Optimism and Self-Selection Models:  
A Theoretical Investigation of the Rothschild-Stiglitz  
Insurance Model*

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Abstract: This paper analyzes the impact of biased beliefs on the structure of informational equilibria in classical self-selection models. Using the intuitive insurance market example of Rothschild-Stiglitz (1976) I show that biases of high-risk individuals have fundamentally different effects on equilibrium contracts than biases of low-risk individuals. Whereas equilibrium contracts of both groups are affected by biases of the high-risk group, equilibrium contracts are robust to moderate biases among the low-risk group. Specifically, optimism of the high-risk group results in underinsurance of their own group and causes negative externalities on the low-risk group by restricting the set of feasible contracts. If optimism is sufficiently strong, it is possible, that a breakdown of the insurance market occurs. This extreme result is more likely to have empirical relevance in insurance markets where losses are relatively small. In addition, I reveal that my analysis is not limited to the insurance market context by outlining a simple education screening model in the spirit of Spence (1972) that produces similar results.

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Introduction

Considerable evidence from the psychology literature suggests that human beings are systematically overconfident. A commonly documented feature of overconfidence is optimism – the tendency of humans to believe that favorable events are much more likely to occur to themselves relative to their peer group (Weinstein 1980, Kunda 1987). For example, Lehman and Nisbett (1985) find that people are convinced that their marriage will not fail even though they are fully aware that nearly 50% of the marriages in the US actually do fail. Similarly, Svenson (1981) shows that 80% of the drivers in Texas think they are above average drivers.

Whether deviations from full rationality, such as optimism, can significantly impact economic equilibria is an important question raised by Akerlof and Yellen (1985). They argue that even small deviations from maximizing behavior by one group can cause first order effects on an economy’s deadweight loss in the presence of externalities. Nonetheless, the importance of these effects has been largely ignored by the neoclassical theory for a long time due to doubts about the persistence of biases. The usual justification for this claim is that correcting market forces (“arbitrageurs”) will identify and exploit these mistakes of irrational individuals and will render them economically insignificant. Whereas the no-arbitrage concept applies well to financial markets where arbitrage transactions can be reasonably well executed – there is no reason to believe why biases should not matter in environments where no arbitrage strategy exists or cannot be executed (Limits of Arbitrage).

This paper analyzes the impact of biases on equilibrium contracts in self-selection models, where agents have asymmetric information and can impose externalities on outside parties. Due to limits of arbitrage, biases have the potential to disrupt the equilibrium that would be obtained otherwise. I will focus on settings where (potentially biased) individuals have private information about their characteristics and rational companies offer a menu of contracts designed such that

1 The reasoning is neatly summarized by the well known phrases “there shouldn't be a $500 bill on the side walk” (Lucas) or “there is no free lunch”.
2 Lamont/Thaler (2003) make the case that even financial markets are sometimes subject to mispricings using the example of Palm and 3Com.
3 See Shleifer/Vishny (1997).
the agent will select the contract that allows the company to break-even\(^4\). In contrast to individuals, firms are assumed to act rationally for two reasons. Firstly, companies with consistently biased assessments are eventually driven out of business due to lack of profitability. Secondly, firms have a higher routine level than individuals as transactions occur more frequently and are therefore able to make better judgments.

This paper contributes to the largely untapped research field of *contract design* by rational firms in the presence of biases among customers. DellaVigna and Malmendier (2004) show how rational firms adjust contracts in order to exploit time-inconsistent preferences of customers. Depending on the type of goods offered by the firms (*leisure goods* vs. *investment goods*), goods are either priced above or below marginal cost. Moreover, firms introduce switching cost and back loaded fees for all types of goods.

For motivational purposes, I choose to sacrifice generality by focusing on the Rothschild-Stiglitz insurance market setup (1976) to analyze the general equilibrium implications of biases among customers. I will later show that my findings can be transferred to other self-selection models, such as the education screening model of Spence (1973). Not only is the Rothschild-Stiglitz model one of the most important and well-known self-selection models, it is also perfectly suited for an analysis of biased beliefs as suggested by the above cited empirical evidence of Svenson (1981). The impact of behavioral distortions in the insurance market has been recently analyzed by Bracha (2004). Building up on the dual process theory from psychology she treats beliefs as choice variables and derives a Nash-equilibrium of beliefs and demand for insurance coverage\(^5\). Thus, her paper explains on the micro level how biased beliefs of individuals are shaped and how they are linked to insurance coverage (in the absence of asymmetric information). My paper takes a broader perspective by analyzing the general equilibrium effects of biases in asymmetric information settings while remaining agnostic about the source of biases among individuals\(^6\). I introduce biases into the Rothschild-Stiglitz model by grouping individuals into cohorts based

\(^4\) Free market entry prevents any firm from obtaining positive profits.

\(^5\) The dual process theory claims that individuals’ decisions are based upon two internal accounts: a rational account and a mental account. “In the insurance model context, the rational account decides on the insurance level that maximizes (perceived) expected utility, while the mental account chooses the risk perception that maximizes expected utility net of mental costs.” Thus, it is assumed that individuals derive utility from beliefs per se.

\(^6\) This question might become relevant, if the model is empirically implemented.
upon beliefs about their riskiness, since the perceived riskiness - as opposed to the actual riskiness - will drive the demand for insurance.

The general implication of biases on insurance demand is that individuals deviate from full insurance – the preferred protection level under rational beliefs – when they are confronted with actuarially fair insurance contracts (ignoring asymmetric information). Optimistic individuals seek less than full insurance, as they perceive their risk to be better than the terms offered by the insurance company\(^7\). However, deviating from full insurance coverage comes at a cost since individuals are assumed to be risk-averse. Thus, the preferred contract is characterized by a tradeoff between perceived mispricings and risk aversion. Less risk-averse individuals are more tilted to exploit perceived mispricings, meaning a greater deviation from the full insurance contract. Since an individual is effectively risk-neutral for small losses, the impact of biases is greater in this case.

The main implication of asymmetric information is that biases among the *high-risk group* fundamentally change the structure of equilibrium contracts (up to a breakdown of the market), whereas biases among the *low-risk group* do not matter for equilibrium contracts unless they are above a certain threshold. In this case, the restrictions caused by the high-risk group on contracts available to the low-risk group are not binding and asymmetric information is not critical. Thus, the theoretical analysis reveals that biases among the high-risk group have a greater impact on contracts. In addition to the greater impact of biases among the high-risk group, I argue that the high-risk group is also more likely to exhibit greater biases due to the following reasoning: By definition optimistic individuals overestimate their own quality. This might cause them to undertake risky actions such as “driving too fast” or “leaving doors unlocked” that they would not do if they were conscious about their true type. If this assertion is valid, biases among the high-risk group should be empirically relevant\(^8\).

The underlying intuition for the results is as follows: Optimism among the *high-risk group* does not only result in underinsurance of their own group, but it also causes externalities for the low-

\(^7\) In contrast, pessimistic individuals seek overinsurance.

