B Online Appendix

B.1 Value Estimation

For each good, Swoopo publishes a visible "worth up to" price, which is essentially the manufacturer's recommended price for the item. This price is one potential measure of value, but it appears to be only useful as an upper bound. In the most extreme example, Swoopo has held nearly 3,000 auctions involving 132 types of "luxury" watches with "worth up to" prices of more than $500. However, the vast majority of these watches sell on Internet sites at heavy discounts (20-40%) from the "worth up to" price. It is difficult, therefore, to justify the use of this amount as a measure of value if the auctioneer or participant can simply order the item from a reputable company at a far cheaper cost. That said, it is also unreasonable to search all producers for the lowest possible cost and use the result as a measure of value, as these producers could be disreputable or costly for either party to locate.

In order to strike a balance between these extremes, I estimate the value of items by using the average price found at Amazon.com and Amazon.de for the exact same item and using the "worth up to" price if Amazon does not sell the item.[45] As prices might have changed significantly over time, I only use Amazon prices for auctions later than December 2007 and scale the value in proportion to any observable changes in the "worth up to" price over time. Amazon sells only 29% of the unique consumer goods sold on Swoopo, but this accounts for 60% of all auctions involving consumer goods (goods that are sold on Amazon are likely to occur more in repeated auctions). For the goods that are sold at Amazon, the adjusted value is 79% of the "worth up to" price without shipping costs and 75% when shipping costs are added to each price (Amazon often has free shipping, while Swoopo charges for shipping). As the adjusted value is equal to the "worth up to" price for the 40% of the auctions for consumer goods that are not sold on Amazon, it still presumably overestimates the true value.[46]

B.2 Definition of Experience

There are multiple potential measures of "experience." For my analysis, I define the experience of a player at a point in time as the number of bids made by that player in

[45] This is a somewhat similar idea to that in Ariely and Simonson (2003) who document that 98.8% of eBay prices for CDs, books, and movies are higher than the lowest online price found with a 10-minute search. My search is much more simplistic (and perhaps, realistic). I only search on Amazon and only place the exact title of the Swoopo object in Amazon's search engine for a result.

[46] The main results of the paper are unchanged when run only on the subset of goods sold at Amazon.

all auctions before that point in time. The qualitative results below are robust to using different experience measures, such as the number of auctions previously played or the total time previously spent on the site. However, the number of bids, rather than these other measures, is a stronger predictor of behavior and profits. Intuitively, unlike a static war-of-attrition, feedback occurs instantly after each bid rather than only at the end of the auction.

Note that players potentially enter my bid-level dataset with prior experience. While I do not know the number of individual bids made by each player prior to the start of the bid-level dataset, the auction-level dataset does contain the number of top-10 appearances of each player in most of the auctions prior to the bid-level data. Using an estimated relationship between the number of appearances in the top-10 lists and the number of bids made by a player in the bid-level data, I (roughly) estimate the number of bids made by players prior to the start of the bid-level dataset using the top-10 lists prior to the start of the bid-level dataset.

B.3 Experience and User Profits

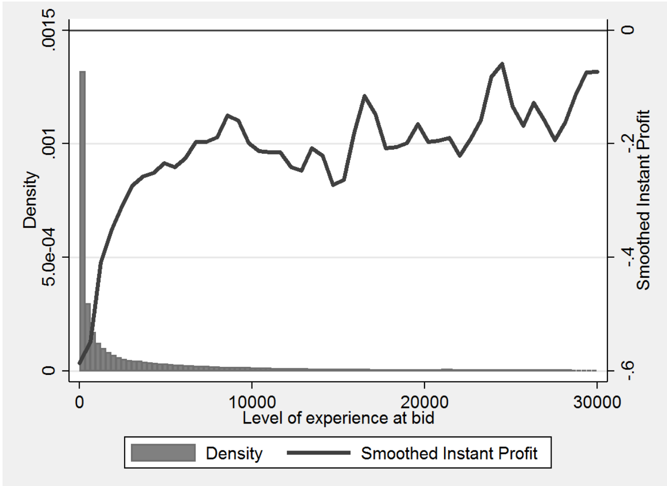
In this section, I examine whether more experienced players make higher expected profits. First, I use a non-parametric regression to show a clear positive relationship between experience and the expected profit from each bid. Then, I parameterize the regression to demonstrate that this relationship is highly statistically significant. Finally, in order to control for potential selection effects, I add user fixed effects, demonstrating that within-individual change (learning) plays a role in the relationship between experience and profits.

I define the concept of auctioneer *instantaneous profits* at time t given leader l_t $\pi(t, l_t)$ in the theoretical setup. Now, consider an analogous definition of the user's instantaneous profits. Clearly, when a user does not have a bid accepted and is not the leader, this user's profits are zero. However, when a user is the leader, if the auctioneer is making \$0.15 on average, the leader must be losing \$0.15 on average: that is, $\pi^U_{it}(t, l_t) = -\ \pi(t, l_t)$.

With this interpretation in mind, I rearrange the individual dataset into an (incomplete) panel dataset in which users are indexed by i and the order of the bids that an individual places is indexed by t, letting π^U_{it} be the payoff of user i's tth bid.[47] Figure B.1 displays a non-parametric regression of user profits on the level of experience of the user at the time of the bid, as well as a histogram of the number of bids made at each experience level for both types of auctions. Clearly, there is a positive concave relationship between the profit

[47]Note that, in an abuse of notation, t represents the bid number of a player, not the auction bid stage, as in the theoretical section.

