Scam the Declined or Decline to be Scammed: A Model of Financial Choice in the Presence of Cognitive Decline*

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Abstract

We develop a multi-period overlapping generations model in which cognitively sound consumers know that their future selves may suffer from cognitive decline. Consumers with cognitive decline are at risk being taken advantage of by fraudsters who sell wealth-reducing financial products, e.g., products that are inappropriate for the consumer, over-priced, or purely fraudulent. Prior to being hit by cognitive decline, consumers can pay to preemptively protect their future selves from fraudsters by constraining their subsequent financial choices. In practice such protection could take many forms such as hiring a fiduciary advisor who is contracted to participate in a consumer's future financial decisions or purchasing an income annuity and thereby limiting the ability of the consumers older self to spend large amounts of wealth.

In equilibrium, more consumers pay for protection from fraudsters and fraudsters enter the market more intensely when the probability of consumers being afflicted by cognitive decline is initially high and increases more significantly and earlier in life, life expectancy is high, average potential losses to consumers are high, and the cost to fraudsters of meeting more consumers is low. The cost of consumer protection and the intensity with which fraudsters enter the market are determined endogenously. When a higher fraction of consumers protect themselves, fraudsters enter the market with lower intensity. Thus when the probability of a consumer being offered a fraudulent product is high, consumers who purchase protection provide a positive externality to other consumers. Since consumers ignore this externality in their individual decisions, in equilibrium fewer consumers purchase protection than is socially optimal.

1 Introduction

Nearly everyone with a telephone or email address has been offered fraudulent financial opportunities. Most of us realize that no Nigerian official truly intends to share $60 million with us, that the IRS does not call threatening to arrest people who won't provide immediate credit card payment over the phone, and that high return investments are never guaranteed. However, not everyone is able to distinguish the real from the fraudulent. Even legitimate financial products, such as home equity loans or variable annuities, may present a hazard to those who are unable to determine whether products are suitable to their needs or reasonably priced.

Over 5.1 million Americans have Alzheimer's disease. By 2050, that number is expected to grow to between 11 million and 16 million (Alzheimer's Association, 2015). Approximately 24% of people between 80 and 89 years old suffer from dementia (Plassman et al., 2007) while 29% have cognitive impairment without dementia (Plassman et al., 2008). It is estimated that 75% of those over 90 have dementia or impairment. Cognitive impairment is a primary risk factor for financial abuse (Heath et al., 2005). Even older persons without dementia but with cognitive decline are more susceptible to financial scams (Boyle et al., 2012); and age-related declines in memory make older adults more susceptible to scams (Jacoby et al., 2005; Jacoby, 1999). Unfortunately, those suffering from cognitive decline usually do not recognize or acknowledge their loss of the ability to make sound financial decisions. Confidence in financial decision-making does not decrease in old age and may even increase (Finke, Howe, and Huston, 2017).

In the cross-section, "some aspects of age related cognitive decline begin in healthy educated adults when they are in their 20s and 30s" (Salthouse, 2009). While household financial decision making is strongly correlated with cognitive ability (Agarwal and Mazumder, 2013), practical knowledge, often gained through experience, is also critical for many financial tasks. Agarwal et al. (2009), propose that the negative relationship of age and cognition coupled with a positive relationship between age and experience, lead to a hump-shaped relationship between age and the ability of adults to make financial decisions. They estimate that for many household finance tasks, decision-making ability peaks, on average, at about 53 years old.

As the US population ages, more wealth is controlled by those at risk of cognitive decline. 23% of U.S. households have a head 65 or older and, in aggregate, these households own 36% of all U.S. household assets and 40% of U.S. household equity in homes (U.S. Census Bureau, 2014). Home equity lines of credit and refinancing make it increasingly easy for consumers to access the wealth in their homes, while the shift from defined benefit to defined contribution pensions gives retirees direct access to their retirement savings. Meanwhile changes in technology have made it increasingly easy for scammers to make contact with the elderly and to defraud them in novel ways (Blanton, 2012; Deem, Nerenberg, and Titus, 2007).