\(^8\) Thus, in the insurance market context, quality (low riskiness) and optimism should be negatively related. This might be different in other contexts. For entrepreneurs or professional athletes, a degree of optimism might increase the chance of succeeding.
risk group by making it more costly for the good risks to separate themselves from the bad risks. Intuitively, optimism reduces the (perceived) opportunity cost of a high-risk individual to mimic a low-risk individual, such that the minimum required signal for separation is increased. Since signals of quality are made by accepting less insurance coverage, the set of feasible separating contracts for the low-risk group becomes smaller. If optimism among the high-risk group is sufficiently strong, they do not insure at all\(^9\). In this case, good risks cannot separate themselves from the bad risks and a breakdown of the insurance market occurs. In contrast, optimism among the low-risk group does not alter the contracts available to the high-risk group. Due to the presence of asymmetric information the low-risk group can only obtain partial insurance anyway. Therefore, biases among the low-risk group will not matter for equilibrium contracts, unless they reach a critical level. Once this critical level has been reached or surpassed, low-risk individuals seek even less insurance than the partial insurance contract that separates them from the high-risk group.

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\(^9\) It is assumed that individuals cannot choose contracts that force them to pay out in the loss state, i.e. the minimum insurance contract is given by no insurance.
The Model Framework

Setup

As outlined above, I consider a generalization of the classical Rothschild-Stiglitz insurance model that allows for behavioral biases and differential risk aversion across types. In order to facilitate readability, I stick to the notation of the original paper as much as possible. Even though the reader is supposed to be familiar with the original Rothschild-Stiglitz model, the main assumptions of the setup are repeated below:

1. Two states: Individuals face a random endowment $\tilde{E}$ which takes on the value $E_1$ in state 1 and $E_2$ in state 2. By convention, $E_1$ is larger than $E_2$, so that the difference of endowments in the two states yields the loss in state 2 ($L = E_1 - E_2$)

2. Insurance companies are risk-neutral and profit maximizing on competitive markets

3. Individuals possess the same “well behaved” von Neumann-Morgenstern utility-of-wealth function $U(W)$ (twice differentiable, strictly increasing, strictly concave) with

$$\lim_{W \to 0} U'(W) = \infty \text{ and } \lim_{W \to \infty} U'(W) = 0$$

4. Individuals are identical in all respects except for their loss probability. High-risk individuals face a loss with probability of $p_H$, low-risk individuals have a loss probability of $p_L$ (where $p_H > p_L$). Each individual knows his type perfectly

Whereas assumptions 1 to 3 are maintained throughout this paper, I will deviate from the fourth assumption in order to allow for biases. Individuals will be divided into two groups based upon their belief about their type, because perceived riskiness drives the demand for insurance coverage, and not the (unknown) true riskiness. Members of each group are assumed to share homogeneous beliefs about their own type. Of course, the belief of the first group is different from the one of the second group. For consistency purposes, I make a monotonicity assumption: The group which perceives itself as better (denoted as Group $L$) has on average a lower objective loss probability than the other group (denoted as Group $H$). I denote the subjective loss
probability as \( q \) and the true loss probability as \( p \)^{10}. The affiliation to the respective groups (low-risk versus high-risk) is denoted by the indices \( L \) and \( H \), respectively. Based on the just introduced notation and the assumptions made, the following relations have to hold:

\[
\begin{align*}
(1) & \quad q_L < q_H \\
(1)' & \quad p_L < p_H
\end{align*}
\]

Note that relation (1) and assumption 3) imply the single crossing property of indifference curves^{11}. It has to be emphasized, that I do not require group members to possess the same true (objective) loss probability. In fact, the objective loss probability of group \( i \) \( (p_i) \) can be interpreted as the conditional expected value of the loss probability given the person is a member of group \( i \). Assuming a continuous underlying probability distribution, \( p_i \) can be formally written as:

\[
(2) \quad p_i = \int_0^1 p \cdot f(p|\text{group } i) dp
\]

where \( f(p|\text{group } i) \) denotes the conditional p.d.f. of the “true” type in the group \( i \).

The average loss probability of the whole population is given by the weighted average of the conditional expectations, where \( \lambda \) denotes the proportion that belongs to the high-risk group.

\[
(3) \quad \bar{p} = \lambda p_H + (1 - \lambda) p_L
\]

Insurance companies are assumed to have knowledge about the population composition of types and the conditional means.

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^{10} Hence, I want to rule out completely hypothetical cases, such that on average „bad types think that they are good types“ and „good types think that they are bad types“. Nonetheless, I allow for such distortions on an individual level.

^{11} The single crossing property implies that indifference curves of both groups intersect at most once. Moreover, through any contract, the absolute value of the slope of the indifference curve will be greater for the low-risk group (see Jehle/Reny (2001)).
Insurance Demand

Lemma 1:
The conditional expected value of the loss probability is the only relevant variable for the decision of an individual under uncertainty about his type. Higher moments of the conditional distribution \( f(p|\text{group} \, i) \) do not matter.

The implication of Lemma 1 is that an individual who knows that his loss probability is 50% will choose exactly the same insurance contract as a person who is uncertain about his type and has either a loss probability of 90% or a loss probability of 10% with equal likelihood. The driver for this result is that the subjective “Expected Utility” concept by Savage (1964) assumes linearity in probabilities\(^{12}\). For sake of completeness, a detailed proof of Lemma 1 in this specific context is shown in Appendix A1.

For the following analysis, it will be convenient to have derived the demand for insurance of an individual with subjective (expected) loss probability \( q \) when being offered insurance contracts priced at loss probability \( p \). Thus, the objective exchange ratio for transferring funds from state 1 (the no-loss-state) to state 2 (the loss-state) is \( (1−p)/p \), i.e. giving up one unit in state 1 results in \( (1−p)/p \) more units in state 2. As in the original Rothschild-Stiglitz paper these contracts are fully determined by the two dimensional vector \( \alpha \), where \( \alpha_1 \) represents the premium for the insurance contract and \( \alpha_2 \) represents the net payment received in state 2 (benefit minus premium). Note, that I do not consider feasibility of these contracts in the presence of asymmetric information here. Rather, I want to derive a formal addendum to the graphical solution (tangency point of indifference curve with zero-profit line). An explanation of the general structure of the graphs is given in the Appendix A6.

\(^{12}\) See Barberis/ Thaler (2003).
Given the pricing of the contract, the customer with belief $q$ solves the following maximization problem.

$$V = \max \left( 1 - q \right) U \left( W_1 \right) + q U \left( W_2 \right) \quad \text{s.t.} \quad W_1 = E_1 - \alpha_1; \quad W_2 = E_2 + \alpha_1 \frac{1 - p}{p};$$

Due to the well behaved nature of the utility function, the first order condition is sufficient.

$$\left| MRS \right| = \frac{\left( 1 - q \right) U' \left( W_1 \right)}{q U' \left( W_2 \right)} = \frac{1 - p}{p}$$

It is possible to rewrite this equation as follows:

$$\frac{U' \left( W_1 \right)}{U' \left( W_2 \right)} = \frac{q}{1 - q} \frac{1 - p}{p} = \frac{\text{odds}_{\text{subjective}}}{\text{odds}_{\text{objective}}} \equiv \eta$$

It can be inferred from equation (6) that the ratio of marginal utilities in the two states is equalized with the ratio of the odds ratios of a loss under the two probability measures (subjective vs. objective) denoted as $\eta$. The parameter $\eta$ can be interpreted as a measure of perceived mispricings. A value greater than 1 indicates that consumers are overestimating their risk ($q > p$) whereas a value less than one indicates that consumers are underestimating their risk ($q < p$). Individuals that overestimate their risk will be referred to as pessimistic people whereas individuals that underestimate their risk will be classified as optimistic\(^{13}\).