Figure B.1: Profit and Experience



Notes: Soild line shows a local polynomial regression of instantaneous user profits (the profit from one bid) on user experience (the number of bids placed by a bidder at the time of the bid) The histogram shows the distribution of player experience at the time of each bid.

of a bid and the level of experience of the bidder. In an auction, a player with no experience can expect to lose $0.60 per each $0.75 bid, while those with very high experience levels have slightly positive expected payoffs per each bid. However, note that this positive effect requires a relatively large amount of experience: raising the expected value of a bid to near zero requires an experience level of nearly 10,000 bids.

Recall that Swoopo runs multiple types of auctions. For example, some auctions allow the use of the automated bidding system (*BidButler* auctions), while others do not allow this option (*Nailbiter* auctions). As these auctions are inherently different, I run the regression analysis separately for these different auction types.[48] Following the shape of the non-parametric regression, I first regress profits on the log of experience, with the results shown in column (1) and (3) of Table B.1 for Nailbiter and BidButler Auctions, respectively. These estimates show that, on average, there is an economically and statistically significant ($p<0.0001$) relationship between experience and profits. Specifically, for both Nailbiter and BidButler auctions, there is an increase in the expected return from each $0.75 bid by $0.05 as the experience of the bidder doubles.

However, it is not clear that this result is due to individual learning. It is possible that individuals with larger coefficients continue in the game for longer, leading t to be positively correlated with the error term. To help mitigate this selection problem, I estimate the model

[48]Interestingly, experience in BidButler auctions has a highly significant negative effect on profits in Nailbiter auctions, and vice versa.

with fixed effects for users, with the results shown in columns (2) and (4) of Table B.1. This specification suggests that, to the extent that the heterogeneity in learning functions is captured by an added constant, there is a selection effect, but that learning alone does play a role in the positive association between experience and profits. The coefficients for both types of auctions are highly significant, with the coefficient on Nailbiter auctions remaining nearly unchanged. This suggests that profits are increasing as players gain experience by placing more bids.

B.4 Details: Comparing Auctions With Different Values

As noted in Section 2.5, it is possible to visually compare auctions with different values of v by using the concept of *normalized time* $\widehat{t} = \frac{t}{v}$. The basic intuition is that, given a constant bidding increment k, an auction with a good of value v is *approximately* as likely to survive past time t as an auction with a good of value $2v$ surviving past time $2t$. This relationship is only approximate when the auction occurs in discrete time. In this section, I note that, as the length of a time period shrinks to zero and the game approaches continuous time, these survival rates converge.

Specifically, let Δt denote a small length of a time and modify the model by characterizing time as $t \in \{0, \Delta t, 2\Delta t, 3\Delta t ...\}$ and changing the cost of placing a bid to $c\Delta t$. With this change in mind, define the non-negative random variable T as the time that an auction ends. I define the continuous survival function $S_{cont}(t, l_t; k, v, c)$, hazard function $h_{cont}(t, l_t; k, v, c)$ for auctions with parameters k, v, c in the normal fashion (as $\Delta t \to 0$ and suppressing dependence on k, v, c):

$$S_{cont}(t, l_t) = \lim_{\Delta t \to 0} \Pr(T > t) \tag{1}$$

$$h_{cont}(t, l_t) = \lim_{\Delta t \to 0} \frac{S(t) - S(t + \Delta t)}{\Delta t \cdot S(t)} \tag{2}$$

Solving for these functions leads to the following proposition:

Proposition 5 *In the equilibrium noted in Proposition 2 (under the simplifying assumption of equally distributed sunk costs), when* $t < F$

$$h_{cont}(t, l_t) = \frac{c}{v + t\frac{1}{n}\theta - tk} \text{ and } S_{cont}(t, l_t) = \left(1 - \frac{t}{v}\left(k - \frac{1}{n}\theta\right)\right)^{\frac{c}{k - \frac{1}{n}\theta}}.$$

Note that, $S_{cont}(t, l_t; v) = S_{cont}(\alpha t, l_t; \alpha v)$.

While Proposition 5 is useful to determine the hazard and survival rates for a specific auction, it is more useful to compare hazard and survival rates across auctions for goods with different values. To that end, define $\widehat{t} = \frac{t}{v}$ as the *normalized time period,* define random variable $\widehat{T}$ as the (normalized) time that an auction ends, define the *normalized Survival and Hazard rates* in a similar way to above:

$$\widehat{S}(\widehat{t}, l_t) = \lim_{\Delta\widehat{t} \to 0} \Pr(\widehat{T} > \widehat{t}) \tag{3}$$

$$\widehat{h}(\widehat{t}, l_t) = \lim_{\Delta\widehat{t} \to 0} \frac{\widehat{S}(\widehat{t}) - \widehat{S}(\widehat{t} + \Delta\widehat{t})}{\Delta\widehat{t} \cdot \widehat{S}(\widehat{t})} \tag{4}$$

With this setup, it is easy to show that:

Proposition 6 *In the equilibrium noted in Proposition 2 (under the simplifying assumption of equally distributed sunk costs), when* $t < F$

$$\widehat{h}_{cont}(\widehat{t}, l_t) = \frac{c}{1+\widehat{t}\frac{1}{n}\theta - \widehat{t}k} \text{ and } \widehat{S}_{cont}(\widehat{t}, l_t) = (1 - \widehat{t}(k - \tfrac{1}{n}\theta))^{\frac{c}{k-\frac{1}{n}\theta}}.$$

Note that these functions are not dependant on v. Given this result, it is possible to combine auctions with goods of different values in the same visual representation of the empirical and theoretical hazard rates by using the normalized time measure, rather than the standard time measure.

