A be the steady-state utility transfer and let $U^s_A := u^s_A/(1 - \delta_A)$ be the agent's steady-state continuation payoff.

To emphasize the role of θ, rewrite the IC constraint $u_{A,t} + \delta_A U_{A,t+1} \geq (1 - \theta)u_{A,t} + d(a^s)$ as

$$\theta u_{A,t} + \delta_A U_{A,t+1} \geq d(a^s). \tag{5}$$

Thus, the parameter θ measures the sensitivity of the current-period's IC constraint to current-period transfers. In particular, when $\theta > 0$, transfers today relax the IC constraint today.

Relative impatience of the agent implies that IC constraints should always bind. Otherwise, moving some of tomorrow's transfer to today would lead to a Pareto-improvement and IC constraints would still be respected. In particular, the IC constraint must bind at the steady state:

$$\theta u^s_A + \delta_A U^s_A = d(a^s). \tag{6}$$

Now, to see why oscillation emerges, first consider an agent payoff $U_A > U^s_A$. To deliver U_A, the principal can, for example, provide the agent with an above-steady-state initial transfer followed by the steady-state continuation payoff. But (6) plus the assumption $\theta > 0$ implies that the IC constraint would be slack. Thus, the principal can do better by further front-loading transfers to the agent until the IC constraint binds. In the end, the agent receives an above-steady-state transfer today, $u_A > u^s_A$, followed by a *below*-steady-state continuation payoff tomorrow, $U^+_A < U^s_A$.

Next, consider the opposite case $U_A < U^s_A$. Mirroring the previous case, the principal could try a below average initial transfer followed by the steady-state continuation payoff. But now (6) plus the assumption $\theta > 0$ implies that the IC constraint is violated. Thus, the initial transfer must be further diminished and the continuation payoff must be increased. In the end, the agent receives a below-steady-state transfer today, $u_A < u^s_A$, followed by an *above*-steady-state continuation payoff tomorrow, $U^+_A > U^s_A$.

We have now shown that if today's payoff is above the steady state, then tomorrow's should be below and if today's is below, then tomorrow's should be above. Oscillation around the steady state results.

Notice how positive incentive effects of current-period transfers when $\theta > 0$ and relative impatience of the agent interact to generate oscillation. Relative impatience makes binding IC constraints uniquely optimal. Then $\theta > 0$ ensures that binding IC constraints plus above- (below-) steady-state payoffs imply below- (above-) steady-state continuation payoffs.

If either feature is missing, the argument for oscillation falls apart. If $\theta = 0$, then to deliver an above- (below-) steady-state payoff with binding IC, the principal can simply provide an above- (below-) steady-state initial transfer followed by the steady-state continuation payoff. As a result, all contracts converge monotonically to the steady state. If the principal and agent are equally patient, then Pareto-optimal contracts no longer need to be maximally front-loaded and IC constraints no longer need to bind. Starting with an oscillating contract, one can always further back-load payments in a payoff neutral way until the contract continuation payoff process no longer oscillates and, instead, converges monotonically to a steady state.

In this primer on oscillation, we have neglected to discuss how participation constraints can distort oscillation and, ultimately, the optimal action sequence if the model possesses a nontrivial action set. Participation constraints matter because the higher is θ, the greater is the amplitude of oscillation. We will show that when $\theta > \delta_A/(1+\delta_A)$, the oscillations become explosive if we were to show no regard for participation constraints. This means that any participation constraint, no matter how low, would eventually be violated. Optimally adjusting the explosive oscillation so as to respect participation constraints leads to nontrivial distortions of the action sequence and the oscillation dynamic itself. We now explore this as part of the formal analysis of the Principal's Problem.

3. ANALYSIS

We start our formal analysis with a preliminary lemma that reveals the limited role of public randomization and implies that we can restrict our large, abstract action set to a subset of *efficient actions*.