In the original Rothschild Stiglitz setup the subjective assessment coincides with the objective risk assessment, so that marginal utilities of wealth are equalized across states. Due to the assumed strict concavity of the utility function, the consumer always seeks full insurance with the associated wealth level $W_F$ in both states. If we allow for biased opinions, it can be immediately verified from equation (6) that optimistic individuals ($\eta < 1$) will seek less than full insurance. Likewise, pessimistic individuals want to overinsure. Note, that this formalizes simply the obvious fact, that customers who assess their risk as better (worse) than the terms offered by the insurance company will want to purchase less (more) than the optimal insurance level.

\(^{13}\) Note, that one can also use equation (6) to determine the “rational” demand if the insurance company does not offer a contract priced according to the risk of the specific person (example: pooling equilibrium).
insurance company will seek less (more) insurance. Graphically, this implies that the tangency point of the indifference curve with the zero-profit line will be below (above) the 45° line from the origin (see Figure 1).

![Insurance Demand with Biased Beliefs](image)

**Figure 1: Insurance Demand with Biased Beliefs**

If the two probability measures (rational beliefs) coincide ($\eta = 1$) the degree of risk aversion is irrelevant for the optimal choice, i.e. full insurance is sought ($W_r$). This is no longer true if there is a wedge between objective and subjective odds. The consumer now has to trade-off his risk aversion with the perceived mispricing of the contract.

**Lemma 2:**

Low risk aversion implies that a consumer’s decision is more tilted towards exploiting perceived mispricings (further away from full insurance contract) whereas high risk aversion dampens the impact of mispricings. A rigorous proof of this proposition is shown in Appendix A2.
It is possible to solve for the deviation $\delta^*_{1}$ (see Figure 1) from the full-insurance contract in state 1 by a first order Taylor approximation of the optimality condition given by equation (6). A derivation is provided in Appendix A4.

\begin{equation}
\delta^*_{1} = \frac{1-\eta}{1+\eta \beta} \frac{1}{\gamma_{A}(W_{F})} \quad \text{where} \quad \beta = \frac{1-p}{p} > 0 \quad \text{and} \quad \gamma_{A}(W_{F}) = -\frac{U^{*}(W_{F})}{U'(W_{F})} > 0.
\end{equation}

The deviation in state 2 has the opposite sign and is given by:

\begin{equation}
\delta^*_{2} = -\frac{1-p}{p} \delta^*_{1}
\end{equation}

This result formally supports the verbal explanations given above. The direction of the deviation is solely determined by the mispricing parameter $\eta$. Optimism ($\eta < 1$) will cause the deviation from full insurance in state 1 to be positive, which implies that the wealth in the loss-state 2 will be smaller than the wealth in state 1. Pessimism ($\eta > 1$) has the opposite effect. Risk aversion – formalized by the absolute risk aversion coefficient $\gamma_{A}(W_{F})$ at full insurance level – decreases the impact of perceived mispricings on insurance demand. In the limit, an infinitely risk-averse consumer ($\gamma_{A} \rightarrow \infty$) will not exploit any (finite) mispricings. Graphically, this result is obvious because the limit value function is of Leontief type: $V = \min(W_{1},W_{2})$. In contrast, insurance demand by consumers that are close to being risk-neutral will be greatly influenced by perceived mispricings.

**Insurance Supply**

Since firms are assumed to be risk-neutral and profit-maximizing on competitive markets they will offer contracts as long these provide non-negative expected profits. The expected profit of a contract chosen by individuals with loss probability $p$ is given by:

\begin{equation}
\pi(p,\alpha) = (1-p)\alpha_{1} + p\alpha_{2} \geq 0
\end{equation}

\footnote{Note that $\eta > 0$ by definition.}
**Equilibrium Concepts**

The analysis of equilibrium contracts requires the definition of a sensible equilibrium concept. I outline three equilibrium concepts presented in the logical and chronological order although only the last equilibrium definition is applied to the subsequent analysis. This approach is chosen because the last equilibrium concept tries to explicitly address shortcomings of the first two concepts. For ease of exposition, individuals are assumed to have unbiased beliefs in the discussion of equilibrium concepts.

**Rothschild-Stiglitz Concept (1976)**

The original equilibrium concept used by Rothschild-Stiglitz is defined as:

1) *No contract in the equilibrium makes negative profits*

2) *There is no contract outside the equilibrium set that, if offered, will make a nonnegative profit*

The analysis of Rothschild-Stiglitz shows that no pooling equilibrium can be a competitive equilibrium, since the second restriction is violated. Thus, there is always a set of profitable contracts that will only attract the low-risk group and will cause the pooling equilibrium contract to incur losses (since only the bad types remain). As a consequence, only separating equilibria are feasible according to their concept. A separating equilibrium requires that it is in the interest of a bad type not to act like a good type (revelation principle). Since the high-risk group can get full insurance at their fair odds (contract $\alpha_H$ in Figure 2), the separating contract for the good type $\alpha_L$ must not provide the bad type with higher utility than the contract $\alpha_H$. Based on this intuition, the equilibrium separating contract is given by the contract on the low-risk zero-profit line that maximizes utility of the low-risk individual subject to the constraint that it is in the interest of the bad type to prefer $\alpha_H$ over $\alpha_L$. Graphically, this contract is given by the intersection point of the indifference curve of the high-risk type through contract $\alpha_H$ with the zero-profit line of the low-risk type (see Figure 2). Thus, the good type can only obtain partial insurance whereas the bad type obtains full insurance.
However, such an equilibrium does not always exist (see Figure 3). Nonexistence of an equilibrium in the Rothschild-Stiglitz sense occurs if there exists a profitable pooling contract that the low-risk type prefers over the separating contract $\alpha_L$. In such a case, it would not be in the interest of the low-risk type to bear the cost of separation. This implies a simple check for the existence of the Rothschild-Stiglitz equilibrium. Let me define the Wilson contract $\alpha_W$ as the contract on the average-risk zero-profit line that provides maximum utility to the low-risk individual. The implication of the analysis in the previous section is that this contract will be below the 45° line and will make zero expected profits. A necessary and sufficient condition for the existence of a competitive equilibrium is that the utility derived from the separating contract $\alpha_L$ is higher than the utility derived from the pooling contract $\alpha_W$.\(^{15}\)

\(^{15}\) If the separating contract dominates the pooling contract that maximizes utility for the low-risk type ($\alpha_W$), it will also dominate all other profitable pooling contracts.
Wilson Concept (1977)

Wilson uses a different equilibrium concept than Rothschild-Stiglitz which causes the contract \( \alpha_w \) to become the equilibrium contract in cases where otherwise no equilibrium exists. “The firms’ expectations are modified by assuming that each firm will correctly anticipate which policies in the offers of other firms will become unprofitable as a consequence of any changes in its own offer”\(^{16}\). Due to this modification the Wilson contract is sustainable as an equilibrium contract even though other profitable contracts (\textit{in the presence of the Wilson contract}) can be offered which only attract the low-risk group. However, these new contracts can only engage in cream skimming, i.e. just attracting the high-quality types, as long as the contract \( \alpha_w \) is offered.