B.5 Naive Equilibrium

In the paper, I discuss a naive sunk cost Markov-Perfect equilibrium. First, note that an equilibrium of the standard game given value v is a collection of vectors of probabilities $p_1, p_2, \ldots$. Define p_i^t as player i's bidding probability at time period t, the vector of other players' bidding probabilities at time t as $\mathbf{p}_{-i}^t$, and the player $i's$ payoff at period t as $\pi_i^t(p_i^t, \mathbf{p}_{-i}^t; v_i)$. Then, for $p_i, \mathbf{p}_{-i}$, an Markov-Perfect Equilibrium must, for all i, satisfy:

$$p_i \in \arg\max E[\sum_{\tau=t}^{\infty} \pi(p_i, \mathbf{p}_{-i}, v)]$$

Define this set of equilibria as $\Theta(v)$.

In the modified game, player i's payoff from receiving the good at time t is $v + \theta s_i^t c$, where $s_i^t c$ represents the sunk costs of the player. Furthermore, the player mistakenly believes that at time t, (1) all other players' perceived value will remain constant at $v + \theta s_i^t c$. and (2) her future perceived value will remain constant at $v + \theta s_i^t c$. I define a *naive* Markov-Perfect Equilibrium as any collection of bidding probabilities $p_i(s_i^t c)$ for each player given the sunk cost expenditures at time t, such that:

$$\begin{aligned} p_i(s_i^t c) &\in \arg\max E[\sum\nolimits_{\tau=t}^{\infty} \pi(p_i(s_i^t c), \widetilde{\mathbf{p}}_{-i}(s_i^t c), v + \theta s_i^t c)] \\ \widetilde{\mathbf{p}}_{-i}(s_i^t c) &\in \Theta(v + \theta s_i^t c) \end{aligned}$$

where $\widetilde{p}_{-i}(s_i^t c)$ is defined as player i's perception of all other player's strategies given sunk costs $s_i^t c$, $\widetilde{\pi}(p_i, \widetilde{p}_{-i})$.

B.6 Structural Model: Alternative

The primary theoretical predictions of hazard rates given the standard risk-neutral model of behavior do not describe the empirical hazard rates well. There are a variety of potential explanations for this deviation. One explanation is the sunk costs fallacy, which I outline in the main section of the paper, leading players to perceive the value of the good as $v + \theta s_i c$, where s_i represents the number of (sunk) bids made by the player at the time of bidding. Under the assumption that sunk costs are distributed equally across players yields a hazard of $\frac{c}{v+\frac{1}{n}\theta s_i c - tk}$. A second explanation is that players receive an additional joy-of-winning that is either *constant* across auctions or is *relative* to the value of a good (that is, a player receives a additional payoff of ψ_c or $v\psi_r$ from winning) or , leading players to perceive the value of the good as $\frac{c}{v+v\psi_r+\psi_c-tk}$. Finally, Platt et al. (2013) have suggested that risk-preferences might explain the results, leading to a hazard of : $\frac{1-e^{-\alpha(v-tk-c)}}{e^{-\alpha(-c)}-e^{-\alpha(v-tk-c)}}$ given a utility function over final wealth of $\frac{1-e^{-\alpha w}}{\alpha}$ Combining the hypothesis leads to a hazard rate of $\frac{1-e^{-\alpha(v+\theta s_i c+v\psi_r+\psi_c-t\cdot k-c)}}{e^{-\alpha(-c)}-e^{-\alpha(v+\theta s_i c+v\psi_r+\psi_c-t\cdot k-c)}}$.

Using the aggregate bid-level dataset constructed from the auction-level dataset (as in Section 4.2), it is possible to estimate each parameter using a maximum likelihood routine. Note that $\frac{1}{n}\theta$, rather than the individual sunk cost parameter θ, is identified. The routine identifies the structural parameters that maximize the log likelihood of observing the realized outcome that the auction ends at each point in time given the auction characteristics. The results are reported in Table B.2:

Controlling for risk-seeking and joy-of-winning, the sunk cost parameter remains robust and intuitively matches the reduced form regressions in the paper. The risk-preference model provides explanatory power, with α (the measure of risk seeking) estimated at -.00026 and

Table B.1: Instantaneous User Profit and User Experience

	Nailbiter		Non-Nailbiter	
	(1) OLS	(2) FE	(3) OLS	(4) FE
Ln[Experience]	0.076***	0.073***	0.070***	0.025***
	(0.0047)	(0.015)	(0.0024)	(0.0054)
Constant	-0.65***	-	-0.87***	-
	(0.020)	-	(0.011)	-
User FE	-	X	-	X
Observations	1,248,482	1,248,482	11,985,502	11,985,502

Notes: Standard errors in parentheses (clustered on users in all regressions). Linear regressions of instantaneous user profits (the profit from one bid) on log user experience (the log of the number of bids placed by a bidder at the time of the bid) for different auction types (nailbiter and non-nailibiter). Columns (2) and (4) include user fixed effects. Constant not reported for regressions with fixed effects. * p<0.05, ** p<0.01, *** p<0.001.