In this articile, we develop a a multi-period overlapping generations model in which consumers decide whether or not to protect their future old selves from bad financial decisions due to cognitive decline. Consumers are born with good cognitive health but do not know whether or when they will suffer from decline. Consumers with cognitive decline are at risk being taken advantage of by fraudsters who sell wealth-reducing financial products, e.g., products that are inappropriate for the consumer, over-priced, or purely fraudulent. The rate at which consumers are afflicted with cognitive decline increases as consumers age. Consumers die stochastically and are replaced by young consumers. Fraudsters pay to increase the probability of meeting consumers but are unable to identify ex-ante the cognitive state of each consumer or whether a consumer has purchased protection. Consumers vary in the losses they will incur if they accept a fraudster's offer. Prior to being hit by cognitive decline, consumers can pay to preemptively protect their future selves from fraudsters by constraining their subsequent financial choices. In practice such protection could take many forms such as hiring a fiduciary advisor who is contracted to participate in a consumer's future financial decisions or purchasing an income annuity and thereby limiting the ability of the consumer's older self to spend large amounts of wealth.

In equilibrium, more consumers protect against bad financial decisions due to cognitive decline when the probability of being afflicted by cognitive decline is initially high and increases more significantly and earlier in life, life expectancy is high, average potential losses are high, the discount rate is low, and the cost to fraudsters of meeting more consumers is low. Fraudsters are more active in the market when the cost of meeting more consumers is low, the probability of consumers being afflicted by cognitive decline is initially high and increases more significantly and earlier in life, life expectancy is high, average potential gains from defrauding consumers are high, and the discount rate is high. The cost of consumer protection and the intensity with which fraudsters enter the market are determined endogenously.

In our model, when a higher fraction of consumers protect themselves, fraudsters enter the market with lower intensity. Thus when the probability of a consumer being offered a fraudulent product is high, consumers who purchase protection provide a positive externality to other consumers. Since consumers ignore this externality in their individual decisions, in equilibrium fewer consumers purchase protection than is socially optimal. In other words, when fraud and cognitive decline are prevalent, a social planner would choose a higher equilibrium rate of protection than consumers choose.

Agents with time inconsistent preferences may anticipate that their future selves will engage in more short-term consumption that the agent believes to be optimal in the long run. Strotz (1956) introduced the use of pre-commitment to deal with time-inconsistent preferences. Laibson (1997) developed a commitment technology in which a person with hyperbolic discounting purchases illiquid assets so as to prevent his future, impatient, self from consuming more that his current self considers optimal. Time inconsistent preferences can also lead to procrastination which an agent realize not to be in his own best long term interests. Bond and Sigurdsson (2018) demonstrate that even with random shocks to wealth or time commitments, contracts can be used to influence the consumption and procrastination behavior of future selves by taking advantage of time inconsistency driven preference reversals.

In our model, we introduce pre-commitment to deal with a different form of intertemporal inconsistency. In our model, consumers' preferences are time consistent. However, consumers realize that their future selves may not have the cognitive ability to make sound financial decisions and may fall prey to fraudsters. Thus, the future self may make bad decisions that the current self would not make. Rather than risk their future selves inflicting self-harm through bad purchase decisions, consumers restrict their own future behavior.

Our model is related to prior work on crime and predators. Cook's (1986) criminal opportunity theory postulates that criminal activity will increase with opportunity and that potential victims will respond to the threat of crime by engaging in self-protection efforts. In our model, fraudsters engage more intensely with consumers when dementia is high, levels of protection low, and potential gains high that is opportunities are plentiful. Consumers respond to the threat of scams through self-protection. Erlich (1996) decomposes the expected net return to a crime into the direct costs of engaging in crime minus the foregone wages minus the probability of convictions times the prospective penalty if convicted. While fraudsters in our model pay a cost to engage in crime (which could include opportunity costs), we do not model the prospect of being caught convicted and punished. Doing so would, of course, reduce the intensity of scams and the demand for protection.

Conlisk (2001) develops a model in one example of which people choose to be "tricksters," "avoiders," or "suckers." It is costlier to become a trickster than an avoider and costlier to become an avoider than a sucker. Each period people are randomly paired and tricksters take money from suckers but not from other tricksters or avoiders. Avoiders neither take money from nor give money to anyone, i.e., are neither perpetrators or victims of scams. Suckers are exploited by tricksters but not by avoiders or other suckers. All three types can co-exist in equilibrium. In our model fraudsters prey on unprotected consumers who have cognitive decline but are not protected. Consumers in our model change types with the unintentional onset of cognitive decline and with the intentional purchase of protection; they also experience changes in the probability of cognitive decline.