Wilson’s argument is that \( \alpha_w \) would be immediately withdrawn once a new contract targeted at

the low-risk group is offered, such that these new contracts would not only attract the low-risk types but also the high-risk types and hence become unprofitable. Therefore, these new contracts are not offered in equilibrium and the Wilson contract represents the equilibrium contract.

A Refinement

In a related working paper (Opp 2005), I propose an equilibrium refinement that tries to remedy the shortcomings of the previously described equilibrium concepts: The Rothschild-Stiglitz definition does not guarantee existence and the Wilson definition relies on the (unrealistic) assumption that insurance companies can “immediately” withdraw contracts. By introducing commitment and non-static expectations (similar to the Wilson concept) of the insurance company the existence of equilibria is guaranteed. Commitment implies that insurance companies cannot “immediately” withdraw their contracts\(^\text{17}\). Non-static expectations imply that companies do not only take into account the contracts currently offered by other companies, but also anticipate the reactions of other players to its own offer\(^\text{18}\).

\[\text{Figure 4: Pooling Equilibria and Cream-Skimming}\]

\(^{17}\) It is required that insurance companies have a sufficiently longer commitment time than customers.

\(^{18}\) The idea is related to Riley’s “reactive” informational equilibrium concept (see Riley (1979)).
In contrast to Wilson, I allow the offer of new contracts as potential reactions\(^{19}\). Based on this rationale, companies do not offer pooling contracts, that are subject to cream-skimming (the risk of losing only the good risks to other companies) because commitment prevents them from immediately withdrawing their contract. The cream-skimming region is given by the yellow shaded area in Figure 4. Suppose that company \( A \) offered the pooling contract \( \alpha_w \) anyways, then it would be in the interest of any other firm \( B \) to offer contracts targeted only at the low-risk individuals (like contract \( \alpha_g \)) as long as company \( A \) is committed to its contract \( \alpha_w \)^{20}. This would render company \( A \) unprofitable. Hence, in order to avoid losses, no company offers pooling contracts that are subject to cream-skimming in the first place. Due to the assumption of an identical utility-of-wealth function of all individuals and the implied single crossing property of indifference curves, all pooling contracts will be subject to cream-skimming in this setup\(^{21}\). This implies that pooling contracts will not be offered and the separating contracts proposed by Rothschild-Stiglitz will surely represent the equilibrium contracts.

**Analysis**

I will analyze the effects of systematic biases on insurance markets using the last equilibrium concept described\(^{22}\). Specifically, I will investigate how the structure of separating contracts (partial insurance for good types, full insurance for bad types) is affected. Even though biases in the form of optimism seem to be empirically more relevant, I will also theoretically examine the effects of pessimism. The effects of these biases will be analyzed separately for each cohort. This approach is not only motivated by expositional purposes but is also theoretically justified: The existence of asymmetric information implies that the presence of bad types causes a negative externality on good types since they can no longer obtain the same contracts as in the case of perfect information. In contrast, the existence of asymmetric information does not reduce the set of available contracts to the high-risk group. This nonsymmetrical structure of externalities

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\(^{19}\) Wilson only allows for withdrawals.  
\(^{20}\) Of course these new contracts are also subject to the same commitment, but this does not matter for these contracts, because they only attract the low-risk group.  
\(^{21}\) Once the assumption of identical utility-of-wealth functions is dropped, it is possible that a pooling contract which is not subject to cream-skimming represents the equilibrium contract. This requires the low-risk group to be sufficiently more risk-averse than the high-risk group. This pooling contract is given by the contract on the average zero-profit line that equalizes the MRS of both cohorts.  
\(^{22}\) This also simplifies the analysis in the sense that I do not have to “worry” about nonexistence of equilibria or different types of equilibria (pooling vs. separating equilibria).
suggests the following logical order for the analysis of separating equilibrium contracts. Firstly, I determine the separating contract of the high-risk group \((a_H)\), i.e. the preferred contract of the high-risk group on their zero-profit line. This choice restricts the set of available contracts to the low-risk group because a high-risk individual must not prefer any contract from this set to \(a_H\). In a second step, I determine the optimal choice of the low-risk individual given the feasible set of separating contracts.

In order to keep the analysis realistic, I will only consider contracts that are between the 45° line (full insurance) and the endowment (no insurance). I rule out the contracts above the 45° line due to moral hazard issues (i.e. consumer would be better off in case of a loss) and the ones below the endowment, since it would imply that individuals have to pay out in the loss state (underwriting insurance based on own risk)\(^{23}\).

**Biases of the High-Risk Group**

**Optimism**

Optimism among the high-risk group causes their preferred contract to shift southeast from full insurance coverage towards the endowment (see Figure 1). Since the high-risk individuals do not have to signal their type, their preferred contract on the high-risk zero-profit line is also feasible. Not only does the high-risk group harm itself (ex post) by underinsurance, but they also cause a negative externality for the low-risk group by further limiting the separating contracts available to the low-risk group\(^{24}\). The intuitive explanation for Proposition 1 is: Overestimating their own quality, it seems less costly for the high-risk group to mimic the low-risk group. Thus, the low-risk group can only credibly reveal their quality by accepting even less insurance.

**Proposition 1:**

*Optimism among the high-risk group reduces the set of available separating contracts to the low-risk group.*

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\(^{23}\) In other institutional setups, it might well be the case that such perverse behavior analogously to “underwriting insurance based on own risk” can occur. In the literature this phenomenon is called a “Texas Hedge”.