Table B.2: Structural Estimation

	(1)	(2)	(3)	(4)
Aggregate sunk cost parameter: $\frac{1}{n}\theta$	.232***	.191***	.079***	.079***
	(.002)	(.002)	(.003)	(.003)
Risk parameter: α	-	-.00026***	-	-.000014*
	-	(.000)	-	(.000)
Joy-of-winning (constant): ψ_c	-	-	-5.31***	-5.23***
	-	-	(.200)	(.221)
Joy-of-winning (relative): ψ_r	-	-	0.35***	0.35***
	-	-	(.005)	(.006)
Number of auctions	147,578	147,578	147,578	147,578
Implied number of bids	94,081,054	94,081,054	94,081,054	94,081,054
Log psuedo-likelihood	-988,915	-988,416	-986,128	-986,127

Notes: T-statistics in parentheses. Structural Estimates of aggregate sunk cost parameter, risk-parameter from an exponential utility function, and an additive and multiplicative joy-of-winning parameter. Standard errors are clustered on auctions in all estimations. * p<0.05, ** p<0.01, *** p<0.001.

-.000014, depending on the specification. For reference, a person with these risk preferences would pay \$112.54 or \$100.63 for a $\frac{1}{10}$ chance at \$1000.

B.7 Robustness of the theoretical model

B.7.1 mod(y-k,c)$\neq 0$

The results in the analytic section relied heavily on the assumption that mod(v-k,c)$= 0$. If this assumption does not hold, there is no equilibrium in which the game continues past period 1. However, as the following proposition shows, strategies that lead to the hazard rates in Proposition 2 form an ϵ equilibrium with ϵ very small and limiting to 0 as the size of time periods shrinks to 0:

Proposition 7 *If* $\text{mod}(v-c,k) \neq 0$, *there is no equilibrium in which the game continues past period 1. Define* $F^* = \max(t|t < \frac{v-c}{k} - 1)$. *There is an* ϵ*-perfect equilibrium which yields the same (discrete) hazard rates as those in Proposition 2 with* $\epsilon = \frac{1}{n}(1 - \frac{c}{v-F^*k})(v - (F^*+1)k - c)[\prod_{t=1}^{F^*-1}(1 - \frac{c}{v-tk})]$. *There is an contemporaneous* ϵ^c*-perfect equilibrium (Mailath (2003)) which yields the same (discrete) hazard rates as those in Proposition 2 with* $\epsilon^c = \frac{1}{n-1}(1 - \frac{c}{v-F^*k})(v-(F^*+1)k-c)$.

To give an idea of the magnitude of the mistake of playing this equilibrium in auctions in my dataset, consider an stylized auction constructed to make ϵ as high as possible, with $v =$\$14.95$, c =$\$.75$, k =$\$.15, and $n = 20$. In this case, $\epsilon =$ \$0.0000000000224 and $\epsilon^c =$\$0.00060. That is, even in the most extreme case and using the stronger concept of contemporaneous ϵ^c-perfect equilibrium, players lose extremely little by following the proposed strategies. This is because their only point of profitable deviation is at the end of the game, where their equilibrium strategy is to bet with low probability, there is a small chance that their bet will be accepted, and the cost of the bet being accepted is small (and, ex ante, there is an extremely small chance of ever reaching this point of the game).

B.7.2 Independent Values

In the model in the main paper, I assume that players have a common value for the item. The equilibrium is complicated if players (1) have values v_i drawn independently from some distribution G of finite support before the game begins or (2) $v_i(t)$ is drawn independently from G at each time t, even when the values are common knowledge. In these equilibria, players' behavior is dependent largely on the exact form of G, with very few clear results about bidding in each individual period (which is confirmed by numerical simulation). However, if these independent values tend to a common value, the distribution of hazard rates approaches the bidding hazard rates in the following way:

Proposition 8 *Assume that (1) v_i is drawn independently from G before the game begins or (2) $v_i(t)$ is drawn independently from G at each time t. For any distribution G, there is a unique set of hazard rates $\{h^G(1), h^G(2), ...h^G(t)\}$ that occur in equilibrium. Let the the support of G_i be $[v - \Delta_i, \overline{v} + \Delta_i]$. For any sequence of distributions $\{G_1, G_2, ...\}$ in which $\Delta_i \to 0$ and the game continues past period 1 in equilibrium, $h^G(t, l_t) \to h(t, l_t)$ from Proposition 2 for $t > 0$. For any sequence of distributions G with $\Delta_i \to 0$ and $\Delta t \to 0$, there exists a sequence of corresponding contemporaneous ϵ^c-perfect equilibria with hazard and survival rates equal to those in Proposition 2 in which $\epsilon^c \to 0$.*

B.7.3 Leader can bid

Throughout the paper, I assume that the leader cannot bid in an auction. This assumption has no effect on the equilibrium noted in Proposition 2 as the leader will not bid in equilibrium even when given the option.

Specifically, consider a modified game in which the leader can bid. Now, a (Markov) strategy for player i at period t is the probability of betting both if a non-leader ($p_t^{i,NL*}$) and, for $t > 0$, when a leader ($p_t^{i,L*}$) (there is no leader in period 0).

Proposition 9 *In the modified game, Proposition 2 still holds.*

However, the assumption that the leader cannot bid does dramatically simplify the exact form of other potential equilibria. Specifically, without this assumption, there exist equilibria in which play continues (slightly) past period 1 without following the equilibrium hazard rate in Proposition 2. That is, the logic of Proposition 3 fails. This occurs because the ability of a leader to bid in period t distorts the incentives of non-leaders in previous periods. To see this, consider the situation in which $h(t+1, l_t) = 1$ and $h(t, l_t) = 0$. When leaders cannot bid, there is no benefit from a non-leader bidding in period $t-1$ as she will not win the object in period t (because the game will continue with certainty) or period $t+1$ (because she will be the leader in period t (who cannot bid in period t) and therefore cannot be a leader at $t+1$), at which point the game will end. However, when leaders can bid, it is possible to construct situations in which non-leaders in period $t-1$ benefit from bidding.

