Is the Electronic Open Limit Order Book Inevitable?

- Prices are continuous
- Market order traders face a marginal price schedule $R'(q)$.
- Traders have a valuation for trade $W(-R(q), q, \omega)$.
- $R(q)$ is the cost of trading $q$ shares, $\omega$ is a private type.
- Strictly increasing in cash position, quasiconcave in $(c, q)$
Example:

- utility defined over consumption $c^*$

$$W(c, q; \omega) = U_o(c^*(c, q, \omega)) + EU(Y_T + (\mu + q)x_T - TC(\mu + q) + (\phi + c - c^*(c, q, \omega))(1 + r_T); Y_T, x_T | S)$$

- MU of cash positive rules out?
• Define the MRS between Cash and shares:

\[ M(q, R(q), \omega) = \frac{W_2(-R(q), q, \omega)}{W_1(-R(q), q, \omega)} \]

• Optimal solution to the investors’ problem characterized by first order condition.

• Barring discontinuities, \( M(q, R(q), \omega) = R'(q) \), gives the optimal trade \( D_{R}(\omega) \).
The limit order trader’s problem

• $X$ is the full liquidation value of the security

• Define:

\[ V(m, q, R) = E[X \mid M(q, R; \omega) \geq m] \]
\[ v(m, q, R) = E[X \mid M(q, R; \omega) \leq m] \]

• $V(m, q, R) \geq E[X \mid M(q, R, \omega) = m] \geq v(m, q, R)$.

• Like affiliation, 2 motives for trade.
Example.

- Market order trader maximizes $-E\exp(-\gamma \text{Wealth})$.
- Endowment $\omega$ drawn from $N$ with mean 0.
- Gets a signal $S = X + \epsilon$, where $X$ and $\epsilon$ are normally distributed.
- Suppose that $\sigma$ is std. deviation of $X$ conditional on $S$, then
  \[ M(q, R; \omega) = E[X \mid S] - \gamma \omega \sigma^2 - rq\sigma^2 \]
- Suppose that $X$ has mean zero, and $r\sigma^2 = 1$.
- Suppose the variance of $\omega = \alpha < 1$, and var of $E[X \mid S] = 1 - \alpha$
- $M(q, R, \omega) = \omega - q$
- $\omega = E[X \mid S] - r\sigma^2 \omega$
• Limit Order Traders maximize $E[P - XQ]$, $P$ is the limit order trader’s proceeds, and $Q$ is the amount that they supply.

• ORDERS ARE EXECUTED PRO RATA

• Zero profit condition at some price:

$$p_i - V(p_i, d, R_{i-1} + p_i(dAQ_{i-1}))$$
$$\Pr(M(d, R_{i-1} + p_i(d - AQ_{i-1})) \geq p_i) = 0$$

• A few results flow from the idea of the upper tail expectation. Positive B/A spread.

• Break even implies that small and large traders are treated differently.

• Relationship to winner’s curse.
Competition

• Because the limit order book makes zero profits, it does at least as well as any other market form.

• in a Nash game, no one would enter

• Also have equilibria in which another exchange enters and the market is split. But, the equilibrium liquidity supply schedule is split.

• Result strongly depends on the pro rata rationing and zero profits.
What did we learn?

• Upper Tail/Lower tail expectations are a powerful idea
• Few people to put the trading question into an asset pricing context.
• Is it reasonable to think that the LO markets are always in equilibrium?
• What about Patrik Sandas’ work?
• What is the difference between dynamic and static markets?