Centralized versus Decentralized Delegated Portfolio Management under Moral Hazard

Job Market Paper

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Abstract

If an investor wants to invest into two asset classes, should he delegate to a single portfolio manager to manage both asset classes (centralized delegation)? Or should he delegate to two managers, each of whom exclusively manages one asset class (decentralized delegation)? Optimal risk sharing and portfolio choice discretion delineate the difference between centralization versus decentralization. Asset classes whose returns are negatively correlated and have high volatilities will favor centralization. But if the two asset classes have very different mean returns, this disfavors centralization: the single manager may disregard portfolios implementing the investor’s desired investments and prefer portfolios in alternative investments. Thus, the investor must pay the single manager high performance fees to disincentivize deviation. Decentralization eliminates this necessity because one manager cannot trade another manager’s asset class, and the investor contracts with each manager individually. But in decentralization, it may be impossible to implement the investor’s desired investments because managers deviate without considering the correlation between the managers’ returns. This last problem can be resolved in a dynamic setting, in which the investor’s wealth “intertemporally glues” together the managers’ wealths to provide the correct incentives.

Keywords: centralized delegated portfolio management, decentralized delegated portfolio management, portfolio choice, optimal contracting, dynamic mean-variance, copula

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1 Introduction

Delegated portfolio management is a core activity in the modern financial markets — but what is the optimal form of delegation? If an investor wants to access, say, an Asian macro strategy and an European macro strategy, should the investor delegate the execution of these two strategies to a single global strategy manager (*centralized delegation*)? Or should be delegate separately to an Asian strategy manager and also an European strategy manager (*decentralized delegation*)? Moreover, suppose there is moral hazard risk that instead of delivering the advertised Asian macro strategy, managers could privately deviate to a passive Asian equity index that the investor could have accessed without delegation. And suppose analogous moral hazard risks could occur in the European asset class. In the presence of such moral hazard problem, which form of delegation is better: centralized delegation or decentralized delegation?

William F. Sharpe was the first to coin the term “decentralized investment management” in his Presidential Address to the *American Finance Association* 1981 Annual Meeting. Furthermore, Sharpe concludes with:

> There is, of course, much more to this problem [of decentralized investment management]. We have assumed away many important aspects of the principal-agent relationships(s). . . . In short, we have clearly provided necessary and sufficient conditions for the traditional final sentence in such a paper: More research on this subject is needed. (Sharpe (1981, page 233))

Clearly the general literature in principal-agent theory, and also to its specific applications to delegated portfolio management, has significantly advanced in the years since 1981. Yet to the best my knowledge, the problem of understanding the similarities and differences between centralized versus decentralized delegation with the presence of moral hazard remains unexplored, and its solution properties remain elusive. In particular, substantial recent empirical evidence (see literature review in Section 2 below) suggests that moral hazard risks are strongly present in hedge funds, via the forms of fraud, operational risk, misrepresentation of investment strategies and conflicting evidence of managerial effort in generating alpha. Thus, the key contribution of this paper is an attempt to explore a question opened by Sharpe from decades ago, and this question is made ever more imperative in the modern financial markets.

In this economy, there are two classes of individuals: a single *Principal* and multiple *Managers*. The Principal is initially endowed with a single unit of wealth, while Managers have zero initial wealth. All individuals are risk averse with mean-variance preferences over terminal wealth. The Principal has a strict desire for Managers to be *compliant* and implement a specific pair of investment strategies (say Asian active macro and European active macro strategies, from the previous example), and the Principal must delegate to Managers to access these strategies. However, implementing these strategies will incur private costs
for the Managers. Moreover, Managers could deviate to alternative *deviant* strategies (say Asian passive market index and European passive market index) that are privately costless but have lower mean returns and different correlation structure than the Principal’s desired pair of strategies. For simplicity, we take an extreme assumption that the Principal would abandon delegation if his desired strategy pair cannot be implemented. Largely for tractability in the model, we emphasize again that the Principal will only want to implement his desired pair of strategies and he will not entertain other strategy pairs.

In the presence of such moral hazard over investment strategies within each of the two asset classes, the Principal needs to decide which form of delegation is best. In the first option, the Principal can choose *centralized delegation*: the Principal will delegate all initial wealth to a single Manager $C$ (say a global strategy manager). Manager $C$ will have two actions: investment strategy choice and portfolio allocation choice. Manager $C$ will first need to select a strategy pair, one strategy from each asset class. Then secondly, taking any offered contract into account, Manager $C$ will construct portfolio weights between this strategy pair. In return, the Principal will compensate Manager $C$ with a linear contract over the net returns of the resulting portfolio.

Alternatively, in the second option, the Principal can choose *decentralized delegation*: the Principal will make a portfolio choice and decide how much of his initial wealth to delegate to Manager $A$ (say, an Asian asset manager) who will exclusively manage one asset class, and delegate the rest to Manager $B$ (say, an European asset manager) who will exclusively manage the other asset class. Both Managers can only pick one strategy from their respective asset classes. The Principal will compensate these two Managers also with linear contracts over the net returns from their respective asset class.

We make clear on the action differences between centralization and decentralization. In centralization, Manager $C$ has both strategy choice and portfolio choice, while the Principal only has contract choice. In decentralization, Manager $A$ and Manager $B$ have strategy choice within their own asset class, while the Principal has both asset allocation choice and contract choice.

### 1.1 Static delegation

We begin with a static delegation model, where the contract begins today and terminates one period later.

In first best with no moral hazard risk, where the Principal can observe and directly contract on the Managers’ strategy choices in each asset class, the comparison of centralized versus decentralized delegation is a simple question of optimal risk sharing. High return correlation of returns between the Principal’s preferred strategy pair will favor decentralization, and low correlation will favor centralization. Given any contract, the single Manager $C$ will pick portfolios between this strategy pair, again with one strategy from each asset.
class. And since the contract is linear over the net returns of the portfolios, then low correlation between the strategies will lead to lower overall portfolio volatility, which then implies lower contract volatility for Manager C. As Manager C is also risk averse, it becomes cheaper for the Principal to compensate him because of reduced risk compensations. In contrast, if the correlation is high, delegating to Manager C will increase his contract volatility, which in turn means the Principal must increase the risk compensation.

In contrast, for decentralization, the Principal picks portfolio weights over his initial wealth to delegate to Manager A and Manager B, in addition to offering separate linear contracts for each Manager over their strategy’s net returns. When the correlation of the Principal’s desired strategy pair is high, the Principal can pick portfolio weights to spread out the risk between himself, Manager A and Manager B to optimally risk share. In contrast, when the correlation is low, since Manager A’s and Manager B’s contracts only depend on their own strategy returns, only the Principal can capture the low correlation diversification benefit and thus the risk sharing benefit is reduced for decentralization. This first best result illustrates the idea that risk management “defines the boundaries” of a firm. In centralization, risk management is handled exclusively by Manager C, since only Manager C picks portfolios between strategies. Whereas in decentralization, the Principal handles risk management himself since only he picks portfolios.

Next, we consider the second best case where moral hazard is distinctly present, in that Managers can privately choose their investment strategies. We highlight three specific components that affect the Principal’s decision for centralized delegation versus decentralized delegation under moral hazard: (i) investment opportunity set; (ii) Managers’ risk aversions; and (iii) Managers’ private costs.

With respect to (i), we claim that a wide investment opportunity set strongly disfavors centralized delegation. For any given contract, Manager C makes portfolio weight choices and also strategy pair choices. When Manager C deviates away from the Principal’s desired strategy pair, the deviant pair of strategies will generically have different mean returns with some correlation level. Given that the Manager C has mean-variance preferences, he will naturally put greater portfolio weights to the deviant strategy of one asset class with a higher mean return and a lower portfolio weight to the deviant strategy of another asset class with a lower mean return. This generates a “long-short” trading profit benefit for Manager C out of the deviant strategy pair that is not enjoyed by the Principal; again, the Principal has a strict desire for Manager C to be compliant and to implement the Principal’s desired strategy pair, and will not entertain any other deviant strategy pairs. Thus to ensure compliance, the Principal must compensate Manager C with higher performance fees as an opportunity cost for Manager C’s foregone long-short trading profits, along with Manager C’s private costs for implementing the Principal’s desired strategy pair. That is to say, if the investment opportunity set is so “wide” that the mean return differences between the two asset classes are large, it will strongly disfavor centralized delegation due to increased performance fees the Principal must
compensate. In contrast, under decentralized delegation, even if Manager A or Manager B deviates, they can only deviate in strategies within their own asset class. So the aforementioned long-short opportunity cost in centralization simply does not exist for them due to restriction in their respective investment opportunity set. Hence, to ensure compliance from the decentralized Managers, the Principal simply needs to compensate for their private costs, and the mean and volatility differences between the compliant and deviant strategies in their respective asset classes.

With respect to (ii) Managers’ risk aversions, lower managerial risk aversion will disfavor centralized delegation. The effect of risk aversion on Manager C is intimately linked to (i) the investment opportunity set restrictions. As Manager C becomes less risk averse, the less he cares about volatility of the overall portfolio and likewise on the correlation between investment strategies across asset classes; the only thing that becomes relevant are simply the mean return differences between the deviant strategy pairs. Thus, in the extreme limit when Manager C becomes risk neutral, for any given contract, Manager C will simply take an infinite long position into the deviant strategy from one asset class with highest possible mean, and take an infinite short position into the deviant investment strategy from another asset class with lowest possible mean. Without any portfolio constraints on Manager C, this potentially infinitely large long-short trading profit would be too high of an opportunity cost for the Principal to compensate, and thereby resulting in the nonexistence of a contract to implement the Principal’s desired strategy pair. We should note that this result is starkly different from standard principal-agent theories\footnote{For instance, the standard references of \textcite{laffont2001principals} and \textcite{bolton2004delegation}.} where it is generically cheaper for a Principal to contract with a less risk averse agent since the Principal then saves on the required risk premium compensation. In our case, the result is completely reversed: a less risk averse Manager C is actually more expensive to compensate, and in the limit when Manager C is risk neutral, it may become infinitely costly to compensate him. These effects are completely driven by Manager C’s relaxed investment opportunity set; in particular, when the Principal is risk averse while Manager C is risk neutral. When Manager C can have less restrictive access to financial markets — namely that he can freely construct portfolio weights between asset classes — he can modify the risk and reward effects of the contract to his desire, and in particular, can offload the contract risks onto the financial markets, and thereby distort the incentive effects of the contract. In contrast, in decentralized delegation and again extending the discussion from (i), Manager A and Manager B cannot trade each other’s asset class, and thus are severely restricted in their respective investment opportunity set. When the two Managers consider a deviation, they are concerned with the differences in the strategies’ means and volatilities within their own asset class. Thus decentralized delegation is much closer to a standard principal-(multi)agent problem whereby compensation to less risk averse agents could
be reduced.

With respect to (iii) Managers’ private costs, high private costs disfavors decentralized delegation. In centralization, by being compliant and implementing the Principal’s desired strategy pair, Manager C must incur a high private cost. But for any given any contract, Manager C is still a risk averse individual; Manager C’s portfolio choice behavior is similar to the Principal if the Principal were to have direct access to his desired strategy pair. Hence, Manager C also prefers portfolio choices that generates a high portfolio mean return and low portfolio volatility to generate an optimal mean-variance trade-off for his performance fees. Thus, Manager C acts like a quasi-Principal for any given contract, and while private costs certainly affect the contracting environment, they only play a second order effect in centralization. In contrast, in decentralization, Manager A and Manager B are restricted in their investment opportunity sets. So when Manager A or Manager B consider a deviation, the private costs play a first order effect as in standard principal-agent theories. Indeed, when private costs are sufficiently high for Manager A and Manager B to implement the Principal’s desired pair of strategies, a contract may fail to exist; in contrast, for those same high private cost levels, a contract may still exist for centralized delegation.

In all, under static delegation, the relaxed investment opportunity set in centralized delegation and the restricted investment opportunity set in decentralized delegation is the critical source of difference between the contracting environment of centralization and decentralization. And indeed, this trickles down to why Managers’ risk aversions and Managers’ private costs have different implications to the contracting environments.

The above discussion illustrates various trade-offs between centralization versus decentralization. However, there is one important special case, which has critical implications for risk management practices for the Principal, whereby decentralization is surely worse than centralization. In decentralization, again, linear contracts are offered over Manager A’s and Manager B’s respective strategy’s net returns. In determining a deviation, Manager A and Manager B are only concerned with the mean return and volatility differences between the Principal’s desired strategy and the deviant strategy within their own asset class. Indeed, these differences are the benefits to Managers for compliance. Nonetheless, the Principal still wants a particular strategy pair from each of the asset classes to be implemented because it correlates favorably with some in-situ background investments that the Principal already holds. However, suppose if these differences are

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2 In the model, we assume that volatilities of all investment strategies are equivalent and hence the risk aversion term will not even appear in the incentive compatibility constraints for decentralized delegation. But from the model, it is evident this will be true when the strategies have different volatilities.

3 Continuing from the previous example with the Asian asset class, one could argue that an Asian macro strategy could have a mean return that is only marginally higher than that of the Asian passive equities index. And likewise for the European asset class. However, the broad passive Asian and European passive indices will tend to have large index weights into constituent members that are conglomerates with global business operations. In contrast, suppose Managers construct the Asian and European macro strategies to have investments in firms whose business scopes are largely confined to their geographies. Furthermore, suppose the Principal already has some background in-situ investments in some developing economies. Then investing an extra
small, then the benefits to the Managers for compliance are small, but the private costs for implementing the
Principal’s desired strategy remain high. In this case, the Managers will surely deviate and thus, no contract
will exist to implement the Principal’s pair of desired strategies in decentralization. Fundamentally, this is
because only the Principal can capture the diversification benefits of the two asset classes, while Managers
completely ignore these benefits when considering a deviation because one Manager’s compensation does not
depend on another Manager’s strategy. Effectively, we need a contracting mechanism that only depends on
the Managers’ own strategy returns, and nothing else, to link the Managers’ wealths despite the presence of
moral hazard.

1.2 Dynamic delegation

A dynamic delegation model with committed reinvestment is a possible solution. The key idea here is through
committed reinvestments, the Principal’s intermediate wealth becomes an “intertemporal glue” that links
the terminal wealths between Manager A and Manager B.

Suppose Manager A’s and Manager B’s strategy choice from their respective asset classes are chosen and
committed to at the initial contracting date $t = 0$. Once the strategies have been committed, subsequent
per-period returns will be generated from this strategy only. Furthermore, the Principal also commit to future portfolio policies and contracts. All individuals have mean-variance preferences over terminal wealths at $t = 2$, and there is no intermediate consumption. The Principal allocates portions of his initial wealth to Manager A and Manager B at $t = 0$, then subsequently, one-period returns are generated and performance fees are collected at $t = 1$. Then also at $t = 1$, the Principal collects all the returns from the Managers and aggregates them into a single pot of intermediate wealth. From this pot of intermediate wealth, the Principal reinvests (quite possibly different proportions than that of $t = 0$) into Manager A and Manager B at $t = 1$. Finally, at $t = 2$, one-period returns are generated and performance fees are collected by the Managers. But then $t = 2$ terminal wealths of Manager A and Manager B will depend on the portfolio weights and performance fees the Principal had allocated to them at $t = 1$, which then depends on the level of intermediate wealth that the Principal had at $t = 1$ available for reinvestment. Furthermore, the intermediate aggregated wealth at $t = 1$ depends on the investment strategies that Manager A and Manager B had committed to at $t = 0$. Thus, using the Principal’s intermediate aggregated wealth as an “intertemporal glue”, Manager A’s and Manager B’s $t = 2$ terminal wealth will depend on each other’s committed strategies.

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4 For instance, if an Asian macro strategy was chosen at $t = 0$, the subsequent period returns (from $t = 0$ to $t = 1$, and $t = 1$ to $t = 2$) will be from this Asian macro strategy.
at \( t = 0 \). Moreover, through this “intertemporal glue”, if one Manager deviates from the Principal’s desired investment strategy choices at \( t = 0 \), it could potentially hurt both of them simultaneously at \( t = 2 \). Note that in this mechanism, the fees the Principal pays to Manager \( A \) and Manager \( B \) are still dependent only on the respective Manager’s strategy returns. To have a comparison against dynamic decentralized delegation, we also consider the dynamic model for centralized delegation whereby the Principal reinvests back into Manager \( C \) in an analogous fashion.

1.3 Overview

A literature review is in Section 2. We introduce the model setup in Section 3. The first best results are in Section 4. The second best results, which are the core contributions of the paper, are in Section 5. From there, we motivate and discuss the dynamic decentralized delegation model in Section 6. The first best results for the dynamic decentralized delegation are in Section 7, while the second best results are in Section 8. All proofs are in the Appendix. Moreover, we also have an Online Appendix for additional results for the dynamic centralized model. All the proofs and additional results on the dynamic models are also in the Online Appendix. We conclude in Section 9.