\(^{24}\) Note that the analysis of optimism does not require each member of the group to be optimistic. This has just to be true on average.
**Proof of Proposition 1**

Let me introduce the following notation:

- $W_o^*$: Wealth vector associated with the preferred contract on the high-risk zero profit line under optimistic beliefs
- $W_r^*$: Wealth vector associated with the preferred contract on the high-risk zero profit line under rational beliefs (full insurance)
- $I_o(W)$: Indifference curve through wealth vector $W$ under optimistic beliefs
- $I_r(W)$: Indifference curve through wealth vector $W$ under rational beliefs

It suffices to show that the intersection point of $I_o(W_o^*)$ with the low-risk zero profit line lies further to the southeast than the intersection point of $I_r(W_r^*)$ with the low-risk zero profit line. The intersection points, that determine the maximum insurance contract available to the low-risk group, are labeled $\alpha(o)_{\text{max}}$ and $\alpha(r)_{\text{max}}$. Since $W_r^*$ is the preferred contract on the high-risk
zero profit line under rational beliefs, it is true that \( I_r(W_o^*) \) will always lie below \( I_r(W_r^*) \), which implies that \( I_r(W_o^*) \) will intersect the low-risk zero profit line further to the southeast than \( I_r(W_o^*) \). Moreover, we know that the absolute value of the MRS through \( W_o^* \) under rational beliefs will be smaller than under optimistic beliefs, (precisely, we have
\[ \text{MRS}_r(W_o^*) = \frac{q_H}{p_H(1-q_H)} \text{MRS}_o(W_o^*). \] In combination with the single crossing property, we thus obtain that \( I_o(W_o^*) \) will lie above \( I_o(W_o^*) \) to the right of the intersection at \( W_o^* \). This implies that \( I_o(W_o^*) \) will intersect the low-risk zero profit line further to the southeast than \( I_r(W_o^*) \) and hence further to the southeast than \( I_r(W_r^*) \), too. \( \text{Q.E.D.} \)

If optimism among the high-risk group is sufficiently strong, it is even possible that a high-risk individual does not want to insure at all and remains at the endowment \( E \). The critical threshold belief is denoted \( q_H^* \), and is defined as the belief which causes the tangency point of the indifference curve with the high-risk zero-profit line to be at the endowment. For all beliefs \( q_H \leq q_H^* \) a high-risk individual seeks no insurance. The specific value of \( q_H^* \) can be calculated after rearranging equation (6).

\[ q_H^* = \frac{U'(E_1)}{1 - p_H \cdot U'(E_2) + U'(E_1)} \approx \frac{p_H}{1 + L \cdot (1 - p_H) \cdot \gamma_A(E_i)} \text{ where } \gamma_A(E_i) = -\frac{U^*(E_1)}{U'(E_1)}. \]

The approximation that follows the definition is derived in Appendix A4. Moreover, it is shown in Appendix A5 that the critical belief about the loss probability that causes non-insurance \( q_H^* \) is smaller than the objective loss probability \( q_H \leq p_H \) (definition of optimism) and that it is a decreasing function of risk aversion and the loss \( L = E_1 - E_2 \)^25. This result simply states that even small biases can result in non-insurance of the high-risk type if risk aversion is relatively low and the losses are not large.

---

^25 Note that \( q_H^* \) could be smaller than \( q_L \) and would violate the monotonicity assumption stated in equation (1) in these instances.
Of course, non-insurance of the high-risk group implies that the low-risk group will be unable to separate themselves from the bad cohort. Thus, the only feasible equilibrium is given by non-insurance of both types or a failure of an insurance market \textit{irrespective of the beliefs of the low-risk group.}

\textbf{Pessimism}

Pessimism among the high-risk group has the opposite general equilibrium effect (see Figure 6). As stated in the introduction, I rule out overinsurance due to moral hazard reasons so that pessimistic persons will seek full insurance at fair odds. Regardless of this restriction, pessimism among the high-risk group will cause positive externalities on the low-risk group, as the set of feasible separating contracts for the low-risk type is increased. The proof of this statement follows the same reasoning as before and is therefore omitted. The contracts $\alpha(p)_{\text{max}}$ and $\alpha(r)_{\text{max}}$ characterize the maximum insurance coverage available to the low-risk group under pessimistic and rational beliefs, respectively.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{positive_externalities_pessimism_group_h.png}
\caption{Positive Externalities of Pessimism among the High-Risk Group}
\end{figure}
The general equilibrium effect of risk aversion is similar to the effect of biases because both affect the opportunity cost of pretending to be a low-risk person. Less risk-averse high-risk individuals are much more likely to mimic low-risk individuals. This causes a negative externality on the low-risk group. Of course, the effects of risk aversion and biasedness are interactive. For example, low risk aversion exacerbates the effect of optimism among bad types on the contracts available to the good types. The opposite is true for high risk aversion which limits the desire to exploit perceived mispricings (see result of Lemma 2).

**Biases of the Low-Risk Group**

The results of the previous section imply that the set of available contracts to the low-risk group is limited to partial insurance contracts. It has been shown, that the separating contract that provides maximum insurance to the low-risk group $\alpha_{\text{max}}$ is influenced by the beliefs and degree of risk aversion of the high-risk group due to externalities in the presence of asymmetric information. Whether a low-risk individual chooses this contract or even less insurance depends on his degree of optimism. Only highly optimistic individuals deviate from the contract $\alpha_{\text{max}}$.

Such optimistic beliefs are characterized by a greater MRS through the contract $\alpha_{\text{max}}$ than the slope of the zero-profit line. By rearranging equation (5), I obtain the critical belief $q_L^{**}$:

\[
q_L^{**} = \frac{U'(I_1)}{1 - \frac{p_L}{p_L} U'(I_2) + U'(I_1)} \quad \text{where} \quad I \equiv E - \alpha_{\text{max}}
\]

Since $q_L^{**}$ is a function of $\alpha_{\text{max}}$ the critical belief of the low-risk group depends on the beliefs and risk aversion of the high-risk group. For more optimistic beliefs than $q_L^{**}$, i.e. $q_L \leq q_L^{**}$, the individual seeks less insurance than $\alpha_{\text{max}}$ and the restrictions caused by the high-risk group are non-binding. This implies that asymmetric information is not really an issue here, because the resulting equilibrium contracts are equivalent to the case where the insurance company can tell which cohort a customer belongs to.

---

26 This is why the case of pessimism is not analyzed separately.
The low-risk person will seek no insurance at all if \( q_L < q_L^* \) where \( q_L^* \) is defined analogously to equation (10) (belief that causes tangency point of indifference curve to be at the endowment).

\[
q_L^* \equiv \frac{U'(E_i)}{1-p_L U'(E_2)+U'(E_i)} \approx \frac{p_L}{1+L \cdot (1-p_L) \cdot \gamma_A(E_i)} < q_L^{**}
\]

In contrast to \( q_L^{**} \) the belief \( q_L^* \) is independent of the beliefs and risk aversion of the high-risk group. The optimal contract choices under beliefs \( q_L^* \) and \( q_L^{**} \) are illustrated in Figure 7. In this graph, the high-risk group is assumed to have rational beliefs.

---

**Figure 7: Effect of Optimism among the Low-Risk Group**
Empirical Implications

Since empirical findings suggest that optimism is much more likely to be present in reality, the equilibrium contracts should be characterized by underinsurance of both groups. Unless the low-risk group is extremely biased, I expect the restriction imposed by the high-risk group to be binding which implies that the high-risk group causes welfare losses for the low-risk group. This typical case is depicted in Figure 8.

![Typical Equilibrium Contracts](image)

Figure 8: Typical Equilibrium Contracts

The extreme implication of biases, given by a breakdown of insurance markets, seems to be most likely in insurance markets, where the typical loss is relatively small. In this case, my analysis suggests that required deviations from fully rational beliefs are small (equations (10) and (12)).

