Bad Associations:
Protecting Trademarks from Dilution

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Abstract

The economic analysis of packaging law is driven largely by the view that packaging provides consumers with credible information about the attributes and origin of goods. However, multiple empirical observations suggest that the nature of packaging is more complex. In this paper, I develop a theoretical decision model that concisely explains these observations and makes a variety of novel predictions. The model provides an economic basis for the use of "anti-dilution" laws, which broadly protect the cues of a firm's trademark. Interestingly, the model suggests that nature's cues can be "diluted" if copied by lower-quality firms, providing justification for anti-dilution protection for nature's cues. Finally, the model suggests that a lack of protection for firms can lead to "fashion cycles," in which one firm copies the cues of another firm, leading the original firm to use new cues, which are then copied, and so on.

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1 Introduction

The economic analysis of packaging law is driven largely by the view that packaging provides information to the consumer about a good's attributes and source of origin (Landes and Posner, 1987). This suggests that packaging law should be designed both to prevent false advertising of attributes and to stop firms from copying other firms' trademarks in a way that confuses consumers about the origin of the good (Carter, 1990; Economides, 1988). However, multiple observations about packaging, as well as trends in trademark law, suggest that the nature of packaging is more complex. Often, firms desire to copy individual "cues" (such as fonts, names, or design elements) from other firm's packaging, even when it does not confuse the consumer about the origin of the good. To counter this action, recent trademark "dilution" laws forbid firms from using similar packaging to other firms' packaging, *even when the goods are sold in different markets and the packaging does not confuse the consumer*. Furthermore, just as firms copy cues from other firm's packaging, they also copy cues from the "packaging" of nature's goods (such as pictures of attractive women or names of places) in a similar way. These observations suggest that individual cues hold value to the consumer beyond their informational content. In fact, many have noted that consumers see trademarks as desired goods in themselves, and courts have recently starting viewing trademarks in this light (Kozinski, 1993; Lemley, 1999).

In this paper, I develop a rational-choice model that concisely captures this behavior. In the model, in addition to making decisions based on the information content of packaging, consumers are affected by the "associations" of the individual cues of the packaging. These associations are formed based on consumers' past experiences with goods in the market. Essentially, if one cue was previously displayed by goods that have consistently higher consumption values than those displaying a second cue, then the first cue has "better associations," and consumers are more attracted to packaging that contains the first cue.

This simple modification yields results that contribute to the literature on decision making and trademark law. With respect to decision making, the model groups a variety of distinct behaviors under one mechanism and makes a variety of predictions (some untested) about consumer behavior. First, the model predicts that arbitrary cues can gain (lose) value if they are displayed on a good along with a high (low) value attribute. For example, if a consumer becomes ill after drinking a dark tea, this leads the consumer to drink less of other dark liquids (such as Coca-Cola), regardless of the consumer's original preferences. Conversely, if a consumer has a positive experience in a city named "Tahoe," this causes the consumer to demand a truck called "Tahoe" more, even if the consumer originally had no preference for that name. Second, when multiple cues provide positive information only when seen together, consumers will overreact when they see each cue individually, acting as if each individual cue holds positive information on its own. Conversely, when multiple cues provide redundant information, consumers will overreact when they see the cues together, acting as if the collection of cues provides more information than it actually does.

The model also provides interesting conclusions about current trademark law. When firms are added to the model, lower value firms copy packaging cues of higher value firms whenever possible, even when it will not confuse consumers as to the origin of the product. This has an effect on consumer's associations with the packaging of each good, which leads consumers to demand more of the lower value good and less of the higher value good. This provides a more formal basis for anti-dilution laws, which protect individual cues of a trademark. These laws, which exist in many countries, apply to situations in which a company uses a mark that is similar to another company's "famous" mark, but there is no confusion about the origin of the product.[1] The basis for this law is that the entrant company is somehow "free riding" off the original trademark, and that the "value" of the original owner's trademark is slowly "whittled away" by this imitation (Landes and Posner, 2003; Schechter, 1927). As Landes and Posner state (2003, p. 161), "if it is irrational for people to be thus influenced by arbitrary associations, it is an irrationality so deeply rooted in human psychology as to make a claim for recognition by law." There is a reasonable debate in the law community about whether this extension is socially good (Kitch, 1990; Schechter, 1927) or bad (Carter 1990, Port 1994).

In addition to providing a formal basis for dilution across companies, the model suggests that *nature's* cues are also subject to dilution. As a result, a *commons* situation with respect to nature's cues arises: as no one has ownership over these cues, low value firms are free display these cues on their packaging, which lowers the demand and utility consumers receive from nature's goods. Therefore, even if consumers rationally prefer packaging that contains nature's goods, there can still be a welfare loss from this practice due to the associational externalities on nature's cues. This provides an interesting basis for trademark law that leads to a more strict allocation of rights to use nature's cues.

If there is limited protection of a firm's cues, the model suggests the potential for "fashion cycles." When a high-value firm uses a cue, it gains positive associations, which lead to higher demand for products that display these cues. If the use of the cue is not protected

[1] There are two types of dilution: "dilution by blurring' and "dilution by tarishment." Dilution by blurring occurs when, by using a mark with cues that are similar to the famous mark, the entrant company "blurs" the association of the trademark cues with the original product. Dilution by tarnishment occurs when the entrant's product is inferior, and therefore causes the cues to be associated with a lower quality good.

by law, low-value firms copy these cues in the following period. This can cause the cue to gain negative associations, leading the high-value firm to abandon it for cues with no associations, starting the cycle over. The model predicts that fashion cycles are more likely to occur with cues that have little inherent value, as the associational value is able to overpower the inherent value. This suggests that cues that appear in fashion cycles over and over are desired mainly for their associative value.

In the economics literature, there are multiple papers in which agents generalize across situations or actions. The model of case based reasoning (Gilboa and Schmeidler, 1995) assumes that, when facing a new problem, an agent chooses an action based on the performance of similar actions in similar problems in the past. Analogy-based expectation equilibrium (Jehiel, 2005) assumes that agents generalize other players' strategic moves across similar decision nodes. Fryer and Jackson (2008) present a model of categorization, in which boundedly rational agents group situations into different coarse categories. A paper on coarse thinking (Mullainathan et al, 2008), in which agents generalize the meaning of messages across situations, is most similar to this one. My paper is significantly different from these papers in a variety of ways, most notably in the fact that consumers generalize across multi-part messages. This allows, for example, arbitrary cues that contain no information to affect an agent's decision.

