



Tariff wars in the Ricardian Model with a continuum of goods

Marcus M. Opp*

University of California, Berkeley (Haas School of Business), 545 Student Services Bldg. #1900, Berkeley, CA 94720, United States

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ABSTRACT

This paper describes strategic tariff choices within the Ricardian framework of Dornbusch, Fischer, and Samuelson (1977) using CES preferences. The optimum tariff schedule is uniform across goods and inversely related to the import demand elasticity of the other country. In the Nash equilibrium of tariffs, larger economies apply higher tariff rates. Productivity adjusted relative size ($\approx GDP$ ratio) is a sufficient statistic for absolute productivity advantage and the size of the labor force. Both countries apply higher tariff rates if specialization gains from comparative advantage are high and transportation cost is low. A sufficiently large economy prefers the inefficient Nash equilibrium in tariffs over free trade due to its quasi-monopolistic power on world markets. The required threshold size is increasing in comparative advantage and decreasing in transportation cost. I discuss the implications of the static Nash-equilibrium analysis for the sustainability and structure of trade agreements.

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1. Introduction

Protectionist policies such as tariffs still represent major impediments to free trade, as exemplified by the recent failure of the Doha Development Round. The temptation to impose tariffs represents a classical problem studied by economists such as Bickerdike (1907): A country can improve its terms-of-trade via unilateral tariffs at the cost of inefficient resource allocation and a reduction in trade volume. The theory of optimum tariffs, which trade off these benefits and costs, has central (empirical) implications for two purposes: On the one hand, it provides testable predictions of actual tariff rates: As such, Broda et al. (2008) provide evidence that non-WTO countries apply tariff rates consistent with the theory. On the other hand, it helps us understand the incentive problems that countries face when they enter legally non-enforceable tariff agreements. According to Bagwell and Staiger (1999) the sole rationale for trade agreements is to escape terms-of-trade driven prisoner's dilemma situations.

For both of these purposes, understanding the determinants and implications of strategic tariff choices within a *general equilibrium production* framework is central, but nonetheless largely unstudied due to the level of complexity such an analysis entails.¹ This paper

aims to bridge that gap within a particularly simple general equilibrium framework—namely the Ricardian Model of Dornbusch et al. (1977) (henceforth labeled DFS). Their setup allows me to study the role of technology in the form of absolute and comparative advantage as well as transportation cost for optimum tariff policies. Moreover, I can characterize the intuitive repercussions of these tariff policies on the allocation of productive resources (efficiency) and the distribution of welfare.

At the heart of the paper is a generalized DFS framework with CES preferences in which tariff rate policies are endogenously determined by benevolent governments.² Within this framework, the optimum tariff rate schedule is uniform across goods. This result holds for different expenditure share parameters (across goods and countries), different elasticities of substitution (across countries) as well as arbitrary specifications of technology. Moreover, it is robust to the inclusion of transportation cost. The result may be surprising to the reader of Itoh and Kiyono (1987) who find that non-uniform export subsidies—interpretable as negative tariff rates—are welfare-improving in the DFS model. The apparent contradiction can be resolved as their carefully designed export-subsidy policy is not proven to be optimal, but solely welfare-enhancing relative to free trade. By reducing the potentially complicated tariff schedule choice to a one-dimensional problem, I am able to derive an easily interpretable optimality condition for the tariff rate: The expression is inversely

* Tel.: +1 510 643 0658.

E-mail address: mopp@haas.berkeley.edu.

¹ The analysis of Alvarez and Lucas (2007) shows how difficult it is to obtain intuitive results within a general equilibrium framework with strategic tariff choices. General equilibrium production models with multiple goods and *exogenous* trade barriers have become standard to study the determinants of trade flows (see Eaton and Kortum, 2002).

² Thus, the original DFS setup with Cobb–Douglas preferences is a special case.

related to an appropriately defined import demand elasticity of the other country. Tariffs can thus be interpreted as optimum markups on export goods. A higher foreign elasticity of substitution among goods increases the foreign import demand elasticity and hence reduces optimum tariff rates.

I prove existence of a unique “trembling-hand-perfect” Nash equilibrium of tariffs in which larger economies apply higher tariffs. Productivity adjusted relative size (\approx GDP ratio) is a sufficient size statistic for optimum tariff rates as higher average absolute productivity impacts the optimum tariff rate exactly as a larger relative size of the labor force. The intuition behind my results is as follows: Small economies are heavily dependent on trade and therefore possess a relatively inelastic import demand function. This can be exploited by larger economies through the lever of tariff rates to achieve terms-of-trade effects (intensive margin) while hardly increasing their already large domestic production (extensive margin). As a result of strategic tariffs, the terms-of-trade will (approximately) only reflect differences in productivity, but not differences in the size of the labor force. In contrast to free trade, a small country will not be able to capture the specialization gains that arise from the focus on the production of goods with the highest comparative advantage.

Both countries apply higher tariff rates if specialization gains (comparative advantage) are high. This is because any given increase in tariffs causes smaller deviations from efficient production. Consider the limiting case, when both countries are very similar and thus comparative advantage is low. Then, even a small tariff can completely exhaust the gains from trade. Transportation cost has the opposite effect of comparative advantage. Higher transportation cost effectively reduces the potential gains from trade. This makes tariffs more costly and reduces equilibrium tariff rates.

The welfare analysis implies that a sufficiently large economy is better off in a Nash equilibrium of tariffs than in a free-trade regime. In such a situation, the small economy bears (more than) the full deadweight loss of the globally inefficient tariff equilibrium. The structure of the DFS framework enables me to show that the threshold size level is an increasing function of comparative advantage and a decreasing function of transportation cost. Hence, if effective gains from trade are high (high comparative advantage, small transportation cost) and therefore both countries apply higher tariff rates in equilibrium a country has to be larger to prefer the Nash-equilibrium outcome over free trade.

The Nash-equilibrium analysis can be used as a stepping stone to study self-enforcing trade agreements in the spirit of Bagwell and Staiger (1990, 2003) and Bond and Park (2002). The static Nash-equilibrium outcome determines the (off-equilibrium path) punishment payoffs for deviating from a trade agreement. I extend the work of Mayer (1981) to a general equilibrium production setting and find that efficient tariff combinations can implement any desired welfare transfer from one country to the other. Free-trade agreements can only be sustained without transfers if both governments are sufficiently patient and size asymmetries are not too large. In contrast, if one government is sufficiently impatient, the short-run temptation to renege on agreements outweighs the long-run cost, so that the static Nash equilibrium occurs on the equilibrium path. To the extent that high discount rates reflect political economy considerations as in Acemoglu et al. (2008) or Opp (2008), political factors determine whether the Nash-equilibrium outcome characterizes the non-observable outside option or the actual equilibrium outcome. Thus, terms-of-trade considerations can be relevant in the sense of Bagwell and Staiger (1999) or Broda et al. (2008).

My paper is related to various lines of research. I follow the traditional economic approach to this subject by not explicitly considering political factors and viewing optimum tariff rates as optimal strategic decisions within a single period non-cooperative game. While government actions may realistically involve political considerations (see Grossman and Helpman, 1995), such a model

does not offer a separate rationale for trade agreements which are designed to escape the terms-of-trade driven prisoner's dilemma in a Nash equilibrium (see Bagwell and Staiger, 1999). However, as pointed out above political factors may matter indirectly through the discount rate for the feasibility of trade agreements.

The dominant part of the existing literature on *tariff games* uses a two-good exchange economy setup to analyze the strategic choice of tariff rates. This approach is largely inspired by Johnson (1953). He finds that a Nash equilibrium of tariffs does not necessarily result in a prisoner's dilemma situation in which both countries are worse off than under a free-trade regime. Most of the subsequent papers focus on generalized preferences (Gorman, 1958), the impact of relative size (Kennan and Riezman, 1988) or existence of equilibria (Otani, 1980). By incorporating a production sector into the analysis, Syropoulos (2002) is able to significantly improve upon the previous literature. Using a generalized Heckscher–Ohlin setup (2 countries, 2 commodities and homothetic preferences), he proves the existence of a threshold size level that will cause the bigger country to prefer a tariff equilibrium over free trade. As opposed to the Ricardian framework employed in my paper the Heckscher–Ohlin theory explains gains from trade by differences in factor endowments rather than technology across countries. My paper can be seen as a response to the concluding remarks of Syropoulos who states that “it would be interesting to examine how technology affects outcomes in tariff wars” and “it would be worthwhile to investigate whether the findings on the relationship between relative country size and tariff war outcomes remain intact in multi-commodity settings”. In addition to technology differences and the multi-commodity framework, I also consider the role of transportation cost for tariff policies. If one added these generalizations to the Syropoulos setup and assumed international capital mobility, the models should produce very similar results.³

General treatments on the optimum tariff structure with multiple goods go back to Graaff (1949) and more recently Bond (1990) and Feenstra (1986). Their analysis yields conditions (e.g. on the substitution matrix) which can generate more complicated tariff structures such as subsidies for some goods. Due to the constant elasticity of substitution the optimum tariff structure turns out to be particularly simple in the DFS setup and consistent with this literature.

McLaren (1997) develops an innovative paper in the spirit of Grossman and Hart (1986) that yields the counterintuitive result that small countries may prefer an anticipated trade war relative to an anticipated trade negotiation.⁴ The key driver for this result is that an irreversible investment in the export sector by the small country in period 1 will reduce the threat point in negotiation talks in period 2 (after the investment is sunk). Despite the different underlying economic intuition, it is confirmed that small size brings about a strategic disadvantage.

My paper is organized as follows. In Section 2 I characterize the equilibrium conditions of the generalized DFS Model with CES preferences. In Section 3 I prove the optimality of a uniform tariff schedule and derive the optimum tariff rate formula. Section 4 presents the Nash-equilibrium analysis using Cobb–Douglas preferences and an intuitive specification of technology. The implications of my static analysis for trade agreements within a dynamic context are discussed in Section 5. Section 6 concludes. For ease of exposition, the proofs of most propositions are moved to the appendix, while the intuition is delivered in the main text. The notation has been kept in line with the original DFS paper.

³ (Neo-) Heckscher–Ohlin models with differences in technology and one mobile factor are reduced to a standard Ricardian model (see Chipman, 1971 for an elegant proof).

⁴ This situation can occur if the production technologies are similar in terms of absolute and comparative advantage.

2. DFS framework

2.1. Setup

The purpose of this section is to compactly describe the general framework of the Ricardian Model with a continuum of goods of DFS. There are two countries (home and foreign) with respective labor endowments L and L^* . Note, that asterisks are used throughout the paper to refer to the foreign country. Tildes refer to the log transformation. Without loss of generality, I normalize the size of the labor force (L^*) and wage rate (w^*) of the foreign country to unity such that the relative size of the home country is simply given by its absolute size L and the wage rate of the home country is equal to the relative wage rate ω .