2 Literature Review

Sharpe (1981) is the seminal paper that coined the term “decentralized investment management”. In particular, he argues an investor would prefer decentralization over centralization for “diversification of style” and “diversification of judgment”. But beyond these two reasons, Barry and Starks (1984) also add that risk sharing is another motive for preferring decentralization over centralization. Elton and Gruber (2004) recognize that decentralized delegation is a very real issue faced by practitioners and offer conditions under which “a central decision maker can make optimal decisions without requiring decentralized decision makers to reveal estimates of security returns”. More recently, van Binsbergen, Brandt, and Kojien (2008) study the decentralization problem in continuous time and derive the optimal wealth that the investor should allocate between decentralized managers. None of the references above have studied a moral hazard problem of any form. The goal of our paper is to study the similarities and differences of centralization versus decentralization under an explicit moral hazard problem, whereby both the contract and the portfolio policies are endogenously determined.

Our problem clearly belongs to the vast literature of delegated portfolio management. Stracca (2006)
offers a survey on the theory findings of delegated portfolio management. The problem of moral hazard in delegated portfolio management, but only to delegation of a single agent, has been studied at least since Bhattacharya and Pliegerer (1953) and Stoughton (1993). These papers usually information based, whereby the principal delegates to an agent because the agent can exert private costly effort to acquire a signal of the future value of a security. Instead, in our paper, we do not take the private costly information acquisition route and rather assume that the principal delegates because the principal has access restrictions to the financial markets. A recent paper by He and Xiong (2013) also assumes the principal has restricted access to the financial markets, and delegates to an agent who will both acquire a signal about an asset’s future return, and also make an investment decision. Our model is not about costly private information acquisition. Nonetheless, the authors reach a similar conclusion that if there is too much flexibility in what an agent can do — like our single Manager C in centralized delegation — it will be more costly to the principal to induce the agent for correct decisions. Indeed, like our paper, the inability of the principal to contract on the agent’s portfolio choice in He and Xiong (2013) is a critical source of moral hazard. But in our paper, the portfolio choice dimension will form a critical difference between centralization versus decentralization, and moreover, we show that there are important cases where centralization is indeed preferred over decentralization.

The empirical question of whether moral hazard is present in investment managers is a subject of substantial research. Although our paper is not specific to the type of funds being delegated to, the prototypical example we have in mind is hedge funds. Getmansky, Lee, and Lo (2015) and Agarwal, Mullally, and Naik (2015) are recent survey papers of the hedge fund industry. In particular, strong empirical evidence suggests that moral hazard is a substantial concern in hedge fund. Patton (2009a) argue that a quarter of funds that advertise themselves as “market neutral” have significant exposures to the market factor. Brown, Goetzmann, Liang, and Schwarz (2008, 2012) and Brown, Goetzmann, Liang, and Schwarz (2009) argue that proper due diligence to the extent of reducing operational risks of hedge funds is a source of alpha. Bollen and Pool (2012) constructs several performance flags based on hedge fund return patterns as indicators of increased fraud risks.

There is a small but growing empirical literature on comparing the effectiveness of centralized versus decentralized delegation. Blake, Rossi, Timmermann, Tonks, and Wermers (2013) document that pension fund managers have gravitated from a centralized delegation model to a decentralized delegation model. In the context of mutual funds, Dass, Nanda, and Wang (2013) compare the performance of sole- and team-managed balanced funds. Similarly, Kacperczyk and Seru (2012) ask whether centrally managed or decentrally managed mutual funds perform better.

Our model also fits into the broad literature of optimal delegation forms. The recent work by Gromb and Martimort (2007) discuss the optimal design of contracts for experts who can privately collect a signal. The
paper there focuses on risk neutral individuals with limited liability and economies of scale of private costs. Whereas in this paper, we explicitly focus on how risk aversion can play a critical role in portfolio choice, and there is no economies of scale in private costs. Some key earlier work on delegation to multiple agents are Demski and Sappington (1984), Demski, Sappington, and Spiller (1988) and Holmström and Milgrom (1991), but these papers do not explicitly consider the issue of portfolio choice and access to financial markets in the moral hazard formulation.

3 Static Model Setup

3.1 Individuals, Assets and Moral Hazard

There are two time periods $t = 0$ and $t = 1$. There are two classes of individuals: a single Principal and three Managers $A$, $B$ and $C$. The Principal is initially endowed with $1$ unit of wealth, and Managers have $0$ initial wealth. Both the Principal and the Managers have mean-variance preferences over their own terminal wealth. The Principal has a risk aversion parameter of $\eta_P > 0$, while the Managers have a risk aversion parameter of $\eta_M > 0$.

There are two risky asset classes, indexed by $\theta$ and $\tau$. Within each asset class, there are two specific investment strategies $\{H, L\}$. Thus, for asset class $\theta$, the specific investment strategies are $\{\theta_H, \theta_L\}$, and for asset class $\tau$, they are $\{\tau_H, \tau_L\}$. We denote the net return of any particular investment strategy to be $R_i$ for $i \in \{\theta_H, \theta_L, \tau_H, \tau_L\}$.

The Principal has no access to the financial markets and must delegate to the Managers for access. Here, we will make an assumption on the investment strategy the Principal strictly prefers from each asset class. Please also see Remark 3.3 for a discussion of the importance and restrictions of this assumption.

**Assumption 3.1.** The Principal has a strict preference to implement the strategy pair $(\theta_H, \tau_H)$ over any other strategy pairs.

Motivated by Assumption 3.1, we will call the “H” investment strategies to be compliant, and the “L” strategies to be deviant. Likewise, we will call the strategy pair $(\theta_H, \tau_H)$ to be the compliant strategy pair, and call any strategy pair $(\theta, \tau) \in S_{\neq}((\theta_H, \tau_H))$ to be deviant strategy pairs. As a concrete example, we may think of $\theta$ as the Asian equities asset class and $\tau$ as European equities. Then $\theta_H$ can represent an active Asian macro equities strategy, while $\theta_L$ is a passive Asian market index. Analogously, the $\tau_H$ strategy can represent an active European macro equities, while $\tau_L$ can represent a passive European market index.

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6 The term compliant refers to the strategies that the Principal strictly prefers the Managers to implement.

7 The term deviant refers the strategies that the Principal strictly prefers the Managers to not implement.
With some abuse of notations, we will also use $\theta$ and $\tau$ to index the investment strategies under their respective asset classes $\theta$ and $\tau$. Thus, we will write $\theta \in \{\theta_H, \theta_L\}$ to denote $\theta$ is an investment strategy from $\{\theta_H, \theta_L\}$ of the asset class $\theta$. Analogous comments for the expression $\tau \in \{\tau_H, \tau_L\}$. And we will write $R_\theta$ to denote the net return of a strategy $\theta \in \{\theta_H, \theta_L\}$ in the asset class $\theta$. Again, analogous comments for the notation $R_\tau$ for $\tau \in \{\tau_H, \tau_L\}$. Thus, the set of all possible strategy pair combinations from these two asset classes is $S := \{(\theta_H, \tau_H), (\theta_H, \tau_L), (\theta_L, \tau_H), (\theta_L, \tau_L)\}$. We will denote the set of strategy pairs that exclude the compliant pair as $S_{-\{\theta_H, \tau_H\}} := S \setminus \{(\theta_H, \tau_H)\}$.

For each asset class, the Managers can privately choose the investment strategy and they incur a private cost for implementing the Principal’s desired strategies. The private cost structure for choosing $(\theta, \tau)$ is,

$$c(\theta) = \begin{cases} c > 0, & \theta = \theta_H, \\ 0, & \theta = \theta_L \end{cases} \quad \text{and} \quad c(\tau) = \begin{cases} c > 0, & \tau = \tau_H, \\ 0, & \tau = \tau_L. \end{cases}$$

(3.1)

We may think of the source of this private as “effort”, in that the Managers need to exert private costs to actively manage a more complex investment strategy for any given asset class.\(^8\)

We will, respectively, denote the means and variances of $\theta \in \{\theta_H, \theta_L\}$ as $\mu_\theta := E[R_\theta]$, $\sigma^2_\theta := \text{Var}(R_\theta)$, with analogous notations for $\tau \in \{\tau_H, \tau_L\}$. And we will denote the correlations of the pairs $(\theta, \tau)$ as $\rho_{\theta\tau} := \text{Corr}(R_\theta, R_\tau)$, for $(\theta, \tau) \in S$.

We make the following assumptions on the moments of the investment strategies.

**Assumption 3.2. Assume that,**

1. **The compliant strategies have identical means,$^9$** $\mu := \mu_{\theta_H} = \mu_{\tau_H}$. Moreover, compliant strategies have higher means than the deviant ones,

$$\Delta \mu_\theta := \mu_{\theta_H} - \mu_{\theta_L} = \mu - \mu_{\theta_L} > 0,$$

$$\Delta \mu_\tau := \mu_{\tau_H} - \mu_{\tau_L} = \mu - \mu_{\tau_L} > 0.$$  

2. **The volatilities of all investment strategies are identical,$^10$**

$$\sigma^2 = \sigma^2_\theta = \sigma^2_\tau, \quad \text{for all } \theta, \tau.$$

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\(^8\) Here, we have assumed that both asset classes $\theta$ and $\tau$ have identical private costs but this can be readily relaxed without affecting the qualitative results.

\(^9\) The equivalent means assumption can be easily relaxed at the expense of more complicated expressions of the results.

\(^{10}\) The equivalent volatility assumption can be easily relaxed at the expense of more complicated expressions of the results.
3. No perfect correlations between the investment strategies,

\[ |\rho_{\theta,\tau}| < 1, \quad (\theta, \tau) \in S. \]

**Remark 3.3.** Assumption 3.1 is a critical assumption of the paper, and it is clearly done with some loss of generality. Furthermore, the assumption has both technical and economic content. From a technical perspective, and as we shall see in the next section, this assumption simplifies the objective function of the Principal. Without this assumption, the Principal will need to cycle through all four possible strategy pairs \((\theta, \tau) \in S\) to compute which pair yields the highest value function for himself. This exercise will then exclusively depend on the parameter values. While this is not difficult to do from a technical perspective, it is not particularly economically insightful. Furthermore, the parameter conditions to ensure that \((\theta_H, \tau_H)\) is the optimal pair is also not economically interesting.

Economically, however, this assumption can be motivated in one of the two following ways. Firstly, this assumption may be justified if the Principal has some in place background investments that correlate favorably with the compliant pair \((\theta_H, \tau_H)\). Thus, he wants to delegate to Managers that will implement \((\theta_H, \tau_H)\) for him.

Secondly, this motivation can be economically justified if the Principal actually has partial access to the financial markets. Suppose the Principal can actually directly and costlessly access both of the deviant strategies of each asset class, so \(\theta_L\) and \(\tau_L\). Continuing from the opening example with Asian equities and European equities, \(\theta_L\) would represent a passive Asian index and \(\tau_L\) would represent a passive European index that the Principal could access directly without delegation. Thus, the Principal would only want to delegate to implement his preferred strategy pair \((\theta_H, \tau_H)\), which are, respectively, the Asian macro and European macro strategies from the opening example. Thus, for instance, if the parameters are such that the strategy pair \((\theta_H, \tau_L)\) yields higher value for the Principal than \((\theta_H, \tau_H)\), then the Principal only needs one outside Manager to manage the asset class \(\theta\); analogous comments also apply for the other deviant strategy pairs. If this were the case, we would have no meaningful discussion of centralized versus decentralized delegation as in our context.

Thus, for the remainder of the paper, Assumption 3.1 is strictly enforced.

### 3.2 Delegation forms

In the presence of moral hazard over investment strategy choices for each asset class, how should the Principal delegate? For the rest of the paper, we will focus on two forms of delegation — *centralized* delegation and *decentralized* delegation. In all forms of delegation, the Principal will offer a linear contract over the portfolio’s
net returns.

### 3.2.1 Centralized delegation

In centralized delegation, the Principal delegates all initial wealth to a single Manager $C$. In the previous example, Manager $C$ can be a global strategy manager who manages both the Asian and European asset class. Manager $C$ will be responsible for managing both asset class $\theta$ and $\tau$. Given any contract, Manager $C$ will have both a strategy choice and a portfolio choice. Firstly, from each of the two asset classes, Manager $C$ will pick a strategy $\theta \in \{\theta_H, \theta_L\}$ and a strategy $\tau \in \{\tau_H, \tau_L\}$. Secondly, for each chosen strategy pair $(\theta, \tau)$, Manager $C$ will pick portfolio weight $1 - \psi$ into strategy $\theta$, and portfolio weight $\psi$ into strategy $\tau$. The resulting portfolio $(1 - \hat{\psi}(\theta, \tau), \hat{\psi}(\theta, \tau))$ will have a net return $\hat{R}(\theta, \tau)$. In return for Manager $C$’s services, the Principal offers a linear contract $x_C + y_C \hat{R}(\theta, \tau)$ over the portfolio net return, where $(x_C, y_C) \in \mathbb{R} \times [0, 1]$. Thus, $x_C$ is a fixed (percentage) fee, while $y_C$ is a performance (percentage) fee. See Figure 1 for a timeline.

![Figure 1: Centralized delegation timeline. Manager $C$ has a strategy choice from each asset class, and also has a portfolio choice between those selected pair of strategies. The Principal only has a contract design choice.](image)

- **Manager $C$ accepts** or rejects the contract
- **Manager $C$ makes** investment strategy choices
  - $\theta \in \{\theta_H, \theta_L\}$
  - $\tau \in \{\tau_H, \tau_L\}$
- **Principal receives managed** portfolio returns
  - $\hat{R}(\theta, \tau) := (1 - \hat{\psi}(\theta, \tau))R_\theta + \hat{\psi}(\theta, \tau)R_\tau$
  - pays $x_C + y_C \hat{R}(\theta, \tau)$ to the Manager $C$

$t = 0$

$t = 1$
Thus, the optimization problem under centralized delegation is as follows.

\[
\sup_{(x_C, y_C) \in \mathbb{R} \times [0,1]} \mathbb{E}[W_{eC}^{(\theta_H, \tau_H)}] - \frac{\eta_M}{2} \text{Var}(W_{eC}^{(\theta_H, \tau_H)}), \\
\text{(Cen)}
\]

subject to,

\[
W_{eC}^{(\theta, \tau)} := 1 + \hat{R}_{(\theta, \tau)} - (x_C + y_C \hat{R}(\theta, \tau)), \\
W_{C}^{(\theta, \tau)} := -(c(\theta) + c(\tau)) + x_C + y_C \hat{R}(\theta, \tau), \\
\tilde{W}_{C}^{(\theta, \tau)} := -(c(\theta) + c(\tau)) + x_C + y_C ((1 - \psi)R_{\theta} + \psi R_{\tau}), \\
\hat{\psi}_{(\theta, \tau)} := \arg \sup_{\psi \in \mathbb{R}} \mathbb{E}[^{\hat{W}_{C}^{(\theta, \tau)}}] - \frac{\eta_M}{2} \text{Var}(^{\hat{W}_{C}^{(\theta, \tau)}}), \\
\hat{R}_{(\theta, \tau)} := (1 - \hat{\psi}_{(\theta, \tau)})R_{\theta} + \hat{\psi}_{(\theta, \tau)} R_{\tau}, \\
0 \leq \mathbb{E}[W_{C}^{(\theta_H, \tau_H)}] - \frac{\eta_M}{2} \text{Var}(W_{C}^{(\theta_H, \tau_H)}), \\
\theta_H, \tau_H = \arg \max_{(\theta', \tau')} \mathbb{E}[W_{C}^{(\theta', \tau')}}] - \frac{\eta_M}{2} \text{Var}(W_{C}^{(\theta', \tau')}). \\
\text{(3.2f, 3.2g)}
\]

In (3.2f), the Principal maximizes his mean-variance utility over terminal wealth (3.2g), which is equal to the return from the Manager C managed portfolio, less the fees that the Principal pays. In the maximization, the Principal needs to pick the optimal fixed fees \(x_C\) and optimal performance fees \(y_C\) as part of the linear contract. Given the linear contract, Manager C will construct the optimal portfolio, as in (3.2d), out of the two strategies, one each from the two asset classes, to obtain the portfolio returns in (3.2e). In return for Manager C’s service, his terminal wealth is (3.2b). The contract must be such that Manager C is willing to participate and so (3.2c) is Manager C’s individual rationality constraint. In the second best case, the Principal’s desired strategy pair \((\theta_H, \tau_H)\) must also be incentive compatible for Manager C, which is (3.2g).

3.2.2 Decentralized delegation

In decentralized delegation, the Principal delegates wealth to two different individuals, Manager A and Manager B. Manager A is responsible for only managing asset class \(\theta\), and Manager B is responsible for only managing asset class \(\tau\). Thus, following the earlier example, Manager A is an Asian asset class manager, while Manager B is an European asset class manager. The Principal will allocate \(1 - \pi\) portion of his initial wealth to Manager A and \(\pi\) proportion to Manager B. In return for the two individuals services, the Principal will offer a linear contract \((1 - \pi)(x_A + y_A R_{\theta})\) to Manager A, and \(\pi(x_B + y_B R_{\tau})\) to Manager B,
Managers $A, B$ make investment strategy choices

\( \theta \in \{ \theta_0, \theta_B \} \) and

\( \tau \in (\tau_L, \tau_R) \), resp.