I will conclude this section with an interpretation of the results from a slightly different perspective. As explained above, members of each group solely share the same belief but do not necessarily possess the same type. Thus, my analysis is still totally valid if only a subgroup of
each cohort is systematically biased while the rest has completely rational beliefs. It is obvious that an individual \( j \) who overestimates his own risk \( (q_j > p_j) \) causes a positive external externality to his own cohort, because he drives down the average objective risk of the cohort (relative to the belief). Likewise, individuals that underestimate their risk \( (q_j < p_j) \) cause a negative externality to their own group. However, as the analysis above reveals, the externalities of individual biases in the high-risk group are not limited to the own cohort, but will also matter for the equilibrium contracts of the whole low-risk group. As such, optimistic individuals among the high-risk group reduce the welfare of all other individuals, whereas pessimistic individuals have welfare enhancing effects on others.

**Implications for Other Self-Selection Models**

The purpose of this section is to reveal that the implications of my analysis are not limited to the institutional context of insurance markets. As claimed above, any asymmetric information environment that is characterized by private information of individuals can be affected by biases.

I want to outline the simplest education screening model in the spirit of Spence (1973). I assume that the employer moves first by setting a specific hurdle for a job (example: MBA required) and potential employees have to self-select whether the cost of education is worth acquiring the signal. For simplicity, I do not account for important institutional details such as abandoning or failing the education program. Two groups of potential employees with following characteristics exist:

<table>
<thead>
<tr>
<th>Group</th>
<th>True Marginal Productivity</th>
<th>Subjective Marginal Productivity</th>
<th>Proportion of Group</th>
<th>Perceived Cost of Education</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( p_1 )</td>
<td>( q_1 )</td>
<td>( 1 - \lambda )</td>
<td>( y / q_1 )</td>
</tr>
<tr>
<td>II</td>
<td>( p_2 )</td>
<td>( q_2 )</td>
<td>( \lambda )</td>
<td>( y / q_2 )</td>
</tr>
</tbody>
</table>

Table 1: Setup of the Education Screening Model

---

27 This is true because the cohort is still biased on average.
28 Recall that cohorts are formed on beliefs.
I assume that the second group is more productive than the first group. Moreover, I make the same monotonicity assumption as above: Individuals which perceive themselves as better are also of higher quality on average. Higher productivity translates into lower cost of education. Note, that in contrast to the insurance model a higher value of $p$ implies higher quality. Thus, I require:

\[(13) \quad p_2 > p_1, \quad q_2 > q_1\]

Firms are assumed to maximize profits and act competitively. Hence, they pay out the (expected) marginal productivity as wages $(w)$. In a separating equilibrium, individuals of group 1 and 2 get paid according to their respective marginal productivities. In a pooling equilibrium, wages are based on the average productivity.

\[(14) \quad \bar{p} = p_1(1-\lambda) + \lambda p_2 = \lambda (p_2 - p_1) + p_1\]

Individuals are assumed to maximize their net payoff based on their wages and the perceived cost of education:

\[(15) \quad U_i = w_i - \frac{y}{q_i}\]

A separating equilibrium relies on the idea that it is not in the interest of the low-quality group (group 1) to mimic the education level of the high-quality group. Thus, it is required, that

\[(16) \quad p_1 \geq p_2 - y / q_i \quad \leftrightarrow \quad y \geq (p_2 - p_1)q_i\]

Hence, the education level $y^* = (p_2 - p_1)q_i$ is the minimum required education level to signal high productivity. Therefore, the two groups can achieve following utility levels in a separating equilibrium:

\[(17) \quad U_1^{sep} = p_1\]

\[(17)' \quad U_2^{sep} = p_2 - \frac{y^*}{q_2} = p_2 - (p_2 - p_1)\frac{q_1}{q_2}\]

Since the difference in marginal productivities of group 2 and group 1 is positive, i.e. $p_2 - p_1 > 0$, we can immediately infer from equation (16) that optimism $(q_i > p_i)$ of the low quality group 1
will cause a negative externality on the high-quality group 2 because it increases the minimum required signal $y^*$. In contrast, pessimism among the less productive group will cause a positive externality on the high-quality group. It can be seen that the minimum required signal only depends on the belief of the low-quality group 1. These statements are true conditional on the existence of a separating equilibrium. However, if the cost of separation for the productive group is too high, a pooling equilibrium with the following payoff for both groups will occur:

(18) $U_{pool}^1 = U_{pool}^2 = \bar{p}$

The condition for the sustainability of a separating equilibrium can be obtained by comparing the payoff for the high-quality person in both equilibrium candidates (equation (17)' and (18)):

(19) $\frac{q_1}{q_2} \leq (1 - \lambda)$

Optimism of good types (group 2) and (or) pessimism of bad types will make a separating equilibrium more likely. If beliefs become closer, i.e. $q_1 / q_2 \to 1$, we can observe a tendency to pool; if they diverge, we can observe a tendency to separate.

Note, that this implies a difference to the insurance market model, where pooling equilibria are not feasible. Other differences arise from the restrictions on the set of available signals in the insurance setup (contracts are assumed to lie between full insurance and the endowment). For sake of simplicity, I have decided against a more realistic treatment of the education model, where restrictions on the signal $y$ could also be motivated. For example, a Ph.D. is usually the highest form of education required which would imply the existence of a $y_{\text{max}}$. Nonetheless, the general principle of the analysis should be clear. Biases affect all self-selection models in quite the same way. Conditional on the existence of separating equilibria, biases of the low-quality group will impose externalities on the high-quality group. Optimism of the low-quality group results in negative externalities, as the minimum required signal is increased. Pessimism has the opposite effects. The education model example reveals an additional interesting implication of biases: Once the existence of pooling equilibria is not ruled out (as in

29 Of course, this depends on the equilibrium concept applied.
the described insurance market setup), biases of the high-quality group matter for the type of equilibrium obtained (pooling vs. separating equilibrium). Intuitively, if beliefs of the two groups converge relative to rational beliefs, a pooling equilibrium is more likely.

**Conclusion**

My analysis of the Rothschild-Stiglitz insurance market model provides a framework of how to incorporate biases into other self-selection models as shown by its application to Spence’s education screening model. It would be interesting to augment the analysis of this paper by characterizing the most general class of models my ideas apply to and deriving the structural solution for any member of this class\(^30\).

Any empirical test of my predictions will be challenged by data limitations. For example, nonexistence of certain insurance markets – the most extreme outcome of my analysis – is not reflected in any dataset for obvious reasons. A potentially more interesting way to empirically implement my model is to try to measure welfare losses of the low-risk group caused by optimism of the high-risk group\(^31\).