2.1.1. Production technology

There exists a continuum of goods (z) which are indexed over the interval $[0, 1]$ and produced competitively with a linear technology in labor. The labor unit requirement functions $a(z)$, $a^*(z)$ specify how many labor units are required to produce one unit of good z in the home country and foreign country, respectively. Both $a(z)$ and $a^*(z)$ are assumed to be twice differentiable and bounded over the domain $[0, 1]$. The ratio of $a(z)$ and $a^*(z)$ is defined as $A(z)$:

$$A(z) \equiv \frac{a^*(z)}{a(z)}. \tag{1}$$

Goods are ordered in terms of decreasing comparative advantage of the home country, which implies that the function $A(z)$ is decreasing in z over the domain $[0, 1]$. The log transformation of $A(z)$ —denoted as $\tilde{A}(z)$ —yields the productivity (dis)advantage of the home country in percentage terms:

$$\tilde{A}(z) \equiv \log[A(z)]. \tag{2}$$

2.1.2. Preferences

The representative agent of each economy possesses a demand function generated by CES preferences. Moreover, I extend the original DFS analysis—which relied on a Cobb–Douglas specification—by allowing the utility function to differ across countries:

$$U(c) = \left(\int_0^1 \pi(z)^{\frac{1}{\theta}} c(z)^{\frac{\theta-1}{\theta}} dz \right)^{\frac{\theta}{\theta-1}}$$

$$U^*(c^*) = \left(\int_0^1 \pi^*(z)^{\frac{1}{\theta^*}} c^*(z)^{\frac{\theta^*-1}{\theta^*}} dz \right)^{\frac{\theta^*}{\theta^*-1}}. \tag{3}$$

Thus, the share parameters $\pi(z)$, $\pi^*(z)$ and the elasticities of substitution θ , θ^* need not be common. I make the technical assumption that $\pi(z)$ and $\pi^*(z)$ are Lebesgue measurable.

2.1.3. Trade barriers

I allow for two types of trade barriers: Import tariffs ($t(z)$, $t^*(z)$) and exogenous symmetric iceberg transportation cost δ (see Samuelson (1952)).⁵ Thus, only a fraction $\exp(-\delta)$ of exported goods eventually arrives in the other country.⁶ Import tariffs are applied to all goods that

arrive in the import country and redistributed to the consumers. I define gross import tariffs as:

$$T(z) \equiv 1 + t(z) \tag{4}$$

$$T^*(z) \equiv 1 + t^*(z). \tag{5}$$

2.2. Equilibrium conditions

2.2.1. Production

Perfect competition and the linear production technology imply that final consumption prices of domestically produced goods $p_D(z)$ are equal to production cost whereas prices of import goods $p_I(z)$ also reflect transportation cost and tariffs.

$$p_D(z) = \omega a(z)$$

$$p_I(z) = a^*(z) \exp(\delta) T(z) \tag{6}$$

Let I denote the set of import goods and D denote the set of domestically produced goods, i.e.:

$$I = \{z | p_D(z) \geq p_I(z)\} \tag{7}$$

$$D = \{z | p_D(z) < p_I(z)\}. \tag{8}$$

For ease of exposition, I assume that the currently exogenous tariff rate policies are given by Lebesgue-measurable functions that divide the set of goods into two connected sets, i.e. I and D .⁷ Hence, the set I and D are completely determined by a cut-off good \bar{z} such that all goods $z < \bar{z}$ are produced domestically and all goods $z \geq \bar{z}$ are imported. Analogously, the foreign country imports goods in the interval $I^* = [0, \bar{z}^*]$ and produces goods domestically in the interval $D^* = (\bar{z}^*, 1]$.⁸ Therefore, efficient production specialization implies the following two equilibrium conditions:

$$A(\bar{z}) \leq \frac{\omega}{\exp(\delta) T(\bar{z})} \tag{9}$$

$$A(\bar{z}^*) \geq \omega \exp(\delta) T^*(\bar{z}^*).$$

2.2.2. Consumption and balance of trade

It is well known that the optimal consumption schedule $\bar{c}(z)$ with CES preferences is given by:

$$\bar{c}(z) = y \frac{\pi(z)}{p(z)^\theta P^{1-\theta}} \tag{10}$$

where y represents the per-capita income of the home country and P an appropriately defined price index:

$$P \equiv \left(\int_0^1 \pi(z) p(z)^{1-\theta} dz \right)^{\frac{1}{1-\theta}}. \tag{11}$$

Note that unless preferences are of Cobb–Douglas type ($\theta = 1$), the expenditure share $b(z)$ does not only depend on the exogenous share parameter $\pi(z)$ but also on the endogenous price of good z and the price index P^θ :

$$b(z) \equiv \pi(z) \left(\frac{p(z)}{P} \right)^{1-\theta}. \tag{12}$$

⁵ The author has verified that the analysis of this paper extends one-to-one to export tariffs. Thus, Lerner's "Symmetry-Result" holds in this setup with a continuum of goods.

⁶ δ is assumed to be sufficiently small, such that trade in at least some products would take place in the absence of import tariffs.

⁷ As in Itoh and Kiyono (1987), this assumption is made for ease of exposition.

⁸ Prices of domestically produced goods in the foreign country are given by $p_D^*(z) = a^*(z)$ whereas import good prices are given by: $p_I^*(z) = \omega a(z) \exp(\delta) T^*(z)$.

⁹ P is well specified since $\pi(z)$, $a(z)$, $a^*(z)$ and $T(z)$ are measurable functions.

Given efficient production specialization the share of income spent on domestically produced goods ϑ is:

$$\vartheta(\bar{z}, p(z), P) \equiv \int_0^{\bar{z}} b(z) dz. \tag{13}$$

Balance of trade requires that the value of imports by the home country at world market (pre-tariff) prices must be equal to the value of imports by the foreign country:

$$Ly \frac{1-\vartheta}{\bar{T}} = y^* \frac{1-\vartheta^*}{\bar{T}^*} \tag{14}$$

where $\bar{T} = \frac{1-\vartheta}{\int_{\bar{z}}^1 \frac{b(z)}{T(z)} dz}$ and $\bar{T}^* = \frac{1-\vartheta^*}{\int_0^{\bar{z}^*} \frac{b^*(z)}{T^*(z)} dz}$ can be interpreted as average tariff rates. Using the definitions of after-tax per-capita income $y = \omega \frac{\bar{T}}{1+\vartheta\bar{T}}$ and $y^* = \frac{\bar{T}^*}{1+\vartheta^*\bar{T}^*}$, the balance of trade equilibrium condition can be rewritten as in the original DFS paper:

$$\omega = \frac{1+\vartheta\bar{T}}{1-\vartheta} \frac{1-\vartheta^*}{1+\vartheta^*\bar{T}^*} \frac{1}{L}. \tag{15}$$

3. Optimum tariff rates

So far, the analysis has treated tariff rates similar to transportation cost, i.e. as exogenously given trade barriers. However, in contrast to transportation cost, import tariffs are choice variables for the respective government and affect national income.¹⁰ Due to the CES demand structure, the indirect utility function of the home country's representative agent $V(\{p(z)\}, y)$ can be written as:

$$V(\{p(z)\}, y) = \frac{y}{P}. \tag{16}$$

It is assumed that each government tries to maximize the real income of the representative agent $\frac{y}{P}$ given the tariff rate decision of the other country and subject to the equilibrium conditions on ω, \bar{z} and \bar{z}^* given by balance of trade (Eq. (15)) and the cut-off good definitions (see Eq. (9)).

Proposition 1. *In the generalized DFS framework with CES preferences, the optimum tariff rate is uniform across all import goods.*

Proof. It is useful to apply the log transformation of the government's objective function. Using basic algebra the problem can be stated as:

$$\max_{\{T(z)\}} \tilde{V}(\{T(z)\}) = \tilde{\omega} - \frac{\theta}{1-\theta} \log(F_D + F_I) - \log(F_D + F_{I_x}) \tag{17}$$

subject to:

$$g_1 = \tilde{L} + \tilde{\omega} + \log(F_{I_x}) - \log(F_D + F_{I_x}) + \log(F_D^* + F_{I_x}^*) - \log(F_{I_x}^*) = 0$$

$$g_2 = \tilde{\omega} - \delta - \log(T(\bar{z})) - \tilde{A}(\bar{z}) \geq 0$$

$$g_3 = \tilde{\omega} + \delta + \log(T^*(\bar{z}^*)) - \tilde{A}(\bar{z}^*) \leq 0$$

where the functions F_D and F_D^* are proportional to the share of domestically produced goods ϑ, ϑ^* and F_I, F_I^*, F_{I_x} are related to the share of imported goods inclusive of tariffs and ex-tariffs (indicated with x):

$$F_D(\omega, \bar{z}) = \int_0^{\bar{z}} \pi(z) p_D(z)^{1-\theta} dz \tag{18}$$

$$F_I(\{T(z)\}, \bar{z}) = \int_{\bar{z}}^1 \pi(z) p_I(z)^{1-\theta} dz \tag{19}$$

$$F_{I_x}(\{T(z)\}, \bar{z}) = \int_{\bar{z}}^1 \pi(z) \frac{p_I(z)^{1-\theta}}{T(z)} dz. \tag{20}$$

Definitions are analogous for the foreign country. The Lagrangian $\tilde{\Pi}$ can be written as:

$$\tilde{\Pi} = \tilde{\omega} - \frac{\theta}{1-\theta} \log(F_D + F_I) - \log(F_D + F_{I_x}) - \sum_{i=1}^3 \kappa_i g_i \tag{21}$$

where κ_i represents the Lagrange multiplier on equilibrium constraint g_i . Note, that only the terms F_I and F_{I_x} depend directly on the home country tariff schedule $T(z)$. Thus, the argument of these functionals F_I, F_{I_x} is itself a function. It is useful to define the functions $f_j(z) = \pi(z) p_j(z)^{1-\theta}$ such that $F_j = \int_{\bar{z}}^1 f_j(z) dz$. Now, the Euler-Lagrange equations for this problem can be written as:

$$-\frac{\theta}{1-\theta} \frac{1}{F_D + F_I} \frac{\partial f_I(z)}{\partial T(z)} - \frac{1}{F_D + F_{I_x}} \frac{\partial f_{I_x}(z)}{\partial T(z)} - \kappa_1 \left(\frac{1}{F_{I_x}} - \frac{1}{F_D + F_{I_x}} \right) \frac{\partial f_{I_x}(z)}{\partial T(z)} = 0 \tag{22}$$

where the partial derivatives of f_I and f_{I_x} with respect to the tariff rate $T(z)$ are given by:

$$\frac{\partial f_I(z)}{\partial T(z)} = \frac{1-\theta}{T(z)} f_I(z) \tag{23}$$

$$\frac{\partial f_{I_x}(z)}{\partial T(z)} = -\frac{\theta}{T(z)^2} f_{I_x}(z). \tag{24}$$

As $\frac{\partial f_I(z)}{\partial T(z)} = -T(z) \frac{1-\theta}{\theta}$ we can rewrite the Euler-Lagrange equations as:

$$T(z) = (F_D + F_I) \left(\frac{1-\kappa_1}{F_D + F_{I_x}} + \frac{\kappa_1}{F_{I_x}} \right). \tag{25}$$

The right hand side is independent of z . So the left hand side must be independent of z as well, i.e. $T(z) = T \forall z \geq \bar{z}$. The solution that satisfies the Euler-Lagrange equation is a global maximum since the objective function is quasi-concave and the constraint set is convex.¹¹ This completes the proof. \square

Proposition 1 is important because it justifies the focus on uniform tariff rates in this framework. It has to be stressed that uniformity has been proven under general labor unit requirement functions, different preferences across countries and with possibly different expenditure share parameters $\pi(z)$ across goods. Due to this result, tariff rate policies of each country can be simply summarized by only two variables t and t^* , respectively.