Manager $A$ receives returns
\[
(1 - \pi)(x_A + y_A R_\theta) - c(\theta);
\]
Manager $B$ receives returns
\[
\pi(x_B + y_B R_\tau) - c(\tau);
\]

Principal makes portfolio choices:

- (i) \( 1 - \pi \in \mathbb{R} \) to Manager $A$; and
- (ii) \( \pi \) to Manager $B$

$$
(3.3a)
$$

$$
(3.3b)
$$

$$
(3.3c)
$$

$$
(3.3d)
$$

$$
(3.3e)
$$

$$
(3.3f)
$$

The Principal’s objective is to pick the optimal linear contracts to compensate the two Managers, and also to pick the optimal portfolio policy to decide how much wealth to allocate to the Managers’ strategies. The Principal’s terminal return \((1 - \pi, \pi)\) is equal to the portfolio \((1 - \pi)\) that the Principal decides to allocate

\( W_{P \theta} := 1 + \pi R_\tau + (1 - \pi)R_\theta - \pi(x_B + y_B R_\tau) - (1 - \pi)(x_A + y_A R_\theta) \)

\( W_A^\theta := (1 - \pi)(x_A + y_A R_\theta) - c(\theta) \)

\( W_B^\tau := \pi(x_B + y_B R_\tau) - c(\tau) \)

\[
0 \leq \mathbb{E}[W_A^{\theta_H}] - \frac{\eta}{2} \text{Var}(W_A^{\theta_H}), \quad \text{and} \quad 0 \leq \mathbb{E}[W_B^{\tau_H}] - \frac{\eta}{2} \text{Var}(W_B^{\tau_H})
\]

\[
\theta_H = \arg \max_{\theta \in \{ \theta_0, \theta_B \}} \mathbb{E}[W_A^{\theta_H}] - \frac{\eta}{2} \text{Var}(W_A^{\theta_H})
\]

\[
\tau_H = \arg \max_{\tau \in (\tau_L, \tau_R)} \mathbb{E}[W_B^{\tau_H}] - \frac{\eta}{2} \text{Var}(W_B^{\tau_H})
\]

Thus, the optimization problem under decentralized delegation is as follows.

\[
\sup_{(x_A, y_A), (x_B, y_B) \in \mathbb{R} \times [0, 1]} \sup_{\pi \in [0, 1]} \mathbb{E}[W_P^{(\theta_H, \tau_H)}] - \frac{\eta}{2} \text{Var}(W_P^{(\theta_H, \tau_H)})
\]

subject to,

(Figures are not shown here and would be included in the original document.)
to Manager A and B’s strategy returns \((R_\theta, R_\tau)\), less the fees owed. Both \((3.3b)\) and \((3.3c)\) represent, respectively, Manager A and Manager B’s terminal wealth. The two Managers’ participation constraints are in \((3.3d)\). To induce Manager A and Manager B to pick the Principal’s desired strategy pair \((\theta_H, \tau_H)\), the Managers’ incentive compatibility constraints are in \((3.3e)\) and \((3.3f)\).

4 First Best

Let us begin by considering the first best setup, whereby the Principal can directly observe and contract on the private investment strategy choices of the Managers.

4.1 Centralized Delegation in First Best

For the first best centralized delegation case, consider problem \((\text{Cen})\) without the incentive compatibility constraint \((3.2g)\).

Proposition 4.1. Consider the first best centralized delegation problem; that is, problem \((\text{Cen})\) without the incentive compatibility constraint \((3.2g)\). Fix any strategy pair \((\theta, \tau) \in S\).

(a) Given any linear contract \((x_C, y_C) \in \mathbb{R} \times [0, 1]\), the optimal portfolio weight to strategy \(\tau\) of Manager C is,

\[
\hat{\psi}_{(\theta, \tau)} = \frac{1}{2} \left( 1 + \frac{1}{y_C} \frac{\mu_{\tau} - \mu_{\theta}}{\eta_M \sigma^2 (1 - \rho_{\theta\tau})} \right).
\] (4.1)

(b) For any given contract \((x_C, y_C)\), the mean and variance of the portfolio return are given by,

\[
E[\hat{R}_{(\theta, \tau)}] = \frac{1}{y_C} \frac{(\mu_{\theta} - \mu_{\tau})^2}{2 \eta_M \sigma^2 (1 - \rho_{\theta\tau})} + \frac{\mu_{\theta} + \mu_{\tau}}{2},
\]

\[
\text{Var}(\hat{R}_{(\theta, \tau)}) = \frac{1}{y_C^2} \frac{(\mu_{\theta} - \mu_{\tau})^2}{2 \eta_M \sigma^2 (1 - \rho_{\theta\tau})} + \sigma^2 (1 + \rho_{\theta\tau}).
\]

(c) The optimal fixed and performance fees are, respectively,

\[
\hat{x}_C((\theta, \tau), y_C) = (c(\theta) + c(\tau)) - y_C E[\hat{R}_{(\theta, \tau)}] + \frac{\eta_M}{2 y_C} \text{Var}(\hat{R}_{(\theta, \tau)}), \quad \text{for any } y_C \in [0, 1]
\] (4.2)

\[
\hat{y}_C^{FB} = \frac{\eta_P}{\eta_P + \eta_M}.
\]

If one needs the value of \(\hat{\psi}_{(\theta, \tau)}(y_C)\) at \(y_C = 0\), we will define it via the limit; that is, \(\hat{\psi}_{(\theta, \tau)}(0) := \lim_{y_C \to 0} \hat{\psi}_{(\theta, \tau)}(y_C)\). However, as we shall see, the optimal performance fee \(y\) generically will not be reached at 0 (i.e. due to individual rationality of Manager C), and hence the point 0 is not really of concern. For subsequent expressions in this proposition that involves \(y_C\) in the denominator, define it through the same limiting argument.
and so under the optimal contract, the optimal portfolio is,

\[ \hat{\psi}(\theta, \tau) = \hat{\psi}(\theta, \tau)(\hat{y}_C) = \frac{1}{2} \left( 1 + \frac{\eta^p + \eta_M \mu - \mu_\theta}{\eta^p \eta_M} \right). \]

(d) The Principal’s value function for implementing \((\theta_H, \tau_H)\),

\[ \mathbb{E}[W_{cP}^{(\theta_H, \tau_H)}] - \frac{\eta_M}{2} \frac{\eta^p}{\eta_M} \var(W_{cP}^{(\theta_H, \tau_H)}) = -2c + 1 + \mu - \frac{1}{4} \frac{\eta^p \eta_M}{\eta^p + \eta_M} \sigma^2 (1 + \rho_{\theta_H, \tau_H}). \]

(e) For any contract \((x_C, y_C)\), the Manager \(C\)’s utility for implementing investment strategy pair \((\theta, \tau)\) is,

\[ \mathbb{E}[W_{cP}^{(\theta, \tau)}] - \frac{\eta_M}{2} \frac{\eta^p}{\eta_M} \var(W_{cP}^{(\theta, \tau)}) = -(c(\theta) + c(\tau)) + x_C + \frac{(\mu_\theta - \mu_{\tau})^2}{4 \eta_M \sigma^2 (1 - \rho_{\theta\tau})} + \frac{1}{2} (\mu_\theta + \mu_{\tau}) y_C - \frac{1}{4} \eta_M \sigma^2 (1 + \rho_{\theta\tau}) y_C^2. \]

and in particular for \((\theta_H, \tau_H)\), it is,

\[ \mathbb{E}[W_{cP}^{(\theta_H, \tau_H)}] - \frac{\eta_M}{2} \frac{\eta^p}{\eta_M} \var(W_{cP}^{(\theta_H, \tau_H)}) = -2c + x_C + \mu y_C - \frac{1}{4} \eta_M \sigma^2 (1 + \rho_{\theta_H, \tau_H}) y_C^2. \]

For any given contract, the portfolio weight \(\hat{\psi}(\theta, \tau)\) into strategy \(\tau\) made by Manager \(C\) will clearly be independent of the fixed fees \(x_C\) and only be dependent on the performance fee \(y_C\). For instance, suppose strategy \(\tau\) has a higher mean return than strategy \(\theta\), so \(\mu_{\tau} > \mu_\theta\) (the converse case is analogous). Then in this case, naturally Manager \(C\) will allocate higher portfolio weights \(\hat{\psi}(\theta, \tau)\) to strategy \(\tau\) and less to strategy \(\theta\); and if the strategies have high correlations \(\rho_{\theta\tau}\), it induces the Manager \(C\) to almost take a “long-short” strategy whereby even more weights are allocated to \(\tau\) and less are to \(\theta\).

Indeed, for any contract \((x_C, y_C)\) and any strategy pair \((\theta, \tau) \in S\), we observe the expected wealth and risk aversion adjusted wealth volatility for Manager \(C\), after he has chosen the optimal portfolio, are
respectively,

\[ \mathbb{E}[W_C^{(\theta, \tau)}] = -(c(\theta) + c(\tau)) + x_C + y_C \cdot \left( \frac{1}{y_C} \frac{(\mu_\theta - \mu_\tau)^2}{2\eta_M \sigma^2(1 - \rho_{\theta \tau})} + \frac{\mu_\theta + \mu_\tau}{2} \right) \]

\[ = -(c(\theta) + c(\tau)) + x_C + \frac{(\mu_\theta - \mu_\tau)^2}{2\eta_M \sigma^2(1 - \rho_{\theta \tau})} + \frac{\mu_\theta + \mu_\tau}{2} - y_C, \]

\[ \frac{\eta_M}{2} \text{Var}(W_C^{(\theta, \tau)}) = \frac{\eta_M}{2} \text{Var}(y_C \hat{R}(\theta, \tau)) \]

\[ = \frac{\eta_M}{2} y_C \cdot \left( \frac{1}{y_C} \frac{(\mu_\theta - \mu_\tau)^2}{2\eta_M \sigma^2(1 - \rho_{\theta \tau})} + \sigma^2(1 + \rho_{\theta \tau}) \right) \]

\[ \frac{\eta_M}{2} \sigma^2(1 + \rho_{\theta \tau}) y_C^2. \]

Firstly, the risk-adjusted long-short trading profits for Manager C will be the term \( \left( \frac{1}{2} - \frac{1}{4} \right) \frac{(\mu_\theta - \mu_\tau)^2}{2\eta_M \sigma^2(1 - \rho_{\theta \tau})} = \frac{(\mu_\theta - \mu_\tau)^2}{4\eta_M \sigma^2(1 - \rho_{\theta \tau})} \), and these trading profits are completely independent of any contract \((x_C, y_C)\) the Principal offers. Secondly, there are contract mean and volatility effects for implementing the strategy pair \((\theta, \tau)\). The contract mean effect is \( \frac{\mu_\theta + \mu_\tau}{2} - y_C \), which is the expected performance fee payoff to Manager C. The contract volatility effect, adjusted for Manager C’s risk aversion, is \( \frac{\eta_M}{2} \sigma^2(1 + \rho_{\theta \tau}) y_C^2. \) In particular, for any contract and any strategy pair, the expected wealth of Manager C is \( \mathbb{E}[W_C^{(\theta, \tau)}] \) and its risk adjusted wealth volatility is \( \frac{\eta_M}{2} \text{Var}(W_C^{(\theta, \tau)}). \) However, the performance fees \( y_C \) only enter through the contract mean and contract volatility effect as discussed above, but is independent of the long-short trading profits for Manager C. These effects are related to Manager C’s “relaxed investment opportunity set” and we will have more to say about incentivization in Section 5.3.1 when moral hazard is present.

The optimal fixed fees \( \hat{x}_C \) are to simply compensate for Manager C’s private costs for taking on investment strategy pairs \((\theta, \tau)\), less Manager C’s share of the returns, and plus a volatility adjustment. The performance fees \( \hat{y}_C \) is equal to the ratio of Principal’s risk aversion \( \eta_P \) over the sum of both the Principal and Manager C’s risk aversion \( \eta_P + \eta_M \). This performance fee form is directly from optimal risk sharing of the linear contract form.

### 4.2 Decentralized Delegation in First Best

Next, we consider the first best decentralized delegation case. That is, consider the problem \((\text{Dec})\) without the incentive compatibility constraints (4.3e) and (4.3f).

**Proposition 4.2.** Consider the first best centralized delegation problem; that is, problem \((\text{Dec})\) without the
incentive compatibility constraints (3.3a) and (3.3d). For any investment strategy \( \theta, \tau \), define the quantities:

\[
\pi_{\theta, \tau}^o := \frac{1}{2} \left[ 1 + \frac{\mu - \mu_\theta (\eta_M + \eta_V (1 - \rho_{\theta V}))}{\eta_V \eta_M \sigma^2 (1 - \rho_{\theta V})} \right],
\]

\[
y_{A, \theta, \tau}^o := \frac{\eta_V \left[ (\mu - \mu_\tau) (1 - \rho_{\theta V}) (\eta_M + \eta_V (1 + \rho_{\theta V})) + \eta_V \eta_M \sigma^2 (1 - \rho_{\theta V}^2) \right]}{\eta_V \eta_M \sigma^2 (1 - \rho_{\theta V})},
\]

\[
y_{B, \theta, \tau}^o := \frac{\eta_V \left[ (\mu - \mu_\tau) (1 - \rho_{\theta V}) (\eta_M + \eta_V (1 + \rho_{\theta V})) - \eta_V \eta_M \sigma^2 (1 - \rho_{\theta V}^2) \right]}{\eta_V \eta_M \sigma^2 (1 - \rho_{\theta V})}.
\]

Then,

(a) For any portfolio \( \pi \) allocated to Manager B and performance fees \( y_A, y_B \), the optimal fixed fees are,

\[
\hat{x}_A(\theta, \pi, y_A) = \frac{1}{1 - \pi} \left[ c(\theta) - (1 - \pi) y_A \mu_\theta + \frac{\eta_M}{2} y_A^2 \eta^2 \sigma^2 \right] \quad \text{(4.3a)}
\]

\[
\hat{x}_B(\tau, \pi, y_B) = \frac{1}{\pi} \left[ c(\tau) - \pi y_B \mu_\tau + \frac{\eta_M}{2} y_B^2 \eta^2 \sigma^2 \right] \quad \text{(4.3b)}
\]

(b) If \( (\pi_{\theta, \tau}^o, y_{A, \theta, \tau}^o, y_{B, \theta, \tau}^o) \) \( \in (0, 1)^3 \), then the optimal portfolio policy and optimal performance fee policy of the Principal for implementing strategy \( (\theta, \tau) \) are \( (\pi_{\theta, \tau}^o, y_{A, \theta, \tau}^o, y_{B, \theta, \tau}^o) \).

(c) In particular, for implementing \( (\theta_1, \tau_1) \), the optimal portfolio and performance fee policies are,

\[
(\hat{x}^{FB}, \hat{y}_{A}^{FB}, \hat{y}_{B}^{FB}) = \left( \frac{1}{2}, \frac{\eta_V (1 + \rho_{\theta_1, \tau_1})}{\eta_M + \eta_V (1 + \rho_{\theta_1, \tau_1})}, \frac{\eta_V (1 + \rho_{\theta_1, \tau_1})}{\eta_M + \eta_V (1 + \rho_{\theta_1, \tau_1})} \right),
\]

and the Principal’s value function is,

\[
\mathbb{E}[W^{(\theta_1, \tau_1)}_P] - \frac{\eta_V}{2} \text{Var}(W^{(\theta_1, \tau_1)}_P) \bigg|_{FB} = -2c + \mu - \frac{1}{4} \frac{\eta_V \eta_M \sigma^2 (1 + \rho_{\theta_1, \tau_1})}{\eta_M + \eta_V (1 + \rho_{\theta_1, \tau_1})}.
\]

In decentralization, the Principal will allocate equal amount of wealth into both Manager A and Manager B, and moreover, the performance fees the Principal will pay to them will be equal as well. This result is immediate since from Assumption 3.3d, we had assumed that the compliant strategy pair \( (\theta_1, \tau_1) \) have identical means and identical volatilities. Unlike the performance fees of centralization in Proposition 3.2, where the performance fees are simply \( \eta_V / (\eta_M + \eta_V) \), the performance fees in decentralization must take into account the correlations \( \rho_{\theta_1, \tau_1} \) of the strategies. Thus, in centralization, risk management is internalized by the single Manager C, and hence the resulting performance fees only need to depend on the risk aversions of the individuals. However, with decentralization, the Principal must handle risk management himself and
thus the performance fees must reflect the correlations of strategies, in addition to the individuals’ respective risk aversions.