Although there is still no chance that the non-leader in period $t-1$ will win the object in period t by bidding, she will be able to bid (as a leader) in period t, leading to the possibility that she will win the object in period $t+1$. Therefore, non-leaders will potentially bid in this situation in equilibrium not to win the object in the following period, but simply to keep the game going for a (potential) win in the future.

B.7.4 Allowing Multiple Bids to Be Accepted

Allowing multiple bids to be accepted significantly complicates the model, especially in a declining-value auction. Consider a player facing other players who are using strictly mixed strategies. If the player bids in period t, there is a probability that anywhere from 0 to $n-2$ other non-leading players will place bids, leading the game to immediately move to anywhere from period $t+1$ to period $t+n$. In each of these periods, the net value of the object is different, as is the probability that no player will bid in that period and the auction will be won (which is dependent on the equilibrium strategies in each of the periods).

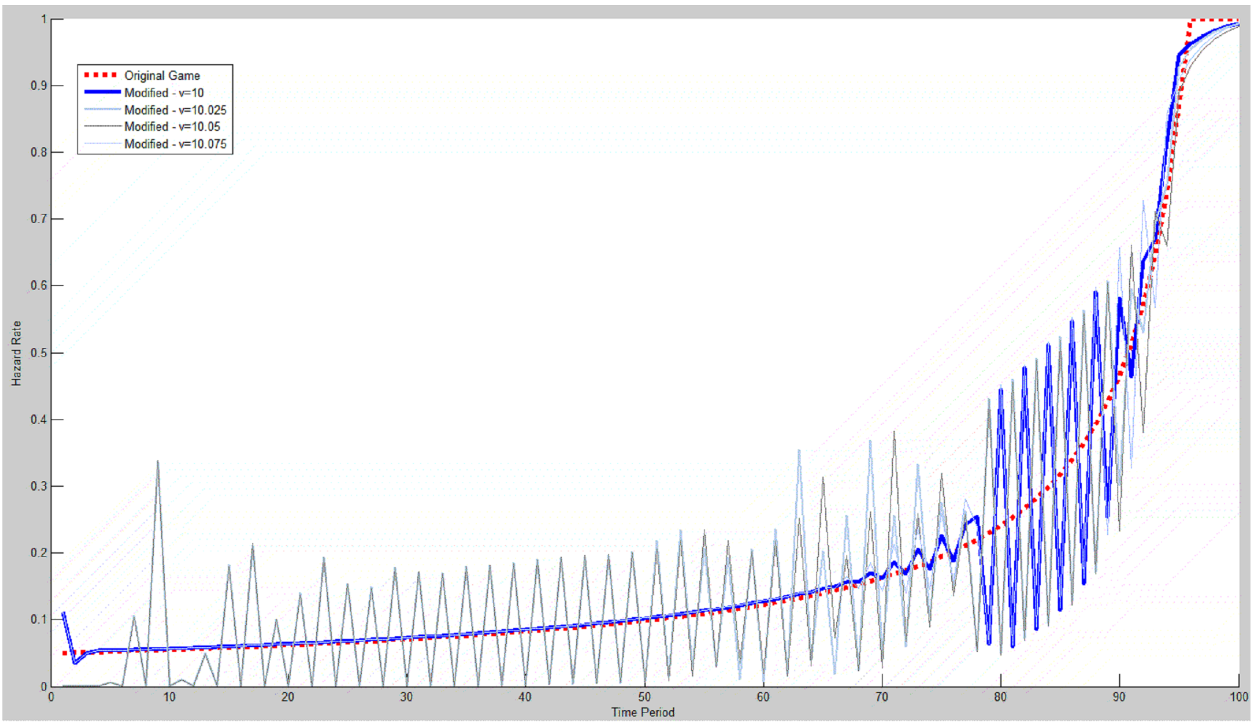
It is possible to solve the model numerically, leading to a few qualitative statements about the hazard rates. Figure B.2 shows the equilibrium hazard rates (with $k = .1, c = .5, n = 10$) given small changes in the value of the good ($v = 10, 10.25, 10.5, 10.75$), as well as the analytical hazard rates from Proposition 2. These graphs demonstrate three main qualitative statements about the relationship between the equilibria in the modified model and the original model:

1. The hazard rates of the modified model are more unstable locally (from period-to-period) than those from Proposition 2, especially in later periods. As n increases, this instability decreases (I do not present graphs for lack of space).

2. The hazard rates of the modified model closely match those from Proposition 2 when smoothed locally.

3. The hazard rates of the modified model are more stable globally to small changes in parameters in the model. Recall that the hazard rates in Proposition 2 were taken from an equilibrium when $\mathrm{mod}(v-c, k) = 0$. When $\mathrm{mod}(v-c, k) \neq 0$, the hazard rates oscillated radically (although they were smooth in an ε-equilibrium with very small ε). The modified model is much more globally robust to these changes.