2 The Model

2.1 Setup and Assumptions

We consider a discrete-time infinite-horizon model in which each of a mass one of *consumers* face the threat of being taken advantage of by sellers of dubious financial products. These products can be characterised by the fact that they are inappropriate for a given consumer (e.g., a high-risk fund for an old individual), they carry excessive fees (e.g., high fees for an index fund), or they are pure frauds or scams. We refer to the sellers of these products as *fraudsters*.

In every period, each consumer (of any age) faces a constant probability $q \in (0,1)$ of dying, and all deceased consumers are replaced with the same mass of newborn consumers aged zero. In each period t, every living consumer encounters a fraudster with probability $\phi_t \in [0,1]$, a quantity to be endogenized later. The fact that ϕ_t is the same for every consumer simply means that fraudsters cannot target consumers based on their age or their state.[1] When a consumer encounters a fraudster, his ability to turn down the fraudster's offer depends on whether or not this consumer suffers from cognitive decline and whether or not he has protection against such eventualities. We assume that the probability that a consumer i enters cognitive decline in period $t \in \{0, 1, \dots\}$ is given by

$$\tilde{d}_{i,t} = \begin{cases} \tilde{d}_{i,t-1}, & \text{prob. } 1-\alpha \\ \delta, & \text{prob. } \alpha, \end{cases} \tag{1}$$

where $\tilde{d}_{i,0} = 0$, $\delta \in (0,1)$, and $\alpha \in (0,1)$. That is, when they are born, consumers do not initially face any risk of suffering from cognitive decline; we refer to them as *low-risk*. The probability of decline goes up to $\delta > 0$ in any given period with probability α; after they experience this shock, consumers are referred to as *high-risk*. In this sense, α represents the speed at which consumers' cognitive skills age as they grow older.

Consumers who have not yet been affected by cognitive decline can identify fraudsters and fend off their advances; we refer to them as *competent* (and they can be low-risk or high-risk consumers). Consumers who are affected by cognitive decline (we refer to them as *compromised*), however, can no longer resist the products that fraudsters offer them: in each period for as long as they live, these consumers have a probability ϕ_t of being taken advantage of.

Consumers are all born with the same wealth of $W_0 > 0$, and consume the interest from it for their entire life. Specifically, the wealth W_t they have at age t generates rW_t over the next period, where $r > 0$ is the exogenous annual interest rate. A consumer's wealth can only go down during their lifetime, and can do so for two reasons: buying protection from fraud, and being

[1] We will see later that consumers go through various stages as they age and as they make financial decisions.

defrauded.[2] At the beginning of any period, a consumer can choose to preemptively protect a fraction of his wealth from fraud by entrusting it to an *insurer*. In that event, the insurer charges the consumer a fraction $p_t \in (0, 1)$ (to be endogenized later) of the wealth that the consumer chooses to protect. From then on, this wealth is insulated from fraudsters; that is, only the fraction of a consumer's wealth that is not protected is subject to fraud. The fraction of a consumer's wealth that is not protected disappears in the event that he is offered a fraudulent financial product while compromised. That is, from that point on, only the fraction of the consumer's wealth that is protected generates interest and allows him to consume.

Note that the protection that the insurer offers can take many forms in reality. For example, the fact that the protection contract involves a lump sum transfer from the consumer to the insurer in return for constant annual payments for as long as the consumer is alive makes the arrangement similar to an annuity. Another interpretation is that the consumer hires a financial advisor who shields him from potentially harmful investment decisions; in this case, the lump sum transfer might represent the present value of future fees that the advisor charges his clients.

Fraudsters choose the intensity with which they target consumers in every period t. Specifically, we assume that fraudsters can increase the probability with which they meet each consumer to ϕ_t at a cost of $c(\phi_t)$, where $c(0) = 0$, $c'(\phi) > 0$, $c''(\phi) \geq 0$, and $c(1)$ is sufficiently large to ensure that the optimal ϕ_t is always strictly smaller than one. For simplicity, we initially assume that $c(\phi) = \frac{k}{2}\phi$, with k sufficiently large to ensure interior solutions. In their choice of ϕ_t, fraudsters seek to maximize the total (expected) amount they can appropriate from consumers through fraud, net of search costs.