4.3 Risk management defines “the boundaries of a firm”

The first best results of centralized delegation in Proposition 4.1 and that of decentralized delegation in Proposition 4.2 illustrate the idea that risk management defines the “boundaries of a firm”. Indeed, the boundaries of a firm are central ideas in economics since Coase (1937), and the contemporary resurgence of these ideas can be traced to Williamson (1975, 1985). See Holmstrom and Roberts (1998) and Williamson (2002) for excellent surveys.

In centralization, the boundary of a “firm” is fixed and wide. Here, we interpret a “firm” to be Manager C, and its boundaries to be the activities that Manager C could take on. The Principal delegates all wealth to Manager C, and in this sense, the boundary of a “firm” is fixed. But the boundary of a firm is wide in the sense that given any contract, Manager C handles portfolio choice between the investment strategy pair desired by the Principal. In first best centralization, while the Principal can observe and contract on the investment strategy choices made by Manager C, the Principal cannot contract on the specific portfolio choices made by Manager C. Thus, risk management is, in effect, non-contractible. In our setup, largely because it will help in analytical tractability for the second best case of Section 5, we assume that the strategy pair \((\theta_H, \pi_H)\) have equivalent mean returns \(\mu\) and also equivalent volatility \(\sigma\); see again Assumption 3.2. Again, both the Principal and Manager C have mean-variance preferences. And suppose, hypothetically, the Principal could have direct access to the capital markets. Then both the hypothetical Principal and Manager C will load equal portfolio weights into the two strategies, regardless of their risk aversions \(\eta_M\) and \(\eta_P\); in this special case, the Principal and Manager C will identically agree on portfolio choice. However, it is not difficult to see that if the mean returns are different, or that the volatilities are different, Manager C’s portfolio choice would differ to that of the hypothetical Principal. Moreover, even in first best, while the Principal can contract on the strategy pair \((\theta, \pi)\) Manager C implements, the Principal cannot contract on Manager C’s portfolio choice. Moreover, this difference in the portfolio weight choices between Manager C and this hypothetical Principal stems from the difference of their risk aversion and also the performance fees offered to Manager C. In all, without ability to contract on portfolio choice (or more broadly speaking, risk management), the boundary of Manager C is effectively fixed and wide, and Principal will use contracts to simply optimally risk share.

The discussion for decentralization is far simpler. In decentralization, the boundary of a “firm” is flexible and narrow. Manager A and Manager B can only operate within their own asset class, and in this sense, their
boundaries are narrow. In contrast to centralization, wealth allocations and hence also risk management, is exclusively handled by the Principal. The amount of wealth the Principal allocates to each Manager thus dictates the influence each Manager has on the Principal’s welfare, and it is in this sense that the boundary of a “firm” is flexible.

As we shall see in Section 5 where moral hazard is present, these two core differences between the boundary of a “firm” in centralization versus decentralization will have key implications for their optimal contracts and its existence.

4.4 Comparison between Centralized Delegation versus Decentralized Delegation in First Best

Now we can compare centralized delegation versus decentralized delegation under first best.

Proposition 4.3. The difference between the Principal’s value function in first best decentralized delegation and first best centralized delegation is,

\[
\left( \mathbb{E}[W_p^{(\theta_{H}, \tau_H)}] - \frac{\eta_P}{2} \text{Var}(W_p^{(\theta_{H}, \tau_H)}) \right)_{FB} - \left( \mathbb{E}[W_c^{(\theta_{H}, \tau_H)}] - \frac{\eta_P}{2} \text{Var}(W_c^{(\theta_{H}, \tau_H)}) \right)_{FB} = \frac{\eta_P \rho_{\theta_{H}, \tau_H} (1 + \rho_{\theta_{H}, \tau_H})^2}{4(\eta_P + \eta_M)(\eta_M + \eta_P(1 + \rho_{\theta_{H}, \tau_H}))}.
\]

Thus, decentralized delegation is better than centralized delegation if and only if the returns of the Principal’s desired strategy pair \((\theta_{H}, \tau_H)\) are strictly positively correlated \(\rho_{\theta_{H}, \tau_H} > 0\); conversely, centralized delegation is better than decentralized delegation if and only if the strategies are strictly negatively correlated \(\rho_{\theta_{H}, \tau_H} < 0\); and the two forms of delegation are equivalent when the strategies are uncorrelated \(\rho_{\theta_{H}, \tau_H} = 0\).

When the correlation between the Principal’s desired strategy pair \((\theta_{H}, \tau_H)\) is strictly negative, \(\rho_{\theta_{H}, \tau_H} < 0\), delegating to a single Manager \(C\) is beneficial. Given that Manager \(C\) will be putting long positions into both investment strategies \(\theta_{H}\) and \(\tau_H\), and since Manager \(C\) is also risk averse, a strictly negative correlation \(\rho_{\theta_{H}, \tau_H}\) lowers the contract volatility for Manager \(C\), and thereby it is cheaper for the Principal to risk share with Manager \(C\). But when the correlations become strictly positive, \(\rho_{\theta_{H}, \tau_H} > 0\), the reverse happens, and decentralized delegation is more appealing to the Principal. When the correlations become positive, delegating to a single Manager \(C\) actually increases Manager \(C\)’s contract volatility, and thereby making it more expensive for the Principal to risk share. However, with decentralized delegation, neither Manager \(A\) nor Manager \(B\) directly absorb the positive correlation effects. Thus, the Principal, via use his own portfolio choice, can make it cheaper to risk share with the decentralized Managers. And finally, in the case when the strategies are uncorrelated, \(\rho_{\theta_{H}, \tau_H} = 0\), both centralized and decentralized delegation are identical, since
neither the centralized or decentralized Managers(s) are affected by the correlation structure directly for the purpose of risk sharing.

5 Second Best

We come to one of the core results of the paper. Here, we assume the Principal cannot direct observe nor contract on the specific investment strategies that the Managers choose within each asset class. In both the second best centralized delegation (Proposition 5.1) and second best decentralized delegation (Proposition 5.2), the key emphasis will be, respectively, the performance fees and the optimal portfolio policies. In contrast, the fixed fees (i.e. $x_C$ in centralization; and $x_A, x_B$ in decentralization) are actually relatively straightforward. In both cases, the optimal fixed fees will ensure the Managers will participate and accept the contract. Furthermore, the fixed fees will compensate the Managers for their private costs, less the expected performance fee amount payoff, plus a volatility adjustment. This fixed fee form is standard in all linear contracting setups, and hence we will focus our paper on the performance fees and the portfolios.

5.1 Centralized delegation

Let’s first state the second best results for centralized delegation.

**Proposition 5.1.** Consider the second best centralized delegation problem (Cen). Then:

(a) For any performance fee $y_C \in [0, 1]$, the optimal fixed fees has the same form as that of first best in (4.2) of Proposition 4.1.

(b) For any performance fees $y_C \in [0, 1]$ and investment strategy pair $(\theta, \tau) \in S$, the optimal portfolio $\hat{\psi}(\theta, \tau)$ chosen by Manager $C$ is the same as the first best form (4.1) of Proposition 4.1. Indeed, the portfolio weight Manager $C$ will allocate to strategy $\tau$ in the strategy pair $(\theta, \tau)$ is,

$$
\hat{\psi}(\theta, \tau) = \frac{1}{2} \left( 1 + \frac{1}{y_C \eta_M \sigma^2 (1 - \rho_{\theta', \tau'})} \right), \quad (\theta', \tau') \in S_{-(\theta, \tau)}
$$

(c) The (three) incentive compatibility constraints on the performance fees $y_C$ for inducing Manager $C$ to
implement the strategy pair \((\theta_H, \tau_H)\) are,

\[
-2c + \frac{1}{2}(\mu_\psi + \mu_\tau)y_C - \frac{1}{4} \eta_M \sigma^2 (1 + \rho_{\psi H, \tau H}) y_C^2 \geq -c(\theta') + c(\tau') + \frac{1}{4} \frac{(\mu_\psi - \mu_\tau)^2}{\eta_M \sigma^2 (1 - \rho_{\psi H, \tau H})} + \frac{1}{2} (\mu_\psi + \mu_\tau)y_C - \frac{1}{4} \eta_M \sigma^2 (1 + \rho_{\psi H, \tau H}) y_C^2,
\]

(5.1)

for \((\theta', \tau') \in S_-(\theta_H, \tau_H)\). The three incentive compatibility constraints (5.1) can be equivalently written as a single constraint,

\[
0 \geq \max_{(\theta', \tau')} \left\{ -c(\theta') + c(\tau') - 2c + \frac{1}{4} \frac{(\mu_\psi - \mu_\tau)^2}{\eta_M \sigma^2 (1 - \rho_{\psi H, \tau H})} + \frac{1}{2} (\mu_\psi - \mu_{\theta H} + \mu_\tau - \mu_{\tau H}) y_C - \frac{1}{4} \eta_M \sigma^2 (\rho_{\psi H, \tau H} - \rho_{\theta H, \tau H}) y_C^2 \right\},
\]

(5.2)

where the maximum is taken over the possible deviant strategy pairs \((\theta', \tau') \in S_-(\theta_H, \tau_H)\). Recall from Assumption 7B, \(\mu = \mu_{\theta H} = \mu_{\tau H}\).

(d) If the net cost for Manager C to comply and implement the Principal’s desired strategy pair \((\tau_H, \tau_H)\) is sufficiently low, the Principal will achieve first best. That is, if

\[
0 > \max_{(\theta', \tau')} \left\{ -c(\theta') + c(\tau') - 2c + \frac{1}{4} \frac{(\mu_\psi - \mu_\tau)^2}{\eta_M \sigma^2 (1 - \rho_{\psi H, \tau H})} + \frac{1}{2} \frac{\eta}{\eta + \eta_M} (\mu_\psi - \mu_{\theta H} + \mu_\tau - \mu_{\tau H}) - \frac{1}{4} \left( \frac{\eta}{\eta + \eta_M} \right)^2 \eta_M \sigma^2 (\rho_{\psi H, \tau H} - \rho_{\theta H, \tau H}) \right\}
\]

(5.3)

then the optimal performance fee is \(\hat{y}_C = \hat{y}_C^{FB}\).

(c) Suppose the net costs for compliance to Manager C is sufficiently high; that is, replace \(>\) in (5.3) with \(\leq\). If an optimal performance fee \(\hat{y}_C \in [0, 1]\) exists, there necessarily exists some (unique) pair of deviant strategy pair \((\theta^b, \tau^b) \in S_-(\theta_H, \tau_H)\) that yields the highest net deviation benefit for Manager C. Furthermore, consider the following two conditions on \((\theta^b, \tau^b)\).

(i) For any strategy pair \((\theta', \tau') \in S_-(\theta_H, \tau_H)\), define \(i.e.\) the quadratic discriminant,

\[
D(\theta', \tau') := \frac{1}{4} (\mu_\psi - \mu_{\theta H} + \mu_\tau - \mu_{\tau H})^2 + \left[ -c(\theta') + c(\tau') - 2c + \frac{1}{4} \frac{(\mu_\psi - \mu_\tau)^2}{\eta_M \sigma^2 (1 - \rho_{\psi H, \tau H})} \right] \eta_M \sigma^2 (\rho_{\psi H, \tau H} - \rho_{\theta H, \tau H}).
\]

(5.4)

Suppose the strategy pair \((\theta^b, \tau^b)\) is such that,

\[
D(\theta^b, \tau^b) \geq 0.
\]
(ii) For the pair \((\theta^b, \tau^b)\), define (i.e. the positive quadratic root),

\[
\hat{y}_{+, (\theta^b, \tau^b)} := \frac{1}{2} \left( \frac{\mu_{\theta^b} - \mu_{\theta_H} + \mu_{\tau^b} - \mu_{\tau_H}}{2} + \sqrt{D_{(\theta^b, \tau^b)}} \right) \\
\times \left[ -(c(\theta^b) + c(\tau^b) - 2c) + \frac{(\mu_{\theta^b} - \mu_{\tau^b})^2}{4 \eta_M \sigma^2 (1 - \rho_{\theta^b, \tau^b})} \right]^{-1}
\]

(5.5)

Suppose the pair is \((\theta^b, \tau^b)\) is such that,

\[
\hat{y}_{+, (\theta^b, \tau^b)} \in [0, 1].
\]

If both conditions (i) and (ii) hold, then the second best performance fee is \(\hat{y}_C = \hat{y}_{+, (\theta^b, \tau^b)}\). If neither condition (i) nor (ii) hold, then no second best contract will exist for centralized delegation.

The right hand side of the incentive compatibility condition (5.2) is the “net cost” for Manager \(C\) for being compliant instead of being deviant. Firstly, we have the standard private costs effect: by being compliant and picking \((\theta_H, \tau_H)\), Manager \(C\) needs to incur private costs of \(c(\theta_H) + c(\tau_H) = 2c\), but by deviating to \((\theta', \tau') \in S_{-(\theta_H, \tau_H)}\), the private costs are strictly lowered to \(c(\theta') + c(\tau')\); hence, \(2c - (c(\theta') + c(\tau'))\) represents the net private costs for complying instead of deviating. These effects would be standard in practically all standard principal-agent models. However, there are three additional effects that arise solely because of Manager \(C\)’s ability to take an arbitrary contract offered by the Principal, and then subsequently trade upon it.

Secondly, incentive compatibility for Manager \(C\) also comes in the form of the net return differences from implementing the compliant investment strategy pair \((\theta_H, \tau_H)\) versus the deviant pair \((\theta', \tau') \in S_{-(\theta_H, \tau_H)}\). From implementing the compliant pair and recalling the optimal portfolio choices, Manager \(C\) gains an expected performance fee payoff of \((\mu_{\theta_H} + \mu_{\tau_H})y/2 = \mu y\), whereas by implementing a deviant pair, Manager \(C\) has the expected performance fee payoff of \((\mu_{\theta'} + \mu_{\tau'})y/2\). But recall for the compliant investment strategies, \(\mu = \mu_{\theta_H} > \mu_{\theta'}\) and \(\mu = \mu_{\tau_H} > \mu_{\tau'}\). Thus, by being compliant, Manager \(C\) enjoys a net gain of \((2\mu - \mu_{\theta'} - \mu_{\tau'})y/2\) in higher performance fee payoffs.

Finally, the Principal wants to incentivize Manager \(C\) as cheap as possible, which is equivalent to binding the incentive compatibility constraints with the minimal net costs to Manager \(C\) across all possible strategy deviations.

For the remaining terms in the incentive compatibility constraint (5.2) we will discuss them in Section 5.3, when we compare centralization versus decentralization. See also Corollary B.1 for the explicit conditions on the parameters under which which strategy pair \((\theta^b, \tau^b)\) is the most profitable deviation for Manager \(C\).
5.2 Decentralized delegation

Next, let’s state the second best result for decentralized delegation.

**Proposition 5.2.** Consider the second best decentralized delegation case; that is consider, problem \((Dec)\) in its entirety.

(a) For any portfolio \(\pi\) and performance fee policies \((y_A, y_B)\), the optimal fixed fees have the form \((\ref{eq:optimal-fees})\) of first best decentralization in Proposition 4.2.

(b) The incentive compatibility conditions to induce the Principal’s desired strategy pair \((\theta_H, \tau_H)\) are,

\[
0 \geq c - (1 - \pi)y_A \Delta \mu_\theta, \quad (5.6a)
\]
\[
0 \geq c - \pi y_B \Delta \mu_\tau. \quad (5.6b)
\]

(c) Suppose the private costs \(c\) are sufficiently high\(^{13}\), then the second best decentralized optimal policies are,

\[
(\hat{\pi}, \hat{y}_A, \hat{y}_B) = \left(\frac{1}{2} \left[ 1 + \frac{\Delta \mu_\tau - \Delta \mu_\theta}{\Delta \mu_\theta \Delta \mu_\tau} \right], \frac{2 \Delta \mu_\tau c}{c(\Delta \mu_\tau - \Delta \mu_\theta) + \Delta \mu_\theta \Delta \mu_\tau}, \frac{2 \Delta \mu_\theta c}{c(\Delta \mu_\theta - \Delta \mu_\tau) + \Delta \mu_\theta \Delta \mu_\tau} \right).
\]

We will defer discussing the decentralized contracting environment and the incentive compatibility constraints \((\ref{eq:ic-conditions})\) in the next section, Section 5.3, when we compare centralization versus decentralization.

5.3 Comparison between Centralization versus Decentralization

Now, we can compare the similarities and differences in contracting between centralization and decentralization.

5.3.1 Investment opportunity set

The relaxation or restriction in the investment opportunity set is the key incentive difference between centralization and decentralization.