Proposition 2. *The optimum response tariff rate in the DFS model t_R can be expressed as follows:*

$$t_R = \frac{1 + \vartheta^* t^*}{T^*} - \frac{1}{\frac{b^*(\bar{z}^*)}{\tilde{A}(\bar{z}^*)} \frac{1}{1-\vartheta^*} + \vartheta^*(\theta^*-1)} \tag{26}$$

¹⁰ If tariff rebates were simply lost—as in the case of iceberg transportation cost—the optimum tariff rate would be zero.

¹¹ Uniqueness is subject to a technical qualification. Any tariff rate $T(\bar{z}) < T$ that satisfies equilibrium constraint 2 would be optimal as well since good \bar{z} has measure 0. Generally, deviations from the optimum tariff rate policy for a finite number of goods will not affect welfare. Tariff rates for *non-import* goods are also not uniquely pinned down. For each good $\bar{z} < \bar{z}$, there exists a lower bound $T_L(\bar{z}) \geq T$ such that for any $T \geq T_L(\bar{z})$ good \bar{z} is not imported. For non-import goods tariff rates can differ from the optimum tariff rate even on a set of goods with positive mass.

$$t_R^* = \frac{1 + \vartheta t}{T} \frac{1}{-\frac{b(\bar{z})}{\bar{A}(\bar{z})} \frac{1}{1-\vartheta} + \vartheta(\theta-1)}. \tag{27}$$

Proof. See Appendix A.2. □

The optimum tariff rate trades off the terms-of-trade improvements with the inefficient expansion of domestic production and the costly reduction in trade volume. The economic intuition is as follows: By imposing a tariff on import goods, the home country increases the final consumption prices of foreign goods which in turn reduces demand for those goods in the home country. At the old equilibrium prices (before imposing the tariff) the balance of trade condition will no longer hold as the foreign country will demand too many import goods. In order to eliminate the excess demand for import goods in the foreign country, the terms-of-trade have to improve from the perspective of the home country (intensive margin). Since the elasticity of the equilibrium wage rate with respect to tariffs is smaller than one ($\frac{d \log(\omega)}{d \log(T)} < 1$) the home country will inefficiently expand home production to products with a lower comparative advantage (extensive margin).

In order to characterize the optimum tariff rate in terms of demand elasticities ($\varepsilon, \varepsilon^*$), it is useful to interpret the per-capita import demand m as a composite good for the continuum of import goods. The real per-capita import demand of the home country is essentially given by the left hand side of the balance of trade condition (see Eq. (14)):

$$m = y \frac{1-\vartheta}{T} = \omega \frac{1-\vartheta}{1+\vartheta t}. \tag{28}$$

The “price” of an import good in local currency is proportional to the inverse of ω such that we can define the import demand elasticity as:

$$\varepsilon = \left| \frac{d \log m}{d \log \omega^{-1}} \right| = \frac{d \log m}{d \tilde{\omega}}. \tag{29}$$

It can be shown that the optimum tariff rate of the foreign country (see Proof of Proposition 2 in Appendix A.2) can be written as $t_R^* = \frac{1}{\varepsilon-1}$ and analogously for the home country $t_R = \frac{1}{\varepsilon^*-1}$. This pricing formula is identical to the optimal markup that a monopolistic firm charges over marginal cost MC , i.e. $p = \left(1 + \frac{1}{\varepsilon-1}\right)MC$. To see this analogy, it is useful to apply Lerner’s symmetry result with regards to import and export taxes (see Lerner, 1936): The optimum import tariff is equivalent to the optimal export tariff. While goods are priced competitively within each economy each country behaves like a monopolist for the goods it exports. In the absence of retaliation a government can improve country welfare through tariffs because it can effectively enforce markup pricing on world markets despite perfect competition within its country. Intuitively, the optimal tariff t_R is smaller the greater the degree of substitutability between goods (θ^*) (see Eq. (26)). In the limit, as preferences of the trade partner become linear, the optimum tariff rate approaches 0.

4. Equilibrium analysis

4.1. Specification

The derived optimum tariff rate formulae (see Eqs. (26) and (27)) allow us to calculate optimum tariff rate policies for arbitrary technology and taste specifications. For ease of exposition, I use simple specifications of technology and preferences to be able to make sharper predictions about comparative statics in the DFS setup.¹²

¹² A previous draft of the paper considered a more general specification which I have dropped for simplicity. The results hold more generally for bounded technology specifications.

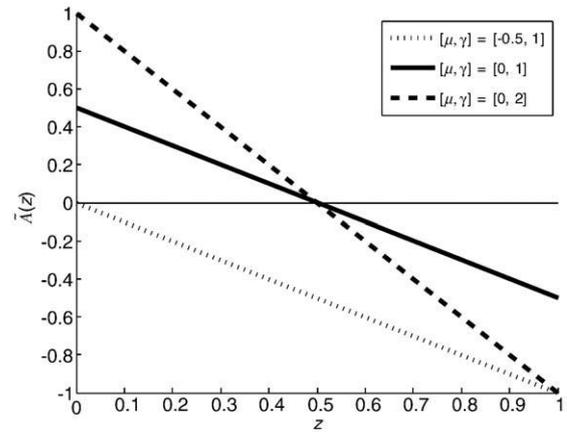


Fig. 1. Relative productivity function for various parameter values.

Preferences are of Cobb–Douglas type ($\theta = \theta^* = 1$) and all goods are valued equally ($b(z) = \pi(z) = 1$).¹³ These preferences imply that the optimum tariff rate only depends on the relative labor unit requirement function $A(z)$, but not separately on $a(z)$ and $a^*(z)$.¹⁴ The share of income spent on domestically produced goods ϑ, ϑ^* is directly determined by the respective cut-off goods:

$$\vartheta = \bar{z} \tag{30}$$

$$\vartheta^* = 1 - \bar{z}^*. \tag{31}$$

The relative labor unit requirement function is assumed to be exponentially affine, which should be interpreted as a linear projection of the true relative productivities:

$$\tilde{A}(z) \equiv \mu + \gamma / 2 - \gamma z, \mu \in \mathbb{R} \text{ and } \gamma \in \mathbb{R}^+. \tag{32}$$

Thus, the two parameters μ and γ are sufficient determinants of technology. It can be readily verified that the function is strictly positive and bounded over the specified domain $[0, 1]$. The effect of different parameter choices for μ and γ is illustrated in Fig. 1. The parameter μ can be interpreted as the absolute productivity advantage of the home country averaged over all goods, i.e. $\int_0^1 (\mu + \gamma / 2 - \gamma z) dz = \mu$.¹⁵ In contrast, the parameter γ measures the degree of comparative advantage (dispersion parameter). Larger values of γ generate a greater diffusion of relative productivities across goods which implies greater gains from trade.¹⁶

The specification generates easily interpretable expressions for the cut-off goods and the size of the non-traded sector $N = \bar{z} - \bar{z}^*$:

$$\begin{aligned} \bar{z} &= \frac{1}{2} + \frac{\mu + \delta + \log(T) - \tilde{\omega}}{\gamma} \\ \bar{z}^* &= \frac{1}{2} + \frac{\mu - \delta - \log(T^*) - \tilde{\omega}}{\gamma} \\ N &= \frac{2\delta + \log T + \log T^*}{\gamma} \end{aligned} \tag{33}$$

¹³ Empirical evidence (see Weder, 2003) suggests that countries have a taste bias in favor of the goods they export, i.e. goods with a higher comparative advantage. In contrast to Opp et al. (2009) I do not account for this empirical fact.

¹⁴ This is because the expenditure share $b(z)$ is given by the exogenous share parameter $\pi(z)$ for Cobb–Douglas preferences.

¹⁵ Due to the simple affine structure this coincides with the productivity advantage at the median good ($z = 0.5$), i.e. $\tilde{A}(0.5) = \mu$.

¹⁶ The parameter γ is closely related to the variance parameter of individual productivities in the Eaton–Kortum Model (see also Alvarez and Lucas, 2007).

Intuitively, the size of the non-traded sector N solely depends on the ratio of exogenous and endogenous trade barriers ($2\delta + \log T + \log T^*$) to the potential gains from trade γ . Thus, barriers are prohibitive if $2\delta + \log T + \log T^* > \gamma$. In the following analysis, I assume that the exogenous transportation cost satisfies $2\delta < \gamma$.

4.2. Optimum response function

In this specific setup the optimum tariff rate expression becomes:

$$t_R = \gamma \bar{z}^* \frac{1 + (1 - \bar{z}^*)t^*}{T^*} \tag{34}$$

$$t_R^* = \gamma(1 - \bar{z}) \frac{1 + \bar{z}t}{1 + t} \tag{35}$$

Note that \bar{z} and \bar{z}^* are functions of the tariff rates as well as the parameters for size and technology. It can be shown (see Appendix A.1) that productivity adjusted size $c = \mu + \tilde{L}$ is a sufficient statistic for the parameters L and μ in determining the cut-off goods \bar{z} and \bar{z}^* . Therefore, tariff rates are also just a function of effective relative size c .¹⁷

In order to make sharper predictions about the properties of the best response functions, I need to make a mild technical assumption.

Assumption 1. The foreign country's tariff rate satisfies: $t^*(2\bar{z}^* - 1) < 1$.

A violation of Assumption 1 necessarily requires an unrealistic combination of high foreign tariff rates (more than 100%) and a large foreign import sector (more than 50% of the goods are imported).¹⁸

Proposition 3. The optimum response function t_R has the following properties:

- Decreasing in the other country's tariff rate $\frac{dt_R}{dt^*} < 0$
- Decreasing in transportation cost $\frac{dt_R}{d\delta} < 0$
- Increasing in the dispersion parameter $\frac{dt_R}{d\gamma} > 0$
- Increasing in a country's relative production capacity $\frac{dt_R}{dc} > 0$.

Proof. See Appendix B.3. □

Thus, tariffs are strategic substitutes (see Fig. 2) with an elasticity $\frac{d \log t_R}{d \log T^*}$ smaller than 1 (see Appendix B.4). The intuition behind this result is that higher foreign tariff rates increase trade barriers that exogenously increase the size of the non-traded sector $N = \frac{2\delta + \log T + \log T^*}{\gamma}$. Hence, the room for imposing additional own trade restrictions without heavily reducing or even shutting down trade is limited.

From the perspective of the home country, transportation cost δ is very similar to the foreign country's tariff rate t^* : Both effectively represent exogenous barriers to trade. Consequently, increases in transportation cost must reduce the optimum tariff rate of the home country. A flatter relative labor unit requirement function (smaller γ) implies that any given tariff rate will result in a greater (inefficient) expansion of the domestic sector, i.e. the cut-off good is more responsive to tariffs. Smaller specialization gains thus render domestic and

¹⁷ Effective relative size is closely related to the amount of goods that can be produced under autarky, i.e. the size of the economy (production capacity). For any specific good the relative production capacity (number of available labor units/number of required labor units) of both countries is given by $C(z) = \frac{L}{a(z)} / \frac{L^*}{a^*(z)} = LA(z)$. Taking logarithms and averaging over all goods implies that we can define $c \equiv \log(L) + \mu$.

¹⁸ Recall, that $t, t^* > 0$ and $0 < z^* < z < 1$ and that $1 - \bar{z}$ and \bar{z}^* can be interpreted as the size of the respective import sectors. Numerical results show that $\gamma < 7.4$ is a sufficient condition for the validity of Assumption 1. A value of $\gamma = 7.4$ would imply that the ratio of the relative labor unit requirement function at the two endpoints $A(0)/A(1)$ is equal to $\exp(7.4) \approx 1635$.