My model will follow the framework of the seminal advertising model of Becker and Murphy (1993), although I focus on the differing effects of using different forms of packaging on the consumer and other producers. Following their model, I assume that consumers actually receive more utility from goods that display cues with better associations, as opposed to simply being attracted to these goods. This assumption is supported by recent neurological experiments (McClure et al, 2004; Plassman et al, 2008) in which measured brain responses that correlate highly with the experience of pleasure were significantly affected by the inclusion of trademark or price cues on a good, suggesting that people experience more pleasure from a product when it displays these cues.

In the psychology literature, there are many papers discussing methods in which decision makers develops associations with cues. For example, there are multiple models (Rescorla & Wagner, 1972; Gluck & Bower, 1988) that posit specific dynamics concerning the formation of decision weights on different cues depending on a consumer's experience with goods that display those cues. Finally, there is a significant experimental literature on brand identity and marketing (Janiszewski & van Osselaer, 2001; van Osselaer, 2008) demonstrating that people act in ways that are consistent with associative models when asked to evaluate goods displaying different trademarks.

The paper is organized as follows. The second section presents the theoretic model. The third section discusses the individual consumer behavior associated with the model. The fourth section analyzes the effect of the addition of strategic firms into the model. Finally, the fifth section concludes.

2 The Model

The full model describes a multi-period strategic interaction between firms and a single consumer, who has a non-standard utility function. The consumer has little strategic power in the model, as her best response to the firms' choices is simply choose the utility-maximizing bundle of goods in each individual period. However, firms are more strategic, as all of firms' decisions today affect the consumer's decisions both today and in the future. I will first introduce the model without firms, in order to focus on the consumer's non-standard decision rule. I will discuss general behavior of a consumer with this non-standard rule. Then, I will add firms into the model. Finally, I will discuss the exact timing and equilibria of the game.

2.1 Consumers

2.1.1 Basic setup

The model takes place in discrete time, indexed by $t \in \{1, 2, ...T\}$. There are m goods, which are indexed by $i \in \{1, 2, ...m\}$. In each period, a single consumer chooses a consumption vector $x(t) = [x_1(t), ..., x_m(t)] \in (\mathbb{R}^+)^m$ over the m goods from perisable income y. In period t, good i has an (unknown) *consumption value* $v_i(t) \in \mathbb{R}$. Before making consumption decisions in each period t, consumers observe each good's price $p_i(t) \in \mathbb{R}^+$ and a *message* about the consumption value for each good i : $\overrightarrow{c_i(t)} = [c_i^1(t), ..., c_i^n(t)] \in C^1 \times ... \times C^N$, which consist of n parts, indexed by $j \in \{1, 2, ...n\}$. For simplicity, I will assume that $\bigcap_{j=1}^{n} C^j = \emptyset$. Let $c_\emptyset^j$ denote a cue that represents the absence of cue j. Define $C \equiv \bigcup_{j=1}^{n} C^j$, $c(t) \equiv [c_1(t), ..., c_m(t)]$, $v(t) \equiv [v_1(t), ..., v_m(t)]$ and $p(t) \equiv [p_1(t), ..., p_m(t)]$.

Rather than speaking of messages in the abstract, I will often speak as if the n parts of a message are different types of packaging characteristics (such as color, shape, type, or font) and as if the set C^j are *cues* that can describe characteristic j (such as red and yellow or round and square). So, two goods might *display* the cues [4 Inch Screen, Black, Curved Edges, Apple logo] or [Democrat, Female, Senator].

In this light, I will allow for some structure on the way that individual parts of messages (cues) relate to consumption value. For example, some cues (such as the size of the screen) might describe attributes that directly affect the consumption value of a good, perhaps only in the presence of other attributes. Other cues (such as a logo) might provide information about the company that produces the good, indirectly representing an unobserved quality parameter which affects the consumption value. Finally, some cues (perhaps such as shape or color) might be *arbitrary*, and have no direct or indirect information on consumption value.

Importantly, I will assume that there is **no** uncertainty about how each possible message $\overrightarrow{c_i(t)}$ relates to the consumption value of a good $v_i(t)$. This is so that I can avoid discussing equilibrium inference of the consumption value and focus on the way that associations affect an agent's decisions, which is the main contribution of this paper. This is not a significant assumption, as I will later require that goods not display cues representing attributes not found on the good, and I will assume that each company has a unique "protected" cue (such as a trademarked name). Given this, a consumer can correctly infer a good's attributes and infer the company that produced the good from the cues.

2.1.2 Associations and Associative Value

With a standard utility function, the consumer in this model solves a trivial utility optimization problem, as each good has a known consumption value once the good's message is observed. The non-standard assumption in the model is that a consumer will make decisions not only based on the consumption value $v(t)$ of the goods, but also on the *associative value* $a(t) = [a_1(t), a_2(t), ..., a_m(t)] \in \mathbb{R}^m$ of the goods (the precise utility function is discussed in the next section).

Rather than focusing on the information content of *entire messages*, the system that produces associative values focuses on the information content of the *individual parts* (or cues) of the message. One could imagine that this would be useful when trying to choose between goods that display *unknown messages* made up of *familiar cues* (when there is some structure to the messages). If previous observations consist of many cues and relatively few consumption values, there is no clear solution and many methods to solve this problem. I will not suggest an exact method in this paper, only a (seemingly reasonable) requirement on the system: *if messages containing cue 1 have led to "consistently higher" consumption values those containing cue 2, then a message containing cue 1 holds a higher associative value than the same message containing cue 2.*

To formalize this idea, I will define a cue's *associations* (the consumption values that have followed messages containing that cue), define when a cue has "consistently higher" or *better associations* than another cue, and discuss how a good's displayed cues' associations affect the good's associative value. For simplicity, I will assume that associations only form based on experiences from the previous period.

For each cue $c \in C$, define cue c's *associations* at time t as:

$$A^c(t) \equiv \coprod_{\text{All } i \text{ such that } c \text{ is an element of } \overrightarrow{c_i(t-1)}} v_i(t-1)$$

This is the disjoint union[2] of the consumption value of the goods that displayed cue j in period $t-1$. Note that the order of the listings of associations does not matter.