LEMMA 1: *Fix a model* $(\theta, \delta_A, \delta_P, \mathcal{A}, d, \pi)$. *Any alternate model* $(\theta, \delta_A, \delta_P, \hat{\mathcal{A}}, \hat{d}, \hat{\pi})$, *where* $\mathrm{Im}(\hat{\mathcal{A}}) = \mathrm{Conv}(\mathrm{Im}(\mathcal{A}))$, *generates the same Pareto frontier with the same Pareto-optimal continuation payoff processes*.

Certainly the alternate model $\hat{\mathcal{A}}$ can achieve any payoff the original model can achieve. To prove the converse, suppose there was a contract in the alternate model that called for action $\hat{a}_t$, transfer $\hat{u}_{A,t}$, and continuation payoff $\hat{U}_{A,t+1}$. First, for any action $\hat{a} \in \hat{\mathcal{A}}$, there exists a random action $\tilde{a} \in \mathcal{A}$ satisfying $\mathbb{E}\pi(\tilde{a}) = \pi(\hat{a})$ and $\mathbb{E}d(\tilde{a}) = d(\hat{a})$. Note, however, the IC constraint (1) must now be satisfied for any random realization $d(\tilde{a})$, and not just for the average realization $d(\hat{a})$. This can be achieved by fine-tuning *only* the transfers

$\tilde{u}_{A,t} = \hat{u}_{A,t} + \frac{d(\tilde{a}_t)-d(\hat{a}_t)}{\theta}$ and leaving the continuation payoff fixed at $\hat{U}_{A,t+1}$. By construction, payoffs are unaffected since $\mathbb{E}\tilde{u}_{A,t} = \hat{u}_{A,t}$, and the continuation payoff process is identical. This proves Lemma 1.

When a model's $\text{Im}(\mathcal{A})$ is convex, public randomization provides no benefits. In particular, all Pareto-optimal payoffs can be delivered by contracts with deterministic actions, transfers, and continuation payoff processes. In addition, since Lemma 1 implies that with respect to Pareto-optimal continuation payoff processes, it is without loss of generality to focus on models where $\text{Im}(\mathcal{A})$ is convex, we have now proved the following corollary.

COROLLARY 1: *Any Pareto-optimal payoff of any model can be delivered by a contract with a deterministic continuation payoff process*.

In particular, this is true even if we are in a model where any such contract must involve random actions and transfers. Thus, in our setting, public randomization is only used to complete the static action space and plays no role in contract dynamics. This fact is important for the interpretation of our results as it allows us to highlight that oscillation of continuation payoffs is not an artifact of randomization.

Our analysis can be further simplified by noting that any Pareto-optimal contract must only use *efficient* actions. An action a is efficient if for any other action a', $\pi(a') < \pi(a)$ or $d(a') > d(a)$ or $(\pi(a'), d(a')) = (\pi(a), d(a))$. Let $\hat{\mathcal{A}}^*$ be the set of efficient actions. Then it is without loss of generality to focus on models with action space of the form $\hat{\mathcal{A}}^*$. Figure 1 shows a representative $\text{Im}(\mathcal{A})$, its convex hull $\text{Im}(\hat{\mathcal{A}})$, and the efficient frontier $\text{Im}(\hat{\mathcal{A}}^*)$. By construction, $\text{Im}(\hat{\mathcal{A}}^*)$ is a concave, strictly increasing function over $[d_{\min}, d_{\max}]$; π is an implicit function of d; the action space can be identified with the interval $[d_{\min}, d_{\max}]$; and $\text{Im}(\hat{\mathcal{A}}^*)$ is just the graph of $\pi(d)$. From now one, for the sake of simplicity, we will refer to actions as d and surpluses as $\pi(d)$, and we will, without loss of generality, disallow public randomization.

The set of contract payoffs is compact. Any continuation contract of a Pareto-optimal contract must be Pareto optimal. Thus, when dealing with Pareto-optimal contracts, we may write $(U_{A,t}, V(U_{A,t}))$ for $(U_{A,t}, U_{P,t})$. From now on, we will refer to the Pareto frontier as V and to Pareto-optimal contracts as V-contracts.