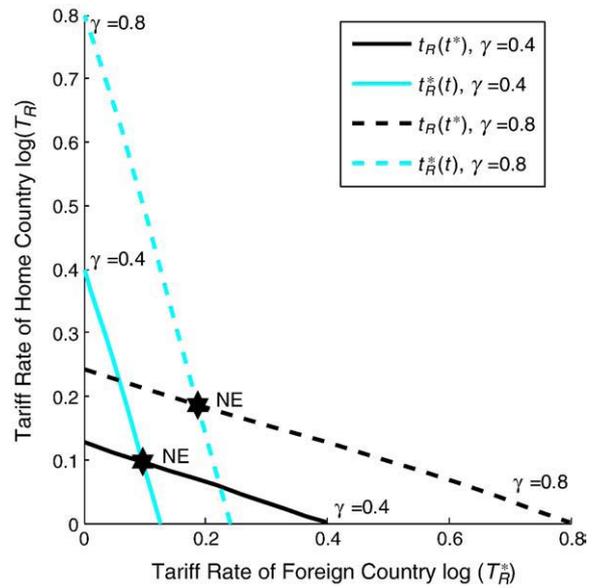


Fig. 2. Optimum response functions and Nash equilibria ($c = 0$).

foreign production to be more substitutable. Therefore, the optimum tariff rate must be smaller.

The positive marginal impact of the size (c) reflects that larger economies have a smaller import demand elasticity. The relationship is monotone, as the harmful side-effects of tariffs (inefficient expansion of home production and price increase of import products) weigh smaller the greater the economic weight of one country. Consider the case when the home economy is very large such that almost all goods are produced domestically even without tariffs. In this case, the cost of tariffs—inefficient expansion of home production—is only of second-order concern. In contrast, the infinitesimally small country will not impose a tariff as the import demand elasticity of the large country approaches infinity.¹⁹

4.3. Nash equilibrium

4.3.1. Existence

In a Nash equilibrium both economies' tariff rate choice represents an optimum response to the tariff rate of the other country. The optimum response functions for two parameter constellations are depicted in Fig. 2. The intersection point constitutes a Nash equilibrium in tariff rates which is characterized by strictly positive trade flows. No-trade Nash equilibria can occur, if both countries choose a prohibitively high tariff rate $t_{\text{proh}} > \gamma - 2\delta$, i.e. a tariff rate that exhausts the gains from trade adjusted for transportation cost. There exists a continuum of No-trade equilibria all of which are uninteresting, as applying a prohibitive tariff rate is weakly dominated. Formally, these equilibria could be ruled out using the “trembling-hand-perfection” refinement. In the following analysis I will only consider interior Nash equilibria.

Proposition 4. There exists a unique interior Nash equilibrium in tariffs that Pareto dominates any No-trade Nash equilibrium.

Proof. See Appendix B.4. □

The existence of a unique interior Nash equilibrium is established by applying the Contraction Mapping Theorem. The contraction property follows directly from the fact that the slope of the optimum response function is less than one (see Fig. 2).

¹⁹ This follows from the boundedness of the technology specification: A violation of this property such as in the Eaton-Kortum specification of technology can imply strictly positive tariff rates even for the infinitesimally small country.

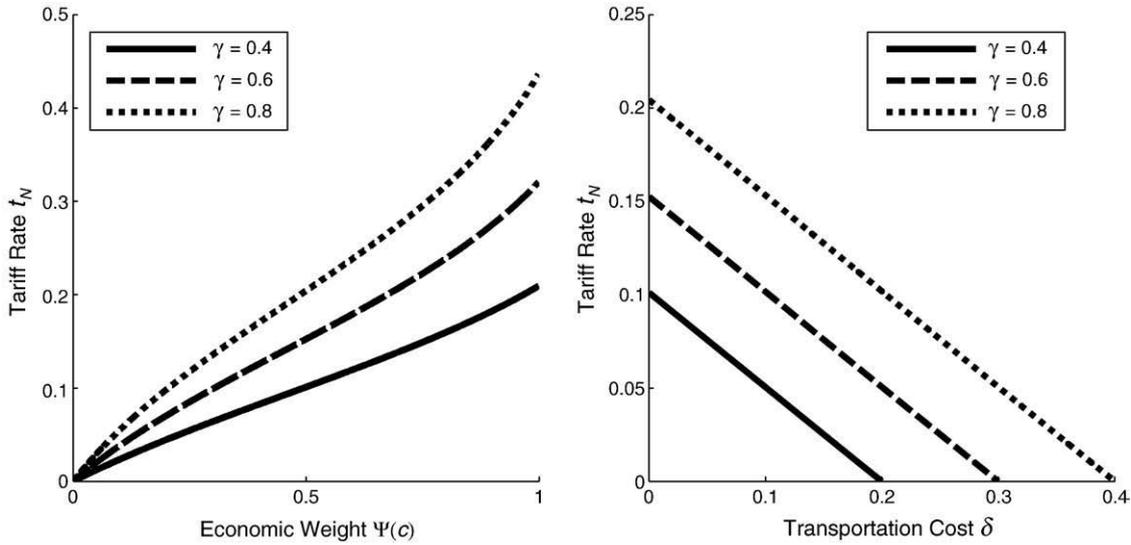


Fig. 3. Comparative statics Nash equilibrium (left panel: $\delta=0$, right panel: $c=0$).

4.3.2. Tariff rates

The comparative statics of the optimum response function with respect to the exogenous parameters c , δ and γ are preserved in the Nash equilibrium of tariffs:

Proposition 5. *The Nash-equilibrium tariff rate t_N has the following properties:*

- Decreasing in transportation cost $\frac{dt_N}{d\delta} < 0$
- Increasing in the dispersion parameter $\frac{dt_N}{d\gamma} > 0$
- Increasing in a country's relative production capacity $\frac{dt_N}{dc} > 0$.

Proof. See Appendix B.5. □

Intuitively, the comparative statics of the Nash-equilibrium tariff rate can be decomposed into the direct effect of the optimum response function and the feedback effect through the strategic tariff choice of the other country. In case of the size parameter c , the feedback effect amplifies the direct effect: As the home economy becomes relatively larger the foreign economy will apply lower tariff rates which in turn increase the tariff rate of the home country due to strategic substitutability (see Proposition 3). In the case of the dispersion parameter γ and transportation cost δ , the feedback effect counters the direct effect, but is outweighed by the direct effect.

Fig. 3 plots the comparative statics where $\psi: \mathbb{R} \rightarrow [0, 1]$ denotes a normalized measure of size c :

$$\psi(c) \equiv \frac{\exp(c)}{1 + \exp(c)}. \tag{36}$$

This measure ψ can be interpreted as the economic “weight” of a country, as the weights of each country $\psi(c)$ and $\psi(c^*) = \psi(-c)$ sum up to one. The infinitesimally small country has a weight of zero, the infinitely large country a weight of one.²⁰ The model theoretic foundation for this measure is based upon the first-order Taylor series approximation of the Nash-equilibrium tariff rate about the point $\gamma=0$ (see Appendix B.6).²¹

$$t_N \approx \psi(c) \left(\frac{\gamma}{2} - \delta \right) \tag{37}$$

²⁰ The careful reader will notice that the function $\psi(x)$ is identical to the CDF of the logistic distribution.

²¹ Formally, I take the limit as γ approaches 0 fixing the ratio of $\frac{\delta}{\gamma}$ at some value $r < \frac{1}{2}$.

The (almost) linear graphs in Fig. 3 reveal that the first-order approximation captures the relevant characteristics of the comparative statics.

4.3.3. Terms-of-trade

Since terms-of-trade effects are central to our analysis, it is helpful to revisit a key result from the DFS model under free trade: Countries with a relatively small size of the labor force L face significantly better terms-of-trade, a measure of relative welfare under free trade. The intuition for this result is that small countries can specialize their production on goods with the highest comparative advantage whereas the large country needs to supply low-comparative-advantage-goods as well.

In a Nash equilibrium of tariffs, the small country can no longer focus on the production of goods with the highest comparative advantage as endogenous tariff rates create a sizeable non-traded sector (see upper panel of Fig. 4). In addition, the small country's

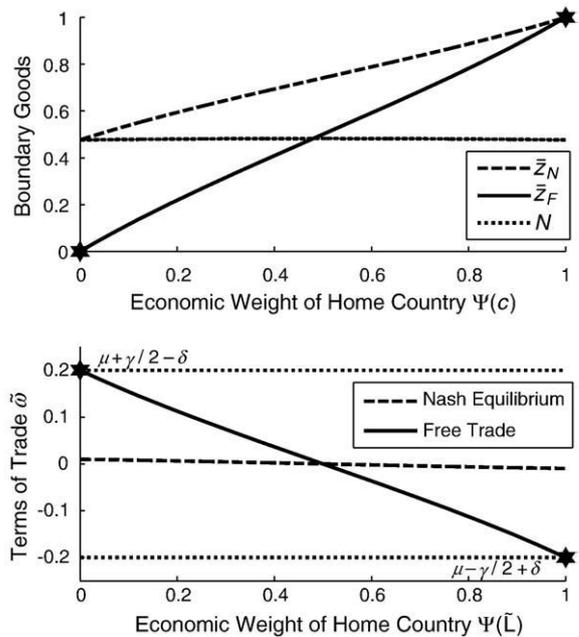


Fig. 4. The effect of size on cut-off goods and terms-of-trade: Nash equilibrium vs. free trade ($\mu=0$, $\gamma=0.4$, $\delta=0$).

terms-of-trade deteriorate as it applies disproportionately lower tariff rates (see Proposition 5). In brief, the specialization benefits that can be reaped under free trade are almost perfectly offset by the strategic choice of tariff rates in a Nash equilibrium (see lower panel of Fig. 4). The exact equilibrium outcome can be well approximated using the first-order approximation of tariffs (see Appendix B.6):

$$\tilde{\omega} \approx \mu \tag{38}$$

$$N \approx \frac{1}{2} + \frac{\delta}{\gamma}. \tag{39}$$

The terms-of-trade only reflect differences in absolute productivity μ , whereas the size of the non-traded sector only depends on the ratio of transportation cost δ to comparative advantage γ .²²

4.3.4. Welfare

Since the Nash equilibrium of tariffs generates globally inefficient production patterns the representative agent of the world economy must be worse off than under free trade. In order to measure welfare losses for each country I determine the required rate of consumption growth Δ that equalizes the utility obtained in the Nash equilibrium (N) and free trade (F).²³ Therefore Δ is defined as:

$$\tilde{V}_F(e^{\Delta} c_F | t = 0, t^* = 0) \equiv \tilde{V}_N(c_N | t = t_N, t^* = t_N^*). \tag{40}$$

Due to the simple form of the utility function the required growth rate of consumption is given by the difference of the utility levels:

$$\Delta V = \tilde{V}_N - \tilde{V}_F. \tag{41}$$

The exponentially affine specification generates intuitive expressions for the derived utilities (see Appendix B.2):

$$\tilde{V}_N = \frac{\gamma}{2}(1 - \vartheta_N)^2 + \log\left(\frac{T_N}{1 + \vartheta_N t_N}\right) \tag{42}$$

$$\tilde{V}_F = \frac{\gamma}{2}(1 - \vartheta_F)^2. \tag{43}$$

The free-trade welfare \tilde{V}_F interacts comparative advantage γ (gains from trade) with the size of the import sector $(1 - \vartheta_F)$. This provides another illustration how small economies benefit from specialization. In the absence of transportation cost, the infinitesimally small economy's derived utility approaches $\frac{\gamma}{2}$. The derived Nash-equilibrium welfare \tilde{V}_N consists of two components. The first component is similar to the free-trade welfare. However, as the import sector is smaller in the Nash equilibrium ($\vartheta_N > \vartheta_F$), the first term must be strictly smaller than under free trade. The second term $\log\left(\frac{T_N}{1 + \vartheta_N t_N}\right)$ is strictly positive and reflects gains from imposing import tariffs.