Figure B.2: Robustness to Multiple Bids Being Accepted



Notes: Numerical Analysis of the hazard rate of auctions for different values (solid lines) when the multiple bids are accepted at each time period vs. the predicted hazard rate (dotted line) when only one bid is accepted. The hazard rates with mutliple bids are much more locally unstable, but follow the path of the predicted hazard rate with only one accepted bid

B.7.5 Timer

In the model in the paper, unlike that in the real world implementation of the model, there is no timer within each period. Consider a game in which, in each discrete period t, players can choose to place a bid at one sub-time $\tau \in [0, T]$ or not bid for that period. As in the original game, if no players bid, the game ends. If any players bid, one bid is randomly chosen from the set of bids placed at the smallest τ of all bids (the first bids in a period). Now, a player's (Markov) strategy set is a function for each period $\chi_t^i(\tau) : [0, T] \to [0, 1]$, with $\int_0^T \chi_t^i(\tau) d\tau$ equaling the probability of bidding at some point in that period. This following proposition demonstrates that, while the timer adds complexity to the player's strategy sets, it does not change any of the payoff-relevant outcomes.

Proposition 10 *For any equilibrium of the modified game, there exists an equilibrium of the original game in which the distribution of the payoffs of each of the players is the same.*

B.8 Proofs of Statements in Online Appendix

Propositions 5 and 6

Proof:

As all functions mentioned do not vary with the leader, l_t, I suppress the dependence on l_t. Let $S_{cont}(t) = p$. Consider the discrete hazard rate at time t for any leader l_t : $h_{cont}(t) = \frac{c\Delta t}{v+t(\frac{1}{n}\theta - k)}$ for $t \leq F$. Then the likelihood of the auction surviving to time period $t + \Delta t$ is: $S_{cont}(t + \Delta t) = (1 - \frac{c\Delta t}{v+(t+\Delta t)(\frac{1}{n}\theta-k)})p$. Therefore, the continuous hazard rate at time t : $h_{cont}(t) = \lim_{\Delta t \to 0} \frac{S_{cont}(t)-S_{cont}(t+\Delta t)}{\Delta t \cdot S_{cont}(t)} = \lim_{\Delta t \to 0} \frac{p(\frac{c\Delta t}{v+(t+\Delta t)(\frac{1}{n}\theta-k)})}{\Delta t \cdot p} = \lim_{\Delta t \to 0} \frac{c}{v+(t+\Delta t)(\frac{1}{n}\theta-k)} = \frac{c}{v+t(\frac{1}{n}\theta-k)}$. Define the continuous cumulative hazard function in the standard way: $H_{cont}(t) = \int_0^t h_{cont}(\widetilde{t})d\widetilde{t}$. As $H_{cont}(t) = \int_0^t \frac{c}{v+\widetilde{t}(\frac{1}{n}\theta-k)}d\widetilde{t}$, $H_{cont}(t) = \frac{c(\ln(v)-\ln(v+t(\frac{1}{n}\theta-k)))}{\frac{1}{n}\theta-k}t$ if $t \leq F$.Note that $H_{cont}(t) = \int_0^t \lim_{\Delta t \to 0} \frac{S_{cont}(\widetilde{t})-S_{cont}(\widetilde{t}+\Delta t)}{\Delta t \cdot S_{cont}(\widetilde{t})}d\widetilde{t} = -\int_0^t \frac{1}{S_{cont}(\widetilde{t})}(\frac{d}{d\widetilde{t}}S_{cont}(\widetilde{t}))d\widetilde{t} = -\ln S_{cont}(t)$.Therefore $S_{cont}(t) = e^{-H_{cont}(t)}$.That is, $S_{cont}(t) = (1 - \frac{t}{v}(k - \frac{1}{n}\theta))^{\frac{c}{k-\frac{1}{n}\theta}}$ if $t \leq F$. To see that the survival rate of a good with value v at time t is equal to the survival rate of a good with value αv at time αt, note that $S_{cont}(t; v) = (1 - \frac{t}{v}(k - \frac{1}{n}\theta))^{\frac{c}{k-\frac{1}{n}\theta}} = (1 - \frac{\alpha t}{\alpha v}(k - \frac{1}{n}\theta))^{\frac{c}{k-\frac{1}{n}\theta}} = S_{cont}(\alpha t; \alpha v)$. Now, consider the normalized time survival rate $\widehat{S}_{cont}(\widehat{t})$. For a good with value v, the odds of surviving to period t is $(1 - \frac{t}{v}(k - \frac{1}{n}\theta))^{\frac{c}{k-\frac{1}{n}\theta}}$. Therefore, for any value v and time t, the odds of surviving to normalized period $\widehat{t}$ is $\widehat{S}(\widehat{t}) = (1 - \widehat{t}(k - \frac{1}{n}\theta))^{\frac{c}{k-\frac{1}{n}\theta}}$. Similar logic shows:

$\widehat{h}(\widehat{t}) = \frac{c}{1+\widehat{t}(\frac{1}{n}\theta - k)}$.