LEMMA 2: *The value function $V(U_A)$ is a concave, strictly decreasing function over its domain $[U_A^{\min}, U_A^{\max}]$, and satisfies $V(U_A^{\max}) = O_P$.*

PROOF: Concavity follows from concavity of $\pi(d)$. Suppose $V(U_A^{\max}) > O_P$. Take the V-contract that delivers $U_A^{\max}$ and increase the initial transfer by $V(U_A^{\max}) - O_P$. This contract still satisfies (1) and (2), and delivers $> U_A^{\max}$ payoff to the agent. Contradiction. *Q.E.D.*

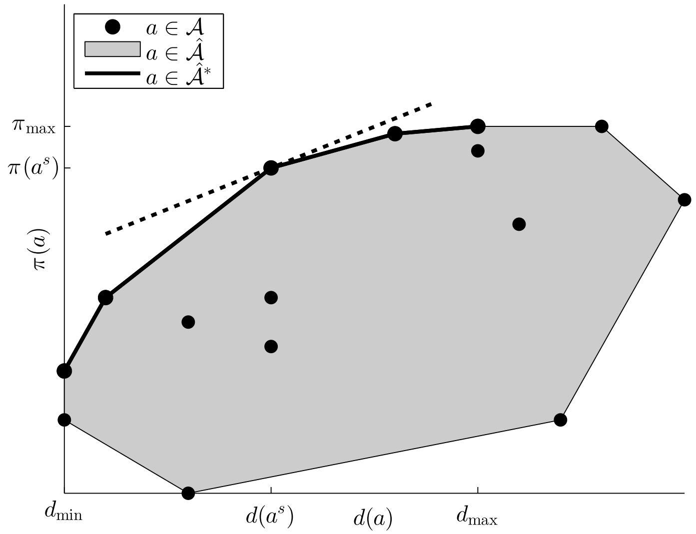

FIGURE 1.—Plots of the images of the original action set $\mathcal{A}$, $\hat{\mathcal{A}}$, and the set of efficient actions $\hat{\mathcal{A}}^*$. The latter set defines π as a function of d. The dotted line has slope $\frac{\delta_P-\delta_A}{\theta\delta_P+(1-\theta)\delta_A}$ and identifies the steady-state action (see Lemma 3).

ASSUMPTION 1: *The principal's outside option satisfies*

$$O_P < \left(\pi(d_{\max}) - \frac{1-\delta_A}{\theta(1-\delta_A)+\delta_A} d_{\max}\right) \Big/ (1-\delta_P).$$

The right hand side of the inequality is the principal's payoff under the unique stationary contract that sustains the surplus maximizing action with the smallest possible stationary transfer to the agent. Paired with Lemma 3 below, this assumption highlights the fact that even though the surplus maximizing action $d^{\max}$ is sustainable, dynamic trading gains may cause the principal and agent to prefer a steady state with a lower static surplus.

LEMMA 3: *A stationary contract* $\{(u_{A,t} = u_A^s, d_t = d^s)\}_{t=0}^{\infty}$ *is a* V*-contract if and only if*

$$d^s \in \arg\max_{d \in \hat{\mathcal{A}}^*} \pi(d) - \frac{\delta_P - \delta_A}{\theta\delta_P + (1-\theta)\delta_A} d, \tag{7}$$

$$u_A^s = \frac{d^s}{\theta + \dfrac{\delta_A}{1-\delta_A}}. \tag{8}$$

In particular, d^s *is generically smaller than* $d_{\max}$.