For example, if at time $t-1$, good 1 displays [Red Curved] and good 2 displays [Red Square], and the consumption values at $t-1$ are 6 and 8 respectively, then $A^{RED}(t)=\{\{6\},\{8\}\}$, $A^{CURVED}(t)=\{\{6\}\}$, and $A^{SQUARE}(t)=\{\{8\}\}$. In this case, I will say that the cue "Red" is *associated with* 6 and 8 (at time t), the cue "Curved" is associated with 6, and the cue "Square" is associated with 8. If there are no goods that displayed "blue" at time $t-1$, then $A^{BLUE}(t)=\{\{\}\}$ and I will say that the cue "Blue" has *no associations.* Importantly, note that a consumer gains associations based on the cues of all goods, regardless of her consumption choices[3]. The initial associations are set in the obvious way: $A^c(1)=\{\{\}\}$ for all $c \in C$.

I will now describe a relation $\geq_A$which defines a partial ordering over the possible associations of cues[4]. I will say that a cue $c_A \in C^j$ has *better associations* than cue $c_B \in C^j$ at time t if $A^{c_A}(t) \geq_A A^{c_B}(t)$ and *strictly better associations* if $A^{c_A}(t) >_A A^{c_B}(t)$. To describe the ordering, consider the mapping Ψ from possible associations into (discrete) probability density functions over $\mathbb{R}$ such that $\psi(x; A^{c_A})$ is the distribution formed if each of the members of A^{c_A} occurs with equal probability. For example, if $A^{c_A}=\{\{6\},\{6\},\{8\}\}$, then $\psi(x; A^{c_A}) = \left\{ \begin{array}{l} \frac{2}{3} \text{ for } x=6 \\ \frac{1}{3} \text{ for } x=8 \end{array} \right\}$. Then, let c_A have *better associations* than c_B if $\psi(x; A^{c_A})$ first-order stochastically dominates $\psi(x; A^{c_B})$ (and strictly better if $\psi(x; A^{c_A}) \neq \psi(x; A^{c_B})$).

[2]The disjoint union is used to differentiate situations in which a cue is displayed on multiple goods with the same quality from when a cue is displayed on one good with that quality. That is, {{4},{6},{6}} is kept distinct from {{4},{6}}.

[3]This is a simplifying assumption. If it was not made, it is possible that a consumer would avoid consuming certain goods to avoid gaining associations in the future that would affect her utility. While this is possible, this paper is largely concerned with the basic behavior that arises as a result of associations and resulting behavior of firms to capitalize on these preferences, so I will remove that possibility.

[4]The (simple) proof that this is indeed a partial ordering is in the appendix.

Letting $\Psi(A^{c_A})$ denote the cumulative distribution function of $\psi(x; A^{c_A})$, then:

Definition 1 $A^{c_A} \geq_A A^{c_B} \Leftrightarrow \Psi(x; A^{c_A}) \leq \Psi(x; A^{c_B})$ *for all* $x \in \mathbb{R}$. $A^{c_A} >_A A^{c_B}$ *if* $A^{c_A} \geq_A A^{c_B}$ *and* $A^{c_A} \leq_A A^{c_B}$

For example, using the definition, it must be that $\{\{1\}\} <_A \{\{1\},\{3\}\} <_A \{\{3\}\}$. While there is no ordering over $\{\{1\},\{3\}\}$ and $\{\{1\},\{2.9\},\{3\}\}$, it must be that $\{\{0\},\{1\},\{3\}\} <_A \{\{1\},\{3\}\} <_A \{\{1\},$ For cue c_A with no associations ($A^{c_A} = \{\{\}\}$), let $\psi(v^*; A^{c_A}) = 1$ for some $v^* \in \mathbb{R}$. That is, for a cue c_A with no associations, $\psi(x; A^{c_A})$ is equivilant to the distribution of a cue with an association of $\{\{v^*\}\}$.

Given this definition, I can discuss the determination of a good's associative value $a_i(t)$ given its displayed cue's associations at time t. Specifically, I will assume that $a_1(t) > a_2(t)$ if and only if $A^{c_1^j}(t) \geq_A A^{c_2^j}(t)$ for every j, with $A^{c_1^j}(t) >_A A^{c_2^j}(t)$ for at least one j.

As the partial ordering defining "better associations" and its relation to associative value is central to this paper, it requires further discussion. The intuition behind this ordering is best accomplished by again considered the problem of determining how parts of a message (the cues) relate to an outcome (the consumption values), given many cues and relatively few outcomes (a severly underidentified model). Consider the following (intuitive) process to solve this problem: First, individual cues are *isolated* and matched with their associated consumption values. Within each cue, the values are used to produce an *average value associated with each cue.* Then, these values are averaged across the cues in the good's message (weighted by the cue's importance) to create an *estimated consumption value* of the good with a certain message. The ordinal ranking I use is *much stronger* than the ranking produced using this process. In the first stage, rather than ordering cues using the expected value of the associations, the ordering above uses first order stochastic dominance. In the second stage, rather than using arbitrary weights to average across cues, the ordering requires that comparison is consistent across *any* non-zero weights.

2.1.3 Utility Function

I assume that, all else equal, the consumer chooses to consume more of a good with higher consumption value and higher associative utility. To formalize this, we assume the consumer's time separable (decision) vNM utility function takes the form:

$$\sum_{t=0}^{T} z(x(t), a(t), v(t), \lambda) = \sum_{t=0}^{T} u(x(t), v(t)) + \lambda w(x(t), a(t))$$

Each period, the consumer receives *consumption utility* $u(x(t), v(t))$, and *associative utility* $w(x(t), a(t))$. The consumption utility function is consistent with a standard utility function as it satisfies the following assumptions:

$$\frac{\partial u}{\partial x_i} > 0, \frac{\partial^2 u}{\partial x_i^2} < 0, \frac{\partial u}{\partial v_i} > 0, \frac{\partial^2 u}{\partial x_i \partial v_i} > 0.$$

That is, the consumer's consumption utility is increasing and concave in quantity, and an increase in the consumption value of a good increases a consumer's consumption utility and marginal consumption utility from that good.

The associative utility function satisfies the similar assumptions:

$$\frac{\partial^2 w}{\partial x_i^2} < 0, \frac{\partial w}{\partial a_i} > 0, \frac{\partial^2 w}{\partial x_i \partial a_i} > 0$$

That is, the consumer's associative utility is concave in quantity, and an increase in the associative value of a good increases a consumer's associative utility and marginal associative utility from that good. An increase in associative value has a similar effect on associative utility as an increase in consumption value has on consumption utility. Note that there is no assumption that $\frac{\partial w}{\partial x_i} > 0$, so it is possible that consuming more of good with low associative values can be harmful to associative utility.