Proposition 7

Proof: Consider the strategies noted in the proof of Proposition 2 with $F = F^*$. For the standard ϵ-perfect equilibrium, we consider the ex ante benefit of deviating to the most profitable strategy, given that the other players continue to follow this strategy. Following the proof of Proposition 2, it is easy to show that there is no profitable deviation in periods $t > F^*$ and $t < F^*$. Therefore, the only profitable deviation is to not bet in $t = F^*$. This will yield a continuation payoff of 0 from period F^*. The ex ante continuation payoff from betting is $\epsilon = \frac{1}{n}(1 - \frac{c}{v-F^*k})(v - (F^*+1)k - c)[\prod_{t=1}^{F^*-1}(1 - \frac{c}{v-tk})]$. (To see this, note that there is a $\prod_{t=1}^{F^*-1}(1 - \frac{c}{v-tk})$ change that the game reaches period F^*. In period F^*, there is a $(1 - \frac{c}{v-F^*k})$ probability that at least one player bets. As strategies are symmetric, this means that, ex ante, a player has a $\frac{1}{n}(1 - \frac{c}{v-F^*k})$ probability of her bet being accepted in this period, given that the game reaches this period. If the bet is accepted, the player will receive $(v - (F^*+1)k - c))$. Therefore, the ex ante benefit from deviating to the most profitable strategy is ϵ. For the contemporaneous ϵ^c-perfect equilibrium, we consider the benefit of deviating to the most profitable strategy once period F^* is reached, given that the other players continue to follow this strategy. This is $\epsilon^c = \frac{1}{n-1}(1 - \frac{c}{v-F^*k})(v - (F^*+1)k - c)$ (To see this, note that in period F^*, there is a $(1 - \frac{c}{v-F^*k})$ probability that at least one player bets. As strategies are symmetric, this means that, ex ante, a non-leader has a $\frac{1}{n-1}(1 - \frac{c}{v-F^*k})$ probability of her bet being accepted in this period (as there are only $n-1$ non-leaders). If the bet is accepted, the player will receive $(v - (F^*+1)k - c))$.

Proposition 8

Proof: In referring to the hazard function $h(t, l_t)$, I refer to $h(t)$ as all results are true regardless of the leader. In case 1, I will refer to $v_i(t) = v_i$. The proof is simple (backward) induction on the statement that there is a unique hazard rate that can occur in each period in equilibrium. By the same logic in the proof to Proposition 1, $h(t) = 1$ for all $t > \frac{v+\Delta_i-c}{k} - 1$. Consider periods $t \leq F^* = \max\{t | t \leq \frac{v+\Delta_i-c}{k} - 1\}$ where $h(t+1)$ is unique in equilibrium by induction. If $h(t+1) = 0$, then $h(t) = 1$ as any player with finite $v_i(t)$ strictly prefers to not bid. If $h(t+1) > 0$, a player with cutoff type $v^*(t) = \frac{c}{h(t+1)} + (t+1)k$ is indifferent to betting at time t given $h(t+1)$. Therefore, $h(t) = G(\max(\min(v^*, v+\Delta), v-\Delta))$ and the statement is true. Suppose G_i is such that the game continues past period 1. Claim 1: If $\Delta < k$,then (1) $h(t) = 0$ for $t \leq F \Rightarrow h(t-1) = 1$ and (2) $h(t) = 1$ for $t \leq F \Rightarrow h(t-1) = 0$. Statement (1) is true as a bidding leads to $-c$, a lower payoff than not bidding. Statement

(2) is true as if $h(t) = 1$, then the payoff of bidding for a player with value $\widetilde{v}$ at period $t-1$ is Pr[Bid Accepted]($\widetilde{v}-$ $tk - c$). Note that Pr[Bid Accepted] > 0 if a player bids. Note that $t \leq F \Rightarrow t \leq \frac{v-c}{k} - 1 \Rightarrow 0 \leq v - c - (t+1)k \Rightarrow 0 < v - \delta - c - tk$ as $\delta < v$. Therefore, $0 <$ Pr[Bid Accepted]($\widetilde{v}-$ $(t+1)k - c$) for every $\widetilde{v} \in [v - \delta, \overline{v} + \delta]$ and therefore $h(t) = 0$ and the claim is proved. Claim 2: If $\delta < k, h(t) \in (0, 1)$ for every $0 < t \leq F$. Suppose that $h(t) = \{0, 1\}$ for some $0 < t \leq F$. If $h(1) = 1$, then game ends at period 1, leading to a contradiction. If $h(1) = 0$, then $h(0) = 1$, and game ends at period 0, leading to a contradiction. If $h(t) = 1$ (alt: 0) for $0 < t \leq F$, then $h(t-1) = 0$ (alt: 1), $h(t-2) = 1$ (alt: 0) by claim 1. But, then $h(1) = \{0, 1\}$, which is leads to a contradiction as above. Claim 3: By the same logic in the proof to Proposition 1, $h(t) = 1$ for all $t > \frac{v+\delta_i-c}{k} - 1$. Therefore, $h^G(t) = 1$ for $t > \frac{v-c}{k} - 1 = F$ as $\delta_i \to 0$. For $0 < t \leq F$, note that for some i^*, $\delta_i < k$ for all $i > i^*$ and therefore claim 1 holds for all $i > i^*$. If claim 1 holds, $h(t-1) \in (0, 1)$ implies a cutoff value $v^*(t) \in (v - \delta, v + \delta)$ from above, which by the definition of $v^*(t)$ implies that $h(t) \in (\frac{c}{v-\delta-(t+1)k}, \frac{c}{v+\delta-(t+1)k})$, and therefore $h^{G_i}(t) \to \frac{c}{v-tk}$ for periods $0 < t \leq F$ as $\delta_i \to 0$. Therefore, $h^G(t) \to h(t)$ from Proposition 2 for $t > 0$.