Lemma 3 characterizes the steady state of V-contracts. The reason d^s is generically smaller than $d_{\max}$ is due to an important trade-off between dynamic trading gains and static surplus: When the agent is more impatient, shifting the steady-state payoff allocation in favor of the principal allows for larger initial transfers to the agent. This front-loading realizes potential gains from trading across time. The trade-off is that shifting the steady state in favor of the principal tightens IC constraints. Since IC constraints were already binding to begin with, a concomitant decrease in the steady-state action d^s, which loosens IC constraints, is required. This leads to a smaller static surplus.[9] The optimal degree of the trade-off is parameterized by the coefficient $\frac{\delta_P-\delta_A}{\theta(\delta_P-\delta_A)+\delta_A}$ in (7), which is proportional to the relative impatience of the agent. When the agent is as patient as the principal, $\delta_A = \delta_P$ and d^s simply maximizes surplus. As δ_A decreases, the relative value of dynamic trading gains increases, and the steady-state action and the surplus decrease relative to the static optimum.

PROOF OF LEMMA 3: The IC constraint (5) requires $\theta u_A^s + \delta_A u_A^s/(1-\delta_A) \geq d^s$ or, equivalently, $u_A^s \geq d^s/(\theta + \delta_A/(1-\delta_A))$. If (8) did not hold, then the IC constraint would be slack each date. One can then easily achieve a Pareto improvement by increasing $u_{A,0}$ slightly and decreasing $u_{A,1}$ slightly. Contradiction. This proves (8). To prove (7), fix a generic V-contract $\{(u_{A,t}, d_t)\}_{t=0}^{\infty}$ and consider two perturbations. First, for a small real ε, let $u_{A,0} \to \hat{u}_{A,0} := u_{A,0} + \varepsilon$ and starting at date 1, enact the V-contract with agent payoff $\hat{U}_{A,1} := U_{A,1} - \theta\varepsilon/\delta_A$. This perturbation is incentive-compatible. The agent's payoff is $U_{A,0} + (1-\theta)\varepsilon$. By definition, the principal's payoff must be weakly smaller than his payoff under the V-contract with the same agent payoff as the perturbation contract:

$$\pi(d_0) - u_{A,0} - \varepsilon + \delta_P V\left(U_{A,1} - \frac{\theta\varepsilon}{\delta_A}\right) \leq V\big(U_{A,0} + (1-\theta)\varepsilon\big).$$

Letting ε be infinitesimally positive and negative, we derive the two fundamental differential conditions linking the payoff U_A and continuation payoff U_A^+ of any V-contract:[10]

$$(1-\theta)V^+(U_A) \geq -1 - \frac{\delta_P}{\delta_A} \cdot \theta \cdot V^-\big(U_A^+\big), \tag{9}$$

$$(1-\theta)V^-(U_A) \leq -1 - \frac{\delta_P}{\delta_A} \cdot \theta \cdot V^+\big(U_A^+\big). \tag{10}$$

[9]See Acemoglu, Golosov, and Tsyvinski (2008), Aguiar, Amador, and Gopinath (2009), and Opp (2012) for examples on investment distortions with heterogeneous discounting. We contribute relative to these papers by highlighting the efficiency of such distortions and being able to characterize the solution for arbitrary action sets.

[10]Technically speaking, the ε-perturbations used to derive (9) apply only to those V-contracts where $U_A < U_A^{\max}$ and the ε-perturbations used to derive (10) apply only to those V-contracts where $U_A^+ < U_A^{\max}$. Similar considerations apply for the second perturbation yielding (12).

Inequalities (9) and (10) plus the fact that $V(U_A)$ is a concave, strictly decreasing function provide a useful necessary condition for when a Pareto-optimal payoff $(U_A, V(U_A))$ can be achieved by a stationary contract:

$$-V^-(U_A) \le \frac{\delta_A}{\theta\delta_P + (1-\theta)\delta_A} \le -V^+(U_A). \tag{11}$$

In the second perturbation, for a small real ε, let $d_0 \to \breve{d}_0 := d_0 + \varepsilon$ and let $u_{A,0} \to \breve{u}_{A,0} := u_{A,0} + \varepsilon/\theta$. Using similar arguments as before, we can establish

$$1 - \theta\pi^-(d_0) \le -V^-(U_A) \le -V^+(U_A) \le 1 - \theta\pi^+(d_0). \tag{12}$$

Inequalities (11) and (12) together imply that if (u_A^s, d^s) is a steady state, then

$$\pi^+(d_s) \le \frac{\delta_P - \delta_A}{\theta\delta_P + (1-\theta)\delta_A} \le \pi^-(d_s). \tag{13}$$

This proves the only if direction of (7). If $\pi(d)$ is strictly concave then we are done. Otherwise, there may be multiple solutions to (7). Let $\{(u_A^s, d^s)\}_{t=0}^{\infty}$ and $\{(\hat{u}_A^s, \hat{d}^s)\}_{t=0}^{\infty}$ be two steady states satisfying (7) and (8). Then the slope between the two steady states is $\delta_A/(\theta\delta_P + (1-\theta)\delta_A)$. Inequality (11) now implies that both are V-contracts. To complete the proof, it suffices to show that any stationary contract $\{(u_{A,t} = u_A^s, d_t = d^s)\}_{t=0}^{\infty}$ satisfying (7) and (8) must satisfy the principal's interim participation constraint (2), that is, $U_A^s < U_A^{\max}$. This is true by Assumption 1. *Q.E.D.*

Lemma 3 establishes the first part of Theorem 1. It implies that the steady state is unique if there is a unique maximizer of (7), which is true outside of the knife-edge case when $\pi(d)$ has an entire edge with slope exactly equal to $(\delta_P - \delta_A)/(\theta\delta_P + (1-\theta)\delta_A)$. In the latter half of the analysis, we will make an assumption that eliminates the knife-edge case. So from now on, we will refer to a unique steady state. While Lemma 3 itself characterizes the steady state, the proof of Lemma 3 contains all the technical ingredients needed to show when and how non-steady-state V-contracts oscillate around the steady state. This will establish the second half of Theorem 1.

We begin the analysis by first supposing that the only available action is the steady-state action d^s. This is exactly the premise of our earlier primer on oscillation in Section 2. In that analysis, we argued that since IC constraints must bind, the continuation payoff $U_A^+(U_A)$ of an above- (below-) steady-state payoff U_A must be weakly below (above) the steady state, resulting in oscillation. The precise relation between the two values is

$$U_A^+(U_A) - U_A^s = -(1+r)\big(U_A - U_A^s\big), \tag{14}$$

where U_A^s is the steady-state agent payoff and $r := (\theta\frac{1+\delta_A}{\delta_A} - 1)/(1-\theta)$ is the growth rate of oscillation for the continuation payoff process. Per-period transfers u_A oscillate around the steady-state value u_A^s analogously.

For the rest of the paper, we will call these contracts described in the primer the benchmark contracts.

DEFINITION 1: Benchmark contracts keep the action fixed at d^s and set transfers to maximally exploit dynamic trading gains by keeping the IC constraint binding at all times.

We now show that when $\theta \le \frac{\delta_A}{1+\delta_A}$ and the growth rate r is nonpositive, V-contracts are essentially the benchmark contracts. But when $\theta > \frac{\delta_A}{1+\delta_A}$ and the growth rate r is positive, the benchmark contracts violate participation constraints and we explain how the true V-contracts become distorted. In particular, the action sequence also oscillates around d^s.

Case $\theta \in [0, \frac{\delta_A}{1+\delta_A}]$. Let U_A be any payoff $\in I := [(U_A^+)^{-1}(U_A^{\max}), U_A^{\max}]$. Because the growth rate r is nonpositive, the benchmark contract delivering payoff (U_A, U_P) does not violate participation constraints and is, therefore, feasible. The resulting payoff frontier is linear and goes through the steady-state payoff point:

$$\frac{U_P - U_P^s}{U_A - U_A^s} = -\frac{\delta_A}{\theta\delta_P + (1-\theta)\delta_A} \quad \forall U_A \in I. \tag{15}$$

Since V is weakly concave, a linear frontier is unimprovable and, therefore, each benchmark contract is a V-contract: $V(U_A) = U_P$ for $U_A \in I$.

In the special case $\theta = 0$, the growth rate is -1 and $U_A^+(U_A) = U_A^s$ for all U_A. Thus, I is the entire domain $(-\infty, U_A^{\max}]$ and every V-contract is a benchmark contract. To deliver U_A, the V-contract always calls for the steady-state action d^s. The agent receives an initial transfer $u_A = u_A^s + U_A - U_A^s$ and then receives the steady-state transfer u_A^s forever.

If $\theta > 0$, then it is possible that $U_A^{\min} < (U_A^+)^{-1}(U_A^{\max})$. For any $U_A \in [U_A^{\min}, (U_A^+)^{-1}(U_A^{\max})]$, the benchmark contract would violate the participation constraint of the principal in the next period. Since the agent continuation payoff is capped by $U_A^{\max}$, the date 0 action must be distorted downward to $d < d^s$, just enough to maintain incentive compatibility. From the next period onward, transfers resume oscillating according to (14). Thus, when $r \le 0$, participation constraints induce only *mild* distortions of V-contracts: by date 1 at the latest, a V-contract becomes a benchmark contract.

When $\theta \in (0, \frac{\delta_A}{1+\delta_A})$, $r < 0$ and oscillations are damped. Every V-contract converges to the steady state. When $\theta = \frac{\delta_A}{1+\delta_A}$, $r = 0$ and the oscillations persist in the long run. This establishes Theorem 1 up through $\theta = \frac{\delta_A}{1+\delta_A}$.

Case $\theta > \frac{\delta_A}{1+\delta_A}$. To see how things change, let us start at the agent's maximum payoff $U_{A,0} = U_A^{\max}$. Suppose the principal still tries to use the benchmark contract. Since oscillation is growing, the principal realizes that this contract will violate his own participation constraint the day after tomorrow. Thus, the agent's date 2 payoff must be adjusted downward. But this violates the IC constraint at date 1. To restore incentive compatibility at date 1, the contract can either stipulate to lower the action $d_1 < d^s$ or increase the agent's date 1 continuation payoff relative to the benchmark contract, so that

$$U_{A,1} = U_A^+\big(U_A^{\max}\big) > U_A^s - \frac{1}{1+r}\big(U_A^{\max} - U_A^s\big).$$

Intuitively, it is optimal to do a little bit of both. But now the date 0 IC constraint is slack and so the principal can increase the date 0 action $d_0 > d^s$ and reap the extra surplus. We have now shown $d_0 > d^s > d_1$. The degree to which actions are distorted depends on the degree to which participation constraints negatively impact the principal's payoff. Formally, the relationship is captured by (12), which relates the action to the slope of V. To ease the exposition and give contracts the ability to fine-tune action distortions, we assume the following differentiability condition.

ASSUMPTION 2: *The function $\pi(d)$ is a strictly concave, continuously differentiable function tracing out an interval of slopes that includes $[0, \frac{1}{\theta}]$ as a subset.*[11]

Assumption 2 simplifies the exposition substantially. Combined with (12), it ensures differentiability of the value function with $V'(U_A)$ ranging from 0 down to the lower bound -1. Also, the cumbersome directional derivative inequalities in the proof of Lemma 3 become simple derivative equations. In particular, (12) simplifies to

$$\pi'(d) = \frac{1 + V'(U_A)}{\theta}.$$

Since V is concave, (12) implies that action d and surplus $\pi(d)$ are weakly increasing in U_A, formalizing the intuition that distortions from the benchmark are optimally stronger the further is the distance from the steady state (with $V'(U_A^s) = -\frac{\delta_A}{\theta\delta_P + (1-\theta)\delta_A}$). Combined with (10) and (9), (12) also reveals how an

[11]Smaller surplus actions where $\pi'(d) \geq \frac{1}{\theta}$ can certainly be part of the action set, but they are never chosen in equilibrium.

action distortion today is optimally balanced against an opposing distortion tomorrow:[12]

$$\delta_P\left[\pi'\left(d^+\right)-\pi'\left(d^s\right)\right]=-\frac{1}{1+r}\left[\pi'(d)-\pi'\left(d^s\right)\right]. \tag{16}$$

The impact of tomorrow's distortion is naturally discounted by the principal's time preference, while the impact of today's distortion is discounted by the growth rate r of oscillation. Intuitively, the higher is r, the more severe are the distortions imposed by participation constraints in the future. As a result, the optimal action adjusts more today.

As long as $d < d_{\max}$, the associated optimal transfer sequence can be obtained from the action sequence and binding IC. The resulting implications of the action adjustments are separately analyzed in two subcases.

When $\theta \in (\frac{\delta_A}{1+\delta_A}, \frac{\delta_A}{\delta_A+\delta_P})$, action adjustments today are still relatively small compared to tomorrow's adjustments. As a result, transfers and action distortions explosively oscillate according to (16). The participation constraint of the principal will be reached in a finite number of periods. From then on, d and u_A perpetually oscillate between two distinct points, exhibiting long-run fluctuations.

When $\theta > \frac{\delta_A}{\delta_A+\delta_P}$, so that $\delta_P > \frac{1}{1+r}$, today's adjustment become so strong compared to tomorrow's that action distortions and transfers damped-oscillate and, hence, converge to the steady state. The economic environment exhibits no long-run fluctuations. In the limit, as $\theta \to 1$ ($r \to \infty$), the damped oscillation of V-contracts becomes trivial and there is monotonic convergence to the steady state. This establishes the second half of Theorem 1.

To demonstrate the rich set of possible contract dynamics described in Theorem 1, we compute a model based on Example 3. See Figure 2 for a panel summary. Effort of the worker, $e \in [0, 1]$, generates surplus $\pi(e) = 2e - e^2$. We vary the fraction of the wage, θ, that the worker loses upon shirking to illustrate the three relevant parameter regions. In the upper panels, $\theta \in (0, \frac{\delta_A}{1+\delta_A})$, so that benchmark contracts converge to the steady state. Transfers damped-oscillate while effort remains fixed at the stationary level e^s after date 0. In the intermediate region, $\theta \in (\frac{\delta_A}{1+\delta_A}, \frac{\delta_A}{\delta_A+\delta_P})$, both transfers and effort choices explosively oscillate until the participation constraint is reached for the first time in period 8. From period 8 onward, they each perpetually oscillate between two values. Finally, for $\theta > \frac{\delta_A}{\delta_A+\delta_P}$, both effort choices and transfers damped-oscillate to the steady state.

[12]This first-order condition holds if the participation constraint does not bind in the subsequent period.