I have chosen this separable form of the utility function to emphasize the effect of associations (clearly, I could have made the assumptions above directly about the non-separable utility function $z(\cdot)$): The parameter $\lambda \in [0, 1]$ represents the relative effect of associative utility. Specifically, if $\lambda = 0$, associations do not affect the consumer's decisions.

2.1.4 Mistakes

If a consumer develops associations with a certain cue, and it causes them to buy more of a good that displays that cue, is that a mistake? While I will remain agnostic with respect to this question, I will lead the possibility open. Recall that the period utility of a consumer is (dropping the time subscripts) $w(x, q, a, \lambda) = u(x, q) + \lambda v(x, a)$. From now on, I will speak of this as the *decision utility.* The *experienced utility* will be defined as $w(x, q, a, \alpha) = u(x, q) + \alpha v(x, a)$ with $\alpha \leq \lambda$. If $\alpha = \lambda$, decision utility and experienced utility are equal. If $\alpha < \lambda$, then the person makes decisions as if they experience more utility than they actually do from associations. For simplicity, this paper will focus on the cases in which $\alpha = 0$ or $\alpha = \lambda$.

3 Discussion of predicted behavior

Before I add firms into the model, it is useful to briefly discuss the effect of the utility modification on consumer behavior. As associations are formed from consumption experiences in the previous period, it is only necessary to discuss the effect of experience in period $t-1$ on decisions in period t. To capture the important effects, I introduce a small amount of structure and notation to links the displayed cues to the consumption value of a good. Let the *added value* of cue c_A be $\widehat{c_A} \in \mathbb{R}$ and let the *complement added value* of any multiple cues c_A, $c_B, ...$ be $\widehat{c_A, c_B, ...} \in \mathbb{R}$ (with $\widehat{c_A, c_B, ...}$ consistent for any ordering of c_A, $c_B, ...$). Then, assume that if good i displays message $[c_i^1, ..., c_i^n]$, it has consumption value $v_i = \sum_{k=1}^{n} \sum_{i=1}^{C_{(n,k)}} \widehat{M_{(n,k)}(i)}$ where $M_{(n,k)}(i)$ is the combinadic over the $C_{(n,k)}$ $k-$combinations of n cues. For example, the value of good i that displays message $[c_i^1.c_i^2]$ is $\widehat{c_i^1} + \widehat{c_i^2} + \widehat{c_i^1, c_i^2}$. I will let the absence of cue add no value, such that $\widehat{c_\emptyset^j} = 0$. Notice that any mapping from a good's cues into consumption values can be represented for some set of *added values* and *complementary values.* Therefore, the main restriction of this setup is that these added and complementary values are consistent over time.

The first observation concerns the efficiency of the relation of associative values and consumption values. Note that the system that produces associative values coorelates cues with their associated outcomes to determine a choice rule over new messages. The rule is somewhat simple: a good is chosen more if its message's individual parts were previously contained in messages that previously led to higher outcomes. By using this rule, the system is implicitly *attributing some part of the past high outcomes to the cue* and then *generalizing this effect across messages.* This assumes that individual parts of messages contain information about consumption value, and that this information is consistent across messages and time. The first proposition demonstrates that, if these assumptions are met and there are enough goods to "identify" the effect of different cues, the system places higher associative values on goods with higher consumption values (all proofs are in the appendix):

If there are no complementary values and cues are combined randomly in goods, then as the number of goods increases, a higher associative values implies a higher consumption values.

Proposition 1 *Let the complement added value of any combination of cues be zero. Let there be a good that displays each message in* $C^1 \times ... \times C^N$ *and let the* m *goods in time* $t-1$ *be chosen randomly from this set. Then, as* $m \to \infty$, $\Pr(v_i(t) > v_j(t) | a_i(t) > a_j(t)) \to 1$

When these assumptions are not met and there is less structure on the messages, the ranking of associative values diverges from the ranking of consumption values. Empirically, these are the situations in which people will diverge from predictions made by utility functions without associative utility. There are three basic situations in which this can occur, which are discussed below.

1) If a cue is (by chance or design) consistently paired with higher value cues in observed messages, the associative value of a good that contains that cue increases in the following period.

Proposition 2 *Let the complement added value of any combination of cues be zero. If* c_A *and* c_B *appear in the same message in period* $t-1$ *and* c_A *appears in good* i *period* t, *then* $\frac{\partial a_i(t)}{\partial \widehat{c_B}} > 0$.

When a cue is consistently paired with higher value cues, it is consistently seen in messages that lead to higher consumption values. Intuitively, the system that produces associative values is attributing some of the cause of the higher values to the cue (even though it was the selection of pairings, and not the cue itself, that caused the higher outcomes). For example, consider a situation in which one cue $c_A \in C^j$ is paired with consistently higher value cues than $c_B \in C^j$ in period 0. If $\widehat{c}_A = \widehat{c}_B$ then, in period 0, the person would have no preference over products that display these cues, all else equal, because the cues add the same consumption value and don't yet have any associations. However, in period 1, the consumer would prefer a good that displays c_A over a good that displays c_B. For example, in period 0, if a person consumes goods that display the color black (c_A) that are unpleasant and consumes goods that display the color white (c_B) that are pleasant, the person will prefer white goods over black goods in period 1, even though she had no preference in period 0. Clearly, if the use of black and white were switched, this effect would also switch. Note that even if $\widehat{c}_1 > \widehat{c}_2$ (goods displaying black were prefered to those displaying white in period 0), the person can prefer goods that display white over those that display black in period 1 if associative utility has a relatively large effect on total utility. Note that it is possible that $\widehat{c}_1 = \widehat{c}_2 = 0$. Then, even though black and white are *arbitrary* cues, the person develops a preference over goods that display these cues in period 1 due to her experience with goods that display black and white in period 0.[5]

[5]Interestly, this example had a particular impact on Coca-Cola in China recently. According to the Economist magazine (March 1st, 2007), Coca-Cola was not selling its Cola well in China's poorer interior, "where the dark colour of colas is associated with the dark tea traditionally used to mask the sediment in the local water." Coca-Cola therefore introduced an orange-colored drink, which has become the first or second most popular drink in every region of the country.