\(^{13}\) The precise conditions for this are in Proposition B.2(aiv). See also Proposition B.2 for further details of the optimal policies of second best decentralized delegation.
Let’s first consider centralized delegation. Firstly, Manager C enjoys a relaxed investment opportunity set. The term $\frac{1}{4}\eta_M \sigma^2(1 - \rho_{\theta', \tau'})$ in Manager C’s incentive compatibility constraint (5.2) represents the long-short trading benefit for Manager C by deviating to $(\theta', \tau')$. Under the compliant strategy pair $(\theta_H, \tau_H)$, we had assumed that they have equivalent means $\mu$ and also equivalent volatility $\sigma$, and thus Manager C would put equal weights into both investment strategies, and hence the optimal portfolio weights would be independent of Manager C’s risk aversion, strategies’ volatility $\sigma$, and the correlation $\rho_{\theta_H, \tau_H}$. In contrast, under the deviant investment strategy pairs $(\theta', \tau') \in S_{-(\theta_H, \tau_H)}$, they have potentially different means, and hence the correlation structure $\rho_{\theta', \tau'}$ and the volatility $\sigma$ contribute to a long-short strategy motive for Manager C; see again the portfolio form $\hat{\psi}(\theta', \tau')$ of Proposition 4.1. This constitutes a benefit for Manager C that is foregone by being compliant, and hence is an opportunity cost for Manager C that the Principal needs to compensate for in the form of higher performance fees.

Secondly, incentive compatibility for Manager C also comes in the form of differences in the contract volatility under the compliant strategy pair and that of deviant strategy pairs. For any investment strategy pair $(\theta, \tau)$, the contract volatility for Manager C is $\sigma^2(1 + \rho_{\theta, \tau})y_C^2$. And thus, adjusting for Manager C’s risk aversion, the term $-\frac{1}{4}\eta_M \sigma^2(\rho_{\theta', \tau'} - \rho_{\theta_H, \tau_H})y_C^2$ is the net change in contract volatility for Manager C from taking the compliant pair $(\theta_H, \tau_H)$ versus a deviant pair $(\theta', \tau') \in S_{-(\theta_H, \tau_H)}$. The signs of the correlations matter. If $\rho_{\theta', \tau'} - \rho_{\theta_H, \tau_H} > 0$, that is the deviant strategy $(\theta', \tau') \in S_{-(\theta_H, \tau_H)}$ has a strictly higher correlation than the compliant strategy pair, then this represents a net benefit for Manager C; that is, since Manager C is risk averse, picking the compliant strategy pair with a lower correlation is beneficial, so being compliant reduces the contract volatility. For the reverse case, when $\rho_{\theta', \tau'} - \rho_{\theta_H, \tau_H} < 0$, being compliant increases the contract volatility.

Finally, there is an interaction between the contract volatility and the long-short trading benefit for Manager C. On the one hand, a higher correlation $\rho_{\theta', \tau'}$ increases the contract volatility for Manager C when consideration a deviation to $(\theta', \tau')$, and is thus detrimental to Manager C. But on the other hand, a higher $\rho_{\theta', \tau'}$ also increases the long-short trading benefit for Manager C, and is thus beneficial for Manager C. These interaction effects, again, are only afforded to Manager C due to his ability to modify the contract via his relaxed investment opportunity set.

In contrast, Manager A and Manager B under decentralization face a far more restricted investment opportunity set as compared to Manager C under centralization. And accordingly, the incentive compatibility constraints of decentralization take on a far more simpler form than centralization. Given any contract, Manager A and Manager B will only care about the mean and volatility differences between the strategies’

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14 From Assumption 3.2, we assumed that all strategies have identical volatility $\sigma$. We further discuss this implication in Section 5.3.2 below.
returns in their respective asset classes. And these differences in the moments between the Principal’s desired strategy versus that of the deviant strategy are the benefits for Manager A and Manager B for deviation. We will see the restricted investment opportunity set in decentralization will translate to different implications than centralization for the managers’ risk aversions and their private costs.

5.3.2 Managers’ risk aversions

Related to the investment opportunity set, the Managers’ risk aversion also play an opposite role in contracting under centralization versus decentralization.

In centralization, suppose Manager C becomes less risk averse, so $\eta_M \downarrow 0$. Firstly, when this happens, Manager C becomes less concerned with the volatility difference of the contract $\frac{1}{2} \eta^2 \sigma^2 (\rho_{\theta', \tau'} - \rho_{\theta, \tau}) \gamma_C^2 \rightarrow 0$, for all deviant strategy pairs $(\theta', \tau') \in S_{-(\theta_H, \tau_H)}$. Secondly, when Manager C considers a deviation, Manager C cares less about the volatilities of the deviant strategy pairs $(\theta', \tau')$ and also less of the correlation of their returns $\rho_{\theta', \tau'}$. And indeed, as Manager C becomes less risk averse, he only cares about the absolute difference $|\mu_{\theta'} - \mu_{\tau'}|$ between the deviant strategies. In the limit when Manager C becomes risk neutral, he will take an infinitely large long position into strategy with highest mean, and take an infinitely large short position into the strategy with the lowest mean. Thus as $\eta_M \downarrow 0$, the long-short trading profit would become infinitely large, $\frac{1}{4} \frac{(\mu_{\theta'} - \mu_{\tau'})^2}{\eta_M \sigma^2 (1 - \rho_{\theta', \tau'})} \uparrow \infty$. When this happens, the cost for the Principal to compensate Manager C to ensure his compliance will be excessively high, and thus a contract to implement the Principal’s desired investment strategy pair $(\theta_H, \tau_H)$ will fail to exist.

The above result is completely the opposite of standard principal-agent theories. The literature suggests that it should be cheaper for a principal to compensate a less risk averse agent, because of the lower risk premium the principal needs to pay the agent for bearing risk. Here it is the reverse — the less risk averse Manager C becomes, the more expensive it is to compensate him. This is again due to the relaxed investment opportunity set in centralized delegation. For any given contract, Manager C can simply use the financial markets to modify the intended incentives of the contract.

In contrast, in decentralized delegation, as Manager A and Manager B become less risk averse, an optimal contract may still exist. Indeed in this regard, unlike centralization, decentralization is much closer to a standard principal-(multi)agent problem. In the model, we have assumed that the volatility $\sigma$ of all investment strategies are equivalent. Thus, the incentive compatibility constraints under decentralization do not involve the Managers’ risk aversion. And even if we were to assume the volatilities of investment strategies are different, it is straightforward to see that the right-hand side of (5.17) would simply have additional

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16 Admati and Pfleiderer (1994, Section V) makes a related point that benchmarked compensations are not relevant to soliciting effort.
terms $+ \frac{m}{2} (1 - \pi)^2 y_2^2 (\sigma_{\theta H}^2 - \sigma_{\theta L}^2)$ and $+ \frac{m}{2} \pi^2 y_2^2 (\sigma_{\tau H}^2 - \sigma_{\tau L}^2)$ for Manager A and Manager B, respectively. Depending on the sign of $\sigma_{\theta H}^2 - \sigma_{\theta L}^2$ and $\sigma_{\tau H}^2 - \sigma_{\tau L}^2$, the Principal either pays additional fees for increased volatility risk imposed on the Managers, or get savings in fees for decreased volatility risk. Regardless, as $\eta_M \downarrow 0$, we collapse back to our current case of $\text{(5.6)}$. Thus, given Managers have restricted investment opportunity sets, Managers’ risk aversion $\eta_M$ play the standard role in the usual principal-agent literature under decentralization.

### 5.3.3 Managers’ private costs

Managers’ private costs play a differentiating effect on centralization and decentralization.

Decentralized delegation cannot tolerate high levels of private costs $c$ by both Manager A and Manager B before no contract to implement the Principal’s desired strategy pair $(\theta_H, \tau_H)$ can exist. Unlike centralized delegation where the Manager C can take an arbitrary contract and trade it to maximize the risk-return trade-offs for himself first, this is distinctly not the case for decentralized delegation. In decentralized delegation, both Manager A and Manager B have completely dedicated themselves to one particular strategy from their respective asset classes, and cannot further form portfolios to maximize risk-return trade-offs. Thus, although Manager A and Manager B are truly risk averse, from the perspective of incentive compatibility, they behave like risk neutral individuals. That is to say, both Manager A and Manager B only care about the private costs $c$ and also the mean return differences $\Delta \mu_{\theta}$ and $\Delta \mu_{\tau}$ between the compliant strategy and the deviant strategy in their own asset class, and do not care about second moment effects of volatility nor correlation and even their own risk aversions. And due to this “risk neutrality” in determining incentive compatibility, contracting with decentralized individuals with high private costs could become prohibitively costly, and so much so that a contract to implement the Principal’s desired strategy pair $(\theta_H, \tau_H)$ could fail to exist.

In contrast, centralized delegation can tolerate a higher level of private costs $c$ before no contract can exist. Given any contract, since Manager C is risk averse, he will pick portfolios that generate a high portfolio mean return and a low portfolio volatility. Indeed, save for the differences in risk aversion levels between the Principal and Manager C, the portfolio choice behavior of Manager C is analogous to that of the Principal, were the Principal to have direct access to the financial markets. Thus, Manager C behaves like a “quasi-Principal” and hence, private costs $c$ only play a second order effect. This is why for moderately high levels of private costs $c$, the compliant Manager C must pay $2c$ and yet a centralized contract will still exist for Principal to implement his desired strategy pair $(\theta_H, \tau_H)$. In sharp contrast, for these same moderately

---

17 As discussed earlier, we had assumed all strategies have equivalent volatilities. But it is not difficult to see that even if strategies in each of the asset classes have different volatilities, the fact that private costs $c$ will still play a first order effect in Manager’s consideration for deviation in decentralization.
high levels of private costs $c$, decentralized contracts may fail for Manager $A$ and Manager $B$.

### 5.4 Numerical illustrations

To gain a fuller understanding of the differences and similarities between second best centralized delegation and second best decentralized delegation, we now turn to some numerical illustrations of our results. As one can surmise from Proposition 5.1 and Proposition 5.2, it is easiest to display these results in a numerical and graphical fashion. Thankfully, despite the perhaps complex structures of the optimal portfolios and fees, and thus extending to their respective Principal’s value functions, in that they are often highly nonlinear in the economic parameters of interest, the results are nonetheless rather straightforward to compute numerically; especially since we actually do have closed form analytical answers for all of the results. A more explicit analytical solution to the difference in value functions between centralization and decentralization is available under the extreme case when there is only moral hazard over mean returns; see Section B.1.

The base parameters that we will use in the numerical illustrations are given in Table 1, unless plotted otherwise. In the figures below, we need to distinguish between two different types of “better”. The first type

| Principal’s risk aversion parameter | $\eta_P$ |
| Managers’ risk aversion parameter  | $\eta_M$ |
| Compliant investment strategies’ mean returns, $\mu \equiv \mu_{\theta H} = \mu_{\tau H}$ | $\mu$ |
| Mean return on deviant strategy $\theta_L$ | $\mu_{\theta L}$ |
| Mean return on deviant strategy $\tau_L$ | $\mu_{\tau L}$ |
| Volatility of all strategies | $\sigma$ |
| Correlation coefficient of compliant pair ($\theta_H, \tau_H$) | $\rho_{\theta H, \tau H}$ |
| Correlation coefficient of deviant pair ($\theta_H, \tau_L$) | $\rho_{\theta H, \tau L}$ |
| Correlation coefficient of deviant pair ($\theta_L, \tau_H$) | $\rho_{\theta L, \tau H}$ |
| Correlation coefficient of deviant pair ($\theta_L, \tau_L$) | $\rho_{\theta L, \tau L}$ |
| Managers’ private costs | $c$ |

**Table 1:** The base parameter assumptions used in Section 5.4.

is when contracts for implementing $(\theta_H, \tau_H)$ exist for both centralization and decentralization; the darker colors indicate which form of delegation is better under this circumstance. The second type is when contracts for implementing $(\theta_H, \tau_H)$ does not exist under one form of delegation, while it does exist for another form of delegation. In this second type, the form of delegation that has contract existence is better, by default; this is indicated by the lighter colors.

In Figure 3, we see that high correlation $\rho_{\theta H, \tau H}$ for the compliant strategy pair $(\theta_H, \tau_H)$ will favor decentralization, while low correlation will favor centralization. This is inherited from the optimal risk sharing result of first best in Proposition 4.3. However, in the presence of moral hazard, when $\rho_{\theta H, \tau H}$ is sufficiently high, a centralized contract to implement $(\theta_H, \tau_H)$ for the Principal will not exist. Recalling the discussion on the investment opportunity set in Section 5.3.1 for any performance fee $y_C \in [0, 1]$, the term
$-\frac{1}{2} \sigma^2 (\rho \tau - \rho_{\theta_H, \tau_H}) y_C^2$ is the difference between the contract volatility for Manager $C$ implementing a deviant strategy pair $(\theta', \tau') \in S_{-(\theta_H, \tau_H)}$, versus that of the compliant strategy pair $(\theta_H, \tau_H)$. As the correlation $\rho_{\theta_H, \tau_H}$ of the compliant strategy pair increases, Manager $C$ will incur a high contract volatility for being compliant, whereas a low contract volatility for being deviant. Thus, when the correlation $\rho_{\theta_H, \tau_H}$ is sufficiently high, Manager $C$ will surely deviate for any performance fee $y_C$ to lower the contract volatility for himself, which then leads to nonexistence of a contract in centralization to implement the Principal’s desired strategy pair $(\theta_H, \tau_H)$. For decentralization, high private costs $c$ will also lead to contract nonexistence for implementing the Principal’s desired strategy pair $(\theta_H, \tau_H)$; this effect is as discussed in Section 5.3.3.

In Figure 3, we again see the effects of the relaxed investment opportunity set of Section 5.3.1 under centralization. In this example, consider the deviant strategy $\tau_L$ of the asset class $\tau$ (the case for the strategy $\theta_L$ of the asset class $\theta$ is similar). Recall the long-short opportunity cost under centralization is $\frac{1}{2} \rho \sigma^2 (\mu_{\tau} - \mu_{\tau'})(1 - \rho \tau', \tau)$ for the deviant strategy pairs $(\theta', \tau') \in S_{-(\theta_H, \tau_H)}$. When the deviant strategy is $\tau' = \tau_L$, if its mean return $\mu_{\tau_L}$ is low, Manager $C$ can take small long or even short positions in $\tau_L$ to finance large positions in strategies in the asset class $\theta$. So, as $\mu_{\tau_L}$ decreases, the long-short opportunity cost for Manager
Figure 4: Comparing the Principal’s value function under centralization versus decentralization: deviant strategy mean return $\mu_{r_l}$ versus private costs $c$.

$C$ increases, making centralized delegation unfavorable. In contrast, this opportunity cost does not exist in decentralization. Furthermore, recall that Manager $B$ is responsible for managing asset class $\tau$. As $\mu_{r_l}$ decreases, the expected performance fee payoff $y_{B}\mu_{r_l}$ for Manager $B$ when he deviates from the compliant strategy $\tau_H$ to the deviant strategy $\tau_L$ also decreases, and thereby making deviation less profitable for him. Thus when this happens, the performance fees for Manager $B$ could reach that of the first best result, and thereby making decentralization favorable. However, we note that as $\mu_{r_l}$ increases and as it approaches the mean return $\mu_{r_H} = \mu$ of the compliant strategy $\tau_H$, the payoff in performance fees for Manager $B$ to be compliant and deviant become similar. However, Manager $B$ still needs to incur a private cost $c$ to implement the Principal’s desired strategy; in such a case when the net benefit for being compliant rather than deviant is small, while Manager $B$ still needs to incur private costs $c$, Manager $B$ will for sure deviate. As a result, a decentralized contract for implementing the Principal’s desired strategy pair $(\theta_H, \tau_H)$ could fail to exist, as Manager $B$ will for sure deviate. This observation here will motivate our discussion of the dynamic model in Section 6.

In Figure 4 we see the effects of strategy volatility $\sigma$ and the correlation $\rho_{\theta_H, \tau_H}$ of the compliant pair
on the contracting environment. For low correlations, as volatility $\sigma$ increases, it will favor centralization because of the optimal risk sharing effect as discussed even in the first best setup of Proposition 4.3. As already discussed in Figure 3, high correlation $\rho_{h_1, h_2}$ of the compliant strategy pair will increase the contract volatility for Manager $C$. Here, volatility also brings about another perspective on this long-short opportunity cost. As volatility $\sigma$ decreases across all strategies, Manager $C$ will care even more about the mean difference between the deviant strategy pairs, and thus place more extreme long and short positions. This increases the opportunity cost for Manager $C$ to be compliant, and thereby making centralization unfavorable.

In Figure 6, we study the effects of the Principal risk aversion $\eta_P$ and the Managers’ risk aversion $\eta_M$ on the contracting environment. As discussed in Section 5.3.2, in centralization when Manager $C$ becomes less risk averse, he will more extreme long-short profits in the deviant strategy pairs, and it will become ever more costly for Principal to induce Manager $C$ to be compliant. In decentralization, thanks to Assumption 3.2 that volatilities are identical across all strategies, Manager $A$ and Manager $B$ will not factor in their risk aversion in a deviation. Note that in one extreme when Manager $C$ is highly risk averse while the Principal is relatively less risk averse, centralization will be favored.
6 Dynamic Model

6.1 Motivation

In Section 5, we discussed the various effects of moral hazard on static centralized delegation and decentralized delegation under second best. However, in spite of our discussions, there is however one distinct important case where decentralized delegation is particularly fragile compared to centralization.