Proposition 9

Proof: Set $p_t^{i,NL*} = p_t^{i*}$ from the Proposition 2 and set $p_t^{i,L*} = 0$ for all i and all t. Note that, as in the proof of Proposition 2. these strategies yield the hazard rates listed in the Proposition 2. The same proof for Proposition 2 shows that, if strategies are followed, the continuation payoff from entering period t as a non-leader is 0 and there is no profitable deviation for a non-leader. Now, consider if there is a profitable deviation for a leader. For the subgames starting in periods $t > F$, refer to the proof of Proposition 1 for a proof that there is no profitable deviation for a leader in these periods. For the subgames starting in period $0 < t \leq F$, the proof continues using (backward) induction with the lack of profitable deviation already proved for all periods $t > F$. In period t, by not bidding, the leading player will receive v with probability $\frac{c}{v-tk}$ (with the game ending) and 0 as a continuation probability as a non-leader in period $t+1$ with probability $1 - \frac{c}{v-tk}$, yielding an expected payoff of v $(\frac{c}{v-tk}) > 0$. By bidding, the game will continue to period $t+1$ with certainty, with some positive probability that her bid is accepted. If her bid is accepted, she receives $-c$ in period t and receives $v - (t+1)k$ in $t+1$ with probability $\frac{c}{v-(t+1)k}$ and 0 as a continuation probability as a non-leader in period $t+2$ with probability $1 - \frac{c}{v-tk}$, leading to a continuation payoff of $-c+$ $(v-(t+1)k)(\frac{c}{v-tk}) = 0$. If her bid is not accepted, she will receive a continuation probability of 0 as a non-leader in period $t+1$. Therefore, the payoff from not bidding in period t is strictly higher than the payoff from bidding.

Proposition 10

Proof: Consider a vector of bidding probabilities $x = [x^1, x^2, ...x^n] \in [0,1]^n = X$ in some period. Let $\Psi : X \to \Delta^n$ be a function that maps x_t into a vector of probabilities of each player's bid being accepted, which I will denote $a = [a^1, a^2, ...a^n]$. Claim: For any $a^* \in \Delta^n$, $\exists\, x \in X$ such that $\Psi(x) = a^*$.

Consider the following sequence of betting probabilities, indexed by $j = \{1, 2, 3...\}$. Let $x^i(1) = 0$. Define $a(j) = \Psi(x(j)) = \Psi([x^1(j), x^2(j), ...x^n(j)])$. Define $\widetilde{a}_i(j) = \Psi([x^1(j-1), x^2(j-1), ...x^i(j), ..., x^n(j-1)])$ and let $x^i(j)$ be chosen such that $\widetilde{a}_i^i(j) = a^{i*}$. Claim: $x(j)$ exists, is unique, $x(j-1) \leq x(j)$ and $a_i(j) \leq a^*$ for all j. This is a proof by induction, starting with $t = 2$. As $x(1) = 0$, $x(2) = a^*$ by the definition of $\widetilde{a}_i(j)$. Therefore, $x(2)$ exists, is unique, $x(1) \leq x(2)$ and $a_i(2) \leq a^*$ as $\frac{\partial \Psi_i}{\partial x^k} < 0$ for $k \neq i$. Now, consider $x^i(j)$. Note (1) $x^i(j) = 0 \Rightarrow \widetilde{a}_i^i(j) = 0$, (2) $x^i(j) = 1 \Rightarrow \widetilde{a}_i^i(j) \geq 1 - \sum_{k \neq i} a^k(j-1) \geq 1 - \sum_{k \neq i} a^{k*} \geq a^{i*}$ where $1 - \sum_{k \neq i} a^k(j-1) \geq 1 - \sum_{k \neq i} a^{k*}$ follows by $a_i(j-1) \leq a^*$, which follows by induction (3) $\widetilde{a}_i(j)$ is continuous in $x^i(j)$ and $\frac{\partial \Psi_i}{\partial x^i} > 0$. Therefore, there is a unique solution $x^i(j)$ such that $\widetilde{a}_i^i(j) = a^{i*}$. As $a_i(j-1) \leq a^*$ by induction, it must be that $x_i(j) \geq x_i(j-1)$ as $\frac{\partial \Psi_i}{\partial x^i} > 0$. Finally, note that if $\widetilde{a}_i^i(j) = a^{i*}$,then as $a_i^i(j) \leq a^{i*}$ as $\frac{\partial \Psi_i}{\partial x^k} < 0$ for $k \neq i$ and $x_k(j) \geq x_k(j-1)$ for $k \neq i$.

Set $x^* = \lim_{j \to \infty} x(j)$. Claim: x^* exists and $\Psi(x^*) = a^*$. First, $\lim_{j \to \infty} x(j)$ must exist as $x^i(j)$ is bounded above by 1 and weakly increasing. Next, note that $\sum_i x^{i*}$ must also exist with $\sum_i x^{i*} \leq n$. Now, suppose that $\Psi(x^*) \neq a^*$. Then, as $\Psi(x(j)) = a(j) \leq a^*$ for all j, $\Psi(x^*) \leq a^*$ and there must be some i such that $\Psi^i(x^*) - a^{i*} = z > 0$. Choose L such $|\sum_i x^{i*} - \sum_i x^i(j)| < \frac{z}{2}$ for all $j > L$. By the definition of $x^i(j+1)$, it must be that $x^i(j+1) \geq x^i(j) +\, z$. But, as $x(j+1) \geq x(j)$, then $\sum_i x^i(j+1) \geq \sum_i x^i(j) + z \geq \sum_i x^{i*} + \frac{z}{2}$, which is a contradiction of L. Therefore, the claim is proved.