The following lemma describes intuitive properties of the welfare measure ΔV (see Fig. 5).

Lemma 1. ΔV is a function of c , δ and γ and has the following limit properties:

- 1) $\lim_{c \rightarrow 0} \Delta V(c, \gamma, \delta) < 0$
- 2) $\lim_{c \rightarrow \infty} \Delta V(c, \gamma, \delta) = 0$
- 3) $\lim_{c \rightarrow \infty} \frac{\partial \Delta V(c, \gamma, \delta)}{\partial c} < 0$.

Proof. See Appendix B.7. □

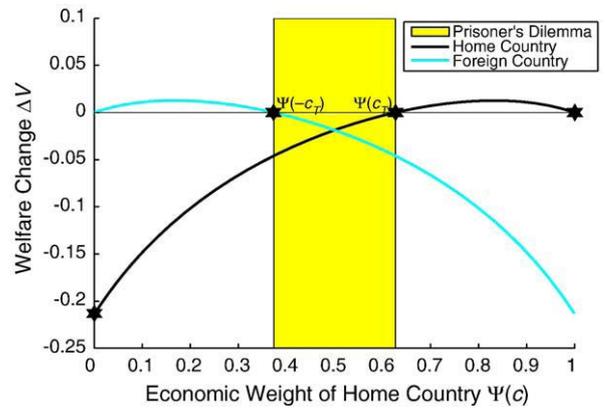


Fig. 5. Welfare and effective size: Nash equilibrium vs. free trade ($\gamma = 0.6, \delta = 0$).

Any welfare change between free trade and the Nash equilibrium of tariffs must be caused by the endogenous choice of tariff rates. As tariff rates only depend on size L and productivity μ through their effect on the parameter c , so does the welfare measure ΔV . The first limit property states that the infinitesimally small economy will surely be worse off in the Nash equilibrium than under free trade where it faces the best terms-of-trade. In contrast, the infinitely large economy will be equally well off in the Nash equilibrium as under free trade because welfare converges to the autarky level in both situations. The last property states that the infinitely large country would benefit from an expansion of the economy of its trading partner (from which monopoly rents can be extracted).

In conjunction with continuity, these limit properties are sufficient to establish existence of a unique threshold size level c_T at which a country is indifferent between the Nash-equilibrium outcome and free trade (see Syropoulos, 2002).

Proposition 6. A country prefers the Nash-Equilibrium outcome over free trade if its effective relative size c exceeds the threshold level $c_T(\gamma, \delta) > 0$.

Since the structure of the proof is essentially identical to Syropoulos' treatment, a rigorous proof of existence and uniqueness is omitted. A graphical illustration is provided in Fig. 5. The idea is as follows: Properties 2) and 3) ensure that it is possible to be better off in the Nash equilibrium than under free trade (i.e. $\Delta V > 0$ for c very large). By property 1), the small country will be worse off in the Nash equilibrium such that ΔV is negative for small c . The existence of the threshold size level follows by continuity.

By symmetry, the threshold size level of each economy is identical. The threshold size level is greater than 0, because a Nash equilibrium induces Pareto inefficient production. Hence, when both economies are of equal size ($c = 0$) they will be both worse off than in the free-trade scenario. Such a prisoner's dilemma situation will always occur provided that the size asymmetries are not too great, i.e. whenever $|c| < c_T$.

Due to the simple general equilibrium structure it is possible to analyze how the threshold size level is influenced by transportation cost and comparative advantage:

Proposition 7. The threshold size level c_T is an increasing function of γ and a decreasing function of δ .

Proof. See Appendix B.8. □

Recall that if gains from trade increase, either through higher specialization benefits γ or lower transportation cost δ , Nash-equilibrium tariffs of both countries increase (see Proposition 5). On the margin, an increase in γ (decrease in δ) will make a country with threshold size c_T prefer free trade over the Nash-equilibrium outcome.

²² The accuracy of these approximations is greater as γ approaches 0. In the limit, the statements hold exactly.

²³ This welfare measure has been proposed by Lucas (1987) and Alvarez and Lucas (2007).

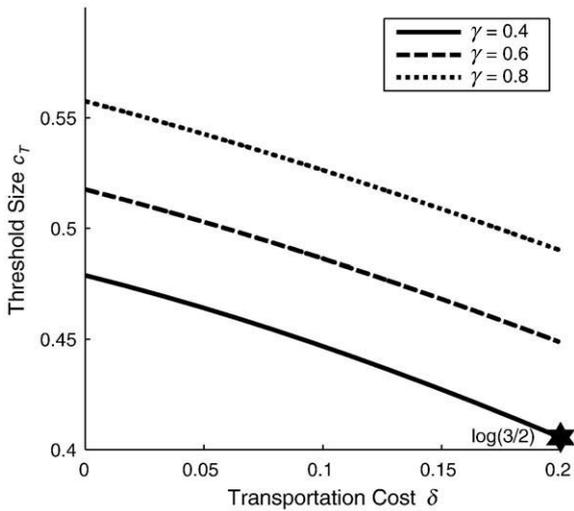


Fig. 6. Effects of transportation cost and comparative advantage on the threshold size.

The graph of the threshold size as a function of γ and δ is plotted in Fig. 6. In the limit, as the gains from trade vanish ($\gamma \rightarrow 0$), the home country prefers the Nash-equilibrium outcome over free trade if its economy is 50% larger than the one of the country or equivalently if its economic weight ψ is greater than $\frac{3}{5}$ ²⁴

$$\lim_{\gamma \rightarrow 0} c_T = \log\left(\frac{3}{2}\right) \tag{44}$$

5. Implications for self-enforcing trade agreements

The goal of this section is to discuss implications of the static Nash-equilibrium analysis for cooperative trade agreements within a dynamic context. Rather than solving for the set of optimal dynamic contracts, I want to highlight conjectures that can be obtained from the static analysis and point to the relevant extensions in a dynamic setup.²⁵

It is well understood, that any trade agreement has to be self-enforcing due to the lack of international courts with real enforcement power. Thus, a sustainable (subgame perfect) agreement requires that the short-run benefit from imposing an optimum tariff rate (see Eq. (26)) must be overwhelmed by the long-run cost resulting from a tariff war.²⁶ Due to the lack of commitment such a punishment by the other country has to be itself credible, i.e. subgame perfect. The static Nash-equilibrium outcome studied in this paper can be used as such an off-equilibrium path threat point to sustain efficient equilibrium allocations.²⁷ In the first part of this section I characterize efficient allocations and show that any desired redistributive transfer can be implemented through efficient tariff combinations in the spirit of Mayer (1981). This insight simplifies the subsequent discussion of the dynamic implications of my static analysis.

²⁴ In the limit ($\gamma \rightarrow 0, \delta = 0$) the cut-off goods evaluated at the threshold size level $c_T = \log\left(\frac{3}{2}\right)$ are given by $z_N = \frac{4}{5}, z_N^* = \frac{3}{10}, z_F = z_F^* = \psi(c_T) = \frac{3}{5}$. See Appendix B.8 for the results when $\delta > 0$.

²⁵ A complete characterization of the Pareto efficient dynamic contracts using the recursive machinery of Abreu et al. (1990) is beyond the scope of this paper.

²⁶ A summary of this literature is found in chapter 6 of Bagwell and Staiger (2002).

²⁷ In his analysis of oligopolies Friedman (1971) suggested this punishment equilibrium first. More sophisticated penal codes (see Abreu, 1988) may exist. Bagwell and Staiger (2002) show that there is also a limited role for on-equilibrium path retaliation within the GATT framework. This idea is not explored further in this paper.

5.1. Efficient allocations and transfers

In the absence of transportation cost, the Pareto-efficient free-trade production allocation is fully characterized by the boundary good \bar{z}_F which satisfies:

$$\bar{z}_F = \frac{1}{2} + \frac{1}{\gamma} \left(c - \log \frac{\bar{z}_F}{1 - \bar{z}_F} \right). \tag{45}$$

Cooperation gains from efficient free-trade production could be split up between countries through good transfers.²⁸ Proposition 8 implies that these transfers can also be implemented by efficient tariff combinations:

Proposition 8. Efficient tariff combinations $T_E = \frac{1}{T_E^*}$ imply:

- a) Efficient free-trade production: $\bar{z}_E = \bar{z}_F$
- b) Terms-of-trade satisfy: $\tilde{\omega}_E = \log(T_E) + \tilde{\omega}_F$
- c) Relative welfare satisfies: $\tilde{V}_E - \tilde{V}_E^* = \log(T_E) + \gamma \left(\frac{1}{2} - \bar{z}_F \right)$.

Proof. Efficient tariffs $T_E = \frac{1}{T_E^*}$ generate identical cut-off goods for both countries:

$$N_E = \bar{z}_E - \bar{z}_E^* = \frac{\log T_E + \log T_E^*}{\gamma} = \frac{\log T_E + \log \frac{1}{T_E}}{\gamma} = 0. \tag{46}$$

Substituting the terms-of-trade expression (Eq. (15)) into the definition of the cut-off good \bar{z}_E (Eq. (33)) implies:

$$\bar{z}_E = \frac{1}{2} + \frac{1}{\gamma} \left(c - \log \frac{\bar{z}_E}{1 - \bar{z}_E} \right) = \bar{z}_F. \tag{47}$$

Therefore, the terms-of-trade satisfy:

$$\tilde{\omega}_E = \log \left(\frac{\bar{z}_F}{1 - \bar{z}_F} \frac{1 + \bar{z}_F(T_E - 1)}{1 + (1 - \bar{z}_F) \left(\frac{1}{T_E} - 1 \right)} \right) - \tilde{L} = \tilde{\omega}_F + \log(T_E). \tag{48}$$

The indirect utility of both countries can be written as:

$$\tilde{V}_E = \frac{\gamma}{2} (1 - \bar{z}_F)^2 + \log \left(\frac{T_E}{1 + \bar{z}_F t_E} \right) \tag{49}$$

$$\tilde{V}_E^* = \frac{\gamma}{2} \bar{z}_F^2 + \log \left(\frac{T_E^*}{1 + (1 - \bar{z}_F) t_E^*} \right) \tag{50}$$

Simple algebra generates the desired result. □

The proposition implies that positive tariffs of the home economy ($T_E > 1$) are isomorphic to real transfers of export goods from the foreign country to the home country under free-trade production. Therefore, the dynamic decision problem of each government can be framed solely as a sequence of tariff rate decisions $\{T_s\}_{s=1}^{\infty}$. Tariff induced transfers are uniform across export goods due to a proportional decrease of the relative wage rate.

5.2. Tariff agreements

The folk-theorem (see Fudenberg and Maskin, 1986) implies that as the discount factor of all players approaches unity any individually rational allocation becomes incentive compatible. Applied to this setup, a trade agreement with efficient production (see previous

²⁸ Note, that within this general equilibrium framework side payments must involve physical transfers of goods from one country to the other.

section) can always be achieved provided that both governments are sufficiently patient. Thus, the Nash-equilibrium outcome only matters through the bounds it imposes on the distribution of rents from efficient production. However, reciprocal free trade without transfers ($T_E = T_E^* = 1$) can only be sustained if both countries are below the threshold size. This is important to the extent that there are political or other constraints (outside of the model) that rule out transfers from the small to the large economy. In this case Proposition 7 implies that free-trade agreements are more likely to occur if specialization benefits γ are higher or countries are closer (δ lower).²⁹

Gradualism in tariff agreements can be analyzed as optimal dynamic contracts (see also Bond and Park, 2002). Contract dynamics can either be driven by exogenous dynamics in model parameters or can result endogenously even in a stationary environment. An example of the former case is expected relative growth (change in c) of one economy (such as China or India). Since a growing economy's outside option of a tariff war is becoming relatively more attractive over time tariff agreements are expected to become more favorable to ensure dynamic sustainability. Differences in discount factors such as in Acemoglu et al. (2008) or Opp (2008) can give rise to non-trivial dynamics even in stationary settings. In the political economy literature self-interested governments with short time horizons are often modeled as effectively impatient. On the one hand, it is desirable to grant the impatient government initially more favorable contract terms ("teaser rates"). On the other hand, the dynamic provision of incentives requires that the agreement cannot become too unfavorable over time to ensure incentive compatibility. The trade-off between impatience and incentive compatibility is the cornerstone of the optimal dynamic contract.

Free-trade agreements are not sustainable if one economy is sufficiently impatient. In this case the short-run temptation to renege on the agreement is greater than the (discounted) long-run benefit from free trade. Therefore, the static Nash-equilibrium outcome should characterize the actual behavior of governments, not just the off-equilibrium path threat point. Empirical studies such as Broda et al. (2008) and Magee and Magee (2008) compare their results relative to optimum static tariffs. This benchmark equilibrium selection can be theoretically justified if governments are sufficiently impatient.

6. Conclusion

This paper has analyzed the strategic choice of tariff rates in a generalized Ricardian Model of Dornbusch et al. (1977) with CES preferences. These standard preferences in the international trade literature generate a particularly simple tariff structure: The optimum tariff schedule is uniform across goods. While this result cannot explain the observed dispersion of tariff rates across goods it is useful from a theoretical perspective even for different underlying models of trade.

The general equilibrium framework of DFS generates novel comparative statics predictions with regards to the role of comparative advantage and transportation cost in strategic settings. Higher gains from trade (higher comparative advantage, lower transportation cost) increase Nash-equilibrium tariff rates as any given tariff rate will lead to smaller deviations from efficient production specialization. Within the model this implies that the cut-off good for domestic production is less responsive to tariffs in settings with high comparative advantage or low transportation cost. Sufficiently large economies (in terms of productivity adjusted size) are better off in a Nash equilibrium than under free trade. The required threshold size is higher when Nash-equilibrium tariffs of both countries are higher, i.e. when comparative advantage is high or transportation cost is low.

The results of this paper suggest two lines of future research. On the empirical side, it would be interesting to examine the testable

predictions with regards to the exogenous parameters in the model in the spirit of Broda et al. (2008). The advantage of using a general equilibrium framework is that demand/supply elasticities are endogenously determined and need not be estimated.³⁰

A rigorous understanding of the static Nash-equilibrium outcome can be viewed as a stepping stone to the more complicated analysis of self-enforcing trade agreements within a dynamic context. For example, it would be interesting to study the effect of growth and increases in specialization gains on trade agreements.³¹ Likewise, it seems worthwhile to analyze the impact of business cycle shocks by extending the partial equilibrium framework of Bagwell and Staiger (2003) to a general equilibrium setup. This line of research might produce interesting implications about the relationship between (free) trade, growth and technology diffusion.

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Appendix A. General proofs

A.1. Partial derivatives of equilibrium conditions

The equilibrium conditions with uniform tariffs can be written as:

$$\begin{aligned} g_1 &= \tilde{L} + \tilde{\omega} + \log(F_{I_x}) - \log(F_D + F_{I_x}) + \log(F_D^* + F_{I_x}^*) - \log(F_{I_x}^*) = 0 \\ g_2 &= \tilde{\omega} - \delta - \log(T) - \tilde{A}(\bar{z}) = 0 \\ g_3 &= \tilde{\omega} + \delta + \log(T^*) - \tilde{A}(\bar{z}^*) = 0 \end{aligned}$$

where the functions F_D , F_I and F_{I_x} are defined in the Proof of Proposition 1 (Eqs. (18)–(20)). The partial derivative of equation i with respect to variable x is denoted by g_{ix} :

Term	Expression	Term	Expression
g_{1z}	$-b(\bar{z}) \frac{T}{(1 + \theta T)(1 - \theta)}$	g_{2z}	$-\tilde{A}'(\bar{z})$
g_{1z^*}	$-b^*(\bar{z}^*) \frac{T^*}{(1 + \theta^* T^*)(1 - \theta^*)}$	g_{2t}	$-\frac{1}{T}$
g_{1t}	$-\frac{\theta \delta}{1 + \theta t}$	g_{3z^*}	$-\tilde{A}'(\bar{z}^*)$
$g_{1\omega}$	$1 - (1 - \theta) \frac{\theta T}{1 + \theta T} - (1 - \theta^*) \frac{-\theta^* T^*}{1 + \theta^* T^*}$	g_{3t^*}	$\frac{1}{T^*}$

The total derivatives of $\tilde{\omega}$ and \bar{z} with respect to the tariff rate choice can be obtained using the implicit function theorem:

$$\frac{d\tilde{\omega}}{dt} = \frac{(g_{1z}g_{2t} - g_{1t}g_{2z})g_{3z^*}}{DD} \tag{51}$$

$$\frac{d\bar{z}}{dt} = \frac{g_{1z^*}g_{2t} + (g_{1t} - g_{2t})g_{3z^*}}{DD} \tag{52}$$

²⁹ This may explain the regional focus of most free trade agreements such as the NAFTA or within the EU.

³⁰ Different estimation methods for elasticities can lead to drastic differences in results. For example, Magee and Magee (2008) obtain very small elasticity estimates relative to Broda et al. (2008).

³¹ Predictable trends in specialization gains may result from production technologies that feature "learning by doing" such as in Devereux (1997).

where: $DD = -g_{1\bar{z}}g_{2\bar{z}} - g_{1\bar{z}}g_{3\bar{z}^*} + g_{2\bar{z}}g_{3\bar{z}^*} > 0$. Moreover, the partials with respect to effective relative size are given by:

$$\frac{d\bar{z}}{dc} = \frac{g_{3\bar{z}^*}}{DD} \tag{53}$$

$$\frac{d\bar{z}^*}{dc} = \frac{g_{2\bar{z}}}{DD} \tag{54}$$

A.2. Tariff rate formula: Proof of Proposition 2

The derived utility function can be expressed as (see Eq. (17)):

$$\tilde{V} = \tilde{\omega} - \frac{\theta}{1-\theta} \log(F_D + F_I) - \log(F_D + F_{I^*}) \tag{55}$$

where the functions F_D , F_I and F_{I^*} are defined in the Proof of Proposition 1 (Eqs. (18)–(20)). The partials of the objective function with respect to the relevant variables are given by:

$$\left[\frac{\partial \tilde{V}}{\partial t} \quad \frac{\partial \tilde{V}}{\partial \bar{z}} \quad \frac{\partial \tilde{V}}{\partial \tilde{\omega}} \right] = \left[-\frac{\theta}{T} \frac{\partial t}{\partial t} \frac{1-\theta}{1+\theta t} \quad -\frac{b(\bar{z})t}{1+\theta t} (1-\theta) \frac{1+\theta \partial t}{1+\theta t} \right] \tag{56}$$

where I have used $\tilde{\omega} - \delta - \log(T(\bar{z})) - \tilde{A}(\bar{z}) = 0$. The first-order condition implies:

$$\frac{\partial \tilde{V}}{\partial t} + \frac{\partial \tilde{V}}{\partial \bar{z}} \frac{d\bar{z}}{dt} + \frac{\partial \tilde{V}}{\partial \tilde{\omega}} \frac{d\tilde{\omega}}{dt} = 0. \tag{57}$$

Using the comparative statics results for $\frac{d\bar{z}}{dt}$ and $\frac{d\tilde{\omega}}{dt}$ from Appendix A.1 simple algebraic manipulations yield:

$$t_R = \frac{1 + \theta^* t^*}{T^*} \frac{1}{-\frac{b^*(\bar{z}^*)}{\tilde{A}(\bar{z}^*)} \frac{1}{1-\theta^*} + \theta^*(\theta^*-1)}. \tag{58}$$

The optimum tariff rate for the foreign country follows by symmetry. It can also be derived via the elasticity formula:

$$t_R^* = \frac{1}{\varepsilon - 1}. \tag{59}$$

The import demand elasticity of the home country $\varepsilon = \frac{d \log m}{d \log \omega}$ is given by:

$$\varepsilon = \frac{d \log \left(\omega \frac{F_{I^*}(\bar{z})}{F_D(\omega, \bar{z})} + F_{I^*}(\bar{z}) \right)}{d \tilde{\omega}} = 1 - \frac{\partial \log(F_D(\omega, \bar{z}) + F_{I^*}(\bar{z}))}{\partial \tilde{\omega}} \tag{60}$$

$$+ \frac{\partial \log \left(\frac{F_{I^*}(\bar{z})}{F_D(\omega, \bar{z}) + F_{I^*}(\bar{z})} \right)}{\partial \bar{z}} \frac{\partial \bar{z}}{\partial \tilde{\omega}}$$

$$= 1 + (\theta - 1) \frac{F_D}{F_D + F_{I^*}} - \frac{g_{1\bar{z}}}{g_{2\bar{z}}} = 1 + (\theta - 1) \theta \frac{T}{1 + \theta t} \tag{61}$$

$$- \frac{b(\bar{z})}{\tilde{A}(\bar{z})} \frac{T}{(1 + \theta t)(1 - \theta)}$$

where I have used the fact that:

$$\frac{F_D}{F_D + F_{I^*}} = \frac{\theta T}{1 + \theta t}. \tag{62}$$

Appendix B. Cobb–Douglas specification

B.1. Equilibrium conditions

For the specification used in Section 4, we obtain the following equilibrium conditions:

$$g_1(x, q) = \log(\omega) + \log\left(\frac{1 + \theta^* t^*}{1 - \theta^*}\right) + \log\left(\frac{1 - \theta}{1 + \theta t}\right) + \log(L) = 0$$

$$g_2(x, q) = \log(\omega) - \log(T) - \delta - \left[\mu + \frac{\gamma}{2} - \gamma \bar{z}\right] = 0$$

$$g_3(x, q) = \log(\omega) + \log(T^*) + \delta - \left[\mu + \frac{\gamma}{2} - \gamma \bar{z}^*\right] = 0 \tag{63}$$

$$g_4(x, q) = t - \gamma(1 - \theta^*) \frac{1 + \theta^* t^*}{T^*} = 0$$

$$g_5(x, q) = t^* - \gamma(1 - \theta) \frac{1 + \theta t}{T} = 0.$$

The partial derivatives are given by:

Term	Expression	Sign	Term	Expression	Sign
g_{1z}	$-\frac{T}{(1 + \theta t)(1 - \theta)}$	<0	g_{3t^*}	$\frac{1}{T^*}$	>0
g_{1z^*}	$-\frac{T^*}{(1 + \theta^* t^*)(1 - \theta^*)}$	<0	$g_{3\gamma}$	$-\frac{1}{2} + \bar{z}^*$?
g_{1t}	$-\frac{\theta}{1 + \theta t}$	<0	g_{4z^*}	$-\gamma \frac{1 - t^*(1 - 2\theta^*)}{T^*}$	<0*
g_{1t^*}	$\frac{\theta^*}{1 + \theta^* t^*}$	>0	g_{4t^*}	$\frac{1 - \theta^*}{1 + t^*} \frac{t}{1 + \theta^* t^*}$	>0
g_{2z}	γ	>0	$g_{4\gamma}$	$\frac{f_{z^*}}{\gamma f_{Iz^*}} = -\frac{t}{\gamma}$	<0
g_{2t}	$-\frac{1}{T}$	<0	g_{5z}	$\gamma \frac{1 - t(1 - 2\theta)}{T}$	>0*
$g_{2\gamma}$	$-\frac{1}{2} + \bar{z}$?	g_{5t}	$\frac{1 - \theta}{1 + t} \frac{t^*}{1 + \theta t}$	>0
g_{3z^*}	γ	>0	$g_{5\gamma}$	$\frac{f_{z^*}}{\gamma f_{Iz^*}} = -\frac{t^*}{\gamma}$	<0

The signs of g_{4z^*} and g_{5z} follow directly from Assumption 1.

B.2. Derived utility

In the Cobb–Douglas case, we can rewrite the derived utility (Eq. (17)) as:

$$\tilde{V} = (\tilde{\omega} - \delta)(1 - \theta) - \int_{\bar{z}}^1 \tilde{A}(z) dz + \theta T - \log(1 + \theta t) - \int_0^1 \log(a(z)) dz. \tag{64}$$

Realize that $\int_0^1 \log(a(z)) dz$ is just a constant and therefore irrelevant. Moreover, use $\omega = \log(T) + \delta + \tilde{A}(\bar{z})$ to obtain equivalent preferences \tilde{v} :

$$\tilde{v} = \tilde{A}(\bar{z})(1 - \theta) - \int_{\bar{z}}^1 \tilde{A}(z) dz + \log\left(\frac{T}{1 + \theta t}\right). \tag{65}$$

Using the exponentially affine production technology generates the derived utility expressions in the Nash equilibrium and free trade.

$$\tilde{v} = \frac{\gamma}{2} (1 - \bar{z})^2 + \log\left(\frac{T}{1 + \theta t}\right) \tag{66}$$

$$\tilde{v}_F = \frac{\gamma}{2} (1 - \bar{z}_F)^2 \tag{67}$$

B.3. Proof of Proposition 3: Properties of optimum response function

Let x denote the vector of endogenous variables $x = [\tilde{\omega} \bar{z} \bar{z}^* t]^T$ and q the vector of exogenous variables $q = [\tilde{L} \mu \gamma \delta t^*]^T$. The first four

elements of the function g (see Appendix B.1) define the equilibrium conditions.³² Moreover, let $D_x g$ represent the Jacobian of the function g with respect to x and $D_q g$ the Jacobian with respect to q .

$$D_x g = \begin{bmatrix} 1 & g_{1\bar{z}} & g_{1\bar{z}^*} & g_{1t} \\ 1 & g_{2\bar{z}} & 0 & g_{2t} \\ 1 & 0 & g_{3\bar{z}^*} & 0 \\ 0 & 0 & g_{4\bar{z}^*} & 1 \end{bmatrix} \text{ and } D_q g = \begin{bmatrix} 1 & 0 & 0 & 0 & g_{1t^*} \\ 0 & -1 & -1 & g_{2\gamma} & 0 \\ 0 & -1 & 1 & g_{3\gamma} & g_{3t^*} \\ 0 & 0 & 0 & \frac{g_{3\bar{z}^*}}{\gamma g_{1\bar{z}^*}} & g_{4t^*} \end{bmatrix} \quad (68)$$

$$\det(D_x g) = \frac{-g_{1\bar{z}^*} g_{2\bar{z}}}{<0} - \frac{g_{1\bar{z}} g_{3\bar{z}^*}}{<0} + \frac{g_{2\bar{z}} g_{3\bar{z}^*}}{>0} - \frac{g_{1\bar{z}} g_{2t} g_{4\bar{z}^*}}{<0} + \frac{g_{2\bar{z}} g_{1t} g_{4\bar{z}^*}}{>0} > 0 \quad (69)$$

The derivative of x with respect to q is given by the implicit function theorem:

$$D_q x = -(D_x g)^{-1} D_q g. \quad (70)$$

The fourth row vector yields the comparative statics of the best response tariff rate:

$$\begin{aligned} \frac{dt_R}{dc} &= \frac{-g_{4\bar{z}^*} g_{2\bar{z}}}{\det(D_x g)} > 0 \\ \frac{dt_R}{d\delta} &= \frac{(-2g_{1\bar{z}} + g_{2\bar{z}})g_{4\bar{z}^*}}{\det(D_x g)} < 0 \\ \frac{dt_R}{dt^*} &= \frac{g_{4t^*} g_{2\bar{z}} g_{3\bar{z}^*} \pi + g_{4\bar{z}^*} \left(g_{1\bar{z}} g_{3t^*} - g_{2\bar{z}} \frac{1-\theta^*}{(1+\theta^*t^*)(1+t^*)} \right)}{\det(D_x g)} < 0 \\ \frac{dt_R}{d\gamma} &= \frac{\frac{\varepsilon}{\varepsilon^*-1} + \gamma((\varepsilon-1)g_{2\gamma} - g_{3\gamma}\varepsilon)g_{4\bar{z}^*} + 1}{\det(D_x g)} > 0 \end{aligned} \quad (71)$$

where $\varepsilon, \varepsilon^* > 1$ represent the respective import demand elasticities and π the Marshall–Lerner condition:

$$\varepsilon = 1 - \frac{g_{1\bar{z}}}{g_{2\bar{z}}} > 1 \quad (72)$$

$$\varepsilon^* = 1 - \frac{g_{1\bar{z}^*}}{g_{3\bar{z}^*}} > 1 \quad (73)$$

$$\pi = \varepsilon + \varepsilon^* - 1 > 0. \quad (74)$$

B.4. Proof of Proposition 4: Existence of Nash equilibrium

Lemma 2. $\sigma_{t,t^*} = \left| \frac{d \log(T_R)}{d \log(T^*)} \right| < 1$.

Proof. We can rewrite $\det(D_x g)$ (see Appendix B.3) as:

$$\det(D_x g) = g_{2\bar{z}} \left[g_{3\bar{z}^*} \pi - \frac{g_{4\bar{z}^*}}{T} \left(\varepsilon - 1 + \frac{\partial T}{1 + \partial t} \right) \right] \quad (75)$$

$$\sigma_{t,t^*} = \frac{dT_R T^*}{dT^* T_R} = \frac{t g_{3\bar{z}^*} \pi - g_{4\bar{z}^*} \left(\frac{1 + \theta^* t^*}{1 - \theta^*} (\varepsilon - 1) + 1 \right)}{\frac{1 + \theta^* t^*}{1 - \theta^*} T g_{3\bar{z}^*} \pi - g_{4\bar{z}^*} \left(\frac{1 + \theta^* t^*}{1 - \theta^*} (\varepsilon - 1) + \frac{1 + \theta^* t^* \partial(1+t)}{1 - \theta^*} \right)} < 1. \quad (76)$$

The numerator is strictly smaller than the denominator, so that the ratio is less than one. □

³² The fifth equilibrium condition is only relevant for the Nash equilibrium analysis in which the foreign country's tariff rate is endogenous.

Let (S, ρ) define the complete metric space with $S = [0, \gamma - 2\delta]$ and $\rho = |x - y|$ and define the operator $\Pi^*: S \rightarrow S$ with $\Pi^* x = \tilde{\tau}^*(\tilde{\tau}(x))$ where $\tilde{\tau}$ represents the optimum response function for log tariff rates $\log(T)$. Π^* is a contraction since the optimum response functions $\tilde{\tau}(\log(T^*))$ and $\tilde{\tau}^*(\log(T))$ are continuous functions with slope uniformly less than one in absolute value (by Lemma 2). Hence, we can invoke the Contraction Mapping Theorem which guarantees the existence of a unique fixed point in S . This fixed point constitutes the Nash-equilibrium tariff rate of the foreign country. An analogous argument with $\Pi x = \tilde{\tau}^*(\tilde{\tau}(x))$ yields the unique Nash-equilibrium tariff rate $\log(T_N)$ of the home country. An interior Nash-equilibrium Pareto dominates the No-trade Nash equilibrium (autarky), since this option (no trade) is available in the action set of each country (choosing a prohibitive tariff rate of $\gamma - 2\delta$).

B.5. Proof of Proposition 5: Properties of Nash-equilibrium tariffs

The proof is analogous to the one of Proposition 3. The tariff rate of the foreign country t^* is now endogenous and given by the 5th equilibrium condition (see Appendix B.1). The comparative statics are again obtained by the implicit function theorem.

B.6. First-order approximation

B.6.1. Limit Nash equilibrium

I consider the limit as γ approaches 0 setting $\delta = r\gamma$ where $r < \frac{1}{2}$ is constant. Nash-equilibrium tariff rates are zero in the limit (see Eqs. (26) and (27)). In the limit, scaled tariff rates are given by:

$$\lim_{\gamma \rightarrow 0} \frac{\log(T_N)}{\gamma} = \bar{z}^* \text{ and } \lim_{\gamma \rightarrow 0} \frac{\log(T_N^*)}{\gamma} = 1 - \bar{z}. \quad (77)$$

The non-traded sector satisfies: (see Eq. (33)):

$$\lim_{\gamma \rightarrow 0} \bar{z} - \bar{z}^* = \lim_{\gamma \rightarrow 0} \frac{\log T_N - \log T_N^* + 2\delta}{\gamma} = 1 - \lim_{\gamma \rightarrow 0} (\bar{z} - \bar{z}^*) + 2r. \quad (78)$$

Rearranging yields:

$$\lim_{\gamma \rightarrow 0} \bar{z} - \bar{z}^* = \frac{1}{2} + r. \quad (79)$$

As comparative advantage γ approaches 0, the terms of trade just reflect absolute productivity advantage:

$$\lim_{\gamma \rightarrow 0} \tilde{\omega} = \mu. \quad (80)$$

Substituting Eq. (80) into the balance of trade condition implies:

$$\mu = \lim_{\gamma \rightarrow 0} \log \left(\frac{\bar{z}^*}{1 - \bar{z}^*} \right) + \log \lim_{\gamma \rightarrow 0} \left(\frac{1 + \bar{z}\tilde{t}}{1 + (1 - \bar{z}^*)\tilde{t}^*} \right) - \tilde{L} \quad (81)$$

which can be simplified to:

$$c = \log \lim_{\gamma \rightarrow 0} \left(\frac{\bar{z}^*}{1 - \bar{z}^*} \right). \quad (82)$$