6.1.1 Failure of static decentralized delegation

Suppose the strategy returns within the asset class $\theta$ and $\tau$ have similar mean returns. That is, suppose $\Delta \mu_\theta \approx 0$ and $\Delta \mu_\tau \approx 0$. To fix ideas on when this is possible, recall again the motivating example discussed in Footnote 3 of the Introduction. Here, we can be more concrete with that example. Recalling the centralized delegation incentive compatibility constraint (5.1), in this case when the mean strategy returns within each asset class are similar, a contract to induce the centralized Manager $C$ to take the Principal’s desired strategy
pair \((\theta_1, \tau_1)\) over other deviant strategy pairs will exist. Indeed in this case when \(\Delta \mu_0 \approx 0\) and \(\Delta \mu_\tau \approx 0\), the opportunity cost for foregone long-short trading profits are actually reduced, and thereby making centralized delegation even more attractive. However, in this case the incentive compatibility constraints (5.10) for Manager A and Manager B, respectively, in decentralized delegation are,

\[
0 \geq c - (1 - \pi)y_A \Delta \mu_0 \approx c \quad \text{and} \quad 0 \geq c - \pi y_B \Delta \mu_\tau \approx c.
\]

The condition \(0 \geq c\) is clearly impossible to satisfy unless the private costs are trivially small, \(c \approx 0\). For Manger A, the difference in mean returns, \(\Delta \mu_0\), between the compliant strategy \(\theta_1\) and the deviant strategy \(\theta_L\) represents the benefit to Manager A for compliance in terms of higher expected performance fees. And when when \(\Delta \mu_0 \approx 0\), the benefit for being compliant is small while Manager A still needs to incur a private cost \(c\). In this situation, it is impossible for the Principal to incentivize Manager A to implement the Principal’s desired strategy \(\theta_1\), as Manager A will surely deviate. Similar observations hold for Manager B with respect to his asset class \(\tau\). Economically, this is because neither of the decentralized Managers are affected by the strategies’ correlations when considering a deviation. Indeed, in a static decentralization, only the Principal reaps the diversification benefits via the strategies’ return correlation.

This issue hints at a severe loss of efficiency for the decentralized delegation form. Here, we have a setup whereby the decentralized Managers cannot be motivated and coordinated to take on the compliant strategy pair for the Principal. An obvious and correct response to this is to simply declare that for these asset classes with such mean return properties, centralized delegation is by default better than decentralized delegation. However, this is a rather unsatisfying response, and it is of interest to study mechanisms whereby we can still correctly incentivize Managers in decentralized delegation.

Remark 6.1. When dynamics are available, a conceivable natural solution to this problem is through the use of benchmarks to ensure that Managers had indeed implemented the desired strategy. Indeed, van Binsbergen et al. (2008) have argued that a carefully designed benchmark can be used to align the incentives of multiple decentralized delegated portfolio managers. However, the use of benchmarks is problematic in light of the discussion of Admati and Pfleiderer (1997), who argue that benchmarks are not useful in providing incentives to Managers, and also echoed in the discussion earlier in Section 5.3.1, allowing the Managers to access the financial markets could allow them to modify the effects of the contract. And most importantly, from an empirical perspective, direct benchmarked compensation contracts are simply not prevalent in hedge funds and private equity type investments whereby the contracts are usually based on the raw returns of the

\(\footnote{One should note, however, van Binsbergen et al. (2008) is not a model of moral hazard. And thus, what they call as “aligning incentives” is really simply better risk sharing in a first best case.} \)
managed assets; see Getmansky, Lee, and Lo (2013) for a recent review. Heinikel and Stoughton (1994) also consider a dynamic two-period delegated portfolio management problem with linear contracts to incentivize managers to acquire information; however, the authors critically assume the managers are risk neutral, and hence is silent to our intertemporal income hedging motive of the Managers that we emphasize as a key economic channel, in both centralized and decentralized delegation.

Furthermore, from a contracting theory perspective, this problem has two obvious candidate solutions even in a static setup. The first one is via a team based contracting scheme\(^\text{19}\) where the two decentralized Managers take on disjoint actions but their compensation is from a common source; for instance, consider when Manager A’s and Manager B’s compensations are dependent not on their own fund returns but actually on the Principal’s terminal wealth. The second one is via a tournament contracting scheme\(^\text{20}\) where Manager A’s compensation depends on not only his own fund performance but also that of Manager B, and vice-versa. However, neither the first type nor the second type of compensation are observed in practice. The current practice remains that a fund’s compensation is only dependent its own return performance.\(^\text{21}\)

In all, a direct expansion of the static contracting space will not solve the problem of coordinating decentralized Managers return strategy dependence in the presence of moral hazard.

6.1.2 Joint incentivization via reinvestments

The use of reinvestments in a dynamic model is a potential mechanism to induce the correct incentivization in decentralized delegation. Suppose the Principal commits to contracting with the Managers for one additional period. For each asset class, strategies chosen by Managers have a committed long term effect: once strategies \((\theta, \sigma)\) have been chosen and committed to at the beginning of the contract, the same set of strategies will be executed in both the first period and in the subsequent period.

It should be strongly emphasized that by contracting for another period, we are not relying on the arguments of repeated interactions as in the repeated dynamic principal-agent literature.\(^\text{22}\) The strategy choice is long term so the Principal need not perform statistical inference over time to learn the true action taking by the Managers and thereby punish or reward accordingly in subsequent periods. Moreover, the key mechanism that we require is the path dependence of Principal’s wealth on the Managers’ long term compensation. In decentralization, once we allow for a subsequent period of contracting, the more wealth

\(^{19}\)See Marschak (1955) and Marschak and Radner (1972).

\(^{20}\)See Vafea and Stiglitz (1983) and more recently for an explicit example in portfolio delegation and competition for fund flows, Basak and Makarov (2013).

\(^{21}\)It is beyond the scope of the paper to study why is it that the current practice does not incorporate these alternative compensation schemes as suggested by contracting theory. However, this author speculates that these team based and tournament based compensation schemes strongly depend on all agents in the game to know of other agents’ existence and characteristics. This condition could be difficult to execute in practice.

\(^{22}\) Say, for instance, Mailath and Samuelson (2000).
the Principal has at the end of the initial period, the more wealth the Principal can allocate to both Managers for investing in the subsequent period. Thus, the next period compensation for the multiple Managers would heavily depend on the Principal’s wealth level from the previous period. Thus, this path dependence in next period compensation would induce a hedging motive for the multiple Managers in the initial period. In particular, this hedging motive comes from the intertemporal covariance between a particular Manager’s future wealth and the Principal’s wealth, of which this compounds in all Managers’ strategy pair correlations. Through this mechanism, the decentralized Managers will become concerned about the correlation of strategies amongst each other when considering a deviation.

6.2 Additional assumptions in dynamics

The economic setup of the dynamic model is essentially identical to that of the static model in Section 3, but we will need a few adjustments to account for the temporal nature of the dynamic problem. The individuals in this economy are still equivalent to before. Instead of contracting for just periods \( t = 0, 1 \), the contracting period is now extended to \( t = 0, 1, 2 \).

The asset class \( \theta \) with investment strategies \( \{\theta_L, \theta_H\} \) now has per-period-returns \( R_{\theta,t} \), for \( t = 1, 2 \); and likewise, the asset class \( \tau \) with strategies \( \{\tau_L, \tau_H\} \) now has per-period-returns \( R_{\tau,t} \) for \( t = 1, 2 \). We should note that this is not a dynamic nor repeated moral hazard model in the usual principal-agent literature. In particular, the Managers could not make private choices on the strategies \( \theta, \tau \) in both periods \( t = 0 \) and \( t = 1 \). Rather, this is a model where the Managers commit to a particular strategy at \( t = 0 \) but simply contracts with the Principal for two periods.

In what follows, we will denote \( E_t \) as the time \( t \) conditional expectation, \( \text{Var}_t \) as the time \( t \) conditional variance, \( \text{Cov}_t \) as the time \( t \) conditional variance, and \( \text{Corr}_t \) as the time \( t \) conditional correlation, for \( t = 0, 1 \).

We will make two assumptions.

**Assumption 6.2.** Let \( R_{\theta,t} \) and \( R_{\tau,t} \) be the period \( t = 1, 2 \) returns of investment strategies \( \theta, \tau \). We assume the time \( t = 1 \) returns of all investment strategies are independent of their \( t = 2 \) counterparts.

**Assumption 6.3.** Assume Assumption 6.2. Furthermore assume the following.

(i) The conditional means of all strategies are equivalent across time; that is, \( \mu_\theta \equiv E_t[R_{\theta,t+1}] \) and \( \mu_\tau \equiv E_t[R_{\tau,t+1}] \) for \( t = 0, 1 \).

(ii) The conditional volatility of all strategies are equivalent across time; that is, \( 0 < \sigma^2 \equiv \text{Var}_t(R_{\theta,t+1}) = \text{Var}_t(R_{\tau,t+1}) \) for all strategies \( \theta, \tau \) and all time \( t = 0, 1 \).
(iii) The conditional correlations of strategy pairs are equivalent across time; that is, 
\[ \rho_{\theta,\tau} = \text{Corr}_t(R_{\theta,t+1}, R_{\tau,t+1}) \]
for all strategies \( \theta, \tau \) and all time \( t = 0, 1 \).

As we begin to discuss dynamic portfolio choice and dynamic contracting, it becomes quite clear the fashion in which state variables statistically relate to each other across time are critically important. Assumption 6.2 immediately rules out stochastic and time varying means, volatility and correlation between investment strategies, which is admittedly the strongest assumption above. While we do impose the assumption of independent distributions across time, we do not need to impose identical distributions. In particular, it is not difficult to actually extend our current model to allow non-identical (but independent) distributions across time. But allowing some moments (i.e. Assumption 6.3) to be equivalent across time, makes some of the discussion easier, and can be extended to be more general case at the cost of some loss of tractability.

### 6.2.1 Dynamic Decentralized Delegation

In all, the optimization problem for decentralized delegation is as follows. Please see Figure 7 for the time line.

<table>
<thead>
<tr>
<th>Principal makes</th>
<th>Managers A, B make investment strategy choices ( \theta ) and ( \tau )</th>
<th>Principal gets aggregated wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 0, 1 ) portfolio weight choices: (i) ( 1 - \pi_1 \in \mathbb{R} ) to Manager A; and (ii) ( \pi_1 ) to Manager B</td>
<td></td>
<td>( w_{P,1}^{(\theta,\tau)} = w_{P,1}^{(\theta,\tau)} )</td>
</tr>
<tr>
<td>Principal offers contracts ( x_A, (y_{A,0}, y_{A,1}) \in \mathbb{R} \times [0, 1]^2 ) to Manager A, and ( x_B, (y_{B,0}, y_{B,1}) \in \mathbb{R} \times [0, 1]^2 ) to Manager B for ( t = 0, 1 )</td>
<td>Managers accepts or rejects the contract</td>
<td>Principal receives returns ( \pi_0 R_{\tau,1} + (1 - \pi_0) R_{\theta,1} ); pays ( (1 - \pi_0) y_{A,0} R_{\theta,1} ) to Manager A, and pays ( \pi_0 y_{B,0} R_{\tau,1} ) to Manager B</td>
</tr>
<tr>
<td>( t = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Principal reinvests ( w_{P,1}^{(\theta,\tau)}(1 - \pi_1) ) to Manager A ( w_{P,1}^{(\theta,\tau)} \pi_1 ) to Manager B</td>
<td>Principal receives ( w_{P,1}^{(\theta,\tau)}(1 + \pi_1 R_{\tau,2} + (1 - \pi_1) R_{\theta,2}) ); pays ( w_{P,1}^{(\theta,\tau)}(1 - \pi_1) y_{A,1} R_{\theta,2} ) to Manager A, and pays ( w_{P,1}^{(\theta,\tau)} \pi_1 y_{B,1} R_{\tau,2} ) to Manager B</td>
<td>Manager A receives (-c(\theta) + x_A + w_{A,1}^{(\theta,\tau)} + w_{P,1}^{(\theta,\tau)}(1 - \pi_1) y_{A,1} R_{\theta,2} ); Manager B receives (-c(\tau) + x_B + w_{B,1}^{(\theta,\tau)} + w_{P,1}^{(\theta,\tau)} \pi_1 y_{B,1} R_{\tau,2} )</td>
</tr>
<tr>
<td>( t = 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Figure 7: Dynamic decentralized delegation time line.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[
\sup_{x_A, x_B \in \mathbb{R}, y_{A, t} \in [0, 1], y_{B, t} \in [0, 1]} \sup_{\pi_0, \pi_1} -x_A - x_B + E_0[W_{P, 2}^{(\theta_0, \tau_0)}] - \frac{\eta P}{2} \text{Var}_0(W_{P, 2}^{(\theta_0, \tau_0)}), \tag{DynDec}
\]

subject to,

\[
W_{P, 1}^{(\theta, \tau)} := 1 + (1 - \pi_0)R_{\theta, 1} + \pi_0 R_{\tau, 1} - (\pi_0 y_{B, 0} R_{\theta, 1} + (1 - \pi_0) y_{A, 0} R_{\theta, 1}), \tag{6.1a}
\]

\[
W_{P, 2}^{(\theta, \tau)} := W_{P, 1}^{(\theta, \tau)} [1 + (1 - \pi_1)R_{\theta, 2} + \pi_1 R_{\tau, 2} - ((1 - \pi_1) y_{A, 1} R_{\theta, 2} + \pi_1 y_{B, 1} R_{\tau, 2})], \tag{6.1b}
\]

\[
W_{A, 1}^{(\theta, \tau)} := (1 - \pi_0) y_{A, 0} R_{\theta, 1}, \tag{6.1c}
\]

\[
W_{A, 2}^{(\theta, \tau)} := W_{A, 1}^{(\theta, \tau)} + W_{P, 1}^{(\theta, \tau)} (1 - \pi_1) y_{A, 1} R_{\theta, 2}, \tag{6.1d}
\]

\[
W_{B, 1}^{(\theta, \tau)} := \pi_0 y_{B, 0} R_{\tau, 1}, \tag{6.1e}
\]

\[
W_{B, 2}^{(\theta, \tau)} := W_{B, 1}^{(\theta, \tau)} + W_{P, 1}^{(\theta, \tau)} \pi_1 y_{B, 1} R_{\tau, 2}, \tag{6.1f}
\]

\[
0 \leq x_A - c + E_0[W_{A, 2}^{(\theta, \tau_0)}] - \frac{\eta M}{2} \text{Var}_0(W_{A, 2}^{(\theta, \tau_0)}), \tag{6.1g}
\]

\[
0 \leq x_B - c + E_0[W_{B, 2}^{(\theta, \tau_0)}] - \frac{\eta M}{2} \text{Var}_0(W_{B, 2}^{(\theta, \tau_0)}), \tag{6.1h}
\]

\[
\theta_H = \arg \max_{\theta} x_A - c(\theta') + E_0[W_{A, 2}^{(\theta', \tau_0)}] - \frac{\eta M}{2} \text{Var}_0(W_{A, 2}^{(\theta', \tau_0)}), \tag{6.1i}
\]

\[
\tau_H = \arg \max_{\tau'} x_B - c(\tau') + E_0[W_{B, 2}^{(\theta_0, \tau')} - \frac{\eta M}{2} \text{Var}_0(W_{B, 2}^{(\theta_0, \tau')}). \tag{6.1j}
\]

See also Remark 6.3 for notes on the slight differences between the economic setup here of the dynamic model and that of the static model in Section 3.

In the decentralized objective function of the Principal in (DynDec), the Principal needs to choose the optimal fixed fees \(x_A, x_B\) and the \(t = 0\) and \(t = 1\) performance fees \(y_{A, 0}, y_{A, 1}\) and \(y_{B, 0}, y_{B, 1}\), respectively for Manager A and B, and the portfolio weights \(\pi_0, \pi_1\) for Manager B. After Manager A and Manager B are paid their performance fees \(y_{A, 0}, y_{B, 0}\) at \(t = 1\), we assume for simplicity that neither Manager will reinvest their collected fees. The budget constraints for the Principal are (6.1a), (6.1b), and the budget constraints for Manager A and B are respectively (6.1c), (6.1d) and (6.1e), (6.1f). Given that both Manager A and Manager B are initially endowed with zero amount of wealth, (6.1g) and (6.1h) are their individual rationality constraints. Likewise, (6.1i) and (6.1j) are their respective incentive compatibility constraints for the Principal to induce the two Managers to take on the Principal’s strict preferred \((\theta_H, \tau_H)\) investment strategy pair.

**Remark 6.4.** Let’s clarify some of the notations and the interpretations of the timing.

- This is a model of commitment and thus the Principal commits to both Manager A and Manager B on
his current and future portfolio and performance fee policies.