As another example, consider the situation above in which c_A is Arial font, c_B is Times New Roman font, and c_C is the name of a company. If firms have no trademark on font, a company name provides information on unobservable quality, but the font does not. If the company consistently uses the font c_A in the logo displayed on their good, then c_A becomes consistently paired with c_C. The utility function above predicts that a consumer who buys this good in period 0 and has a relatively good experience will have a relatively better associations with c_A than c_B in period 1. Then, the consumer will buy relatively more of a good that display c_A in period 1, even when this good display *a different company name*. The consumer has developed a preference for goods that display Arial font, based on her past experiences. Furthermore, in period 1, the consumer buys more of goods that display both c_A and c_C. So, if a company extends its product line to other products, the consumer buys more of these products than if she did not have the experience in period 0. The consumer will also buy more counterfeit goods that display c_A and c_C in period 1, even when she knows that these goods do not have the same unobservable quality as those in period 0. The consumer has developed a taste for products that display the entire logo, regardless of the informational content of the logo.

This prediction matches with the observations of multiple (Kozinski, 1993; Lemley, 1999) scholars, who have suggested that consumers are interested in buying goods that display certain components of trademarks, such as the Dallas Cowboy's team logo or the Nike swoosh, *regardless of who produces them*. Furthermore, there is a general trend in the courts to view trademarks as property in their own right. In fact, some suggest that this property is some of the most valuable of any business and needs protection (Magid et al, 2006). Others (Lemley, 1999; Lunny, 1999) question why a company should receive a lifetime monopoly on this new good. Our model does suggest that a component of a trademarks can become a commodity in itself. Furthermore, it suggests one reason that this "property" should be protected: it provides a further incentive to a firm to produce higher quality product, as this will result in trademarks with better associations. However, given that firms do not have lifetime patents on over investment-based innovations, it is not clear why they should have a lifetime monopoly on a trademark.

The next proposition discusses the effect of cue complementarities (when two cues add more value when seen together):

2) If the complementary value of two cues in the same message rises, the associative value of a good that only contains one of the cues increases in the following period.

Proposition 3 *If c_A and c_B appear in the same message in period $t-1$ and c_A appears in good i period t, then* $\frac{\partial a_i(t)}{\partial \widehat{c_A, c_B}} > 0$.

This proposition captures a situation in which the agent incorrectly generalizes across messages. c_A might only add value when observed with c_B, but if a consumer consumes a good that displays c_A and c_B, the system that produces associative values generalizes this positive effect to goods that only display c_A. For example, consider if c_A is the cue of Michael Jordan's face and c_B represents all of the cues at a basketball game. When c_A is seen with c_B, it adds a large amount of value, leading to a high consumption value. However, this experience leads to a high associative value on any good that displays c_A in the next period, regardless of whether c_B is present. So, a consumer will consume more of the cereal Wheaties that displays Michael Jordan's face. Perversely, if she does not enjoy the Wheaties in that period, she will consume less basketball games with Michael Jordan in the following period.

Consider a more extreme example, in which the cue c_A loses its added value or has a negative added value when placed in a message with c_C. Let c_A be the Wheaties logo and c_C represents all of the cues of a basketball game. Here, if Wheaties places a banner with its logo at a basketball game, this banner (presumably) does not provide any extra enjoyment for fans at the basketball game at time 0 (and might actually harm the fan's experience). However, if the consumer of the basketball game still had a relatively positive experience, they will place a higher associative value on goods that display the Wheaties logo in period 1. Here, even though the Wheaties logo had no (or negative) effect on the basketball experience, the system that produces associative values can be seen as attributing some of the value of the game to the logo.

Finally, assume that, in addition to the value of the good, there is a shock to the consumption value of a good each period, so that $v_i(t) = \sum_{k=1}^{n} \sum_{i=1}^{C_{(n,k)}} \widehat{M_{(n,k)}(i)} + \varepsilon(t)$ for some random variable $\varepsilon(t)$. Then:

3) If there is a positive unobservable or observable shock to the consumption value of a good that contains a cue, the associative value of goods that contain that cue is higher (lower) in the following period.

Proposition 4 *If c_A appears in a good in period $t-1$ and c_A appears in good i period t, then* $\frac{\partial a_i(t)}{\partial \varepsilon(t-1)} > 0$.

Here, the agent incorrectly attributes the effect of the shock on the value of the good to all of the observable cues. So, for example, if a person does not feel well, the consumption value of eating food is dramatically reduced. Clearly, this is only a temporary (and observable) shock. However, this lowers the associations of the cues (such as [Quiznos, Roast Beef, Bread]) displayed by a food. Therefore, in the following period, the person will consume less of the good [Quiznos, Roast Beef, Bread]. Furthermore, the person will generalize this across all messages containing this cues, so they will consume less [Quiznos, Turkey, Bread] or [Subway, Roast Beef, Baquette]. The converse is true with positive shocks. So, for example, if a person is temporarily aroused and then meets a person of the opposite sex, the cues displayed by that person will have better associations than if the person was not aroused. Therefore, in the following period, the person will value the person of the opposite sex more than if the shock did not occur.

3.1 Comparison with other models

The effect has some clear relations to the effect discussed in the paper on persuasion by MSS, in that value from one situation is transferred to another. In some situations, the model can be seen as providing similar results as MSS, if one thinks of an individual cue as the "message," the cues for other characteristics as the "category," and the addition value from seeing the cue as the "value." For example, in the situation with Michael Jordan, one could think of the cue "Michael Jordan" as the message, the other cues in the basketball game as representing the category "Basketball Games," and the other cues on the cereal box as representing the category "Cereal Boxes." "Michael Jordan" adds value in the "Basketball" category, but not in the "Cereal Boxes" category. However, the consumer transfers some of the value from the basketball category into the cereal box category.

This relationship, however, does not hold for most of the examples above. For example, our model predicts that if the consumer has positive experiences with the color "white" because it is paired with high value cues, then she will place more value on goods that display white in the future. However, in the example, the message "white" holds the same added value no matter what cues it is paired with (that is, no matter what category it appears in). Therefore, if this situation was forced in the MSS model, there would be no transfer of value in this situation. This is even more pronounced in the example of Wheaties placing a banner at the basketball game. Here, the message of Wheaties in the basketball game suggests no added value (in the "category" of cereal, the message Wheaties does suggest positive added value). If this situation was forced in the MSS model, the consumer would than place less value on Wheaties cereal in the future after seeing the basketball game. However, in our model, the consumer ends up placing more value on the Wheaties logo because it was artificially paired with high value cues. Finally, the relationship does not hold in situations with shocks. Here, the person would have to transfer the value from the category "positive shock" to the category "no positive shock."