\(^{30}\) This approach would be very similar to Riley’s model of informational equilibria (1979)

\(^{31}\) This approach would require measures of wealth, risk aversion, riskiness, potential loss and optimism. It seems that manual collection of data (for example telephone interviews) would be required. Moreover, markets with relatively homogeneous individuals in terms of wealth and risk aversion would be preferable.
Appendix:

A1: Proof of Lemma 1

In our simple setup, the individual is either member of group $L$ or group $H$ and faces uncertainty about his own type as described by the conditional p.d.f. of the true type of his group. Thus, the only information available to the consumer is his affiliation to a certain group. Therefore, the optimal contract choice can only depend on this information (and not the true type). If he knew his true type the expected utility of a contract $\alpha$ with premium $\alpha_1$ and net benefit $\alpha_2$ is given by:

\[ (20) \quad EU(\alpha, p) = pU(W_2) + (1-p)U(W_1) \quad W_1 = E_1 - \alpha_1 \quad W_2 = E_2 + \alpha_2 \]

Thus, under uncertainty the expected utility of contract $\alpha$ is given by:

\[ (21) \quad EU(\alpha|\text{group } i) = \int_0^1 f(p|\text{group } i)EU(\alpha, p)dp \]

Recalling the definition of the conditional expected value of the loss probability for each group (see equation (2)) we obtain

\[ (22) \quad EU(\alpha|\text{group } i) = p_iU(W_2) + (1-p_i)U(W_1) \]

Due to von Neumann-Morgenstern preferences and the associated linearity in probabilities, a rational consumer who is uncertain about his own type behaves exactly like a consumer that knows his own type and possesses a loss probability equal to the average loss probability in his cohort group. Thus, a person does not care about uncertainty about his loss probability\(^{32}\).

---

\(^{32}\) Note, that this is true, because the underlying loss does not depend on the type of the person. One could imagine cases where bad risks also face potentially higher losses. In these circumstances, the result would no longer hold.
A2: Proof of Lemma 2:

Preliminary definitions

We can rewrite the optimality condition (equation (6)) as:

\[
\frac{U'(W_1^*)}{U'(W_2^*)} = \frac{U'(W_F + \delta_1^*)}{U'(W_F + \delta_2^*)} = \eta
\]

Since the optimal contract \( W^* \) must be on the zero profit line (tangency point) we can substitute:

\[
\delta_2^* = -\delta_1^* \frac{1-p}{p}
\]

Thus, we obtain:

\[
U'(W_F + \delta_1^*) = \eta U'(W_F - \delta_1^* \frac{1-p}{p})
\]

For simplicity, I will introduce the following notation for the purpose of this proof:

\[
g(x) \equiv U'(x), \quad \gamma(x) \equiv \frac{-g'(x)}{g(x)}, \quad F \equiv W_F, \quad y \equiv \delta_1^*, \quad \beta \equiv \frac{1-p}{p}
\]

The function \( g(x) \) represents marginal utility \( g(x) > 0, g'(x) < 0 \), the function \( \gamma(x) = \frac{-g'(x)}{g(x)} \) stands for the absolute risk aversion, i.e. the normalized curvature at point \( x \), and \( y \) represents the deviation from the full insurance contract in state 1.

Based on this notation, the first order condition (25) becomes:

\[
g(F + y) = \eta g(F - \beta y)
\]

A solution \( y \) to equation (26) is guaranteed under the assumed structure of the utility-of-wealth function. The goal of this proof is to show, that higher risk aversion implies smaller absolute deviations from full insurance. Higher risk aversion of utility functions is formalized by \( \gamma_0(x) > \gamma_1(x) \ \forall x \) for otherwise arbitrary functions \( g_0(x) \) and \( g_1(x) \). Now, it has to be shown that \( \gamma_0(x) > \gamma_1(x) \ \forall x \) implies \( |y_1| > |y_0| \). Without loss of generality, I assume that \( \eta < 1 \).
(optimism). This implies that \( y_i > 0 \) (more wealth shifted to state 1). Hence, the goal of the proof is:

**WTS:** \( \gamma_0(x) > \gamma_1(x) \ \forall x \implies y_i > y_0 \)

For this proof, I will need the following lemma:

**Lemma 3:**
If \( \gamma_0(x) > \gamma_1(x) \ \forall x \) and \( g_o(z) = g_1(z) \) for some \( z > 0 \), then \( |g_0'(x)| > |g_1'(x)| \ \forall x < z \)

(The proof of this Lemma is shown below this proof).

**Proof Strategy:**
I will do a proof by contradiction. Thus, I assume that \( y_0 \geq y_1 \) and show that the relation \( \gamma_0(x) \geq \gamma_1(x) \ \forall x \) cannot hold at the same time.

**Proof**
Since the functions \( g_i \) can be interpreted as marginal utilities, any scaling by a positive constant does not affect the optimal deviation \( y_i \). I will scale them in such a way that:

(27) \( g_o(F + y_0) = g_1(F + y_0) \)

We can rewrite both functions \( g_i(F - \beta y_i) \) in the following way:

(28) \( g_i(F - \beta y_i) = g_i(F + y_i) - \int_{F-\beta y_i}^{F+y_i} g_i'(x) dx \)

Plugging the expression for \( g_i(F - \beta y_i) \) from equation (28) into the optimality condition of equation (26) yields:

(29) \( g_i(F + y_i) = \eta \left[ g_i(F + y_i) - \int_{F-\beta y_i}^{F+y_i} g_i'(x) dx \right] \)

We can rewrite this equation as:
\[ \int_{F - \beta y_i}^{F + y_i} g_i' (x) \, dx = \eta \int_{F - \beta y_i}^{F + y_i} \left| g_i' (x) \right| \, dx \]

It is helpful to introduce the following definition:

\[ \Delta \equiv g_i (F + y_i) - g_i (F + y_0) \]

Under the assumption that \( y_0 \geq y_i \), the term \( \Delta \) will be nonnegative since \( g \) is a decreasing function. Using the definition of \( \Delta \) we can substitute for \( g_i (F + y_i) \) in equation (30):

\[ \left[ \Delta + g_i (F + y_0) \right] (1 - \eta) = \eta \int_{F - \beta y_i}^{F + y_i} \left| g_i' (x) \right| \, dx \]

Now, we can rewrite equation (32) as:

\[ g_i (F + y_0) (1 - \eta) = \eta \int_{F - \beta y_i}^{F + y_i} \left| g_i' (x) \right| \, dx - (1 - \eta) \Delta \]

Moreover, we know from equation (30):

\[ g_0 (F + y_0) (1 - \eta) = \eta \int_{F - \beta y_0}^{F + y_0} \left| g_0' (x) \right| \, dx \]

By the definition of the two functions, the left hand side of equations (33) and (34) is the same (see equation (27)). Thus, the right hand side must also be equal. We obtain after rearranging:

\[ \int_{F - \beta y_0}^{F + y_0} \left| g_0' (x) \right| \, dx - \int_{F - \beta y_i}^{F + y_i} \left| g_1' (x) \right| \, dx = -\frac{(1 - \eta)}{\eta} \Delta \]