- To be clear on the returns notation, we write $R_{\theta,1}$ to be the net return from $t = 0$ to $t = 1$ for strategy $\theta$, and $R_{\theta,2}$ to be the net return from $t = 1$ to $t = 2$. Analogous comments apply for strategy $\tau$ with net returns notation $R_{\tau,t}$.

- We should discuss the notation of the portfolio and performance fee policies. At $t = 0$, the Principal will allocate $1 - \pi_0$ of his initial wealth to Manager A, and $\pi_0$ to Manager B. The Principal will also at $t = 0$ promise performance fees $y_{A,0}$ and $y_{B,0}$ to Manager A and Manager B, respectively, at $t = 1$. Thus for instance, the total performance fee payoff for Manager A at $t = 1$ for implementing strategy $\theta$ is $(1 - \pi_0)y_{A,0}R_{\theta,1}$. We will call $\pi_0$ to be the $t = 0$ portfolio policy, and $y_{A,0}$ be the $t = 0$ performance fee policy to Manager A, even though the performance fees are actually paid at $t = 1$. We keep this timing convention notation because the randomness in the returns are indeed realized at $t = 1$. Thus, the portfolio and performance policy must be known in advance, at $t = 0$, in order for the model to be nontrivial. Hence, we refer to policies at the time of decision, and not at the time of payment. Likewise, we will call $1 - \pi_1$ be the $t = 1$ portfolio policy to Manager A and $y_{A,1}$ be the $t = 1$ performance fee policy, even though the actual performance fee payoff to Manager A is at $t = 2$ for the quantity $w_{P,1}^{(\theta,\tau)}(1 - \pi_1)y_{A,1}R_{\theta,2}$. Analogous comments apply for Manager B.

Remark 6.5. While the economic setup in the dynamic model is largely identical to that of the static model setup in Section 3, there are two minor differences that are worth noting. Firstly, we have assumed that the fixed fees $x_A, x_B$ to Manager A and Manager B, respectively, are a lump sum figure and only paid at the contract terminal date $t = 2$. That is, only performance fees are paid at $t = 1$ and $t = 2$. This is largely to simplify the analysis. Secondly, the private costs $c(\theta)$ and $c(\tau)$ are — akin to the fixed fees — are now paid at $t = 2$. Thus, even though the Managers commit to their investment strategy choice at $t = 0$, the costs are only incurred at $t = 2$. This, again, is largely to simplify the problem. Alternatively, we may think of the private cost not as a wealth cost but as a utility cost incurred at $t = 0$.

Finally, in light of the modifications of the timing of the fixed fees and private costs as discussed above, as opposed to the static model of Section 3, we slightly reinterpret the budget constraints $W_{k,t}^{(\theta,\tau)}$ the individual $k$ at time $t$. Whereas the wealths in the static model are inclusive of the private costs and the fixed fees, in the dynamic model we interpret the wealths are exclusive of the private costs and the fixed fees. This largely is done to simplify the notations.

Remark 6.6. We discuss the analogous dynamic centralized delegation model in Section D. As the core motivation for introducing a dynamic model is to fix an issue specific to decentralized delegation in the static model, we will thus keep the main discussion of a dynamic model to decentralized delegation.
7 First Best in Dynamics

7.1 Dynamic Decentralized Delegation in First Best

Let us begin by considering the first best setup, whereby the Principal can directly observe and contract on the private investment strategy choices of the Managers.

**Proposition 7.1.** Consider the first best dynamic decentralized delegation problem; so consider \((\text{DynDec})\) without the incentive compatibility constraints \((\text{6.1i})\) and \((\text{6.1j})\). Recall that the Principal wants to implement strategy pair \((\theta_1, \tau_1)\).

(a) For any portfolio and performance fee policies \((\pi_0, \pi_1, y_{A,0}, y_{A,1}, y_{B,0}, y_{B,1})\), the optimal fixed fees for Manager A and Manager B, are, respectively,

\[
\hat{x}_{A,1}(\theta_1, \tau_1) = c - \mathbb{E}_0[W^{(\theta_1, \tau_1)}_{A,2}] + \frac{\eta_M}{2} \text{Var}_0(W^{(\theta_1, \tau_1)}_{A,2}),
\]
\[
\hat{x}_{B,1}(\theta_1, \tau_1) = c - \mathbb{E}_0[W^{(\theta_1, \tau_1)}_{B,2}] + \frac{\eta_M}{2} \text{Var}_0(W^{(\theta_1, \tau_1)}_{B,2}).
\]

(b) The \(t = 1\) optimal policies are given as follows.

(i) The optimal \(t = 1\) portfolio to allocate to Manager B is independent of fees and wealth effects,

\[
\tilde{x}_B^1 = \frac{1}{2}.
\]

(ii) The optimal \(t = 1\) performance fee policies chosen by the Principal to compensate Managers A and B are equivalent, and they are,

\[
\tilde{y}_{A,1}^1 = \tilde{y}_{B,1}^1 = \frac{\eta_T(1 + \rho_{\theta_1, \tau_1})}{\eta_T(1 + \rho_{\theta_1, \tau_1}) + \eta_M}.
\]

(c) The \(t = 0\) optimal policies are given as follows.

(i) The optimal \(t = 0\) portfolio to allocate to Manager B is again independent of fees and wealth effects,

\[
\tilde{x}_B^0 = \frac{1}{2}.
\]

(ii) The optimal \(t = 0\) (interior solution) performance fee policies chosen by the Principal to compensate
Manager A are,  
\[ \hat{y}_{A,0}^{FB} = \frac{\hat{Y}_{A,0}^N}{\hat{Y}_{A,0}^D}, \]

provided that \( \hat{y}_{A,0}^{FB} \in (0,1) \), and where,

\[ \hat{y}_{A,0}^N := 2\eta_0^2(1 + \rho_{0\text{H},\tau_H})^2[\eta_0^2 \sigma^2(1 + \rho_{0\text{H},\tau_H}) - 2\mu^2] \]
\[ + \eta_0^2 \eta_M (1 + \rho_{0\text{H},\tau_H})[4 + \mu(5 - \rho_{0\text{H},\tau_H} + \mu(3 + \rho_{0\text{H},\tau_H}))] \]
\[ + \eta_0^2 (\mu(5 - \rho_{0\text{H},\tau_H} + \mu(3 + \rho_{0\text{H},\tau_H})) + \eta_0^2 \sigma^4(1 + \rho_{0\text{H},\tau_H})^2)] + \eta_0^2 [\mu^2 + \eta_0^2(2 + \mu(5 + 4\mu - \rho_{0\text{H},\tau_H}))(1 + \rho_{0\text{H},\tau_H}) \sigma^2 + \eta_0^2 \sigma^4(1 + \rho_{0\text{H},\tau_H})^2], \]
\[ \hat{y}_{A,0}^D := \sigma^2 [2\eta_0^2(1 + \rho_{0\text{H},\tau_H})^3 + 2\eta_M^3 \]
\[ + \eta_0^2 \eta_M^2 (1 + \rho_{0\text{H},\tau_H})^2[6 + \mu(2 - 2\rho_{0\text{H},\tau_H} + \mu(3 + \rho_{0\text{H},\tau_H})) + \sigma^2(1 + \rho_{0\text{H},\tau_H})] \]
\[ + \eta_0^2 \eta_M^2 (1 + \rho_{0\text{H},\tau_H})[6 + 4\mu^2 + 2\mu(1 - \rho_{0\text{H},\tau_H}) + (1 + \rho_{0\text{H},\tau_H}) \sigma^2]]. \]

The performance fees to compensate Manager B are identical, \( \hat{y}_{B,0}^{FB} = \hat{y}_{A,0}^{FB} \).

Observing Proposition \( \text{E.44} \), under the Principal’s desired investment strategy pair \((\theta_H, \tau_H)\), and recalling Assumption \( \text{E.22} \) and Assumption \( \text{E.23} \), given that both strategies have equivalent means, it is no surprise that the \( t = 1 \) portfolio policy \( \hat{\pi}_1^{FB} \) would place equal weights between Manager A and Manager B. And indeed, the \( t = 1 \) optimal performance fees \( \hat{y}_{A,1}^{FB} \) and \( \hat{y}_{B,1}^{FB} \) for Manager A and Manager B, respectively, are identical in form to that of the static first best decentralized delegation problem of Proposition \( \text{E.44} \). For the \( t = 0 \) policies, and again given the return assumptions of Assumption \( \text{E.22} \) and Assumption \( \text{E.23} \), the Principal will still place equal portfolio weights \( \hat{\pi}_0^{FB} \) into Manager A and Manager B. Given that Manager A and Manager B have identical risk aversion \( \eta_M \) and identical outside options, it is no surprise that their \( t = 0 \) performance fees \( \hat{y}_{A,0}^{FB} \) and \( \hat{y}_{B,0}^{FB} \) would be identical. However, even though the portfolio policies of \( \hat{\pi}_0^{FB} \) and \( \hat{\pi}_1^{FB} \) are identical across time, the performance fees are not. The \( t = 0 \) performance fees will affect the amount of wealth \( W_{t=1}^{(\theta_H,\tau_H)} \) available for reinvestment at \( t = 1 \), and hence affecting the terminal \( t = 2 \) wealths of all individuals involved; thus the \( t = 0 \) performance fees \( \hat{y}_{A,0}^{FB} \) and \( \hat{y}_{B,0}^{FB} \) must take into account this intertemporal wealth hedging channel, and hence why these \( t = 0 \) performance fee policies will differ from that of the \( t = 1 \) policy (since one period later at \( t = 2 \) is the terminal contracting date). See Section \( \text{E.45} \) for numerical illustrations of Proposition \( \text{E.44} \).

\(^{23}\) We use “\( N \)” for numerator and “\( D \)” for denominator.
8 Second Best in Dynamics

Now we consider the second best dynamic delegation problem in its entirety.

8.1 Dynamic Decentralized Delegation in Second Best

**Proposition 8.1.** Consider the second best decentralized delegation problem (DynDec) in its entirety.

(a) For any portfolio and performance fee \((\pi_0, \pi_1, y_{A,0}, y_{A,1}, y_{B,0}, y_{B,1})\), the optimal fixed fees for Manager A and Manager B have the same form as that of the first best result of Proposition 7.1.

(b) Consider the \(t = 1\) optimal policies. Suppose the \(t = 1\) realized wealth under strategy pair \((\theta, \tau) \in \mathcal{S}\) is \(W_{t=1}^{(\theta, \tau)} = w_{t=1}^{(\theta, \tau)}\). Then for some constants \(\lambda_A, \lambda_B \in \mathbb{R}\):

(i) The optimal \(t = 1\) portfolio is,

\[
\hat{\pi}_{t=1}^{\lambda_A, \lambda_B} = \frac{\pi_{t=1}^{N, \lambda_A, \lambda_B}}{\pi_{t=1}^{D, \lambda_A, \lambda_B}},
\]

where the analytical forms of the numerator and denominator can be found in (J.8) of the Appendix.

(ii) The optimal \(t = 1\) performance fee to compensate Manager A is,

\[
\hat{y}_{A,1}^{\lambda_A, \lambda_B} = \frac{\hat{y}_{A,1}^{N, \lambda_A, \lambda_B}}{\hat{y}_{A,1}^{D, \lambda_A, \lambda_B}},
\]

where the analytical forms of the numerator and denominator can be found in (J.9) of the Appendix.

(iii) The optimal \(t = 1\) performance fee to compensate Manager B is,

\[
\hat{y}_{B,1}^{\lambda_A, \lambda_B} = \frac{\hat{y}_{B,1}^{N, \lambda_A, \lambda_B}}{\hat{y}_{B,1}^{D, \lambda_A, \lambda_B}},
\]

where the analytical forms of the numerator and denominator can be found in (J.10) of the Appendix.

(c) The \(t = 1\) continuation utilities for the Principal, Manager A and Manager B under the strategy pair \((\theta, \tau) \in \mathcal{S}\) are as follows. For notational simplicity, we will denote \(\hat{\pi}_1 := \hat{\pi}_{t=1}^{\lambda_A, \lambda_B}, \hat{y}_{A,1} := \hat{y}_{A,1}^{\lambda_A, \lambda_B}, \hat{y}_{B,1} := \hat{y}_{B,1}^{\lambda_A, \lambda_B} \).
The optimal constants $\hat{\lambda}_A, \hat{\lambda}_B \in \mathbb{R}$ are the solution to the problem,

$$
\inf_{\lambda_A, \lambda_B \in \mathbb{R}, \text{and at least one of them are nonzero}} \hat{U}_{P,0}^{\lambda_A, \lambda_B}.
$$

The analytical forms (see Section (ii)) of these $t = 1$ optimal policies all have a fractional form, where both
the numerator and denominator are separately nonlinear in the \( t = 1 \) realized wealths of both compliant and deviant strategy pairs. As we explain below, this fractional form of the optimal policies at \( t = 1 \) will drive the value-at-risk (VaR) constraint interpretation of the \( t = 0 \) portfolio and performance policies.

### 8.1.1 The \( t = 1 \) portfolio and performance fee policies

Let’s begin with a discussion of the \( t = 1 \) portfolio and performance fee policies \((\pi_1, y_{A,1}, y_{B,1})\). To make the argument concrete, for any given contract, let \( \tilde{U}_{P,t}^{(\theta_H, \tau_H)} \) be the Principal’s time \( t = 0, 1 \) continuation utility under the compliant investment strategy pair \((\theta_H, \tau_H)\), and let \( U_{A,t}^{(\theta, \tau)} \) and \( U_{B,t}^{(\theta, \tau)} \) be the time \( t = 0, 1 \) continuation utilities of both Manager \( A \) and Manager \( B \) under a general strategy pair \((\theta, \tau) \in S\). In particular, the \( t = 1 \) continuation utilities for the Principal, Manager \( A \) and Manager \( B \), respectively, under the compliant investment strategy pair \((\theta_H, \tau_H)\) are,

\[
\begin{align*}
\tilde{U}_{P,1}^{(\theta_H, \tau_H)} &= E_1[W_{P,2}^{(\theta_H, \tau_H)}] - \frac{\eta_P}{2} \text{Var}_1(W_{P,2}^{(\theta_H, \tau_H)}), \\
U_{A,1}^{(\theta_H, \tau_H)} &= E_1[W_{A,2}^{(\theta_H, \tau_H)}] - \frac{\eta_M}{2} \text{Var}_1(W_{A,2}^{(\theta_H, \tau_H)}), \\
U_{B,1}^{(\theta_H, \tau_H)} &= E_1[W_{B,2}^{(\theta_H, \tau_H)}] - \frac{\eta_M}{2} \text{Var}_1(W_{B,2}^{(\theta_H, \tau_H)}).
\end{align*}
\]

As it is the case with first best, the Principal wants to design portfolio and performance fee policies to optimally risk share at the lowest possible cost possible with Manager \( A \) and Manager \( B \). This implies that, on the one hand, Principal wants to pick policies such that the sum of all individuals’ continuation utilities \( \tilde{U}_{P,1}^{(\theta_H, \tau_H)} + U_{A,1}^{(\theta_H, \tau_H)} + U_{B,1}^{(\theta_H, \tau_H)} \) is maximized.

However, Manager \( A \) and Manager \( B \) could have deviated at \( t = 0 \). If Manager \( A \) was compliant and Manager \( B \) was deviant, the resulting strategy pair would be \((\theta_H, \tau_L)\); if Manager \( A \) was deviant and Manager \( B \) was compliant, the resulting strategy pair would be \((\theta_L, \tau_H)\). Since the Principal only wants to implement \((\theta_H, \tau_H)\), there is no circumstance where Manager \( A \) and Manager \( B \) would deviate to the pair \((\theta_L, \tau_L)\). Thus, the incentive compatibility constraints at \( t = 0 \) to implement \((\theta_H, \tau_H)\) are,

\[
\begin{align*}
-c + U_{A,0}^{(\theta_H, \tau_H)} &\geq U_{A,0}^{(\theta_L, \tau_H)}, \\
-c + U_{B,0}^{(\theta_H, \tau_H)} &\geq U_{B,0}^{(\theta_L, \tau_H)},
\end{align*}
\]

where, for reasonable parameters in equilibrium, both constraints will bind. Now, let \( W_{P,1}^{(\theta_H, \tau_H)} = w_{P,1}^{(\theta_H, \tau_H)} \) and \( W_{P,1}^{(\theta_L, \tau_L)} = w_{P,1}^{(\theta_L, \tau_L)} \) be the Principal’s realized wealths at \( t = 1 \) under the two deviant strategy pairs \((\theta_L, \tau_H)\) and \((\theta_H, \tau_L)\), respectively.