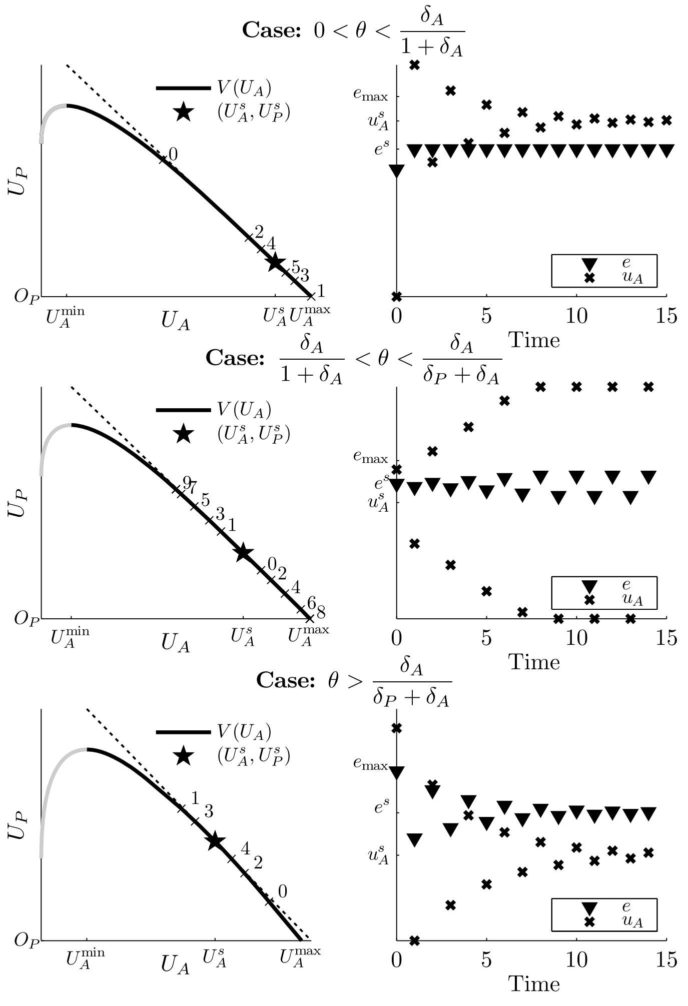

FIGURE 2.—Modified worker example. The left panels of the graphs plot the respective Pareto frontiers and the dynamics of continuation values given $U_{A,0}$. The dotted line through the steady state with slope $-\frac{\delta_A}{\theta\delta_P+(1-\theta)\delta_A}$ characterizes the value derived from benchmark contracts (15). The right panels plot the associated time series of effort choices, e_t, and transfers, $u_{A,t}$, given $U_{A,0}$. The value of $e_{\max} = 1$ corresponds to the surplus maximizing effort level. The steady state is characterized by effort e^s and transfer u_A^s.

A Final Remark

With the proof of Theorem 1 completed, we have now shown that the impatience versus incentives conflict causes V-contracts to oscillate around the steady state. The one requirement is that neither the agent nor the principal's participation constraint binds at the steady state; otherwise, the steady state would be at a corner of V and there would be no room to oscillate. We made sure that this requirement was satisfied in the analysis by assuming that there was no agent participation constraint and that the principal's participation constraint was sufficiently low (Assumption 1). In general, the analysis goes through if both participation constraints are present but do not bind at the steady state. But what if one of them binds? In this case, it is easy to show that when the agent's (principal's) participation constraint binds, all V-contracts monotonically converge to the steady state, which is at the left (right) corner of V.

Fix a model where the steady state is not at a corner of V, and consider an alternate version that shifts the d function up or down by a constant: $d(a) + x$, where $x \in \mathbb{R}$. As x increases, the deviation payoffs increase and one can interpret the incentive force as *getting stronger*. Similarly, as x decreases, the incentive force is *getting weaker*. Lemma 3 implies that a noncorner steady state shifts proportionally with x, which means there exists a bound $\overline{x} > 0$ ($\underline{x} < 0$) such that for all $x \geq \overline{x}$ ($x \leq \underline{x}$), the principal's (agent's) participation constraint will bind at the steady state. Combining this observation with the monotone convergence result of the previous paragraph, we now have a complete picture of the impatience versus incentives conflict.

REMARK 2: When the impatience force dominates the incentives force ($x \leq \underline{x}$), the steady state is the leftmost V-contract and all other V-contracts are front-loaded, monotonically converging leftward to the steady state. When the incentives force dominates the impatience force ($x \geq \overline{x}$), the steady state is the rightmost V-contract and all other V-contracts are back-loaded, monotonically converging rightward to the steady state. When neither force dominates ($\underline{x} < x < \overline{x}$), the impatience versus incentives conflict is nontrivial, and oscillation around the steady state is a generic feature of V-contracts.

Remark 2 helps put into context the opposing predictions of Ray (2002) and Lehrer and Pauzner (1999). As Ray (2002) points out, when "the agent is more impatient than the principal ... the Lehrer–Pauzner findings and the results of [Ray (2002)] tug in different directions. It may be worth exploring if one of the two factors always dominates." Our results not only show what happens when one factor dominates, but also reveal that often times neither factor dominates and oscillation is the natural outcome.

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