3.2 Firms

This paper is largely concerned about the strategic behavior of firms that choose the cues for their goods. To this effect, assume that there are m firms, indexed by $i \in \{1, 2, ...m\}$ where firm i produces the firm-specific good i. For ease of exposition, I will focus on the situation in which there are two firms, which will be called (in a slight abuse of the model) "L" and "H." In the interest of differentiating the firms, let each firm's good have one fixed attribute c_i^*, which represents to the quality of the firm's good. Let a good that displays c_i^* have a value of v_i, with $v_H > v_L$ and $v_H > v^*$ (note that all other cues are arbitrary and hold no value). Furthermore, if good displays cues with associations $\{\{v_H\}\}$, let $\frac{\partial w}{\partial x_i} > 0$ (the high value's firm associations add utility to the consumer). The firms have identical marginal cost $b > 0$. At the beginning of each period t, after observing all decisions of firms and consumers in the past, firms simultaneously choose the cues displayed by their good $\overrightarrow{c_i(t)} \in C_i(t) \subset C^1 \times ... \times C^N$ and its price $p_i(t) \in \mathbb{R}^+$ to maximize profit.

The choice set $C_i(t)$ of each firm i and time t will be restricted in the following ways.

1. (Firms cannot display cues representing attributes not found on their good, which captures the effect of laws that prohibit false advertising). $c_i^1(t) = c_i^*(t)$
2. (Only certain firms will be allowed to use certain subsets of cues, which captures the effect of trademark law). Cue number j is *protected* in time t if, for each firm i, there exists a *protected cue* $\widetilde{c}_i^j \in C^j$ such that $\widetilde{c}_i^j \in C_i^j(t)$ but $\widetilde{c}_i^j \notin C_k^j(t)$ for any $k \neq i$. Cues are *fully protected* in time t, cue number j is *protected* in time t for every $j \in \{1, 2, ...n\}$

3.3 Timing and equilibrium

This is a game of complete information in which each firm's objective is to maximize total profit and the consumer's objective is to maximize (decision) utility. To review, for each period, firms publically choose cues and the price for their goods. After these decisions are made, the consumer publically chooses a consumption vector. I will focus on Markov Strategies, in which firm's decisions can be dependant on the previous period's packaging decisions (these affect associations in the current period), but not on periods previous to that. Therefore, firm $i's$ strategy in period 1 is $\overrightarrow{c_i(1)}$ and firm $i's$ strategy in period $t > 1$ is a mapping $\phi_i(t)$ from previous choices $\bigcup_{\tilde{i}=1}^{m} \overrightarrow{c_{\tilde{i}}(t-1)}$ of all firms into $\overrightarrow{c_i(t)}$. I will focus on pure strategies and, as the game is one of complete information, I will use subgame perfection as the solution concept.

4 Behavior of firms

4.1 Effect of Dilution Law

To discuss the effect of trademark laws on packaging choices and welfare, I will study the effect of creating a dilution law. When there is no dilution law, firms have some protected cues (such as a tradename, which is covered under traditional trademark law), but are not fully protected (as other firms can copy the color or style of their trademark). For exposition purposes, I will first focus on the situation in which $t \in \{1, 2, 3\}$ (the situation with more periods is discussed later). First, consider the situation in which there is no dilution law:

Proposition 5 *(No Anti-Dilution Law):*

If firm H is not fully protected, all equilibrium outcomes involve firm H displaying consistent protected cues for each protected cue number in all periods, and firm L in period 2 displaying firm H's unprotected cues from period 1.

When there is an anti-dilution law, all cues are protected, which leads to the following results and welfare changes:

Proposition 6 *(Anti-Dilution Law):*

If all cues are fully protected, all equilibrium outcomes involve firm H displaying consistent protected cues in all periods. Firm L is indifferent to its possible cue choices.

Proposition 7 *(Welfare Effects of an Anti-Dilution Law):*

The addition of a anti-dilution law yields lower quantity and price for good L in period 2 and higher quality and price for good H in period 2. Consequently, firm H's profits rise and firm L's profits fall. If $\lambda > 0$ and $\alpha = 0$, consumer's utility is increased. If $\lambda > 0$ and $\alpha = \lambda$, the change in consumer's utility is ambiguous.

When a anti-dilution law exists, firm H continues to use consistent cues throughout the game. This is because these cues gain positive associations, while Firm L is not legally allowed to copy these cues. However, when no anti-dilution law exists, firm L copies firm H's unprotected cues in period 2, after the associations form in period 1. As a result, in period 3, the unprotected cues have *strictly worse associations* than in period 2, leading to lower demand for a product with these cues. This captures the idea that firm H's cues have been *diluted* by their use by firm L, even though consumers are fully aware that the products are made by different companies. Note that as v_L falls, the effect of dilution is more pronounced. Also, note that if $\lambda = 0$ (associations do not affect the consumer's decisions), the set of equilibria in the environment with a anti-dilution law is identical to the set without this law, demonstrating that the addition of associative utility is driving this result.

When a consumer makes "mistakes" ($\alpha = 0$), total utility is increased by a anti-dilution law. The use of cues by firm L cause the consumer to have a higher decision utility from consuming good L, leading her to purchase relatively more of good L. However, as her experienced utility is not affected by associations, this is a misallocation of resources toward good L. Interestingly, when the consumer is a rational, the model does not necessarily suggest that a anti-dilution law improves welfare. Consumers receive more utility from (a set quantity of) good L when lower quality firms are allowed to copy these cues in period 2, but receive less utility from (a set quantity of) good H in period 3, after the cues have been diluted. The relative size of these effects determines the sign of the welfare effect.