Insurers set the proportional fee p_t to be charged to consumers for protecting some of their wealth in period t. We assume that the cost of protecting a consumer is proportional to the wealth I_i that each consumer i insures, i.e., ρI_i where $\rho > 0$. Initially, we assume that the insurance industry is competitive and that, as such, prices adjust to marginal cost in equilibrium (i.e., $p_t = \rho$). Later in the paper, we also analyze a monopolistic setting in which one insurer sets the price of insurance to maximize expected profits.

Finally, to make sure that the equilibrium is economically meaningful, we restrict the set of parameters to be used throughout the paper.

Assumption 1. *We assume that*

$$\rho < \frac{\delta}{1 - (1 - q)(1 - \delta)}. \tag{2}$$

This assumption ensures that the insurance market stays active: when insurers can offer insurance at sufficiently low costs (i.e., ρ is small) and consumers are sufficiently fearful about cognitive

[2] The interpretation we have in mind is that age 0 is the start of retirement for a consumer.

decline (i.e., δ is large), consumers are willing to buy some protection and it is profitable for insurers to provide it.

2.2 Steady State

Throughout the paper, we are interested in characterizing the steady-state equilibrium, in which the consumer population's age profile remains the same. As such, from here on, we use $t \in \{0, 1, \ldots\}$ to denote the consumer population of age t. Given the unit mass of consumers, the constant probability of dying and automatic replacement of those who die, and the assumption that one's experience in the financial market does not affect one's life expectancy, the steady-state economy has a mass $n_0 = q$ of consumers aged zero, $n_1 = q(1-q)$ consumers aged one and, more generally, a mass $n_t = q(1-q)^t$ of consumers aged $t \in \{0, 1, \ldots\}$. Because all consumers are born with the same wealth, it is also the case that, in the steady-state equilibrium, the search intensity of fraudsters is constant at ϕ, and that the price of insurance is set at a constant p by insurers.

To summarize, the sequence of events for any consumer i born with wealth W_0 is as follows in each period. Every period starts with the insurers setting the publicly available fee p of insurance and with fraudsters setting the unobservable search intensity ϕ with which they reach consumers. A competent consumer i enters period t learning $\tilde{d}_{i,t}$, as specified in (1). Based on $\tilde{d}_{i,t}$ and p, the consumer decides how much of his wealth, $I_i \in [0, W_0]$, to entrust the insurer. This decision comes with a transfer of pI_i from consumer i to the insurer.[3] Then consumer i suffers a cognitive decline shock with probability $\tilde{d}_{i,t}$ and remains competent with probability $1 - \tilde{d}_{i,t}$. In decline, consumer i becomes susceptible to having his uninsured wealth defrauded in period t and in every subsequent period thereafter, until he dies. If consumer i remains competent and survives the period, then he enters period $t + 1$, and a similar sequence of events is repeated.

3 Equilibrium Analysis

As mentioned above, we start our analysis by assuming that the insurance industry is competitive; that is, we initially assume that the price of insurance automatically adjusts to $p = \rho$ at the beginning of every period. For any such ρ, there is an equilibrium that prevails between consumers and fraudsters: based on the fraudsters' search intensity that they conjecture, consumers determine their optimal protection policy; similarly, based on the consumers' protection policy that they conjecture, fraudsters determine their optimal search intensity. In equilibrium, the conjectured quantity by each party is equal to the actual choice by the other party. As we will show later, the

[3] It will soon be clear that it is optimal for the consumer to insure at most once during his entire lifetime. Since consumers consider insuring only as long as they are competent, this is why I_i does not depend on the consumer's age and that his wealth will still be W_0 at the time he insures.

monopolistic setting will only require us to endogenize the price p instead of letting it adjust to an exogenous constant ρ, as the game played between consumers and fraudsters will remain the same.

3.1 The Consumers' Problem

Suppose that consumers observe $p > 0$ and conjecture the fraudsters' search intensity to be some $\phi \in (0,1)$.[4] As long as a consumer i remains competent and the probability of a cognitive shock remains the same (i.e., $\tilde{d}_{i,t} = \tilde{d}_{i,t-1}$), his decision to insure is the same in periods $t-1$ and t. This, along with the fact that $\tilde{d}_{i,0} = 0$, further implies that consumers only consider protecting some of their wealth in the first period τ in which $\tilde{d}_{i,\tau}$ jumps to $\delta > 0$. And, of course, they only consider the protection if they are still competent, as they can then anticipate the possibility that they will eventually be compromised and potentially defrauded.

Let us denote the expected lifelong consumption of a consumer who has already been defrauded by $V_{\text{DEF}}(p, I_i, \phi)$. This quantity depends on the price p of insurance, the wealth I_i that the consumer had chosen to insure, and on the fraudsters' search intensity ϕ. It must solve

$$V_{\text{DEF}}(p, I_i, \phi) = r(1-p)I_i + (1-q)V_{\text{DEF}}(p, I_i, \phi), \tag{3}$$

which implies that

$$V_{\text{DEF}}(p, I_i, \phi) = \frac{r(1-p)I_i}{q}. \tag{4}$$

That is, for the rest of his life, consumer i receives the interest on the fraction of his wealth that he chose to insure, net of insurance fees.

For a consumer to get defrauded, he must first be compromised. Let us denote the expected lifetime consumption of a compromised consumer i who has yet to be defrauded by $V_{\text{COM}}(p, I_i, \phi)$. This quantity must solve

$$V_{\text{COM}}(p, I_i, \phi) = (1-\phi)\Big\{r\big[(1-p)I_i + (W_0 - I_i)\big] + (1-q)V_{\text{COM}}(p, I_i, \phi)\Big\} + \phi V_{\text{DEF}}(p, I_i, \phi). \tag{5}$$

That is, if the consumer is not defrauded (probability $1-\phi$), he receives a rate r on both the wealth that is protected by the insurer and the wealth that he manages himself. Otherwise (probability ϕ), he gets to consume only from his post-fraud wealth. This solves for

$$V_{\text{COM}}(p, I_i, \phi) = r\,\frac{q(1-\phi)(W_0 - pI_i) + \phi(1-p)I_i}{q\big[1-(1-q)(1-\phi)\big]}. \tag{6}$$

Finally, before a consumer is compromised, he must have become a high risk (i.e., $\tilde{d}_{i,t}$ must

[4] Even though the price will adjust to $p = \rho$ in this competitive equilibrium, we use p to denote the price as this will allow a natural transition to the monopolistic scenario later.

have jumped to $\delta > 0$). Let us denote the expected lifetime consumption of a consumer who is still competent but has become a high risk by $V_{\text{H}}(p, I_i, \phi)$. This quantity must solve

$$V_{\text{H}}(p, I_i, \phi) = (1-\delta)\Big\{r\big[(1-p)I_i + (W_0 - I_i)\big] + (1-q)V_{\text{H}}(p, I_i, \phi)\Big\} + \delta V_{\text{COM}}(p, I_i, \phi), \qquad (7)$$

which in turn implies that

$$V_{\text{H}}(p, I_i, \phi) = r\,\frac{\Big\{\delta(1-\phi) + (1-\delta)\big[1-(1-q)(1-\phi)\big]\Big\}(W_0 - pI_i) + \delta\phi(1-p)I_i}{q\big[1-(1-q)(1-\phi)\big]\big[1-(1-q)(1-\delta)\big]}. \qquad (8)$$

Since consumer i faces no risk of being compromised until $\tilde{d}_{i,t}$ jumps from zero to $\delta > 0$, there is no point for him to buy any protection before that time. Also, once $\tilde{d}_{i,t} = \delta$, only two states are possible for consumer i as he ages: either he remains a high risk (probability $1-\delta$) or he becomes compromised (probability δ). In the former case, his previous protection decision is still optimal; in the latter case, he no longer considers protection. Thus consumer i will potentially choose to buy some protection at one and only one time: the first period in which $\tilde{d}_{i,t}$ jumps to δ. In that period, the value of his lifetime income is given by (8), which he maximizes by choosing I_i. This leads to the following decision.

Lemma 1. *Suppose that the price of insurance is $p \in (0,1)$, and that consumer i conjectures the search intensity of fraudsters to be $\phi \in (0,1)$. Then the amount of money that consumer i chooses to insure is given by*

$$I_i = \Upsilon(\phi) \equiv \begin{cases} W_0 & \text{if } \phi > \bar{\phi}_p \\ \in [0, W_0] & \text{if } \phi = \bar{\phi}_p \\ 0 & \text{if } \phi < \bar{\phi}_p, \end{cases} \qquad (9)$$

where

$$\bar{\phi}_p = \frac{pq\big[1-(1-q)(1-\delta)\big]}{\delta - p(1-q)\big[1-(1-q)(1-\delta)\big]}. \qquad (10)$$

Equation (9) emphasizes the fact that I_i is a function of ϕ: the financial decisions of consumers depend on their expectations about the fraudsters' search intensity. Specifically, consumers prefer to manage their own money if the threat from fraudsters is small ($\phi < \bar{\phi}_p$) and prefer protecting all of it if the threat is large ($\phi > \bar{\phi}_p$). When $\phi = \bar{\phi}_p$, they are indifferent. Notice that $\bar{\phi}_p$ is increasing in p and decreasing in δ: consumers prefer managing their own money when the price of protection is large, and they prefer protecting their money when the risk of cognitive decline is large.

3.2 The Fraudsters' Problem

Fraudsters understand that, although their search targets the entire population of consumers, it is successful only when they come across an uninsured consumer with cognitive decline who has yet to lose his wealth. Thus, from the fraudsters' perspective, the number of such consumers becomes a direct determinant of their search intensity.

As mentioned earlier, there are $n_1 = q(1-q)$ consumers aged 1 in steady state. For a consumer of age 1 to be targeted by fraudsters, he must have become a high risk at age 0, and must have entered cognitive decline at age 1; this happens with probability $\psi_1 \equiv \alpha\delta$. Similarly, there are $n_2 = q(1-q)^2$ consumers aged 2 and, of these, the ones that are targeted by fraudsters are from one of the following three categories.

- They became a high risk at age 0, suffered from decline at age 1, but were not defrauded at age 1. The probability if this event is $\alpha\delta(1-\phi)$.
- They became a high risk at age 0, and suffered from decline only at age 2. The probability of this event is $\alpha(1-\delta)\delta$.
- They became a high risk only at age 1, and suffered from decline at age 2. The probability of this event is $(1-\alpha)\alpha\delta$.

The sum of these three probabilities, $\psi_2 \equiv \alpha\delta\big[(1-\phi)+(1-\delta)\big]+\alpha\delta(1-\alpha)$, is the fraction of age-2 consumers targeted by fraudsters. More generally, one can show that the fraction of age-t consumers targeted by fraudsters is given by

$$\psi_t \equiv \alpha\delta \sum_{\tau=0}^{t-1}(1-\alpha)^\tau \sum_{s=0}^{t-1}(1-\delta)^s(1-\phi)^{t-s}. \tag{11}$$

This leads to the following lemma.

Lemma 2. *In steady state, the total number of consumers targeted by fraudsters is given by*

$$N_{\text{TAR}}(\phi) \equiv \sum_{t=0}^{\infty} n_t\psi_t = \frac{\alpha\delta q(1-q)}{\big[1-(1-\alpha)(1-q)\big]\big[1-(1-\delta)(1-q)\big]\big[1-(1-\phi)(1-q)\big]}. \tag{12}$$

It is easy to verify that $N_{\text{TAR}}(\phi)$ is increasing in α and δ: the consumers are more vulnerable to fraudster attacks when the rates at which they transition to high risk and cognitive decline are high. It is also the case that $N_{\text{TAR}}(\phi)$ is decreasing in ϕ: if fraudsters attack consumers with great intensity, fewer consumers will have any wealth that can be appropriated by the current crowd of fraudsters. This dependence of N_{TAR} on ϕ highlights the fact that the actions of fraudsters in any given period are affected by the actions of fraudsters in all previous periods. To solve for the steady state, we assume that the current set of fraudsters choose their search intensity based on a conjecture $\hat{\phi}_{\rm F}$ about the ϕ that other fraudsters adopt; in equilibrium, this conjecture is correct and the search intensity of all fraudsters, past and present, is the same.

Likewise, when consumers choose how much of their wealth to insure, they base their decisions on a conjecture about the intensity with which fraudsters will pursue them if and when they become compromised. In fact, if we denote this conjecture by $\hat{\phi}_{\rm C}$, we know from (9) that consumers will make their insurance decisions $\Upsilon(\hat{\phi}_{\rm C})$ based on a comparison between $\hat{\phi}_{\rm C}$ and $\bar{\phi}_p$. In equilibrium, their conjectures will also prove to be correct.

When the current period's fraudsters choose the intensity ϕ with which they look for compromised consumers, their choice has no direct impact on the choice of fraudsters in past and future periods. Specifically, based on their conjecture $\hat{\phi}_{\rm F}$ about fraudsters before them, they expect to reach a fraction ϕ of the $N_{\rm TAR}(\hat{\phi}_{\rm F})$ vulnerable consumers. Also, based on this conjecture, they anticipate consumers to insure $I(\hat{\phi}_{\rm F})$ and therefore to leave $W_0 - I(\hat{\phi}_{\rm F})$ on the table. As such, fraudsters maximize their expected profits from defrauding consumers by solving:

$$\max_{\phi} \ \phi N_{\rm TAR}(\hat{\phi}_{\rm F})\big[W_0 - I(\hat{\phi}_{\rm F})\big] - c(\phi). \tag{13}$$

The following lemma shows the solution to this problem under the assumption that $c(\phi)$ is quadratic in ϕ.

Lemma 3. *Suppose that the price of insurance is $p \in (0, 1)$, and that the current period's fraudsters conjecture other fraudsters to choose a search intensity of $\hat{\phi}_{\rm F} \in [0, 1]$. Then the search intensity chosen by the current period's fraudsters is*

$$\phi = \Phi(\hat{\phi}_{\rm F}) \equiv \frac{1}{k} N_{\rm TAR}(\hat{\phi}_{\rm F})\big[W_0 - I(\hat{\phi}_{\rm F})\big]. \tag{14}$$

Equation (14) shows that fraudsters will search more intensively when they think that consumers manage more of their own money (i.e., consumers do not insure much of their money). Also, since $N_{\rm TAR}(\hat{\phi}_{\rm F})$ is decreasing in $\hat{\phi}_{\rm F}$, fraudsters will search less intensively if they think that previous fraudsters are likely to have already robbed a large fraction of compromised consumers of their wealth.

Another interpretation of (14) is as follows. Suppose that there is a mass one of fraudsters, and that each such fraudster must incur a fixed cost of k in order to participate in the search for customers. Under the conjecture that past and future fraudsters choose $\hat{\phi}_{\rm F}$, the total expected profits that are available to the current period's fraudsters are $N_{\rm TAR}(\hat{\phi}_{\rm F})\big[W_0 - I(\hat{\phi}_{\rm F})\big]$. If ϕ of them choose to operate, they each expect to receive $\frac{1}{\phi} N_{\rm TAR}(\hat{\phi}_{\rm F})\big[W_0 - I(\hat{\phi}_{\rm F})\big]$. In equilibrium, it must be that fraudsters are indifferent between operating and staying out. That is, the equilibrium condition is

$$\frac{1}{\phi} N_{\text{TAR}}(\hat{\phi}_{\text{F}}) \big[W_0 - I(\hat{\phi}_{\text{F}}) \big] = k.$$

which is equivalent to (14). Thus ϕ can be interpreted as the mass of fraudsters who are actively seeking consumers who suffer from cognitive decline.

3.3 Equilibrium Between Consumers and Fraudsters

In equilibrium, it must be the case that consumers rationally and correctly anticipate the intensity of the fraudsters' search, and that fraudsters rationally and correctly anticipate the search intensity of frausters in other periods (which also implies that they rationally and correctly anticipate the consumers' insurance choices). That is, in equilibrium, we must have $\hat{\phi}_{\text{C}} = \hat{\phi}_{\text{F}} = \phi$.

As we will show, the equilibrium features either partial or no insurance by consumers (i.e., $I \in [0, W_0)$) and a probabilistic search by fraudsters (i.e., $\phi \in (0,1)$). The result that $I < W_0$ is immediate: if consumers choose to fully insure their wealth (i.e., $I = W_0$), then fraudsters do not have anything to gain from searching for consumers in cognitive decline and so choose $\phi = 0$; but then, consumers do not need to insure any of their wealth at all; as such, this is not an equilibrium. The following proposition solves for the model's equilibrium.

Proposition 1. *Suppose that the price of insurance is $p \in (0,1)$, and that Assumption 1 holds. If*

$$\frac{W_0}{k} > \frac{pq\big[1-(1-\delta)(1-q)\big]^2\big[1-(1-\alpha)(1-q)\big]}{\alpha(1-q)\Big\{\delta - p(1-q)\big[1-(1-\delta)(1-q)\big]\Big\}^2}, \tag{15}$$

then consumers insure

$$I = W_0 - \frac{kpq\big[1-(1-\delta)(1-q)\big]^2\big[1-(1-\alpha)(1-q)\big]}{\alpha(1-q)\Big\{\delta - p(1-q)\big[1-(1-\delta)(1-q)\big]\Big\}^2}, \tag{16}$$

in the first period they become a high risk, and fraudsters search for compromised consumers at an intensity of

$$\phi = \frac{pq\big[1-(1-\delta)(1-q)\big]}{\delta - p(1-q)\big[1-(1-\delta)(1-q)\big]}. \tag{17}$$