Since \( y_0 \geq y_i \) by assumption, we can split up the integral in the following way:

\[ -\frac{(1 - \eta)}{\eta} \Delta = \int_{F - \beta y_0}^{F + y_0} \left| g_0' (x) \right| \, dx - \int_{F - \beta y_i}^{F + y_i} \left| g_1' (x) \right| \, dx \]

\[ = \int_{F - \beta y_0}^{F + y_0} \left| g_0' (x) \right| \, dx + \int_{F - \beta y_i}^{F + y_i} \left| g_0' (x) \right| \, dx + \int_{F - \beta y_0}^{F + y_0} g_0' (x) \, dx - \int_{F - \beta y_i}^{F + y_i} g_1' (x) \, dx \]

\[ = \int_{F - \beta y_i}^{F + y_i} \left| g_0' (x) \right| - g_1' (x) \, dx + \int_{F - \beta y_0}^{F + y_0} \left| g_0' (x) \right| \, dx + \int_{F + y_i}^{F + y_0} g_0' (x) \, dx \]
This yields:

\[
(37) \quad -\left[ \frac{(1-\eta)}{\eta} \Delta + \int_{F-\beta y_1}^{F-\beta y_0} |g_0'(x)| dx + \int_{F+\beta y_0}^{F+\beta y_1} |g_0'(x)| dx \right] = \int_{F-\beta y_1}^{F+\beta y_1} |g_0'(x)| - |g_1'(x)| dx
\]

Now, let us evaluate the sign of each term of the left-hand side under the assumption that \( y_0 \geq y_1 \)

a) \( \frac{(1-\eta)}{\eta} > 0 \) \quad since \( 0 < \eta < 1 \)

b) \( \Delta = g_1(F + y_1) - g_1(F + y_0) \geq 0 \) \quad since \( g \) is decreasing

c) \( \int_{F-\beta y_0}^{F-\beta y_1} |g_0'(x)| dx \geq 0 \) \quad since \( y_0 \geq y_1 \)

e) \( \int_{F+\beta y_0}^{F+\beta y_1} |g_0'(x)| dx \geq 0 \) \quad since \( y_0 \geq y_1 \)

Thus, the left hand side is strictly nonpositive. So the right hand side cannot be positive, either. However, according to lemma 3 it is true that \( \gamma_0(x) \geq \gamma_1(x) \) \( \forall x \) implies that \( |g_0'(x)| - |g_1'(x)| \geq 0 \) \( \forall x < F + y_0 \). Thus, it cannot be true that \( \gamma_0(x) \geq \gamma_1(x) \) \( \forall x \). This is a contradiction, because we assumed in the beginning \( \gamma_0(x) \geq \gamma_1(x) \) \( \forall x \).

Q.E.D.

**A3: Proof of Lemma 3**

If \( \gamma_0(x) > \gamma_1(x) \) \( \forall x \), it must also be true at \( x = z \) such that we have:

\[
(38) \quad -\frac{g_0'(z)}{g_0(z)} > -\frac{g_1'(z)}{g_1(z)}
\]

But since \( g_0(z) = g_1(z) \) by definition, we can simplify the relation in equation (38) to:

\[
(39) \quad |g_0'(z)| > |g_1'(z)|
\]

Thus a small deviation \( \epsilon \) from \( z \) has stronger effects on the function \( g_0 \) than on \( g_1 \). Since the functions are decreasing, there exists a \( \delta > 0 \) such that:

\[
(40) \quad g_0(z) = g_1(z) < g_1(z-\delta) < g_0(z-\delta)
\]
Since $\gamma_0(x) > \gamma_1(x) \; \forall x$ it must also be true for $v = z - \delta$

\[(41) \quad \frac{|g_0'(v)|}{g_0(v)} > \frac{|g_1'(v)|}{g_1(v)}\]

Since $0 < g_1(v) < g_0(v)$ we have:

\[(42) \quad \frac{|g_0'(v)|}{g_1(v)} = \frac{|g_0'(v)| g_0(v)}{|g_0(v)| g_1(v)} > \frac{|g_0'(v)|}{g_0(v)} > \frac{|g_1'(v)|}{g_1(v)}\]

The second inequality follows from relation (41).

We can now multiply by $g_1(v)$ and obtain:

\[(43) \quad |g_0'(z - \delta)| > |g_1'(z - \delta)|\]

Continuing in this fashion, we obtain that:

\[(44) \quad |g_0'(z - h)| > |g_1'(z - h)| \quad \forall h \geq 0\]

**A4: Derivation of Deviation from Full Insurance Contract**

Let us introduce the following notation:

$\alpha_F$ : Full insurance contract on zero profit line (payoff: $W_F, W_F$)

$\alpha^*$ : Chosen contract on zero profit line (wealth payoff: $W^*_1, W^*_2$)

$\delta^*$ : Deviation of chosen contract from Full insurance contract, i.e. $\delta^* = W^* - W_F$

A first order Taylor Expansion of equation (25) around $W_F$ yields:

\[(45) \quad U'(W_F) + \delta^* U''(W_F) = \eta \left[ U'(W_F) - \delta^* \frac{1-p}{p} U''(W_F) \right]\]

After rearranging and substituting, $\gamma_A(W_F) = -\frac{U''(W_F)}{U'(W_F)}$

\[(46) \quad \delta^*_1 = \frac{1-\eta}{1+\eta} \frac{1}{\frac{1-p}{p} \gamma_A(W_F)} \quad \text{and} \quad \delta^*_2 = -\delta^*_1 \frac{1-p}{p}\]
A5: Properties of $q_i^*$:

Let $L = E_1 - E_2$. Due to concavity of the utility function $U''(E_1 - L) < 0$, we can claim the following that $q_i^* \leq p_i$:

$$q_i^* = \frac{U'(E_i)}{1 - p_i U'(E_1 - L) + U'(E_1)} \leq \frac{U'(E_i)}{1 - p_i U'(E_1) + U'(E_1)} = \frac{1}{1 - p_i} = p_i$$

Moreover, $\frac{\partial q_i^*}{\partial L} < 0$:

$$\frac{\partial q_i^*}{\partial L} = \frac{U'(E_i)}{1 - p_i U'(E_1 - L) + U'(E_1)} = \frac{U'(E_i)}{\left(1 - p_i U'(E_1 - L) + U'(E_1)\right)^2} \frac{1 - p_i U'(E_1)}{p_i} < 0$$

Approximation of $q_i^*$

$$q_i^* = \frac{U'(E_1)}{1 - p_i U'(E_1 - L) + U'(E_1)} \approx \frac{1}{1 - p_i U'(E_1 - L) + U'(E_1)} = 1 + \frac{1}{1 - p_i U'(E_1 - L) + U'(E_1)}$$

This approximation suggests that $\frac{\partial q_i^*}{\partial \gamma_A(E_1)} < 0$

A6: Explanation of Figures

Since I will follow Rothschild-Stiglitz in their graphical approach, let me summarize the general structure of the figures. Assumption 1) says that only 2 states of the world can occur (loss/no loss). Therefore the associated wealth vector in both states can be illustrated in a two-dimensional graph with $W_1$ (the wealth in state 1) on the x-axis and $W_2$ (the wealth in state 2) on the y-axis.

Due to the assumptions on the utility function (strictly increasing and concave) indifference curves are convex. Preferred wealth combinations lie to the northeast. The Endowment $E$ will lie below the $45^\circ$ line since the loss occurs in state 2. Zero-profit lines go through the endowment and have the slope $\frac{1 - p_2}{p_1}$. 
References:


