Laying off Credit Risk: Loan Sales versus Credit Default Swaps

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Abstract

After making a loan, a bank finds out if the loan needs contract enforcement (“monitoring”); it also decides whether to lay off credit risk in order to release costly capital. A bank can lay off credit risk by either selling the loan or by buying insurance through a credit default swap (CDS). With a CDS, the originating bank retains the loan’s control rights but no longer has an incentive to monitor; with loan sales, control rights pass to the buyer of the loan, who can then monitor, albeit in a less-informed manner. In a single-period setting, for high levels of base credit risk, only loan sales are used in equilibrium; risk transfer is efficient, but monitoring is excessive. For low levels of credit risk, equilibrium depends on the cost of capital shortfalls. When capital costs are low, only poor quality loans are sold or hedged; risk transfer is inefficient, and monitoring may also be too low. When capital costs are high, CDS and loan sales can coexist, in which case risk transfer is efficient but monitoring is too low. In both cases, if gains to monitoring are sufficiently high, the borrowing firm may choose to borrow more than is needed to finance itself so as to induce monitoring. Restrictions on the bank’s ability to sell the loan expand the range where CDS are used and monitoring does not occur.

In a repeated setting, reputation concerns may support efficient outcomes where CDS are used and the bank still monitors. Because loan defaults trigger a return to inefficient outcomes in the future, total efficiency cannot be sustained indefinitely. Reputational equilibria are most likely for firms that have high base credit quality or for firms where monitoring has a high impact on default probabilities.

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1 Introduction

Over the last fifteen years, there has been a massive increase in the size of all types of markets for credit risk. Increasingly, a bank wishing to free up regulatory capital may either sell a loan directly or buy a synthetic product — a credit default swap (CDS) — that effectively insures it against non-performance.¹ As noted in the literature (which we review below), these markets allow risk transfer at the cost of reducing the monitoring incentives of the banks. There has been little work, however, on the tradeoffs between loan sales and credit default swaps. Broadly, the difference between the two is that in the former cash flows are bundled with control rights, in the latter they are not. Understanding which markets banks trade in is important for regulators trying to evaluate risk exposure and traders who are trying to evaluate credit risk across various securities. Our paper presents a parsimonious model of bank lending that characterizes how the choice between these two credit risk transfer methods affects equilibrium in their respective markets and ultimately the credit worthiness of the underlying loan.

In our model, a firm has a risky, positive NPV project and seeks funding from a bank. After making the loan, the bank receives private information about the project’s success probability. The bank is also hit by a capital shock that makes it costly to continue to hold the credit risk of the loan.² It can lay off this risk either through a CDS or through a loan sale. The critical difference between the two is that, with a CDS, the originating bank retains ownership and thus control rights over the loan that it made, whereas a loan sale transfers these control rights to the buyer of the loan. Control rights matter because the loan’s owner may enforce these control rights (“monitor the loan”) at a cost. In the right circumstances, such monitoring reduces the loan’s probability of default; thus, the originating bank’s private information is valuable as it is required for efficient monitoring.

At first glance, using a CDS might seem optimal, since it allows the originating bank to exercise control in an informed fashion while laying off the risk of the loan when capital shocks occur. Unfortunately, once it has laid off the risk, the bank has little incentive to incur the costs of monitoring. By contrast, after a loan sale, even though a bank will not monitor, the buyer of the loan can do so, albeit in an imperfectly-informed manner. This is because there is no way for the bank to credibly communicate its information about how necessary monitoring is. There is therefore a tradeoff between efficient risk sharing and

¹Briefly, a CDS is a contract written on the solvency of a firm, (referred to variously as the “reference entity” or the “name”). If there is a credit event (default, restructuring) associated with the reference entity, then the protection seller remunerates the protection buyer. In exchange, the latter pays regular premia to the former over the term of the swap.
²Specifically, the bank is faced with a correlated and profitable lending opportunity, which requires that it either raise more costly capital or forego the opportunity.
efficient monitoring.

If credit risk is high, then monitoring is most attractive. In this case, loan sales dominate CDS. However, even though risk transfer is efficient, too much monitoring takes place: loan buyers monitor all loans, even those that do not benefit from it. When credit risk is relatively low, monitoring adds less value. The equilibrium that obtains depends on how eager banks are to lay off the risk. If the capital cost of retaining credit risk is sufficiently high, CDS and loan sales may coexist, in which case credit risk is transferred efficiently, but there is too little monitoring. If instead capital costs are low, only banks with negative information lay off credit risk, leading to too little risk transfer. CDS and loan sales markets may both be active, in which case monitoring is also too low; alternatively, loan sales may dominate, in which case monitoring is efficient. For intermediate capital costs, both types of equilibria are possible, leading to some indeterminacy. In these cases, the firm (which earns all the social surplus) may also choose to borrow more than is needed to finance the project so as to bring about the equilibrium with efficient risk transfer and excessive monitoring. This will be profit maximizing when the expected social gains to monitoring outweigh the costs of monitoring too much.

All of this assumes that the bank is free to sell its loan. If the loan agreement contains restrictions on the bank’s ability to sell loans, then legally the bank cannot transfer material aspects of the business relationship: in other words, the possibility of monitoring by a loan buyer is reduced. Such anti-assignment clauses increase the bank’s incentive to use CDS and to not monitor. Thus, loan sales restrictions may be counterproductive.

This analysis also assumes that the originating bank is the only seller of credit risk. If others sell the firm’s credit risk for portfolio reasons, then the CDS market may be active even when loan sales should dominate. In this case, the originating bank will prefer to lay off risk through CDS rather than loan sales, and efficient monitoring will not occur.

Our discussion thus far has focused on single-period outcomes. We also explore whether reputation concerns in an infinitely-repeated version of the model allow the bank to commit to monitoring even while it makes use of CDS. We show that limited reputation effects are possible if market participants use loan defaults as a noisy signal that the bank has not monitored, and loan defaults are followed by repetition of the single-period equilibrium as a “punishment” for the bank. Because monitored loans have some chance of default, there is always some chance that, even when the bank honors its commitment to monitor, defaults will occur and inefficient behavior will follow. This reputational equilibrium is generally more likely to be feasible the higher the base credit quality of the bank’s loan and the higher the impact of monitoring on default probability. Thus, our single-period result that CDS are most likely to be used when credit risk is relatively low continues to find support
in the case of multiple periods.

Our analysis matches a number of stylized facts about the CDS and loan sales markets. First, the CDS market was initially confined to high-grade credits, whereas loan sales often involve riskier credits — see for example Drucker and Puri (2007). Second, Minton, Stulz, and Williamson (2008) examine banks’ credit risk transfer practices over a sample period of 1999-2003. Of all U.S. bank holding companies with $1 billion or more assets, only 18-20 use credit derivatives during the sample period however, these firms account for over 60% of sample assets. They find some evidence that loan sales and credit derivatives are complementary products, which is consistent with our findings that the two markets may coexist. In addition, they report that banks that do use credit derivatives to hedge loan risk do so for lower risk loans, not higher risk loans; this is consistent with our prediction that credit derivatives will tend to be used for less risky credits. Our analysis suggests that the introduction of CDS is most likely for firms with low or intermediate levels of credit risk, and that such an introduction undermines monitoring, increasing default rates. This is consistent with Ashcraft and Santos (2007), who find that the introduction of CDS tends to have a negative impact on the borrowing rates of firms that are most likely to need monitoring but slightly improves the borrowing rates of firms that are safe and transparent. That the CDS market is informative is a implication of Acharya and Johnson (2006) who find that information from the CDS market adds to stock market information, primarily with respect to bad news.

Empirical evidence on the quality of sold loans is mixed. In the context of our model, because we establish when a loan will be monitored and its credit quality enhanced, it is important to distinguish between the exogenous base credit quality of the loan and the endogenous default probability that obtains after the monitoring decision is made. Gupta, Singh, and Zebedee (2006) point out that loans sold are typically senior, secured, and sold piecemeal. While Drucker and Puri (2007) find that borrowers whose loans are sold are more than 1.5 times the size of borrowers whose loans are not sold, and have higher leverage and lower distance-to-default than borrowers whose loans are not sold, they report that loans that are sold have more (and more restrictive) covenants than those that are not sold. Similarly, tighter covenants increase the probability of sale when the initial lender is less reputable (lower market share, or not in the top-ten lead banks).

Several theoretical papers examine the effect of credit derivatives on relationship banking. Arping (2004) formalizes credit derivatives as a relaxation of the firm’s limited-liability constraint. By paying out cash in the case of a credit event, protection sellers reinforce the bank’s incentive for efficient early liquidation of the borrower’s project. Critically, the maturity of the credit derivative must be observably shorter than that of the underlying loan,
which in turn must be longer than the point at which bank monitoring is efficient; otherwise, monitoring incentives are undermined. Morrison (2005) shows that the unobservability of credit derivative transactions can undermine bank monitoring, causing corporate borrowers to prefer bond market finance; this disintermediation reduces welfare. Parlour and Plantin (2006) considers how the ability to sell loans (or trade in CDS) affects banks’ incentives to monitor. They provide conditions under which a liquid credit risk transfer market can be socially inefficient. Chiesa (2008) examines credit risk transfer when bank monitoring improves returns in downturns. She finds that optimal credit risk transfer takes the form of a securitization of the entire loan portfolio with limited credit enhancement. Proper risk-based capital regulations are needed to implement this; otherwise, the bank provides too much credit enhancement and shirks on monitoring. Note that none of these papers examines the distinction between loan sales and credit default swaps.

Duffee and Zhou (2001) do consider the co-existence of loan sale and credit derivative markets. As in Arping (2004), they emphasize the fact that, unlike a loan sale, credit derivatives allow a bank to shed a loan’s risk of early default risk without shedding the risk of default at maturity. This flexibility can undermine the loan sales market, which can lead to both good or bad outcomes. If high quality banks do not use the loan sales market in order to signal their type, introducing credit derivatives can promote pooling, which enhances welfare. However, if loan sales would otherwise lead to risk-sharing by both high- and low-quality banks, credit derivatives may undermine risk-sharing. Critical assumptions are that bank loan maturities are known, loan sales and credit derivative transactions are jointly observable, and asymmetric information is greater later in the life of the loan rather than earlier. Thompson (2006) adapts the model of Duffee and Zhou (2001) to consider the case of imperfect competition in the CDS market. He finds that CDS can lead to efficient outcomes if the insurance provider has market power. We differ from both of these papers in that we explicitly consider the value of control rights in the loan sales market, and model when and how they are exercised.

2 Model

Consider the following five-date model of an entrepreneur who raises funds from a bank to undertake a risky project. After the loan is originated, the bank can lay off credit risk either through a credit default swap market or a loan sales market. The owner of the loan can exert costly effort and in some cases decrease the default probability.

At $t = 0$ an entrepreneur raises money from a bank, by making a take-it-or-leave-it offer, in order to fund a project of fixed size $1$ which pays off $R$ with some probability and
otherwise. The bank’s contract is characterized by the pair \((R^\ell, C)\), where \(R^\ell \leq R\) is the payoff conditional on the project’s success. Here, \(C\) is the firm’s collateral or liquidation value, and \(R^\ell - C\) is the risky portion of the loan.

At \(t = 1\) the bank gets a private signal about the project’s governance. With probability \(\theta\) it learns that the project is “simple” and succeeds with probability \(p + \Delta\); with probability \(1 - \theta\) the project is “complex.” In this case, the entrepreneur can choose either the simple project or a riskier one that succeeds with probability \(p\) but gives the entrepreneur a private benefit \(B\). Thus, with probability \(1 - \theta\), the project is prey to moral hazard. We refer to a bank’s private information as its “type.” Thus, there are two types of originating banks, \(p\) and \(p + \Delta\), depending on which project the bank expects the entrepreneur to pick.

At \(t = 1\), the bank also receives an opportunity to invest in another project correlated with the existing one. Due to regulatory constraints, (i.e., there is a risk limit and banks have to raise costly external capital), the bank bears a cost of \(\beta > 0\) per unit of outstanding risk.\(^3\) In what follows, we refer to the arrival of a correlated opportunity as a capital shock.

At \(t = 2\), the bank can offload credit risk from its balance sheet. There are two ways in which a bank can do this; first any bank can trade in the CDS market. Second, the bank can sell the loan. If a bank enters into a credit default swap (buys protection), it buys insurance that pays off the face value of the loan if the firm defaults. This is achieved either through physical delivery, in which case the bank delivers the instrument conditional on default (valued at \(C\)) and receives \(R^\ell\), or through cash settlement, in which case the CDS seller pays out \(R^\ell - C\) and the bank retains the existing loan. In both cases after default, the value of the loan plus the CDS is worth \(R^\ell\). The bank’s aggregate trades in either market are not observable, and so we focus on the case where the bank completely hedges its risk.\(^4\)

Both the loan and CDS market participants are risk neutral and competitive. Prices equal the expected value of the loan or CDS contract to the market. The unconditional probability of project success is \(p + \theta \Delta\). However, as different bank types have different preferences over the credit risk transfer markets, participation in each of the markets may be informative about the project’s underlying success probability. Therefore, the market belief of the default probability is an important endogenous variable that affects the payoffs to each of these actions. Let \(p^{CDS}\) denote the market belief about the probability of success of the loan given that the bank participates in the CDS market, and let \(p^{LS}\) be the market belief

\(^3\)For example, suppose that the ex ante variance (beliefs are relative to the public information) of its position is restricted, so that \((R^\ell - C)^2 p (1 - p) \leq \bar{V}\). Another project that is perfectly correlated would violate the bank’s variance constraint: In particular, \(4 (R^\ell - C)^2 p (1 - p) > \bar{V}\). Costly external funds prevent a bank from instantaneously raising more capital.

\(^4\)If the market thought that the bank was hedging less than its total exposure, this would work as a good signal; however, knowing this, the bank would then have incentive to hedge everything at a favorable price.
if the bank sells the loan. For notational ease we sometimes describe $p^{CDS} = p + \phi^{CDS}\Delta$, and $p^{LS} = p + \phi^{LS}\Delta$. Belief about the success probability depends on the equilibrium.

At $t = 3$, the owner of the loan can exert costly effort (“monitor”). The cost of monitoring is $b$. Monitoring represents the use of control levers (e.g., enforcing covenants, threatening to call the loan) to improve the borrower’s financial position. Monitoring is beneficial when the project admits moral hazard. If the loan owner monitors, the entrepreneur is prevented from choosing the riskier project; thus, the project’s probability of success increases from $p$ to $p + \Delta$. By contrast, if the project’s probability of success was already $p + \Delta$, then there is no effect. It follows that information about the firm’s probability of success is valuable for informing the monitoring decision. Finally, at $t = 4$, all claims pay off.

<table>
<thead>
<tr>
<th>Loan originated</th>
<th>Bank learns $p + \Delta$ or $p$</th>
<th>Bank Chooses Loan Sales or CDS or Retains</th>
<th>Owner Exerts Costly Effort</th>
<th>All claims pay off</th>
</tr>
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<tbody>
<tr>
<td>$t = 0$</td>
<td>$t = 1$</td>
<td>$t = 2$</td>
<td>$t = 3$</td>
<td>$t = 4$</td>
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Figure 1: Sequence of Events

To resolve indifference, we assume that when banks are indifferent between laying off risk and retaining it, they retain it on their balance sheets. In addition, if a bank is indifferent between monitoring and not monitoring, we assume that it does not monitor. Effectively, we assume that there is an infinitesimal cost to either laying off risk or monitoring a project.

We impose parameter restrictions to ensure that there is a moral hazard problem and that monitoring is socially efficient. First, malfeasance on the part of the entrepreneur arises if any lending rate $R^\ell$ that lets the bank break even induces the entrepreneur to choose the riskier project. For any $R^\ell$, an entrepreneur will shirk if

$$B + p(R - R^\ell) > (p + \Delta)(R - R^\ell),$$

or, $B > \Delta(R - R^\ell)$. The lowest possible break-even value of $R^\ell$ is the one for which the bank assesses the default probability at $p + \Delta$, for a profit of $(p + \Delta)R^\ell + (1 - p - \Delta)C - 1$. If the bank makes a zero profit, this implies a minimum value of $R^\ell$. Therefore, a sufficient condition for moral hazard is that:

**Assumption 1** $\Delta \left( R - C - \frac{(1-C)}{p+\Delta} \right) < B$, so given the chance, the entrepreneur prefers to choose the riskier project and consume private benefits.

Second, this behavior is undesirable if a social planner, who knows the entrepreneur can
consume private benefits, prefers to monitor the project. Or,
\[ C + (p + \Delta)(R - C) - b > C + p(R - C) + B \quad . \] (2)

Thus, if \( \Delta(R - C) > B + b \), aggregate surplus is higher if the bank monitors the entrepreneur when he is subject to moral hazard. Eschewing the risky project is therefore socially efficient.

**Assumption 2** \( \Delta(R - C) > B + b \) so that it is socially inefficient for the entrepreneur to choose the riskier project.

In our simple framework, risk transfer is efficient if banks lay off their credit risk and thus bear no capital cost. Monitoring is efficient if projects that the originating bank has identified as being subject to moral hazard and thus having a low success probability of \( p \) are monitored. Thus, the ex ante value of a loan if there is efficient monitoring and efficient risk transfer is
\[ \Omega^* = C + (p + \Delta)(R - C) - 1 - (1 - \theta)b \quad . \] (3)

There are two potential sources of inefficiency in the CRT markets. First, if the price of loans is sufficiently low (cost of CDS is sufficiently high), then a bank with a capital shock may not hedge its position, forcing it to raise more costly capital. Second, if there is pooling, there may be inefficient monitoring; monitoring may not take place at all, or monitoring may take place even when the originating bank knows that the loan’s success probability is \( p + \Delta \) so that monitoring is inefficient. We explore these possibilities in the next section.

3 Characterization of Equilibria

We begin by analyzing equilibria taking the loan’s face value \( R^\ell \) as given; then, in the next section, we endogenize the choice of \( R^\ell \). We first establish conditions under which the agent who can monitor the loan at time \( t = 3 \) chooses to do so. It is immediate that a CDS removes the incentive for the originating bank to monitor the loan. Effectively, as the bank bears the private cost of monitoring, \( b \), and the benefits accrue to the seller of protection, it can never be optimal to monitor. Further, a bank that knows that the success probability is \( p + \Delta \) will never monitor, as monitoring has no effect.

However, if the success probability is \( p \), the originating bank that retains the risk of the loan monitors if and only if
\[ C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C) \quad . \] (4)
i.e., if $R^\ell - C > \frac{b}{\Delta}$. Here $\frac{b}{\Delta}$ is the per-unit cost of reducing the default probability.

Clearly, if the credit risk is sufficiently small, so that $R^\ell - C \leq \frac{b}{\Delta}$, the bank would never monitor even if it were exposed to the full risk of the loan. Since the loan buyer knows the firm’s success probability is no less than $p$, it too has no incentive to monitor, and thus control rights are immaterial. By contrast, for $R^\ell - C > \frac{b}{\Delta}$, monitoring can add value.

If the bank sells the loan, then the loan buyer, although less informed than the originator, updates its beliefs about the performance of the loan conditional on the loan being sold. In a similar fashion, the buyer compares the cost of monitoring to the expected benefit.

**Lemma 1**

(i) An originating bank that buys CDS or knows that the project is simple will never monitor.

(ii) An originating bank that knows the project is complex and does not buy CDS monitors if $R^\ell - C > \frac{b}{\Delta}$

(iii) A loan buyer will monitor the loan if $R^\ell - C > \frac{b}{\Delta(1 - p_{CDS})}$.

Notice that $\frac{b}{\Delta(1 - p_{CDS})} \geq \frac{b}{\Delta}$. Therefore, unless there is perfect communication of the originating bank’s private information, there will be a range of credit exposures for which an originating bank would monitor if it retained ownership and did not purchase a CDS, but the buyer of the loan will not. This illustrates that information in this economy is useful because it allows agents to make optimal monitoring decisions.

Because the originating bank sells to a competitive market, the price of credit risk reflects the buyers’ valuation of the project. If a buyer decides to monitor, both the expected benefits and costs are passed on to the originating bank in the price.

**Lemma 2** An originating bank can lay off credit risk:

(i) Through a loan sale at price $C + (p + \Delta)(R^\ell - C) - b$ if the loan buyer plans to monitor;

(ii) Through a loan sale at price $C + pLS(R^\ell - C)$ if the loan buyer does not plan to monitor;

(iii) By entering into a CDS for a payment of $(1 - p_{CDS})(R^\ell - C)$.

It is clear from Lemma 2 that if a bank wants to lay off credit risk, the decision whether to use loan sales or CDS depends on two things: first, the inference that the market draws from the credit risk transfer method and second, the action that the loan purchaser takes (i.e., whether it plans to monitor or not).

Holding beliefs fixed about the bank types that use the two methods, if the loan buyer does not plan to monitor, then there is no difference between CDS and loan sales. However, if the beliefs vary so that, for example, one type of credit risk transfer is associated with high default loans and the other is not, then the first method will be eschewed and the
other will dominate. That is, there will be no trade in the dominated credit risk transfer method. (To simplify matters, in what follows, we assume that if the loan buyer does not plan to monitor, then beliefs are the same across the two credit risk transfer methods.) By contrast, if the loan buyer plans to monitor, then holding beliefs fixed, loan sales are preferred by some banks to CDS. They are strictly preferred if the originating bank knows that the loan should be monitored (i.e., the bank’s type is $p$).

Market beliefs about the quality of the project and therefore the value of the control rights differ depending on the equilibrium that occurs. We now address the possible equilibria, first in the case where the originating bank would never monitor ($R^e - C \leq b\Delta$), then in the case where the originating bank might monitor ($R^e - C > b\Delta$).

**Proposition 1** Suppose that $R^e - C \leq b\Delta$. Then neither the originating bank nor the loan buyer monitor. Two equilibria are possible.

(i) If $\beta > (1 - \theta)\Delta$, then an equilibrium exists where both bank types shed credit risk. Trade is possible in both the CDS and loan sales markets, and market beliefs are $p^{CDS} = p^{LS} = p + \theta\Delta$. The resale price of a loan is $C + [p + \theta\Delta](R^e - C)$.

(ii) If $\beta \leq \Delta$, then an equilibrium exists where only a type $p$ bank sheds credit risk. Trade is possible in both the CDS and loan sales markets, and market beliefs are $p^{CDS} = p^{LS} = p$. The resale price of a loan is $C + p(R^e - C)$.

Intuitively, when monitoring is ruled out, there are two equilibria: a pooling one in which both banks shed credit risk, and a separating one in which only the worse type of bank sheds credit risk. A bank that knows the project is of high quality compares the benefit of laying off risk (which depends on the capital cost) to the adverse selection discount in the market. In the pooling equilibrium, the type $p + \Delta$ bank must prefer to sell or hedge the loan at the pooling price rather than hold the loan and face the capital cost. This accounts for the condition in part (i) of the proposition. By contrast, in the separating equilibrium, the type $p + \Delta$ bank prefers to retain the loan and so the price of a sold loan is lower and the premium on CDS is higher. The condition in part (ii) ensures that the type $p + \Delta$ bank prefers not to use credit risk transfer at these prices.

The upshot is that, when capital costs are sufficiently low ($\beta \leq \Delta$), a bank with a good project will keep it on its balance sheet and the separating equilibrium exists. Whereas, when capital costs are sufficiently high ($\beta > (1 - \theta)\Delta$), the good bank lays off credit risk and so the pooling equilibrium exists. If $(1 - \theta)\Delta < \beta \leq \Delta$, both equilibria are possible.

The conditions in Proposition 1 can also be expressed in terms of $b\Delta$, the per unit cost of decreasing the default probability. (This captures the effectiveness of monitoring.) For example, the condition $\beta > (1 - \theta)\Delta$ is equivalent to the condition that $\frac{b}{\beta} < \frac{b}{\Delta(1 - \theta)}$. Figure
Figure 2: Equilibria when the originating bank would not monitor

When the loan face value is high enough that the originating bank may monitor, matters are somewhat more complex, as we now show.

**Proposition 2** Suppose that \( R^\ell - C > \frac{b}{\Delta} \), so that an originating bank of type \( p \) monitors retained loans.

(i) If \( R^\ell - C \leq \frac{b(1-\theta)}{\Delta(1-\theta)^2} \) and \( \beta > (1-\theta)\Delta \), then an equilibrium exists in which both bank types shed credit risk, trade is possible in both the CDS and loan sales markets, and market beliefs are \( p^{CDS} = p^{LS} = p + \theta\Delta \). Loan buyers do not monitor, and the resale price of a loan is \( C + [p + \theta\Delta](R^\ell - C) \).

(ii) If \( R^\ell - C > \max \left\{ \frac{b}{\beta}, \frac{b(1-\theta)}{(1-\theta)\Delta} \right\} \), then an equilibrium exists where both bank types shed credit risk through loan sales, there is no trade in the CDS market, and market beliefs are \( p^{LS} = p^{CDS} = p + \theta\Delta \). Loan buyers monitor, and the resale price of a loan is \( C + (p + \Delta)(R^\ell - C) - b \).
(iii) If $R^\ell - C \leq \frac{b}{\beta}$ (which requires that $\beta < \Delta$), then an equilibrium exists where only type $p$ banks shed credit risk through loan sales, there is no trade in the CDS market, and market beliefs are $p^{CDS} = p^{LS} = p$. Loan buyers monitor, and the resale price of a loan is $C + (p + \Delta)(R^\ell - C) - b$.

Once monitoring is feasible, more possibilities arise: banks may either pool in the credit risk transfer market or separate, and monitoring may or may not occur. Although at first glance this would suggest there are four types of equilibria, the fact that $R^\ell - C > \frac{b}{\Delta}$ means that if the two bank types separate, a sold loan has probability of success $p$ and thus will be monitored. There is no equilibrium in which the two bank types separate and there is no monitoring. This leaves three types of equilibria, which are dealt with in the three parts of the proposition.

Part (i) of the proposition gives conditions under which the equilibrium with pooling in the credit risk transfer market and no monitoring is feasible. As in Proposition 1(i), capital costs are sufficiently high so that $\beta > (1 - \theta)\Delta$. In this case, a bank that knows a project is of high quality has a strong incentive to sell it. In addition, because the eventual buyer will not monitor the loan, the credit risk $R^\ell - C$ has to be sufficiently low, so that the type $p$ bank prefers shedding credit risk to holding and monitoring the loan. The same condition guarantees that potential loan buyers do not find monitoring attractive. Note that both loan sales and CDS are possible in this equilibrium. Indeed, in the absence of monitoring, there is no difference between the two credit risk transfer method.

As a bank’s exposure to credit risk becomes larger, monitoring becomes more attractive. Part (ii) of the proposition gives conditions under which there is an equilibrium with pooling in the credit risk transfer market and monitoring by loan buyers. This requires that credit risk is high enough that loan buyers prefer to monitor ($R^\ell - C > \frac{b}{(1 - \theta)\Delta}$) and that the type $p + \Delta$ prefers to sell at the pooling price rather than hold and monitor the loan ($R^\ell - C > \frac{b}{\beta}$). In this equilibrium, loan sales dominate CDS: loan sales permit monitoring by buyers of credit risk, which is attractive here, whereas CDS do not permit this. This dominance occurs despite the fact that investor beliefs do not view CDS as a negative signal relative to loan sales.

Finally, if the cost of retaining risky capital is sufficiently small and the cost of monitoring sufficiently large, then there is an equilibrium with separation in the credit risk transfer market and monitoring by loan buyers. Part (iii) of the proposition gives conditions under which the type $p + \Delta$ bank prefers holding the loan to selling it at the market price, which in turn requires that the cost of capital to cover the loan’s risk, $\beta(R^\ell - C)$, cannot be too
high relative to the cost of monitoring $b$ (which reduces the market price of the loan). \(^5\)

Once again, we can portray these results in terms of $\frac{R^\ell - C}{b}$. This is done in Figure 3. Notice that this figure corresponds to the upper half of Figure 2, the region where $R^\ell - C > \frac{b}{\Delta}$.

\[ R^\ell - C = \frac{b}{\Delta} \left( \frac{1}{1 - \theta} \right) \]

\[ R^\ell - C = \frac{b}{\Delta} \]

\[ \frac{b}{\beta (1 - \theta)} \]

\[ \frac{b}{\beta} \]

\[ \frac{b}{\beta (1 - \theta)} \]

\[ \frac{b}{\Delta} \]

Figure 3: Parameter Ranges for different equilibria if the originating bank monitors

Looking at Propositions 1 and 2, we have four kinds of equilibria: loan buyers may monitor or not and the banks may pool or not. Each differs in expected social surplus, which can be expressed in terms of the efficient outcome. Let $\Omega^{ij}$ denote the expected aggregate payoff to a loan where $i \in \{p, s\}$ indicates if there is pooling or separating. If there is pooling, in the CRT market then all banks lay off risk. If there is separation, then only type $p$ lays off risk. In addition, $j \in \{m, n\}$ indicates how the control rights are used in the loan sale market. If $j = m$ then the loan buyer monitors; if $j = n$ the loan buyer does not.

\[^5\] In parts (ii) and (iii) of the proposition, one can make $p^CDS$ slightly greater than $p^LS$ without disrupting the equilibrium. This is because CDS are strictly dominated by loan sales, and slightly raising market beliefs associated with CDS does not change this.
Proposition 3 The expected social benefit to a loan is:

\[
\Omega^{p,n} = \Omega^* - (1 - \theta)[\Delta(R - C) - (b + B)] \\
\Omega^{p,m} = \Omega^* - \theta b \\
\Omega^{s,n} = \Omega^* - (1 - \theta)[\Delta(R - C) - (B + b)] - \theta \beta(R^\ell - C) \\
\Omega^{s,m} = \Omega^* - \theta \beta(R^\ell - C)
\]

for \((1 - \theta)\Delta < \beta, R^\ell - C \leq \frac{b}{(1 - \theta)\Delta}\)

for \(R^\ell - C > \max\left[\frac{b}{(1 - \theta)\Delta}, \frac{b}{\beta}\right]\)

Note that none of the equilibria achieve the maximal social surplus. In each, there
is a tradeoff between efficient monitoring and efficient risk sharing. Specifically, if the
equilibrium is pooling, then there is efficient risk sharing. However, in this case a potential
loan buyer does not have enough information to correctly assess the value of monitoring
and so renders it inefficient.

Corollary 1 If control rights are valuable, then if equilibrium in the credit risk transfer
market exhibits

(i) pooling and monitoring: then risk transfer is efficient, but aggregate monitoring is too
high.

(ii) pooling and no monitoring: then risk transfer is efficient but aggregate monitoring is
too low.

(iii) separating and monitoring: then risk transfer is inefficient, but monitoring is efficient.

(iv) separating and no monitoring: risk transfer is inefficient and monitoring is too low.

Pooling always leads to efficient risk transfer because both bank types lay off credit risk.
However, if there is pooling, the the loan buyer cannot infer the value of monitoring from
the bank’s choice in laying off credit risk. The bundling of the cash flow with the underlying
cash flow mean that either loan buyers monitor excessively, spending resources when the
firm’s project is simple, or they will not monitor at all, which is also inefficient. If there
is separation so that only the \(p\) type bank enters the CRT market, then risk transfer is
inefficient as the high quality bank cannot release regulatory capital.

4 Choosing Debt Values and Equilibria

The loan’s face value, \(R^\ell\), plays a critical role in the type of equilibrium that takes place
and therefore the efficiency properties and the value of control rights. In this section, we
endogenize this choice. Recall that the entrepreneur makes a take-it-or-leave-it offer to the
originating bank, and that the bank’s net opportunity cost of funds is 0. It follows that
the entrepreneur gets all surplus, and so she will choose a loan face value that maximizes
ex ante welfare subject to feasibility constraints. There are two such constraints: the bank must lend at least one unit so as to fund the project, and the bank must expect to break even given the equilibrium that will occur. (If the bank lends more than one unit, the entrepreneur invests 1 in the project and consumes the rest.)

To calculate the bank’s expected payoff for each equilibrium, let \( \pi_{i,j} \) be the expected continuation payoff to the bank, where \( i \in \{p,s\} \) and \( j \in \{n,m\} \) denotes the type of equilibrium: pooling or separating, and no monitoring and monitoring, respectively. The bank must earn at least 1 in expectation, and this break-even condition implies the following minimum feasible loan face values.

**Lemma 3**

(i) If the equilibrium is type \((i,j)\), where \( i \in \{p,s\} \) and \( j \in \{n,m\} \), then the bank’s break-even condition requires that the loan’s face value \( R_{i,j}^\ell \) must weakly exceed \( R_{i,j}^\ell \), where

\[
R_{i,j}^\ell = \begin{cases} 
\frac{1-C}{p+\theta\Delta} + C & \text{if } i, j = p, n \\
\frac{1-C}{p+\Delta} + C & \text{if } i, j = p, m \\
\frac{1-C}{p+\theta\Delta - \theta\beta} + C & \text{if } i, j = s, n; \\
\frac{1-C+(1-\theta)b}{p+\Delta - \theta\beta} + C & \text{if } i, j = s, m
\end{cases}
\]  

(ii) \( R_{i,j}^\ell \) is decreasing in the firm’s collateral value \( C \), base probability of success \( p \), and impact of monitoring \( \Delta \). It is weakly decreasing in the probability, \( \theta \), that there is no moral hazard. It is weakly increasing in the cost of monitoring \( b \) and the cost of capital \( \beta \).

It is immediate that \( R_{i,j}^\ell \) is decreasing in \( C, p, \) and \( \Delta \). These parameters govern the firm’s innate credit risk; an increase in any of them reduces the bank’s expected credit exposure and thus reduces the face value it needs to break even. Thus, for any given type of equilibrium in the credit risk transfer market, feasibility constraints are less binding as the firm’s innate credit risk is lower.

We also show that \( R_{i,j}^\ell \) is weakly decreasing in \( \theta \). If monitoring does not take place, an increase in \( \theta \) increases the expected value of the firm. If monitoring does take place, an increase in \( \theta \) reduces the probability with which monitoring takes place, reducing expected monitoring costs. Either increases the expected value of a loan, reducing the face value the bank needs to break even. Although it is true that, in a separating equilibrium, an increase in \( \theta \) increases the probability that efficient risk transfer does not occur, this effect is dominated by the previous two.

Finally, \( R_{i,j}^\ell \) is weakly increasing in the cost of monitoring \( b \) and the cost of capital \( \beta \). Both factors decrease the bank’s expected profits from making the loan, increasing the bank’s break-even face value.
While minimum credit exposures tend to move with the firm’s innate credit risk, it is also apparent that the exact sensitivity depends on the type of equilibrium that results. As a result, a parameter change that results in a new type of equilibrium may result in a jump in the feasibility requirement. Nevertheless, we can establish that there is continuity between pooling equilibria with and without monitoring, and between separating equilibria with and without monitoring.

Lemma 4 The minimum feasible loan face value, \( R_{i,j}^\ell \) exhibits continuity in that

(i) \( R_{p,n}^\ell - C > R_{p,m}^\ell - C \) if and only if \( R_{p,n}^\ell - C > \frac{b}{(1-\theta)\Delta} \) if and only if \( R_{p,m}^\ell - C > \frac{b}{(1-\theta)\Delta} \).

(ii) \( R_{s,n}^\ell - C > R_{s,m}^\ell - C \) if and only if \( R_{s,n}^\ell - C > \frac{b}{\Delta} \) if and only if \( R_{s,m}^\ell - C > \frac{b}{\Delta} \).

(iii) \( R_{s,m}^\ell - C > R_{p,m}^\ell - C \) if and only if \( R_{s,m}^\ell - C > \frac{b}{\beta} \) if and only if \( R_{p,m}^\ell - C > \frac{b}{\beta} \).

Parts (i) and (ii) of the lemma show that there is continuity between both pooling and separating with no monitoring and pooling and separating with monitoring. Part (iii) shows there is continuity between pooling with monitoring and separating with monitoring equilibria.

As the entrepreneur gets all net surplus, she bears the cost of both inefficient monitoring and inefficient risk sharing. By selecting a loan face value \( R_{i,j}^\ell \), she can to a large extent select the equilibrium that will result and thus the amount of surplus generated. However, in some cases a given \( R_{i,j}^\ell \) can give rise to more than one equilibrium, creating indeterminacy. Based on the results of Propositions 1 and 2, we identify three cases that differ in the level of the capital cost \( \beta \). The simplest case is that in which capital costs are low.

Proposition 4 Suppose that capital costs are high (\( \beta > \Delta \)). Then the only equilibria that are feasible are the pooling equilibria \((p,n)\) and \((p,m)\).

(i) If \( R_{p,n}^\ell - C \leq \frac{b}{(1-\theta)\Delta} \), then the entrepreneur chooses the pooling with no monitoring equilibrium \((p,n)\) if \( b > (1 - \theta)|\Delta(R - C) - B| \), and the pooling with monitoring equilibrium \((p,m)\) otherwise. Loan face values that implement this are \( R_{i,j}^\ell = R_{p,n}^\ell \) for \((p,n)\) and \( R_{i,j}^\ell = \frac{b}{(1-\theta)\Delta} + \epsilon \) (where \( \epsilon \) small) for \((p,m)\).

(ii) If \( R_{p,n}^\ell - C > \frac{b}{(1-\theta)\Delta} \), then the entrepreneur chooses the pooling with monitoring equilibrium \((p,m)\). This can be implemented with \( R_{i,j}^\ell = R_{p,m}^\ell \). This outcome is more likely if collateral \( C \), the firm’s base probability of success \( p \), or the cost of monitoring \( b \) is low. It is also more likely if the probability that monitoring is needed \((1 - \theta)\) is high.

When capital costs are high, banks prefer to pool and get efficient risk-sharing rather than separate. If the firm’s innate credit quality is relatively high, the cost of monitoring is relatively high, or it is likely that moral hazard is not a problem, then the pooling with no monitoring equilibrium may be feasible. The entrepreneur prefers this to pooling with
monitoring if the costs of excessive monitoring outweigh the social gains from monitoring, which leads to the condition in part (i) of the proposition. If the costs of monitoring are relatively low or moral hazard is likely, the entrepreneur prefers the pooling with monitoring equilibrium. She can implement this by choosing a loan face value that exceeds what is needed to fund the project, levering up the firm. Also, if innate credit risk is high, pooling with no monitoring may be infeasible, because the minimum face value required on the loan is so high that loan buyers prefer to monitor. In this case, the entrepreneur must choose pooling with monitoring.

Next, we turn to the case where capital costs are low.

**Proposition 5** Suppose that capital costs are low \((\beta \leq (1 - \theta)\Delta)\). Then the only equilibria that are possibly feasible are \((s, n)\), \((s, m)\), and \((p, m)\).

(i) If \(R_{s,m}^\ell - C \leq \frac{b}{\Delta}\), then the entrepreneur chooses the separating with no monitoring equilibrium \((s, n)\) if \((1 - \theta + \frac{\theta}{2}\theta)b - \theta\beta(R_{s,n}^\ell - C) \geq (1 - \theta)[\Delta(R - C) - B]\), and the separating with monitoring equilibrium \((s, m)\) otherwise. Loan face values that implement this are \(R^\ell = R_{s,n}^\ell\) for \((s, n)\) and \(R^\ell = \frac{b}{\Delta} + \epsilon\) (where \(\epsilon\) small) for \((s, m)\).

(ii) If \(\frac{b}{\Delta} < R_{s,m}^\ell - C \leq \frac{b}{\beta}\), then the entrepreneur chooses the separating with monitoring equilibrium \((s, m)\). This can be implemented with \(R^\ell = R_{s,m}^\ell\).

(iii) If \(R_{s,m}^\ell - C > \frac{b}{\beta}\), then the entrepreneur chooses the pooling with monitoring equilibrium \((p, m)\). This can be implemented with \(R^\ell = R_{p,m}^\ell\).

When capital costs are low, the separating equilibria may be feasible. If the firm’s innate credit risk is low, then part (i) of the proposition applies; both of the separating equilibria (with and without monitoring) and the pooling equilibrium with monitoring are feasible. The pooling equilibrium with monitoring has one welfare cost, namely overmonitoring, which hurts welfare by \(-\theta b\). The separating equilibrium with monitoring has inefficient risk transfer, leading to a welfare cost of \(-\theta\beta(R^\ell - C)\); by the requirements of this equilibrium, \(R^\ell - C \leq \frac{b}{\beta}\), and so this equilibrium dominates the pooling equilibrium with monitoring. Intuitively, capital costs are low, so the losses from inefficient risk transfer are less than those from excessive monitoring.

It follows that the entrepreneur chooses between the two separating equilibria in this case. In both equilibria, welfare is maximized by choosing a loan face value that is as low as is feasible. When the condition in the proposition holds, the separating equilibrium with no monitoring dominates: because it uses a lower face value, it has lower capital costs than the separating equilibrium with monitoring, and these offset the losses from no monitoring if the latter are small enough. Note that this condition is less likely to hold than the similar condition in Proposition 4(i). Also, if the entrepreneur chooses the separating equilibrium...
with monitoring, she deliberately issues more debt than is needed to fund the project so as to induce monitoring; the possibility of credit risk transfer and the undermining effect of CDS on monitoring leads to higher leverage being chosen.

In part (ii) of the proposition, the firm’s innate credit risk is somewhat higher, ruling out the separating equilibrium with no monitoring, but still low enough that the separating equilibrium with monitoring is feasible. As in part (i), this dominates the pooling equilibrium with monitoring, so the entrepreneur chooses the separating equilibrium with monitoring.

Finally, in part (iii) of the proposition, the firm’s innate credit risk is so high that only the pooling equilibrium with monitoring is feasible. In this case, the entrepreneur must choose this equilibrium, leading to efficient risk transfer and excessive monitoring.

The final case to be considered is that where capital costs fall into an intermediate region. As we will see, there is now a possibility of indeterminacy of equilibrium, complicating the entrepreneur’s choice of loan face value.

**Proposition 6** Suppose that capital costs are at intermediate levels \((1-\theta)\Delta < \beta \leq \Delta\).

(i) If \(R_{p,m}^f - C \leq \frac{b}{\beta}\), then any loan face value that would allow a separating equilibrium also allows the pooling equilibrium with no monitoring \((p,n)\), leading to indeterminacy.

(a) If such a loan face value leads to the relevant separating equilibrium, then the entrepreneur chooses between implementing \((s,m)\) with \(R^f - C = \max\{R_{s,m}^f - C, \frac{b}{\Delta} + \epsilon\}\) (where \(\epsilon\) is small) and implementing \((p,n)\) with \(R^f - C \in \left(\frac{b}{\beta}, \frac{b}{(1-\theta)\Delta}\right]\). She chooses equilibrium \((p,n)\) if and only if \((1-\theta)b + \theta\beta \max\{R_{s,m}^f - C, \frac{b}{\Delta}\} \geq (1-\theta)[\Delta(R - C) - B]\).

(b) If such a loan value leads to \((p,n)\), then the entrepreneur chooses between implementing \((p,n)\) with \(R^f - C \in \left[R_{p,n}^f - C, \frac{b}{(1-\theta)\Delta}\right]\) and implementing \((p,m)\) with \(R^f - C \in \left(\frac{b}{(1-\theta)\Delta}, R - C\right]\). She chooses equilibrium \((p,n)\) if and only if \(b > (1-\theta)[\Delta(R - C) - B]\).

(ii) If \(\frac{b}{\beta} < R_{p,m}^f - C \leq \frac{b}{(1-\theta)\Delta}\), then the separating equilibria are infeasible. The entrepreneur chooses between implementing \((p,n)\) and \((p,m)\) as in (i.b).

(iii) If \(R_{p,m}^f - C > \frac{b}{(1-\theta)\Delta}\), then only equilibrium \((p,m)\) is feasible. The entrepreneur can implement this with \(R^f = R_{p,m}^f\).

In part (i) of the proposition, the separating equilibrium with monitoring \((s,m)\) is feasible (this follows from Proposition 3(v)), but any rate that allows that equilibrium or the separating equilibrium with no monitoring \((s,n)\) also allows the pooling equilibrium with no monitoring \((p,n)\). The equilibrium that is realized will depend on the beliefs of market participants, which the entrepreneur cannot control. Also, as noted in the discussion of Proposition 5, \((s,m)\) dominates the pooling equilibrium with monitoring \((p,m)\) whenever \((s,m)\) exists. We can also show that \((p,n)\) dominates \((s,n)\), since both lead to no monitoring.
but \((p, n)\) leads to efficient risk transfer whereas \((s, n)\) does not.

If market participant beliefs favor the separating equilibria, then the entrepreneur effectively chooses between setting a loan face value that leads to separating with monitoring and a higher loan face value that leads to pooling with no monitoring. (For a face value that is high enough, \((s, m)\) is not feasible whereas \((p, n)\) is.) She chooses pooling with no monitoring if the losses from no monitoring are less than the losses from inefficient risk transfer; rearrangement yields the condition in the proposition.

If instead market participants favor the pooling equilibrium with no monitoring, then the entrepreneur must choose between the two pooling equilibria, as in Proposition 4(i), with the same condition for when pooling with no monitoring is preferred to pooling with monitoring. Note that this condition is more likely to be met than that in part (i.a), since \(R_{s,m}^\ell - C \leq \frac{b}{\beta}\), which again implies that welfare under \((s, m)\) exceeds welfare under \((p, m)\).

In part (ii) of the proposition, the firm’s innate credit risk is higher, making the separating equilibria infeasible, but still low enough that the pooling equilibrium with no monitoring is feasible. Again, the entrepreneur chooses between the two pooling equilibria. Finally, in part (iii), innate credit risk is so high that only the pooling equilibrium with monitoring is feasible.

Looking over the results of Propositions 4, 5, and 6, a few patterns emerge. First, sufficiently high innate credit risk (relative to the per unit cost of reducing default probabilities through monitoring, \(\frac{b}{\Delta}\)) tends to favor equilibria with monitoring by making other equilibria infeasible. Since equilibria with monitoring have active loan sales markets and inactive CDS markets, this suggests that CDS will be less frequently used in cases where credit risk is high and monitoring is attractive (\(\frac{b}{\Delta}\) is low).

Second, higher capital costs favor pooling equilibria over separating equilibria, because the cost of inefficient risk-sharing increases. Since pooling equilibria have a larger range where monitoring is not supported (the relevant cut-off is \(\frac{b}{(1-\theta)\Delta}\) rather than \(\frac{b}{\Delta}\)), higher capital costs combined with credit risk transfer tend to undermine monitoring. Also, because the pooling equilibrium with no monitoring exists over a larger region than the separating equilibrium with no monitoring, an increase in capital costs makes it more likely that the CDS market is active.

If gains from monitoring are sufficiently high, or the probability of moral hazard \(1 - \theta\) is sufficiently high, monitoring equilibria are more attractive. The entrepreneur can implement these by choosing a loan face value that is higher than the amount needed to fund the firm’s investment. Thus, in these cases, leverage will be higher, loan sales will dominate CDS, and loan buyers will monitor.

These patterns are not always clear cut. For example, in the case of intermediate
capital costs, increases in innate credit risk may shift the equilibrium from separating with monitoring to pooling with no monitoring and then to pooling with monitoring.

5 Extensions

Our analysis so far has focused on a model with a number of restrictive assumptions: for example, monitoring is always cost efficient, the originating bank can always sell its loan without restriction, and there is only one period. In this section, we consider a number of extensions. We show that the use of “covenant light” loans may enhance the development of CDS markets. Conversely, an active CDS market with nonbank liquidity traders helps undermine efficient monitoring. Surprisingly, loan sales restrictions may also increase the viability of CDS markets and undermine monitoring. Finally, we consider the impact of reputation concerns in an infinite horizon version of our model. We find that reputation concerns may support first-best behavior in the short-run (the originating bank makes use of CDS to lay off credit risk yet still monitor efficiently), but this is more likely for loans of better innate credit quality. Eventually, defaults will still occur and undermine monitoring incentives.

5.1 Monitoring costs and default

In reality, loans vary in the ease with which covenants can be enforced. For example, “covenant light” loans do not include standard protective clauses such as minimum cash flow requirements or restrictions on future debt and therefore do not provide a monitor with levers in the event of financial distress. In the context of our model, this is equivalent to an increase in the cost of monitoring \( (b) \), possibly making monitoring prohibitively expensive. If monitoring is ruled out there is no value in control rights and, in equilibrium, both the loan sales and CDS markets will be active. Effectively, the barriers to monitoring make the CDS market more viable.

The optimal use of control rights naturally translates into default probabilities. Since the only cases in which both the CDS and loan sales markets are active are those in which there is no monitoring, it immediately follows that:

**Corollary 2** If both the CDS and loan sales market are active then the default probabilities are too high relative to the social optimum.

Thus, because CDS do not bundle control rights with the underlying cash flows, their use undermines optimal control. More generally, credit risk transfer often results in a loss of the originating bank’s private information, making monitoring decisions less efficient.
5.2 The microstructure of the CDS and loan markets

Our model makes stark predictions about when the CDS market will be viable, but given that there are portfolio reasons to trade in these instruments, the markets may be active even when banks do not trade. The existence of this “liquidity” or “noise trading” has an effect on how credit risk is priced in this market. The presence of “noise traders” allows banks to trade in CDS markets anonymously. This is because both CDS and loans are traded over the counter, and there is no centralized clearing or record of any economic agent’s full position. Further, through the use of intermediaries it would be very easy to disguise a CDS position. This is not true of loan sales, because due to the control rights bundled with the cash flows, a bank selling a loan cannot be anonymous to the counterparty.

To see this, consider any equilibrium in which the loan buyer monitors. In this case, the future success rate of the loan will be $p + \Delta$. However, it cannot be an equilibrium for the price in the CDS market to reflect the true default probability $p + \Delta$. If risk were priced this way, then any bank could lay off credit risk at this advantageous price and would have no incentive to sell the loan. Alternatively, the bank could retain the loan and monitor. Of course, if it were to do so and there was an active CDS market which priced the credit risk at the true default probability $p + \Delta$, it would be better off just trading in the CDS market. Of course, in this case, the loan is not monitored and $p + \Delta$ will not be the success probability of the project.

In sum, if the CDS market is active, then the price in the CDS market must be less than or equal to the price in the loan sales market. This is not consistent with zero profits for the CDS market makers as their contracts will pay off with the post monitoring probability $p + \Delta$. Therefore, if there is any monitoring of credit risk, prices in the CDS market must generate positive economic profits for the intermediaries. (Effectively, they capture the monitoring cost as profit.) The prices that the intermediaries charge act as a screening device and change both the actions of market participants and those who choose to enter the market.

**Proposition 7** Suppose that a CDS market is always active, then either intermediaries make positive profits or loan buyers do not monitor.

In as much as CDS are synthetic contracts, they should not affect the real economy. However, because these markets are essentially anonymous the existence of the CDS market prevents optimal monitoring.
5.3 Securitization and the retention of risk

In our model, banks do not contract on the amount of credit risk that they retain. However, the right to assign a loan is contractible. Therefore, a natural question to ask is if a bank or issuer would choose ex ante to commit to not selling the loan. (This is known as an anti-assignment clause.) In fact, a recent study by Pyles and Mullineaux (2008) finds that 63% of the loans in their sample of syndicated loans have clauses that require borrower approval before sales are allowed.

Such clauses have two effects. First, in the presence of a CDS market, a loan that cannot be sold (a binding anti-assignment clause) may be monitored less than a loan that permits sales. This is because the originating bank has the option of purchasing a CDS if its desire to lay off risk is sufficiently high. In this case, it is no longer exposed to the economic risk of the underlying loan and therefore has no incentive to monitor.

The second effect is that clauses restricting future sales are frequently not binding. Currently, under Article 9 of the Uniform Commercial Code, a bank may sell participation in a loan even though the underlying loan agreement has an anti-assignment clause. However, in the presence of an anti-assignment clause, the bank is not allowed to transfer collection rights (effectively monitoring rights) to the buyer. In sum, under current law, such clauses do not actually prevent a bank from selling a loan, but do prevent a bank from transferring the business relationship. In the context of our model, this suggests that a bank with a loan that has an anti-assignment clause has effectively committed that the purchaser will not monitor.

Economically, then, an anti-assignment clause followed by loan participation is equivalent to CDS in our model. To the extent anti-assignment clauses either lead a bank to use CDS or are themselves equivalent to CDS because they allow loan participation, their widespread use would have a critical impact on our results. Separating and pooling equilibria would still exist, but any transfer of credit risk would lead to no monitoring, decreasing welfare and increasing loan default rates and thus credit exposures. In fact, Pyles and Mullineaux (2008) find that loans with sales restrictions do pay higher spreads; although they attribute this to the resulting illiquidity of the loans, increased chance of default is another possible explanation.

5.4 The role of reputation

Central to the inefficiency of the credit default swap market is the idea that the originating bank is typically best informed about the benefit to monitoring but has no incentive to

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6A detailed description of this Article appears in Schwartz(1999).
monitor. Of course, this affects the price of credit risk in the market. Ex ante, it is efficient for a bank to be able to retain the loan and monitor when necessary yet lay off credit risk through CDS if it has a capital shock; ex post, this cannot be supported in a single-period setting.

It is reasonable to ask whether reputation concerns could lead a bank to monitor when it used CDS. We now explore this in an infinite horizon setting, where each period the (infinitely-lived) bank faces a new borrower and a new set of investors in the market. As we will see, such a setting may support behavior that is closer to the efficient outcome. This is generally more likely as the bank’s discount factor is larger, the borrowing firm’s base chance of default is smaller, and the impact of monitoring on firm default is higher. Nevertheless, there are cases where the repeated setting does not support better behavior. Also, the first-best is unlikely to be attainable in any case. Too many defaults signal market participants that the bank probably has not monitored, which is followed by a reversion to inefficient behavior. Since even a bank that monitors will face some defaults, some “false positives” are inevitable. This drives behavior away from the first-best even when the bank tries to fulfill its role as a monitor. Gopolan, Nanda, and Yerramilli (2007) find empirical evidence of similar behavior in the loan syndication market: after defaults on a lead loan arranger’s loans, the lead arranger loses loan volume and finds it more difficult to offload loans to other syndicate members.

Suppose that the bank exists for an infinite number of periods, and it discounts profits with a factor $\delta \in (0,1)$. Each period, the bank makes loans to a new borrower that only exists for a single period, and the bank may also engage in credit risk transfer with competitive investors that only live for one period. The question we now explore is whether a Bayesian perfect equilibrium where the bank makes use of CDS yet still monitors when appropriate exists.

If the bank is to “do the right thing” and monitor despite the presence of CDS, it must earn higher profits through such behavior than through using CDS and not monitoring (“shirking”). This requires that market participants have some means of assessing whether the bank has monitored, and some way of rewarding the bank for monitoring and “punishing” it for shirking.

We assume that a bank’s monitoring is not directly observable, but this is not critical for our results.\footnote{Even if monitoring could be observed, third parties might not know whether monitoring is useful or not (i.e., whether the borrower can engage in moral hazard or not). In this case, seeing no monitoring followed by a default is not a guarantee of shirking; instead, the bank might have genuinely seen that monitoring was not useful, and yet through bad luck the borrower subsequently defaulted anyway.} In this setting, only the bank knows whether it has actually monitored when appropriate in the past; investors and the current borrower only know whether the
bank’s past borrowers defaulted or not. Nevertheless, because the bank’s loans are more likely to default if the bank does not monitor, market participants can use past defaults as a noisy signal that the bank has not monitored.

This brings us to the question of how the bank is to be punished. To simplify matters, we assume that if market participants decide that the bank is probably shirking, they revert to single-period equilibrium beliefs and behavior, as does the bank; that is, they all follow a “trigger strategy” equilibrium of the sort first proposed by Green and Porter (1984) in the context of collusive oligopoly. For simplicity, we continue to assume that the bank has no market power in normal circumstances. Thus, the bank’s rents when it is punished for apparent shirking are 0 per period. For this to be punishment, it is critical that the bank earn some form of rents as long as it maintains a reputation for monitoring. We will assume that, if the bank maintains a reputation for monitoring, its borrowers will agree to a loan face value $R^\ell$ that gives the bank the additional surplus $S^*$ it generates vis-a-vis the single-period equilibrium.\footnote{One way of creating such an outcome would be if banks with reputations were relatively rare, whereas banks that behaved as single-period maximizers were common. In this case, single-period maximizing would only obtain zero rents, but a bank with a good reputation could extract rents.}

Note that

$$S^* = (p + \Delta)R^\ell + (1 - p - \Delta)C - (1 - \theta)b - 1 = \Omega^* - \Omega^i,j(i \in \{s, p\}, j \in \{m, n\}).$$

Assuming that the bank gets this entire surplus is the most favorable framework for giving the bank incentive to monitor when using CDS.

First, let us consider the case where a single default is used as a signal that the bank has shirked and is followed by reversion to single-period equilibrium behavior forever. Suppose that the bank has not yet suffered any defaults. Define $V_N$ as the expected present value of the bank’s future stream of profits in this case. Similarly, if the bank’s loan defaults, define $V_D$ as the expected present value of the bank’s future stream of profits. Since a default is followed by single-period Nash equilibria, and the bank’s rents in this case are assumed to be zero per period, we have $V_D = 0$.

Assuming that the bank does not shirk in the current period, we have the following equation for $V_N$:

$$V_N = S^* + (p + \Delta)\delta V_N + (1 - p - \Delta)\delta V_D$$
$$= S^* + (p + \Delta)\delta V_N.$$

Solving for $V_N$, we have

$$V_N = \frac{S^*}{1 - (p + \Delta)\delta}; \quad (7)$$
the bank discounts the stream of potential rents $S^*$ at an effective rate of $(p + \Delta)\delta$. Since the first-best value of rents would be $S^* \left[ \frac{1}{1-\delta} \right]$, there is a loss of efficiency even when this reputation equilibrium can be sustained.

Of course, this assumes that the bank does not find it worthwhile to shirk. If the bank shirks, it still gets $S^*$ in the current period (it locks these profits in with a CDS) and saves an additional $(1 - \theta)b$ by not monitoring. The downside to shirking is that the chance of default in the current period is higher, reducing the odds that the bank will collect rents in the future. Letting $V(\text{shirk})$ be the value of shirking given that there have not yet been any defaults, we have

$$V(\text{shirk}) = S^* + (1 - \theta)b + (p + \theta\Delta)\delta V_N.$$  \hfill (8)

Note that this calculation assumes that the bank’s future value if the current loan does not default is $V_N$, which has been derived under the assumption that the bank will not shirk. Continuing under this assumption, we have that shirking is not attractive if $V_N \geq V(\text{shirk})$, which is equivalent to

$$(1 - \theta)\Delta\delta V_N \geq (1 - \theta)b,$$  \hfill (9)

which is equivalent to

$$\frac{\Delta\delta S^*}{1 - (p + \Delta)\delta} \geq b.$$  \hfill (10)

From this expression, a number of implications follow. Holding the surplus level $S^*$ constant, the bank is more likely to monitor rather than shirk if the discount factor $\delta$ is higher, if the borrower’s base chance of success $p$ is higher, and if the increment $\Delta$ added by monitoring is higher. The first result is standard in such reputational models, but the others are novel. Intuitively, the bank is less likely to face defaults in the future if $p$ is higher; this reduces the chance of false signals of shirking, making monitoring more attractive. Higher $\Delta$ means that monitoring reduces the chance of default by more, increasing the likelihood of collecting future rents.

This analysis ignores changes in $S^*$. The next proposition shows what happens when the impact of parameter changes on $S^*$ is taken into account.

**Proposition 8** Consider the reputational equilibrium in the infinitely-repeated game where the first-best outcome is achieved so long as the bank has not suffered any defaults, but a single default leads to repetition of the optimal single-period equilibrium.

(i) An increase in the discount factor $\delta$ makes it more likely that the reputational equilibrium exists.

(ii) If the single-period equilibrium is pooling (with or without monitoring), an increase in the borrowing firm’s base probability of success $p$ makes it more likely that the reputational
equilibrium exists. If the single-period equilibrium is separating with monitoring, then if \( R^\ell - C = \frac{b}{\Delta} + \epsilon \) or \( \delta[2(p + \Delta) - \theta\beta] > 1 \), an increase in \( p \) makes it more likely that the reputational equilibrium exists. If the single-period equilibrium is separating with no monitoring, then if \( \delta[2p + (1 + \theta)\Delta - \theta\beta] > 1 \), an increase in \( p \) makes it more likely that the reputational equilibrium exists.

(iii) If the single-period equilibrium is pooling, an increase in the impact of monitoring \( \Delta \) makes it more likely that the reputational equilibrium exists. If the single-period equilibrium is separating with pooling, then if \( R^\ell - C = \frac{b}{\Delta} + \epsilon \) or \( (p - \theta\beta)(1 - p\delta) + \Delta^2\delta \geq 0 \), an increase in \( \Delta \) makes it more likely that the reputational equilibrium exists. If the single-period equilibrium is separating with no monitoring, then so long as \( (p - \theta\beta)(1 - p\delta) + \theta\Delta^2\delta \geq 0 \), an increase in \( \Delta \) makes it more likely that the reputational equilibrium exists.

Generally speaking, even when the endogeneity of \( S^* \) is taken into account, the results previously mentioned still hold: the reputational equilibrium is more likely to exist when \( \delta, p, \) or \( \Delta \) is higher. The exception occurs when the single-period equilibrium is separating and the lending face value just meets the bank’s break even constraint (i.e., \( R^\ell = R^\ell_{s,m} \) or \( R^\ell_{s,n} \)), because here \( S^* \) increases with \( R^\ell \), which is decreasing in \( p \) and \( \Delta \). If the credit quality \( p + \Delta \) of the borrower is sufficiently low and \( \theta\beta \) sufficiently high, this can offset the impact on the bank’s effective discount rate of increasing \( p \) and \( \Delta \). However, low \( p + \Delta \) or high \( \theta\beta \) make it more likely that the equilibrium is pooling with monitoring, in which case these caveats do not apply.

One can argue that punishing the bank forever after a single default is fairly draconian. Other reputational equilibria are in fact possible. For example, the market might punish the bank after a default for some finite number of periods \( n \), after which the bank’s reputation is “cleansed” and behavior can revert to the first-best. Since such an equilibrium will attain first-best behavior more often, it will clearly be attractive if it exists. The difficulty is that incentive compatibility is harder to meet for such an equilibrium: by making punishment less severe (loss of rent for \( n \) periods instead of forever), the bank’s incentives to shirk are enhanced. This suggests that, if the bank’s relative rents for monitoring (\( S^* \)) are lower, the punishment period will need to increase so as to maintain incentives.

Another possible objection is that if the borrower’s underlying risk of default is fairly high (\( p \) is low), a single default has a high chance of happening even if the bank does monitor when appropriate. We have explored the case where the bank maintains a reputation for monitoring unless it experiences two defaults in a row, after which market behavior reverts to the single-period equilibrium forever. In the interests of space, we do not present the full results here, but instead offer an overview.

Suppose that this new reputational equilibrium exists. It is easy to show that this is a
more efficient equilibrium than that where a single default leads the bank to lose its reputation forever; intuitively, one default through bad luck is less likely to lead to the inefficient single-period equilibrium in the future. Conditions for incentive compatibility are more complex, however. There are separate conditions governing the bank’s incentives depending on whether it has just experienced one or no borrower defaults. Incentive compatibility is more binding in the case where the bank has not yet experienced a default. Intuitively, since a first default does not lead to an immediate loss of reputation, the bank has less to lose by shirking when it has had no defaults; by contrast, when it has just had one default, shirking increases the chance of another default and consequent loss of reputation. This suggests that, in a more complete model, incentives to maintain a reputation for monitoring credit exposures may be weakest just when performance seems to be highest.

Regardless of whether the reputational equilibrium penalizes one or more defaults, reputation in this setting will be unable to attain the first-best outcome (monitoring by the bank in every period). The reason is that punishment follows too many defaults, and there is always a chance that defaults are generated through bad luck rather than through negligence; thus, large numbers of defaults will be followed by inefficient behavior during the punishment phase. The intuitive idea of extending the time segment during which signals (defaults) are accumulated to get a clearer sense of the bank’s behavior is misleading; here, the bank chooses each period whether to monitor or not, and extending the time over which signals are accumulated makes it easier for the bank to shirk during this time.

6 Conclusion

We have presented a simple model in which a bank with a loan and private information about the loan’s default probability chooses between laying off risk synthetically through a CDS or by selling the loan. In a single-period setting, when the firm’s credit risk innate is high, the only equilibrium has the bank use loan sales to offload credit risk, regardless of its information about the borrowing firm; risk transfer is efficient but monitoring is excessive. When the firm’s innate credit risk is lower, however, other equilibria may be possible. If the capital cost of retaining credit risk is sufficiently high, there is an equilibrium in which both loan sales and CDS coexist, risk transfer is efficient, and monitoring is too low. If the capital cost of retaining credit risk is low, there are equilibria in which only loans in need of monitoring are layed off, inefficiently reducing risk transfer; in some cases, loan sales and CDS coexist and monitoring is too low, while in other cases, only loan sales are used and monitoring is efficient.

We have also analyzed how our results change in a repeated setting. In such a setting,
CDS and monitoring may coexist for some period of time, if bank required returns are not too high ($\delta$ is high) and the probability of the loan’s default is not too high ($p$ and $\Delta$ are high). Nevertheless, the chance of monitored loans defaulting means that it will generally be impossible to support the first-best outcome for an unlimited period of time. A period of high defaults will be followed with a collapse in the bank’s reputation for monitoring and reversion to an inefficient equilibrium outcome.

Our discussion of securitization and the retention of risk suggests some further caveats. To the extent that anti-assignment clauses are common, banks may simply resort to CDS or use loan participations to reduce risk. Since these leave borrower control rights in the hands of the bank, efficient monitoring is totally undercut, unless the bank can maintain a reputation for monitoring. As we have just said, monitoring incentives will be easiest to maintain if the loan’s credit risk is relatively low; for weaker credits, such clauses may prevent useful monitoring by loan buyers in return for no monitoring at all by the originating bank. Similarly, if some of the sellers of credit risk in the markets are not originating banks, their activity makes it easier for the originating bank to hide its trades, especially in the CDS market. This enhances activity in CDS, again undermining monitoring incentives. Overall, our model suggests that activity in CDS markets will be more likely to undermine monitoring than activity in loan sales, especially for weaker credits.

The focus of this paper is on how the use of control rights is affected by the markets in which those rights are traded and the cash flows with which they are bundled. While this is not an investigation of optimal security design, it does suggest that contractual rights can affect the underlying value of cash flows. This has broad implications for valuation. A risky dollar traded in one market is not equivalent to the same dollar of credit risk traded in another market.
7 Proofs

Proof of Lemma 1

(i) If a bank buys CDS then it receives $R^\ell - C$ if the borrower defaults and 0 otherwise. Therefore, the bank’s payoff is $R^\ell$, independent of the default probability. If the bank monitors, it still receives $R^\ell$ but also incurs a cost $b > 0$. Thus, it will not monitor.

(ii) If the bank does not have protection it monitors if and only if

$$C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C).$$

The result follows.

(iii) A loan buyer monitors if and only if

$$C + (p + \Delta)(R^\ell - C) - b > C + (p + \phi^{LS} \Delta)(R^\ell - C).$$

Again, the result follows.

Proof of Lemma 2

Results (i) and (ii) follow from the net value of a loan to investors depending on whether or not it is monitored. Result (iii) follows from the fact that a loan that has a CDS outstanding will not be monitored.

Proof of Proposition 1

Since monitoring will not take place and thus loan sales and CDS are equivalent if market beliefs are the same, we follow our assumption and assume that $p^{CDS} = p^{LS}$.

(i) In a complete pooling equilibrium, the market’s probability that the firm will succeed is the expectation across types $p + \Delta$ and $p$. This yields $p^{CDS} = p^{LS} = p + \theta \Delta$. The resale price of the loan follows immediately. For this to be an equilibrium, the two types of bank must prefer to shed credit risk rather than hold the loan.

Type $p + \Delta$ sells the loan if

$$C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C) < C + p^{LS}(R^\ell - C)$$

$$\implies \beta > (1 - \theta)\Delta.$$

Type $(p, \beta)$ sells the loan if $C + p(R^\ell - C) - \beta(R^\ell - C) < C + p^{LS}(R^\ell - C)$, which holds because $p^{LS} > p$.

(ii) In a no pooling equilibrium, only type $p$ sheds credit risk, so $p^{CDS} = p^{LS} = p$. Again, the resale price of the loan follows immediately. For this to be an equilibrium, we
need two conditions. First, type \( p + \Delta \) holds the loan, so

\[
C + (p + \Delta)(R^e - C) - \beta(R^e - C) \geq C + p^{LS}(R^e - C)
\]

\[
\implies \beta \leq \Delta.
\]

Second, type \( (p, \beta) \) sells the loan, so

\[
C + p(R^e - C) - \beta(R^e - C) < C + p^{LS}(R^e - C),
\]

which holds because \( p^{LS} = p \).

Proof of Proposition 2

There are four possibilities to consider. First, the banks either both take the same action or one retains risk and the other lays it off. In the second case, type \( p + \Delta \) retains the risk (it has the least incentive to shed risk at any price). In addition, buyers in the loan sales market either monitor or do not. Because \( R^e - C > \frac{b}{\Delta} \), if the originating bank is type \( p \) and it retains the loan, then it will monitor.

From Lemma 1, if \( R^e - C < \frac{b}{\Delta(1 - \phi^{LS})} \), then the loan buyer will not monitor the loan. Notice, that if only \( p \) is shedding risk, then the loan buyer will monitor the loan. This rules out the equilibrium where type \( p \) lays off credit risk, \( p + \Delta \) does not, and loans are not monitored; in this case, \( p^{LS} = p \), so \( \phi^{LS} = 0 \), and loan buyers monitor because \( R^e - C > \frac{b}{\Delta} \).

This leaves three potential equilibria.

(i) Both types shed credit risk and loan buyers do not monitor. In this case, loan sales and CDS are equivalent and\( p^{CDS} = p^{LS} = p + \theta\Delta \), so the condition that the loan buyers do not monitor becomes \( R^e - C \leq \frac{b}{(1 - \theta)\Delta} \). The price of a sold loan is \( C + (p + \theta\Delta)(R^e - C) \).

Type \( p + \Delta \) sheds risk if and only if

\[
C + (p + \Delta)(R^e - C) - \beta(R^e - C) < C + p^{LS}(R^e - C)
\]

\[
\implies \beta > (1 - \theta)\Delta.
\]

Type \( p \) sells the loan rather than holding and monitoring it if and only if

\[
C + (p + \Delta)(R^e - C) - \beta(R^e - C) - b < C + p^{LS}(R^e - C)
\]

\[
\implies \frac{b}{R^e - C} + \beta > (1 - \theta)\Delta
\]

which must hold if \( \beta > (1 - \theta)\Delta \). Thus, the two necessary and sufficient conditions for this equilibrium are \( R^e - C \leq \frac{b}{\Delta(1 - \theta)} \) and \( \beta > (1 - \theta)\Delta \).

(ii) Both types shed credit risk and loan buyers monitor. Thus, \( p^{LS} = p + \theta\Delta \). The condition that buyers monitor becomes \( R^e - C > \frac{b}{(1 - \theta)\Delta} \). The price of a sold loan is \( C + (p + \Delta)(R^e - C) - b \).
Type $p + \Delta$ prefers to sell the loan rather than holding it if

$$C + (p + \Delta)(R^\ell - C) - b > C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C)$$

$$\iff R^\ell - C > \frac{b}{\beta}$$

Type $p$ sells the loan if $C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C) - \beta(R^\ell - C)$, which is always true because $\Delta(R^\ell - C) > b$.

Thus we require that $R^\ell - C > \frac{b}{(1 - \phi)^\Theta \Delta}$ and $R^\ell - C > \frac{b}{\Delta}$.

A further requirement of equilibrium is that $p^{CDS}$ must be such that the banks prefer selling the loan to hedging it with CDS:

$$C + (p + \Delta)(R^\ell - C) - b \geq C + p^{CDS}(R^\ell - C),$$

which is equivalent to $p^{CDS} \leq p + \Delta - \frac{b}{R^\ell - C}$. Since $R^\ell - C > \frac{b}{(1 - \theta)^\Theta \Delta}$, the bound on $p^{CDS}$ is greater than $p + \theta \Delta$, so setting $p^{CDS} = p + \theta \Delta$ meets this condition.

(iii) Suppose that only $p$ sheds credit risk, so $p^{CDS} = p^{LS} = p$. In this case, $\frac{b}{(1 - \phi)^\Theta \Delta} = \frac{b}{\Delta} < R^\ell - C$, so loan buyers will monitor the loan. Since loan buyers monitor, the resale price of the loan is $C + (p + \Delta)(R^\ell - C) - b$.

Type $p + \Delta$ prefers to hold the loan rather than sell it if

$$C + (p + \Delta)(R^\ell - C) - b \leq C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C)$$

$$\iff R^\ell - C \leq \frac{b}{\beta}$$

Type $p$ prefers to sell the loan if $C + (p + \Delta)(R^\ell - C) - b > C + p(R^\ell - C) - \beta(R^\ell - C)$; this always holds since $\Delta(R^\ell - C) > b$.

Thus, for $R^\ell - C \leq \frac{b}{\beta}$, only $p$ sheds credit risk.

Again, we need to verify that banks prefer loan sales to CDS in this case. The condition is as in (ii): $p^{CDS} \leq p + \Delta - \frac{b}{R^\ell - C}$. Since $R^\ell - C > \frac{b}{\Delta}$, the bound on $p^{CDS}$ is greater than $p$, so setting $p^{CDS} = p$ meets this condition.

Proof of Proposition 3

Given the results of Propositions 1 and 2, just sum the net welfare (relative to the first-best) under each bank type weighted by their probabilities. For example, in the case of the separating equilibrium when loan buyers do not monitor, type $p + \Delta$ does not lay off its credit risk, causing a loss of $\beta(R^\ell - C)$ with probability $\theta$; also, monitoring does not take place for type $p$, causing a loss of $\Delta(R - C) - (B + b)$ with probability $1 - \theta$.

Proof of Lemma 3
(i) The proof follows by establishing the participation constraint of the bank in each possible equilibrium. We denote the profit of type $p$ bank by $\pi(p)$ with an equivalent expression for type $p + \Delta$.

(a) If the equilibrium is pooling with no monitoring, then

$$\pi(p) = C + (R^\ell - C)p$$
$$\pi(p + \Delta) = C + (R^\ell - C)(p + \Delta)$$

The bank is of type $p$ with probability $1 - \theta$ and type $p + \Delta$ with probability $\theta$. Therefore, we obtain the bank’s expected value, which must exceed 1:

$$C + (p + \theta \Delta)(R^\ell - C) \geq 1$$

The expression for $R^\ell_{p,n}$ follows easily.

(b) If the equilibrium is pooling with monitoring, then

$$\pi(p) = C + (p + \Delta)(R^\ell - C) - b$$
$$\pi(p + \Delta) = C + (p + \Delta)(R^\ell - C) - b,$$

and weighting by the appropriate probabilities and rearranging yields the expression for $R^\ell_{p,m}$.

(c) If the equilibrium is separating with no monitoring, then

$$\pi(p) = C + (p)(R^\ell - C)$$
$$\pi(p + \Delta) = C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C),$$

which yields the expression for $R^\ell_{s,n}$.

(d) If the equilibrium is separating with monitoring, then

$$\pi(p) = C + (p + \Delta)(R^\ell - C) - b$$
$$\pi(p + \Delta) = C + (p + \Delta)(R^\ell - C) - \beta(R^\ell - C),$$

which yields the expression for $R^\ell_{s,m}$. 

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(ii) The results for $C$, $p$, and $\Delta$ are obvious, as are those for $b$ and $\beta$. It is clear that $R^{\ell}_{p,n}$ is decreasing in $\theta$. So is $R^{\ell}_{s,n}$, since $\Delta > \beta$ when that equilibrium exists. $R^{\ell}_{p,m}$ is unaffected by $\theta$. Finally, if the equilibrium is separating with monitoring, we have

$$\frac{\partial R^{\ell}_{s,m}}{\partial \theta} = (p + \Delta - \theta \beta)^{-2} [(p + \Delta - \theta \beta)(-b) - (1 - C + (1 - \theta)b)(-\beta)]$$

But we also have $R^{\ell}_{s,m} - C \leq \frac{b}{\beta}$ if the equilibrium is to exist, so

$$\frac{1 - C + (1 - \theta)b}{p + \Delta - \theta \beta} \leq \frac{b}{\beta} \Rightarrow (1 - C)\beta \leq (p + \Delta - \beta)b.$$

It follows that $R^{\ell}_{s,m}$ is decreasing in $\theta$. ■

Proof of Lemma 4

(i) $R^{\ell}_{p,n} - C > R^{\ell}_{p,m} - C$ is the same as $\frac{1 - C}{p + \theta \Delta} > \frac{1 - C + b}{p + \Delta}$. Multiplying both sides by the product of the denominators and rearranging, we have $(1 - C)(1 - \theta)\Delta > b(p + \theta \Delta)$, which is the same as $\frac{1 - C}{p + \theta \Delta} = R^{\ell}_{p,n} - C > \frac{b}{(1 - \theta)\Delta}$. Also, $(1 - C)(1 - \theta)\Delta > b(p + \theta \Delta)$ is the same as $(1 - C)(1 - \theta)\Delta + b(1 - \theta)\Delta > b(p + \Delta)$, which is the same as $\frac{1 - C + b}{p + \Delta} = R^{\ell}_{p,m} - C > \frac{b}{(1 - \theta)\Delta}$.

(ii) $R^{\ell}_{s,n} - C > R^{\ell}_{s,m} - C$ is the same as $\frac{1 - C}{p + \theta \Delta - \theta \beta} > \frac{1 - C + (1 - \theta)b}{p + \Delta - \theta \beta}$. This can be rearranged to yield $(1 - C)(1 - \theta)\Delta > (1 - \theta)b(p + \theta \Delta - \theta \beta)$, which then yields $\frac{1 - C}{p + \theta \Delta - \theta \beta} = R^{\ell}_{s,n} - C > \frac{b}{\Delta}$. It also yields $(1 - C)\Delta > b(p + \theta \Delta - \theta \beta)$; adding $(1 - \theta)\Delta b$ to both sides yields $(1 - C)\Delta + (1 - \theta)\Delta b > b(p + \Delta - \theta \beta)$, which is the same as $\frac{1 - C + (1 - \theta)b}{p + \Delta - \theta \beta} = R^{\ell}_{s,m} - C > \frac{b}{\Delta}$.

(iii) Analysis similar to that in (iv) shows that all of these conditions are equivalent to $\frac{1 - C}{p + \Delta - \theta \beta} > \frac{b}{\beta}$. ■

Proof of Proposition 4

That only pooling equilibria are feasible follows from Proposition 1(ii) and Proposition 2(iii).

(i) In this case, it is clear that equilibrium $(p, n)$ is feasible. Any loan face value $R^{\ell}$ such that $R^{\ell} - C > \frac{b}{(1 - \theta)\Delta}$ implements equilibrium $(p, m)$. Thus, the entrepreneur chooses between these two. From Proposition 3, $\Omega^{p,n} > \Omega^{p,m}$ if and only if $-(1 - \theta)[\Delta(R - C) - (b + B)] > -\theta b$. Rearranging yields the condition in the text.

(ii) In this case, equilibrium $(p, n)$ is not feasible. The comparative statics results follow from Proposition 3. ■
Proof of Proposition 5

Because the \((p, n)\) equilibrium requires \((1 - \theta)\Delta < \beta\), it is clearly not feasible in this case.

(i) Proposition 3(iv) implies that \(R_{s,n}^\ell - C < \frac{b}{\Delta}\), so \((s, n)\) is feasible and can be implemented by \(R^\ell = R_{s,n}^\ell\). Also, a loan face value of \(R^\ell - C = \frac{b}{\Delta} + \epsilon\) with \(\epsilon\) small feasibly implements \((s, m)\) (note that this rules out \((s, n)\), and \(\frac{b}{\Delta} < R^\ell - C < \frac{b}{(1 - \theta)\Delta} < \frac{b}{\beta}\), ruling out \((p, m)\). This loan face value also implies \(\Omega^{p,m} = \Omega^* - \theta b < \Omega^* - \theta \beta (R^\ell - C) = \Omega^{s,m}\), since \(R^\ell - C < \frac{b}{\beta}\). Thus \((p, m)\) will never be chosen in this case.

The entrepreneur chooses between \((s, n)\) and \((s, m)\). Note that, for each equilibrium, social welfare is maximized by choosing a loan face value that is as low as feasible, so the previously specified values are the best choices. \(^9\) \(\Omega^{s,m} > \Omega^{p,m}\) if and only if \(- (1 - \theta)[\Delta(R - C) - (b + B)] - \theta \beta (R_{s,n}^\ell - C) > - \theta \beta (\frac{b}{\Delta} + \epsilon - C)\). This can be rearranged to yield \((1 - \theta + \frac{\beta}{\Delta})b - \theta \beta (R_{s,n}^\ell - C) + \theta \beta \epsilon > (1 - \theta)\Delta(R - C - B)\); letting \(\epsilon\) go to zero yields the inequality in the proposition.

(ii) In this case, from Proposition 3(iv), \(R_{s,n}^\ell - C > R_{s,m}^\ell - C > \frac{b}{\Delta}\), so \((s, n)\) is not feasible and \(R_{s,m}^\ell\) is the smallest face value implementing \((s, m)\). The same steps as in the proof of (i) above show that \(\Omega^{p,m} < \Omega^{s,m}\), so the entrepreneur chooses \((s, m)\) and maximizes welfare by choosing loan face value \(R_{s,m}^\ell\).

(iii) In this case, \((s, n)\) and \((s, m)\) are not feasible. Proposition 3(v) shows that \(R_{p,m}^\ell - C > \frac{b}{\beta}\) and \(\frac{b}{\beta} > \frac{b}{(1 - \theta)\Delta}\), so a loan face value of \(R_{p,m}^\ell\) implements \((p, m)\).

\[\square\]

Proof of Proposition 6

(i) \(R_{p,m}^\ell - C \leq \frac{b}{\beta} < \frac{b}{(1 - \theta)\Delta}\), so by Proposition 3(iii) and (v), \(R_{p,n}^\ell - C \leq \frac{b}{(1 - \theta)\Delta}\) and \(R_{s,m}^\ell - C \leq \frac{b}{\beta}\). As shown in the proof of Proposition 3(v), \(R_{s,m}^\ell - C \leq \frac{b}{\beta}\) if and only if \(\frac{1 - C}{p + \Delta - \beta} \leq \frac{b}{\beta}\), so we have \(R_{p,n}^\ell - C = \frac{1 - C}{p + \theta \Delta} = \frac{1 - C}{p + \Delta - (1 - \theta)\Delta} < \frac{1 - C}{p + \Delta - \beta} \leq \frac{b}{\beta}\). Thus, any \(R^\ell - C \in \left(\frac{b}{\beta}, \frac{b}{(1 - \theta)\Delta}\right]\) implements \((p, n)\) uniquely and feasibly.

Moreover, from Propositions 1 and 2, any loan face value such that \(R^\ell - C \leq \frac{b}{\beta}\) supports one of the separating equilibria \(((s, n)\) if \(R^\ell - C \leq \frac{b}{\Delta}\) and \((s, m)\) otherwise) and also supports \((p, n)\).

(a) Suppose that \(R^\ell - C \leq \frac{b}{\beta}\) leads to the relevant separating equilibrium. It is easy to show that \(\Omega^{p,n} > \Omega^{s,n}\), and \(\Omega^{s,m} > \Omega^{p,m}\), as shown in Proposition 5(i). So the entrepreneur chooses between \((s, m)\) with \(R^\ell\) as small as possible, i.e., \(R^\ell - C = \max\{R_{s,m}^\ell - C, \frac{b}{\Delta} + \epsilon\}\), and implementing \((p, n)\) with any \(R^\ell - C \in \left(\frac{b}{\beta}, \frac{b}{(1 - \theta)\Delta}\right]\). Substituting in, we have \(\Omega^{p,n} > \Omega^{s,m}\) if and only if \(\theta \beta \max\{R_{s,m}^\ell - C, \frac{b}{\Delta} + \epsilon\} > (1 - \theta)\Delta(R^\ell - C - (b + B))\); rearranging

\[\text{Technically, there is no optimal face value for } (s, m), \text{ since one can always choose a smaller } \epsilon > 0 \text{ that implements this equilibrium. Nevertheless, our analysis shows when some such equilibrium will be chosen and when it won’t.}\]
and letting $\epsilon$ go to 0 yields the condition in the proposition.

(b) If $R^\ell - C \leq \frac{b}{\beta}$ leads to equilibrium $(p, n)$, the separating equilibria cannot be implemented. The entrepreneur chooses between the two pooling equilibria as in Proposition 4(i), leading to the condition given in this proposition.

(ii) $b \beta < R^\ell - C \leq \frac{b}{\beta}$ implies that, by Proposition 3(iii)-(v), $R^\ell_{p,n} - C \leq \frac{b}{\beta}$ and $R^\ell_{s,m} - C > \frac{b}{\beta}$, so $R^\ell_{s,n} - C > \frac{b}{\beta}$ too. Thus neither $(s, n)$ nor $(s, m)$ can be feasibly implemented, but $(p, n)$ is feasible and can be uniquely implemented through $R^\ell - C \in (\max\{R^\ell_{p,n} - C, \frac{b}{\beta}\}, \frac{b}{1-\theta/\Delta}]$, and $(p, m)$ is also feasible and can be implemented through any $R^\ell - C > \frac{b}{\beta}$. The rest of the analysis is as in the proof of (i.b) given above.

(iii) $R^\ell_{p,m} - C > \frac{b}{1-\theta/\Delta}$ implies that $R^\ell_{p,n} - C > \frac{b}{1-\theta/\Delta}$ and (as in the proof of (ii)) $R^\ell_{s,m} - C, R^\ell_{s,n} - C > \frac{b}{\Delta}$. Thus, the only feasible equilibrium is $(p, m)$, which can be implemented by loan face value $R^\ell_{p,m}$.

Proof of Proposition 7

The proof follows from the arguments presented in the text.

Suppose that loan buyers do not monitor. In this case, their valuation of the loan depends on how the banks lay off credit risk. As both markets (CDS and loan sales) are open, then no inferences can be made about the quality of the credit risk, and the posterior value conditional on observing a loan sale is $p + \theta \Delta$. Therefore, loan buyers do not monitor if $(p + \theta \Delta)(R^\ell - C) > (p + \Delta)(R^\ell - C) - b$. Or, $(R^\ell - C) < \frac{b}{\Delta(1-\theta)}$.

Suppose that the loan buyers monitor. Then, a $p + \Delta$ bank sells the loan if $C + (p + \Delta + \beta)(R^\ell - C) - b > C + (p + \Delta)(R^\ell - C)$, or $R^\ell - C > \frac{b}{\beta}$. A $p$ type bank always sells the loan. Therefore, if loan buyers monitor, the default probability of the underlying is $p + \Delta$, and a zero profit market maker would insure default using this probability. However, if he cannot distinguish between the originating bank and noise traders, then both banks have a strict incentive to sell the risk in the CDS market, which ensures that it will not be monitored and therefore yields a default probability of $p$ which generates a loss for the Market Maker.

However a price of credit risk that makes the $p$ type bank indifferent between the loan sales market and the CDS market is consistent with equilibrium. Specifically, consider a $p < p^* < p + \Delta$, so that $(p + \Delta - p^*)(R^\ell - C) \geq b$. This generates a positive profit for the MM, and the $p$ types trader strictly prefers to sell his loan in which case it is monitored.

Proof of Proposition 8

(i) Using Proposition 3, it is easy to see that, for any single-period equilibrium, the surplus $S^* = \Omega^* - \Omega^j$ is not affected by $\delta$, so the left-hand side of condition (10) is increasing in $\delta$.

(ii) For a pooling equilibrium, $S^*$ does not depend on $p$, so the left-hand side of condition
(10) is increasing in $p$.

If the single-period equilibrium is separating with monitoring, we have $S^* = \theta \beta (R^\ell - C)$, where $R^\ell - C = \max \left\{ \frac{1 - C + (1 - \theta) h}{p + \Delta - \theta \beta} \cdot b \right\}$ is the credit exposure that would be chosen in the single-period equilibrium. It follows that condition (10) becomes

$$\frac{\delta \Delta}{1 - (p + \Delta) \delta} \theta \beta (R^\ell - C) \geq b. \tag{11}$$

If $R^\ell - C = \frac{b}{\Delta} + \epsilon$, the left-hand side of condition (11) is increasing in $p$. Otherwise, the $R^\ell = R^\ell_{s,m}$, and the derivative of the left-hand side with respect to $p$ equals

$$\frac{\delta \Delta}{1 - (p + \Delta) \delta} \theta \beta (R^\ell_{s,m} - C) \left[ \frac{\delta}{1 - (p + \Delta) \delta} - \frac{1}{p + \Delta - \theta \beta} \right]. \tag{12}$$

It can then be shown that the sign of the term in brackets equals that of $\delta [2(p + \Delta) - \theta \beta] - 1$, so the left-hand side of (11) is increasing in $p$ if $\delta [2(p + \Delta) - \theta \beta] > 1$.

If the single-period equilibrium is separating with no monitoring, $S^* = (1 - \theta) [\Delta (R - C) - (B + b)] + \theta \beta (R^\ell_{s,n} - C)$. It follows that the derivative of the left-hand side of condition (10) with respect to $p$ is greater than

$$\frac{\delta \Delta}{1 - (p + \Delta) \delta} \theta \beta (R^\ell - C) \left[ \frac{\delta}{1 - (p + \Delta) \delta} - \frac{1}{p + \theta \Delta - \theta \beta} \right]. \tag{13}$$

The sign of the term in brackets equals that of $\delta [2p + (1 + \theta) \Delta - \theta \beta] - 1$, which leads to the condition in the proposition.

(iii) For a pooling equilibrium, $S^*$ is weakly increasing in $\Delta$, so the left-hand side of condition (10) is increasing in $\Delta$.

Suppose the single-period equilibrium is separating with monitoring, so $S^* = \theta \beta (R^\ell - C)$. If $R^\ell - C = \frac{b}{\Delta} + \epsilon$, the left-hand side of condition (10) equals $\frac{\delta b}{1 - (p + \Delta) \delta} \theta \beta$, which is increasing in $\Delta$. If $R^\ell = R^\ell_{s,m}$, the derivative of the left-hand side of condition (10) with respect to $\Delta$ equals

$$\frac{\delta}{1 - (p + \Delta) \delta} \theta \beta (R^\ell - C) \left[ 1 + \frac{\Delta \delta}{1 - (p + \Delta) \delta} - \frac{\Delta}{p + \Delta - \theta \beta} \right]. \tag{14}$$

The sign of the term in brackets equals that of $(p - \theta \beta)(1 - p \delta) + \Delta^2 \delta$.

If the single-period equilibrium is no pooling with no monitoring, then, similar to part (ii), the derivative of the left-hand side of condition (10) with respect to $\Delta$ is greater than

$$\frac{\delta}{1 - (p + \Delta) \delta} \theta \beta (R^\ell - C) \left[ 1 + \frac{\Delta \delta}{1 - (p + \Delta) \delta} - \frac{\theta \Delta}{p + \theta \Delta - \theta \beta} \right]. \tag{15}$$

The sign of the term in brackets equals that of $(p - \theta \beta)(1 - p \delta) + \theta \Delta^2 \delta$. ■
References