If instead (15) *fails to hold, then $I = 0$, and*

$$\phi = \frac{q}{2(1-q)}\left(\sqrt{1 + \frac{4W_0\alpha\delta(1-q)^2}{k\big[1-(1-\delta)(1-q)\big]\big[1-(1-\alpha)(1-q)\big]}} - 1\right). \tag{18}$$

When consumers are born wealthy (i.e., W_0 is large) and it is cheap to search for those who suffer from cognitive decline (i.e, k is small), consumers anticipate fraudsters to be aggressive, and so protect some of their wealth (i.e., $I > 0$). In fact, the fraction $\frac{I}{W_0}$ of their wealth that they protect is increasing in W_0.

3.4 Properties of the Equilibrium

(to be written later)

4 Discussion

The average ability to make household financial decisions declines after age 53 (Agarwal et al., 2009). More that 42% of the households in the United States have a head who is 55 or older and that percentage is rapidly increasing. These households control 62% of household net wealth (U.S. Census Bureau, 2014). As illustrated in our model, one approach to coping with the financial dangers of cognitive decline is to limit older peoples' ability to spend their wealth. However, overall trends tend to be in the opposite direction. Defined-benefit plans are being replaced with defined contribution plans. Defined benefit pensions provide retirees with a regular income, but not with access the present value of that income. Defined contribution plans give retirees access to and discretion over their entire retirement savings. While home equity loans and reverse mortgages benefit some consumers, they can be difficult for consumers with cognitive decline to understand and may provide liquidity that can be easily misspent.

In our model, young consumers rationally assess the probability of cognitive decline and potential loses. Some consumers choose to constrain the ability of their future selves to succumb to fraud. In reality, the financial danger of cognitive decline is a risk for which many people do not prepare. One reason may be the lack of good solutions.

One pre-commitment mechanism for limiting future financial discretion would be to buy an immediate or deferred income annuity and, thereby, providing income for one's old self while limiting the old self's ability to spend large amounts. Of course, the primary function of an income annuity is to provide protection against longevity risk and income annuities are priced to pool that risk. Thus, an income annuity that is fairly priced from a longevity-risk perspective, will have additional value to a consumer concerned about the risk bad financial decisions due to cognitive decline.

In addition to annuities, different commitment devices may be needed to deal with different potential financial mistakes. Tying up one's wealth in an income annuity will protect against making major investments in scams such as Bernie Madoff's. However, annuitization is unlikely to prevent an older person from wiring money to a putative grandson who calls with an emergency request for $3,000. One protection against such scams would be to put one's liquid assets in an account that required authorization from a fiduciary or other trusted person for unusual expenses beyond some threshold.

Alesina and Lusardi (2006) propose that states require people to pass a test and obtain a financial driver's license. People who did not obtain the license so would be restricted to default investments or required to obtain financial advice. Of course there are many practical challenges to using financial driver's tests to screen diminished financial knowledge due to cognitive decline. Our model can be extended to have young consumers commit to protecting their wealth if their old selves do not pass a test of cognitive ability. However, if such a test does not identify cognitive decline perfectly, consumers will choose a lower passing threshold than is socially optimal. This is because, as discussed above, when more consumers with cognitive decline are protected, fewer scammers are active.

5 Conclusion

We add to the literature on the use of pre-commitment to solve time inconsistent choice problems by modeling a market in which young consumers know that their future selves may make bad financial decisions due to cognitive decline. Old consumers with cognitive decline are unable to distinguish financial products that will benefit them from those that will lead to losses. In our model, young consumers can choose to constrain their old selves from buying financial products. When cognitive decline is likely, the cost of protection low, and fraud is prevalent, more consumers choose to protect their old selves. Fraudsters are more likely to enter the market when cognitive decline is high and protection levels low. For this reason, each consumer who protects herself provides a positive externality to other consumers (by discouraging scammers) and the socially optimal level of protection is higher than what consumers choose for themselves. Pre-commitment enables consumers facing the prospect of being scammed when in cognitive decline, to decline to be scammed.

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