Setting $\bar{z}^* = \bar{z} - \left(\frac{1}{2} + r\right)$ implies the closed-form expressions for \bar{z} and \bar{z}^* using $\psi(c) = \frac{\exp(c)}{1 + \exp(c)}$:

$$\bar{z}_N = \frac{1}{2} + r + \psi(c) \left(\frac{1}{2} - r \right) \quad (83)$$

$$\bar{z}_N^* = \left(\frac{1}{2} - r \right) \psi(c). \quad (84)$$

B.6.2. Limit free trade

As γ approaches 0, the free-trade balance of trade condition can be simplified analogously to the Nash-equilibrium limit (see Eq. (82)). Since tariff rates are 0 Eq. (33) now implies that $\bar{z}^* = \bar{z} - 2r$. This implies:

$$\bar{z}_F = \psi(c) + 2r(1 - \psi(c)) \tag{85}$$

$$\bar{z}_F^* = \psi(c)(1 - 2r). \tag{86}$$

B.6.3. First-order approximation

The first-order Taylor series expansion about $\gamma = 0$ for the Nash-equilibrium tariff rate is now given by:

$$t_N(\gamma) = t(0) + \gamma \lim_{\gamma \rightarrow 0} t'_N(0) + O(\gamma^2) \tag{87}$$

$$= \gamma \left(\frac{1}{2} - r \right) \psi(c) + O(\gamma^2) \tag{88}$$

$$t_N^*(\gamma) = \gamma \left(\frac{1}{2} - r \right) (1 - \psi(c)) + O(\gamma^2). \tag{89}$$

Using the first-order approximation the non-traded sector becomes:

$$N = \frac{\log(1 + t_N) + \log(1 + t_N^*) + 2\delta}{\gamma} \approx \frac{1}{2} + \frac{\delta}{\gamma}. \tag{90}$$

The equilibrium log wage rate can be determined via the second equilibrium condition g_2 :

$$\tilde{\omega} = \log(1 + t_N) + \delta + \mu + \frac{\gamma}{2} - \gamma \bar{z} \tag{91}$$

$$\approx \gamma \bar{z}^* + \frac{\gamma}{2} + \delta + \mu - \gamma \bar{z} = \mu + \frac{\gamma}{2} + \delta - \gamma N \approx \mu. \tag{92}$$

B.7. Proof of Lemma 1: Welfare comparison

If we subtract the derived utility under free trade (Eq. (43)) from the Nash-equilibrium utility (Eq. (42)) we obtain:

$$\Delta V(c, \gamma, \delta) = \frac{\gamma}{2} [(1 - \bar{z}_N)^2 - (1 - \bar{z}_F)^2] + \log \left(\frac{T_N}{1 + \bar{z}_N t_N} \right). \tag{93}$$

Since the tariff rate choice T and the threshold goods \bar{z}_N, \bar{z}_F are just a function of effective relative size c , ΔV does not depend separately on μ and L . The first two properties claimed in Lemma 1 are discussed in the text. The third property $\lim_{c \rightarrow \infty} \frac{\partial \Delta V(c, \gamma, \delta)}{\partial c} < 0$ is proved here:

$$\frac{d}{dc} \Delta V = \frac{\partial \Delta V}{\partial \bar{z}_N} \frac{d\bar{z}_N}{dc} + \frac{\partial \Delta V}{\partial \bar{z}_F} \frac{d\bar{z}_F}{dc} + \frac{\partial \Delta V}{\partial T_N} \frac{dT_N}{dc}. \tag{94}$$

Since $\frac{d\bar{z}_N}{dc} > 0$, we obtain:

$$\text{sign} \left(\lim_{c \rightarrow \infty} \frac{\partial \Delta V(c, \gamma, \delta)}{\partial c} \right) = \text{sign} \left(\lim_{c \rightarrow \infty} \frac{\partial \Delta V}{\partial \bar{z}_N} + \lim_{c \rightarrow \infty} \frac{\partial \Delta V}{\partial \bar{z}_F} \frac{d\bar{z}_F}{d\bar{z}_N} + \lim_{c \rightarrow \infty} \frac{\partial \Delta V}{\partial T_N} \frac{dT_N}{d\bar{z}_N} \frac{d\bar{z}_N}{dc} \right). \tag{95}$$

There exists a constant B such that $\max \left(\lim_{c \rightarrow \infty} \left| \frac{d\bar{z}_F}{d\bar{z}_N} \right|, \lim_{c \rightarrow \infty} \left| \frac{dT_N}{d\bar{z}_N} \right| \right) < B$. Moreover, it is easy to show that:

$$\lim_{c \rightarrow \infty} \vartheta_N = \lim_{c \rightarrow \infty} \bar{z}_N = \lim_{c \rightarrow \infty} \vartheta_F = \lim_{c \rightarrow \infty} \bar{z}_F = 1 \tag{96}$$

and

Term	Expression	$\lim_{c \rightarrow \infty}$
$\frac{\partial \Delta V}{\partial \bar{z}_N}$	$-\left(\gamma(1 - \bar{z}_N) + \frac{t_N}{1 + \bar{z}_N t_N} \right)$	$-\frac{t_N}{T_N}$
$\frac{\partial \Delta V}{\partial \bar{z}_F}$	$\gamma(1 - \bar{z}_F)$	0
$\frac{\partial \Delta V}{\partial T_N}$	$\frac{1 - \bar{z}_N}{T(1 + \bar{z}_T)}$	0

(97)

After simple algebraic manipulations we obtain:

$$\text{sign} \left(\lim_{c \rightarrow \infty} \frac{\partial \Delta V(c, \gamma, \delta)}{\partial c} \right) = \text{sign} \left(-\frac{t_N}{T_N} \right). \tag{98}$$

This completes the proof.

B.8. Proof of Proposition 7: Threshold size comparative statics

B.8.1. Threshold size level

In order to obtain closed-form expressions for the threshold size level and for the derivatives of interest I consider the limit as γ approaches 0 and setting $\delta = r\gamma$. It is useful to realize that:

$$\frac{T_N}{1 + \bar{z}_N t_N} = 1 + (1 - \bar{z}_N) \frac{t_N}{1 + \bar{z}_N t_N} \tag{99}$$

which allows us to rewrite Eq. (93) as:

$$(1 - \bar{z}_N)^2 - (1 - \bar{z}_F)^2 + \frac{2}{\gamma} \log \left(1 + (1 - \bar{z}_N) \frac{t_N}{1 + \bar{z}_N t_N} \right) = 0. \tag{100}$$

In the limit, second-order terms can be ignored:

$$\lim_{\gamma \rightarrow 0} [(1 - \bar{z}_N)^2 - (1 - \bar{z}_F)^2] + 2 \lim_{\gamma \rightarrow 0} (1 - \bar{z}_N) \lim_{\gamma \rightarrow 0} \frac{t_N}{\gamma} \lim_{\gamma \rightarrow 0} \frac{1}{1 + \bar{z}_N t_N} = 0. \tag{101}$$

Using $\lim_{\gamma \rightarrow 0} \frac{t_N}{\gamma} = \bar{z}$ and $\lim_{\gamma \rightarrow 0} \frac{1}{1 + \bar{z}_N t_N} = 1$ implies:

$$(1 - \bar{z}_N)^2 - (1 - \bar{z}_F)^2 + 2(1 - \bar{z}_N)\bar{z}_N^* = 0. \tag{102}$$

The derived closed-form expressions for \bar{z}_N, \bar{z}_F and \bar{z}_N^* (see Appendix B.6) allow us to solve for c_T :

$$\lim_{\gamma \rightarrow 0} c_T = \log \left(\frac{3}{2} \right). \tag{103}$$

The associated variables of interest are:

$$\lim_{\gamma \rightarrow 0} C_T = \log(3/2) \quad \lim_{\gamma \rightarrow 0} z_N^*(C_T) = \frac{3}{10} - \frac{3}{5}r \tag{104}$$

$$\lim_{\gamma \rightarrow 0} \psi C_T = \frac{3}{5} \quad \lim_{\gamma \rightarrow 0} z_F^*(C_T) = \frac{3}{5} + \frac{4}{5}r$$

$$\lim_{\gamma \rightarrow 0} z_N(C_T) = \frac{4}{5} + \frac{2}{5}r \quad \lim_{\gamma \rightarrow 0} z_F(C_T) = \frac{3}{5} - \frac{6}{5}r.$$

B.8.2. Comparative statics

In order to obtain the derivatives, we have to use again the implicit function theorem:

$$\frac{dc_T}{d\gamma} = - \left[\frac{d\Delta V(c_T, \gamma, \delta)}{dc} \right]^{-1} \left[\frac{d\Delta V(c_T, \gamma, \delta)}{d\gamma} \right] \tag{105}$$

$$\frac{dc_T}{d\delta} = - \left[\frac{d\Delta V(c_T, \gamma, \delta)}{dc} \right]^{-1} \left[\frac{d\Delta V(c_T, \gamma, \delta)}{d\delta} \right]. \tag{106}$$

From the discussion of Proposition 6 (see also Fig. 5) it must be true that the derivative of ΔV with respect to size c (see Eq. (94)) evaluated at the threshold size c_T is positive. The remaining derivatives of interest are:

$$\frac{d\Delta V}{d\gamma} = \frac{\partial \Delta V}{\partial \gamma} + \frac{\partial \Delta V}{\partial \bar{z}_N} \frac{d\bar{z}_N}{d\gamma} + \frac{\partial \Delta V}{\partial t_N} \frac{dt_N}{d\gamma} + \frac{\partial \Delta V}{\partial \bar{z}_F} \frac{d\bar{z}_F}{d\gamma} \tag{107}$$

$$\frac{d\Delta V}{d\delta} = \frac{\partial \Delta V}{\partial \bar{z}_N} \frac{d\bar{z}_N}{d\delta} + \frac{\partial \Delta V}{\partial t_N} \frac{dt_N}{d\delta} + \frac{\partial \Delta V}{\partial \bar{z}_F} \frac{d\bar{z}_F}{d\delta} \tag{108}$$

where the partials are given by:

$$\begin{aligned} \frac{\partial \Delta V}{\partial \bar{z}_N} &= - \left(\gamma(1 - \bar{z}_N) + \frac{t_N}{1 + \bar{z}_N t_N} \right) < 0 \\ \frac{\partial \Delta V}{\partial \bar{z}_F} &= \gamma(1 - \bar{z}_F) > 0 \\ \frac{\partial \Delta V}{\partial t_N} &= \frac{1 - \bar{z}_N}{T_N(1 + \bar{z}_N t_N)} > 0 \\ \frac{\partial \Delta V}{\partial \gamma} &= \frac{1}{2} \left((1 - \bar{z}_N)^2 - (1 - \bar{z}_F)^2 \right) < 0 \end{aligned} \tag{109}$$

The other terms follow directly from the comparative statics of the Nash equilibrium. Using the limiting expressions of Eq. (104) one obtains the following expressions:

$$\lim_{\gamma \rightarrow 0} \frac{dc_T}{d\delta} = - \left(\frac{19}{75} + \frac{2}{5}r \right) < 0 \tag{110}$$

$$\lim_{\gamma \rightarrow 0} \frac{dc_T}{d\gamma} = \frac{53}{300} + \frac{r^2}{5} > 0. \tag{111}$$

The signs of the derivatives are not affected if $\gamma > 0$ as the absolute value of the derivatives is increasing in γ . This completes the proof.

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