An interesting corollary arises when "nature" is substituted for firm H. Nature's choice of cues is fixed and it has no (currently) protected cues under trademark law. Given the results with firm H, firm L will copy nature's cues whenever possible, as this causes firm L's goods to be more desirable. However, just as firm H's cues ended with strictly worse associations due to firm L's imitation, nature's cues are similarly "diluted" and nature's goods are consequently demanded less:

Corollary 2 *(Welfare Effects of a Anti-Dilution Law for nature):*

*If $\lambda > 0$ and $\alpha = 0$, consumer's utility is increased by an **anti-dilution law for nature's goods**. If $\lambda > 0$ and $\alpha = \lambda$, the change in consumer's utility from this law is ambiguous.*

If consumers make mistakes, then the use of nature's cues leads them to overconsume good L. However, *even if consumer rationally decides to consume more of good L, her overall utility can still be decreased by the use of nature's cues by firms.* This is a situation of the commons: as there is no owner of nature's "trademark," lower quality firms are free to copy it's good's cues, which imposes an externality on the consumer as she desires nature's goods less. If this effect outweighs the positive effect of increasing utility from good L, this behavior can reduce consumer welfare. This suggests that laws either protecting or assigning ownership of nature's cues might be welfare improving, even if consumers prefer to consumer goods with packaging that contains these goods.

4.2 Fashion cycles with no Anti-Dilution Law

The previous section suggested that, without an anti-dilution law, firm L will copy firm H's unprotected cues in period 2. This leads to strictly lower associations on these cues in period 3. How should firm H best respond to this action? If $v_L \simeq v_H$, then the unprotected cues' associations will remain largely unchanged, potentially leading consumers to continue to have higher demand for goods that display these goods. If this is the case, firm H will continue to use these cues in period 3. However, if v_L is extremely low, it is possible that the unprotected cue's associations will lead to consumers to avoid goods that display these cues. If this is the case, firm H would prefer to switch to cues in period 3 that have no associations. Interestingly, (in a model with more than 3 periods) firm L will then copy these new unprotected cues in period 4, leading firm H to switch to different cues in period 5, and so on. These fashion cycles are captured in the following proposition (recall that $\{\{v^*\}\}$ are the associations of a cue that was not displayed in the previous period):

Proposition 8 *(Fashion Cycles):*

Assume that for some v_L^*, $\{\{v_H\}, \{v_L^*\}\} =_A \{\{v^*\}\}$. Then:

1. *If $v_L < v_L^*$, all of the equilibrium outcomes involve firm H displaying different unprotected cues than in the previous period on odd periods and firm L displaying the unprotected cues that firm H displayed in the previous period on even periods.*
2. *If $v_L > v_L^*$, all of the equilibrium outcomes involve firm H displaying the same unprotected cues in every period and firm L displaying these unprotected cues after period 1.*

This proposition suggests that the lack of a full protection can lead to fashion cycles in unprotected cues. This is due to the fact that low-value firms display cues used by high value firms in the previous period, which leads to the dilution of the value of those cues in the following period, leading the high value firms to display to different cues in the following period. Interestingly, this suggests a method to determine the relative consumption value of cues, discussed in the following proposition:

Proposition 9 *Assume that* $v_L < v_L^*$. *Assume that unprotected cue* c_A *has an added value of* $\widehat{c_A}$. *Then, for some value* $\widehat{c_A^*}$:

1. *If* $\widehat{c_A} < \widehat{c_A^*}$, c_A *will be displayed by a firm for a maximum of two consecutive periods.*
2. *If* $\widehat{c_A} > \widehat{c_A^*}$, c_A *will be displayed by firms for no periods or for every period.*

This proposition suggests that cues with low added consumption value are more prone to be involved in fashion cycles. If a cue has a high added value, it will continue to be used even though it might develop low associations as a result of use by low quality firms. If a certain feature empirically continues to be displayed and then go "out of style," the model suggests that the value of this cue is largely due to the associations that the cue develops in this cycle, rather than an "inherent" desire for the cue.

5 Discussion and Conclusion

This paper presents a model of consumer behavior designed to explain a variety of observations in packaging and trademark law that are not easily explainable by a traditional model in which packaging simply provides information about the source or inherent value of the good. In the model, a consumer makes decisions based partially on her past experiences with the individual "cues" that combine to form the packaging of the good. Specifically, if her experiences with packaging containing cue 1 have led to "consistently higher" consumption values those containing cue 2, then she receives more (decision) utility from a good with packaging containing cue 1 instead of cue 2.

This simple rule leads to a variety of interesting testable predictions about behavior when goods are defined by multiple parameters. First, when multiple cues provide positive information only when seen together, the model predicts that consumers will overreact when they see each cue individually. For example, a person will be place a higher value on property in San Francisco, Costa Rica than San Juan, Costa Rica as a result of a positive experience with housing values in San Francisco, California. Conversely, when multiple cues provide redundant information, consumers will overreact when they see the cues together, acting as if the collection of cues provides more information than it actually does.

The model elucidates interesting tradeoffs in trademark and advertising law, which have not been discussed by the current theoretical economic literature. For example, the model predicts why laws preventing “dilution” are needed: in the model, when a lower quality entrant uses design elements similar to those of an established high quality firm, the entrant receives a short-term increase in demand, while "diluting" the associative power of the design elements, hurting the incumbent. However, the model also suggests a reason that the law will hurt consumers: cues associated with high value become goods in their own right, separate from their information content regarding the origin of the good. Therefore, this law is providing monopoly power to the incumbent firm to produce this good, which will directly hurt consumers.

The model broadens these conclusions to cues displayed on goods produced by "nature," such as the shape or look of an attractive person. Just as with the cues of high-value firms, low-value firms will copy these cues in order to increase demand for their goods. This can hurt consumers even if consumers choose purchase more of these goods, because the "dilution" of nature's cues reduces consumers' utility from nature's goods. The model suggests that, if policy makers believe that firms require protection from this phenomenon, they should consider the same protection for nature's goods.

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A Appendix

Proofs to other propositions in progress. Please contact the author for proof sketches.

Proposition 2

Proof: Assume that c_A and c_B are displayed by good j in period $t-1$. Assume that c_A is displayed by good i in period t. By definition, $a_i(t)$ is partially a function of $\psi(x; A^{c_A}(t-1))$, so I will write $a_i(t; \psi(x; A^{c_A}(t-1)))$. Note that $A^{c_A}(t-1) = \{...\{v_j(t-1)\}...\}$ and that $v_j(t-1) = \widehat{c_A} + \widehat{c_B} + ...$ By the definition of ψ, $\Psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + ...\}...\}) < \Psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + \varepsilon...\}...\})$ for each x given any $\varepsilon > 0$. Then by the definition of $a_i(t)$, $a_i(t; \psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + ...\}...\})) < a_i(t; \psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + \varepsilon...\}...\}))$. Therefore, $\frac{\partial a_i(t)}{\partial \widehat{c_B}} = \lim_{\varepsilon \to 0} \frac{a_i(t;\psi(x;\{...\{\widehat{c_A}+\widehat{c_B}+...\}...\}))-a_i(t;\psi(x;\{...\{\widehat{c_A}+\widehat{c_B}+\varepsilon...\}...\}))}{\varepsilon} > 0$.

Proposition 3

Proof: Assume that c_A and c_B are displayed by good j in period $t-1$. Assume that c_A is displayed by good i in period t. By definition, $a_i(t)$ is partially a function of $\psi(x; A^{c_A}(t-1))$, so I will write $a_i(t; \psi(x; A^{c_A}(t-1)))$. Note that $A^{c_A}(t-1) = \{...\{v_j(t-1)\}...\}$ and that $v_j(t-1) = \widehat{c_A} + \widehat{c_B} + \widehat{c_A, c_B} + ...$ By the definition of ψ, $\Psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + \widehat{c_A, c_B} + ...\}...\}) < \Psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + \widehat{c_A, c_B} + \varepsilon...\}...\})$ for each x given any $\varepsilon > 0$. Then by the definition of $a_i(t)$, $a_i(t; \psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + \widehat{c_A, c_B} + ...\}...\})) < a_i(t; \psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + \widehat{c_A, c_B} + \varepsilon...\}...\}))$. Therefore, $\frac{\partial a_i(t)}{\partial \widehat{c_A, c_B}} = \lim_{\varepsilon \to 0} \frac{a_i(t;\psi(x;\{...\{\widehat{c_A}+\widehat{c_B}+\widehat{c_A,c_B}+...\}...\}))-a_i(t;\psi(x;\{...\{\widehat{c_A}+\widehat{c_B}+\widehat{c_A,c_B}+\varepsilon...\}...\}))}{\varepsilon} > 0$.

Proposition 4

Proof: Assume that c_A and c_B are displayed by good j in period $t-1$. Assume that c_A is displayed by good i in period t. By definition, $a_i(t)$ is partially a function of $\psi(x; A^{c_A}(t-1))$, so I will write $a_i(t; \psi(x; A^{c_A}(t-1)))$. Note that $A^{c_A}(t-1) = \{...\{v_j(t-1)\}...\}$ and that $v_j(t-1) = \widehat{c_A} + \widehat{c_B} + ... + \varepsilon(t-1)$. By the definition of ψ, $\Psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + ... + \varepsilon(t-1) + \varepsilon\}...\}) < \Psi(x; \{...\{\widehat{c_A} + \widehat{c_B} + ... + \varepsilon(t-1) + \varepsilon\}...\})$ for each x given any $\varepsilon > 0$. Then by the definition of

$a_i(t)$, $a_i(t;\psi(x;\{...\{\widehat{c_A}+\widehat{c_B}+...+\varepsilon(t-1)\}...\})) < a_i(t;\psi(x;\{...\{\widehat{c_A}+\widehat{c_B}+...+\varepsilon(t-1)+\varepsilon\}...\}))$. Therefore, $\frac{\partial a_i(t)}{\partial \widehat{\varepsilon(t-1)}} = \lim_{\varepsilon \to 0} \frac{a_i(t;\psi(x;\{...\{\widehat{c_A}+\widehat{c_B}+...+\varepsilon(t-1)\}...\}))-a_i(t;\psi(x;\{...\{\widehat{c_A}+\widehat{c_B}+...+\varepsilon(t-1)+\varepsilon\}))}{\varepsilon} > 0$.

Propositions 5 and 6

Proof:

First consider the best response of player L to firm H's choices $c_H^j(t)$. Claim: In periods 2 and 3, for each cue number j, firm L will choose the cue $c_L^j(t)^*$ with the best associations in the choice set $C_i^j(t)$ for any previous choices $\bigcup_{\tilde{i}=1}^{m} \overrightarrow{c_{\tilde{i}}(t-1)}$. In period 1, firm L will choice cues $c_L^j(1)^*$ such that $c_L^j(1)^* \neq c_H^j(1)$ whenever possible. First, consider period 3. The statement must be true as there are no future periods in the game. Now, consider period 2. Consider otherwise, such that firm L chooses $c_L^j(2) \neq c_L^j(2)^*$. Consider a deviation to $c_L^j(2)^*$.

1) If $c_L^j(2)^* \neq c_H^j(2)$ and $c_L^j(2) \neq c_H^j(2)$, then L strictly prefers the deviation as she receives a higher payoff in period 2 and the same payoff in period 3 (as each cue number will have a cue with the same associations).

2) If $c_L^j(2)^* \neq c_H^j(2)$ and $c_L^j(2) = c_H^j(2)$, then L strictly prefers the deviation as she receives a higher payoff in period 2 and period 3 as the cue $c_L^j(2)$ in period 3 will have strictly better associations $\{\{v_H\}\} > \{\{v_H, v_L\}\}$ (and is in the choice set of firm L as $C_i(2) = C_i(3)$)

3) If $c_L^j(2)^* = c_H^j(2)$ and $c_L^j(2) \neq c_H^j(2)$, then L strictly prefers the deviation as she receives a total payoff. Given the strategies in the first period, it must be that $A^{c_L^j(2)^*} = \{\{v_H\}\}$. Then, in period 3, the worst associations for cue $c_L^j(2)^*$ will be either $\{\{\}\}$ or $\{\{v_H, v_L\}\}$. But, as a result of discounting, firm L prefers this to choosing $c_L^j(2)$ in period 2 (which has associations $\{\{v_L\}\}$ or $\{\{\}\}$), and choosing $c_H^j(2)$ (which has associations $\{\{v_H\}\}$) in period 3.

Now consider period 1. Firm L is indifferent between cues in period 1, but strictly prefers to not choose the same cues as firm H in period 1 when possible, as it will strictly increase the associations (and payoff) of cues in period 2.

Now, consider firm H's best response given that firm L is best responding. If firm H chooses a non-protected cue in period 1, firm L must copy that cue in period 2. Either firm H will choose a different cue in period 2 (with associations $\{\{\}\}$) or chose that cue in period 2 and a cue with associations $\{\{\}\}$ or $\{\{v_H, v_L\}\}$ in period 3. These outcomes are strictly worse than simply choosing the protected cue in each period and receiving associations of $\{\{v_H\}\}$ in each period.

This proves both propositions.