\(^{24}\) See [Jorion 2006](#) for a survey of the value-at-risk (VaR) literature.
Here we argue why the Principal’s wealth can be used as an “intertemporal glue” to bridge the payoffs of the two Managers even in the presence of moral hazard. For instance from the budget constraint \((\pi_1, \pi_2)\), for any strategy pair \((\theta, \tau)\), Manager A’s expected performance fee payoffs under the compliant strategy pair, and are also nonlinear in the wealths \(w_{A,1}\) of Proposition \(\pi_1\). The Lagrange multipliers \(\lambda_A, \lambda_B \in \mathbb{R} \setminus \{0\}\) to the equality binding constraints, the Principal’s \(t = 1\) optimization problem will thus be maximizing the \(t = 1\) policies \((\pi_1, y_{A,1}, y_{B,1})\) over, \(^{26}\)

\[
\begin{align*}
U_{P,1}^{\pi_A, \lambda_B} &= \tilde{U}_{P,1}^{(\theta_H, \tau_H)} + U_{A,1}^{(\theta_H, \tau_H)} + U_{B,1}^{(\theta_H, \tau_H)} \\
&\quad - \lambda_A \left( U_{A,1}^{(\theta_H, \tau_H)} - (c + U_{A,1}^{(\theta_H, \tau_H)}) \right) - \lambda_B \left( U_{B,1}^{(\theta_H, \tau_H)} - (c + U_{B,1}^{(\theta_H, \tau_H)}) \right).
\end{align*}
\]

We optimize the \(t = 1\) policies to get \((\tilde{\pi}_A^{\lambda_A, \lambda_B}, \tilde{\pi}_A^{\lambda_A, \lambda_B}, \tilde{\pi}_B^{\lambda_A, \lambda_B})\) as per \((\mathbb{S}1\)), \((\mathbb{S}2\)) and \((\mathbb{S}3\)). The \(t = 1\) portfolio and performance fee policies \((\tilde{\pi}_A^{\lambda_A, \lambda_B}, \tilde{\pi}_A^{\lambda_A, \lambda_B}, \tilde{\pi}_B^{\lambda_A, \lambda_B})\) will be contingent on the realized wealths \(W_{P,1}^{(\theta_H, \tau_H)} = W_{P,1}^{(\theta_H, \tau_H)}\) of the compliant strategy pair \((\theta_H, \tau_H)\), and also the realized wealths \(W_{P,1}^{(\theta_H, \tau_H)} = W_{P,1}^{(\theta_H, \tau_H)}\) (when Manager B deviates) and \(W_{P,1}^{(\theta_H, \tau_H)} = W_{P,1}^{(\theta_H, \tau_H)}\) (when Manager A deviates). The resulting optimized \(t = 1\) aggregated continuation utility is \(\tilde{U}_{P,1}^{\lambda_A, \lambda_B}\) as per \((\mathbb{S}3\)). To ensure that both Manager A and Manager B would be compliant in equilibrium, the Principal must compensate both Manager A and Manager B for the difference in the mean and volatility in their performance fee payoffs under the compliant strategy pair \((\theta_H, \tau_H)\) versus that of the deviant strategy pairs \((\theta', \tau')\). This effect is intuitively quite similar to the opportunity cost effect of foregone alternative wealth realizations as per second best static centralization of Proposition \(\pi_1\). The Lagrange multipliers \(\lambda_A, \lambda_B\) give the appropriate scaling of said compensation difference.

Moreover, the optimal policies \((\tilde{\pi}_A^{\lambda_A, \lambda_B}, \tilde{\pi}_A^{\lambda_A, \lambda_B}, \tilde{\pi}_B^{\lambda_A, \lambda_B})\) will take on a fractional form, where the numerator and denominator are nonlinear in the Principal’s \(t = 1\) realized wealth \(w_{P,1}^{(\theta_H, \tau_H)}\) under the compliant strategy pair, and are also nonlinear in the wealths \(w_{P,1}^{(\theta_H, \tau_H)}\) and \(w_{P,1}^{(\theta_H, \tau_H)}\) that are realized under the deviant

\(^{25}\) See Section \(\mathbb{S}\) for the dynamic programming principle specific to mean-variance preferences so that the optimal policies are time consistent.
strategy pairs \((\theta_H, \tau_L)\) and \((\theta_L, \tau_H)\), respectively. The fractional forms are inherent from the mean-variance preferences of all individuals involved. Even from the classical Markowitz (1952) mean-variance formulation, it is immediate that the optimal portfolio policy there would follow a fractional form, where the numerator involves the mean returns of securities, and the denominator involves the volatilities of the securities and the initial wealth of an investor. A similar effect is at play here. To Manager A, for instance, \(w_1^{(\theta_H, \tau_H)} \cdot (1 - \pi_1^{A, \lambda_B}) \pi_1^{A, \lambda_B} \) is precisely the amount of after performance fees wealth that the Principal allocates to Manager A, and multiplying this term out, we can see that the numerator of this expression involves the risk-adjusted mean payoffs to the Principal, Manager A and Manager B under the wealths of the compliant strategy pair and wealths of the deviant strategy pairs, and the denominator involves the volatility of such payoffs and also the correlations with respect to the return strategies.

One should note that in the first best case of Proposition 7.1, when no moral hazard is present and so \(\lambda_A, \lambda_B = 0\), the optimal \(t = 1\) portfolio and performance fee policies have the simple form,

\[
\left( \hat{\pi}_1^{FB}, \hat{y}_{1,A}^{FB}, \hat{y}_{1,B}^{FB} \right) = \left( \frac{1}{2}, \frac{\eta(1 + \rho_{\theta_H, \tau_H})}{\eta(1 + \rho_{\theta_H, \tau_H}) + \eta_M}, \frac{\eta(1 + \rho_{\theta_H, \tau_H})}{\eta(1 + \rho_{\theta_H, \tau_H}) + \eta_M} \right).
\]

Under first best, the only motive for the Principal is to optimally risk share and indeed, the Principal would invest equally into both Manager A and Manager B, and the resulting performance fees are simply a re-weighting of their respective risk aversions by the correlation of the compliant strategy pair. In particular, in first best, no wealth effects are involved in the \(t = 1\) optimal policies, whereas in second best, the wealth effects driven by strategy pair deviations are distinctly present.

### 8.1.2 The \(t = 0\) portfolio and performance fee policies

We now need to determine the \(t = 0\) portfolio and performance fee policies \((\pi_0, y_{A,0}, y_{B,0})\). At this point, at least from a mechanical perspective, it is apparent why the full joint distribution of compliant and deviant return strategies \((R_{\theta_H, 1}, R_{\tau_H, 1}, R_{\theta_L, 1}, R_{\tau_L, 1})\) will be needed. The \(t = 1\) optimal policies are a fractional form of the \(t = 1\) wealths under compliant and deviant strategies of the Principal, which in turn depends on the \(t = 0\) policies \((\pi_0, y_{A,0}, y_{B,0})\). Given that we are using a mean-variance framework, it implies that we need to compute the \(t = 0\) expectation and variance of \(t = 1\) fractional form random variables to arrive at the \(t = 0\) optimal policies. Given that ratios of expectations are not the same as expectations of ratios, \(\frac{\text{E}[X]}{\text{E}[Y]} \neq \frac{\text{E}[X]}{\text{E}[Y]}\), and \(\text{Var}(X/Y) \neq \text{Var}(X)/\text{Var}(Y)\).
Economically, moral hazard implies a value-at-risk (VaR) type constraint on the \( t = 0 \) portfolio and performance fee policies \( (\pi_0, y_{A,0}, y_{B,0}) \). By the principle of dynamic programming for mean-variance preferences (see Section \( \square \)), the Principal’s \( t = 0 \) policies \( (\pi_0, y_{A,0}, y_{B,0}) \) is the solution to the problem \( (\text{SR}) \). Firstly, the \( t = 0 \) policies will have an intertemporal hedging effect on the individuals’ \( t = 2 \) terminal wealth volatility under the compliant strategy pair \( (\theta_1, \tau_1) \) and these are to be minimized, as seen in

\[
\frac{\partial}{\partial \theta_1} \text{Var}_0(\mathbb{E}_1 W_{P,2}^{(\theta_1, \tau_1)}) + \frac{\partial^2}{\partial \theta_1^2} \text{Var}_0(\mathbb{E}_1 W_{A,2}^{(\theta_1, \tau_1)}) + \frac{\partial^2}{\partial \theta_1^2} \text{Var}_0(\mathbb{E}_1 W_{B,2}^{(\theta_1, \tau_1)}).
\]

In these \( t = 0 \) variance computations, the “intertemporal glue” effect of the Principal’s wealth become apparent; these \( t = 0 \) variance computations will involve the \( t = 1 \) optimal policies, which then affect the wealths \( W_{P,1}^{(\theta_1, \tau_1)} \) under the compliant strategy pair, and the wealths \( W_{P,1}^{(\theta_1, \tau_1)} \) and \( W_{P,1}^{(\theta_1, \tau_1)} \) under the deviant strategy pairs. That is, the Principal’s wealths (both under compliant and deviant strategy pairs) enters into the covariance term of Manager A’s and Manager B’s wealths, and thereby through this “intertemporal glue”, Manager A’s and Manager B’s terminal wealth volatility are connected to each other. This effect is starkly absent in static decentralization. Secondly, there is a direct intertemporal hedging effect from \( t = 0 \) policies that Manager A and Manager B’s \( t = 2 \) terminal wealth volatilities under the compliant strategy pair \( (\theta_1, \tau_1) \) is weakly lower than that of the deviant strategy pairs; this is reflected in the condition \( \lambda_A(\frac{\partial^2}{\partial \theta_1^2} \text{Var}_0(\mathbb{E}_1 W_{A,2}^{(\theta_1, \tau_1)}) + \frac{\partial^2}{\partial \theta_1^2} \text{Var}_0(\mathbb{E}_1 W_{B,2}^{(\theta_1, \tau_1)})) \) for Manager A, and \( \lambda_B(\frac{\partial^2}{\partial \theta_1^2} \text{Var}_0(\mathbb{E}_1 W_{A,2}^{(\theta_1, \tau_1)}) + \frac{\partial^2}{\partial \theta_1^2} \text{Var}_0(\mathbb{E}_1 W_{B,2}^{(\theta_1, \tau_1)})) \) for Manager B. Thirdly and finally, the \( t = 0 \) policies must ensure that the continuation utility \( \mathbb{E}_0[\hat{U}^{\lambda_A, \lambda_B}] \) is maximized. These three points places not only restrictions on the types of \( t = 0 \) portfolios and performance fees the Principal can offer, but also these are also intimately linked with the return distribution — not just moments — of the compliant and deviant strategies.

### 8.1.3 Illustration of VaR constraints effects due to moral hazard

As we can see from Figures \( \square, \square, \square \) and \( \square \) (and see also its caption descriptions), whether a high or low realization in the wealths \( (w_{P,1}^{(\theta_1, \tau_1)}, w_{P,1}^{(\theta_1, \tau_1)}) \) of deviant strategy pairs at \( t = 1 \) makes a substantial difference in the \( t = 1 \) optimal policies. Effectively, if an extremely low realization of wealths under deviant strategy pairs are realized, to maintain incentive compatibility for those affected Managers, the Principal must redirect further extreme wealth allocations and performance fees to ensure that the terminal wealths between the continuation utilities under compliant strategy pair and the continuation utilities under deviant strategy pairs are equal. In contrast, when positive shocks are realized, the Principal need not consider extreme wealth allocation and performance fees for incentive compatibility. Clearly, the \( t = 0 \) optimal policies \( (\pi_0, y_{A,0}, y_{B,0}) \) of the Principal will endogenously alter the distribution of \( t = 1 \) wealths under the compliant and deviant strategy pairs; but as seen in those figures, the actual distribution of the strategy

\[\text{Note these } t = 1 \text{ wealths are random from the perspective at } t = 0.\]
returns also play a critical input.

Thus, due to extreme differences between the $t = 1$ optimal policies response to the positive and negative shocks, which then propagate back to the $t = 0$ optimal policy choices, we can thus see that the incentive compatibility constraints are precisely akin to a VaR constraint on the $t = 0$ portfolio policies, where downside return shocks play a far more prominent role than upside return shocks. Furthermore, this is why extreme downside tail risks and joint probability tail dependence play a critical role in the contracting environment, and this motivates the discussion in Section 8.1.4.
The \( t = 1 \) optimal policies of second best decentralized delegation when the wealths under deviant strategy pairs \((\theta_H, \tau_L)\) and \((\theta_L, \tau_H)\) have a positive shock. The base parameters are equivalent to that of Table 4 and with the amendment Table 5, which simply resets the deviant strategies to have equivalent means. We set the Lagrange multipliers \((\lambda_A, \lambda_B) = (-1.7, -1.7)\) as these are the numerical results associated with this set of base parameters when we actually numerically compute for the value function of the Principal. The vertical axis plots the respective \( t = 1 \) policies \((\bar{\pi}^{\lambda_A, \lambda_B}_1, y^{\lambda_A, \lambda_B}_{A, 1}, y^{\lambda_A, \lambda_B}_{B, 1})\), while the horizontal axis plots the \( t = 1 \) realized values \(W^{(\theta_L, \tau_H)}_{P, 1} = w^{(\theta_L, \tau_H)}_{P, 1}\) of the Principal under the compliant strategy pair \((\theta_H, \tau_H)\). The legended pairs of values correspond to various scenarios of \( t = 1 \) realized wealths \((W^{(\theta_L, \tau_H)}_{P, 1}, W^{(\theta_L, \tau_H)}_{F, 1})\) = \((w^{(\theta_L, \tau_H)}_{P, 1}, w^{(\theta_L, \tau_H)}_{F, 1})\) under deviant strategy pairs; these wealths represent a positive realization to the returns \((R_{\theta_H}, R_{\tau_L})\) and \((R_{\theta_L}, R_{\tau_H})\), such that the realized wealths, respectively, \(w^{(\theta_H, \tau_L)}_{P, 1}\) and \(w^{(\theta_H, \tau_L)}_{F, 1}\) are greater than the Principal’s \( t = 0 \) initial wealth of $1.

As an illustrative example, when the wealth \( w^{(\theta_L, \tau_H)}_{P, 1} \) (i.e. Manager A compliant, Manager B deviant) is higher than \( w^{(\theta_L, \tau_H)}_{P, 1} \) (i.e. Manager A deviant, Manager B compliant) and as the wealth \( w^{(\theta_H, \tau_L)}_{P, 1} \) under the compliant strategy pair increases, the Principal places less wealth \( \bar{\pi}^{\lambda_A, \lambda_B}_1 \) into Manager B and more wealth into Manager A. Furthermore, higher wealths \( w^{(\theta_L, \tau_H)}_{P, 1} \) will increase the \( t = 1 \) performance fees \((\bar{y}^{\lambda_A, \lambda_B}_{A, 1}, \bar{y}^{\lambda_A, \lambda_B}_{B, 1})\) for both Manager A and Manager B. However when the deviant wealth \( w^{(\theta_H, \tau_L)}_{P, 1} \) is high, it uniformly shifts up the performance fee \( \bar{y}^{\lambda_A, \lambda_B}_{A, 1} \) to Manager A, and uniformly shifts down the performance fee \( \bar{y}^{\lambda_A, \lambda_B}_{B, 1} \) to Manager B. Thus, in this case, there is fund in-flow and higher performance fees to the compliant Manager A, and fund out-flow and lower performance fees to the deviant Manager B. This is the case where the Principal is rewarding for compliance and punishing for deviance. But it is not necessarily true that being complaint necessarily implies higher fund flows.

Suppose, however, the situation is reversed in that \( w^{(\theta_H, \tau_L)}_{P, 1} \) is lower than \( w^{(\theta_L, \tau_H)}_{P, 1} \). Then in this case, the Principal will allocate higher portfolio weights \( \bar{\pi}^{\lambda_A, \lambda_B}_1 \) to Manager B, and lower portfolio weights to Manager A. Moreover, the performance fees \( y^{\lambda_A, \lambda_B}_{A, 1} \) to Manager A uniformly decreases, while the performance fees \( y^{\lambda_A, \lambda_B}_{B, 1} \) to Manager B uniformly increases. Thus in this case, even though Manager A was complaint while Manager B was deviant, there is now fund out-flow and lower performance fees to the compliant Manager A, and fund in-flow and higher fee performance fees to the deviant Manager B. This is the case where the Principal is essentially rewarding for luck.
Figure 9: The $t = 1$ optimal policies of second best decentralized delegation when the wealths under deviant strategy pairs $(\theta_H, \tau_L)$ and $(\theta_L, \tau_H)$ have a negative shock. The setup and layout of this figure is identical to that of Figure 8. However, we assume in this case the wealth $w^{(\theta_H, \tau_L)}$ under the deviant strategy pair $(\theta_H, \tau_L)$ had a positive shock due to positive realizations of $(R_{\theta_H}, R_{\tau_L})$, whereas the wealth $w^{(\theta_L, \tau_H)}$ under the deviant strategy pair $(\theta_L, \tau_H)$ had a negative shock due to negative realizations of $(R_{\theta_L}, R_{\tau_H})$.

It is evident that the negative shock case here is significantly different than that of the positive shock case of Figure 8. The discontinuity in the optimal performance fee process marks a cutoff in the wealth realization $w^{(\theta_H, \tau_H)}$ under the compliant strategy pair $(\theta_H, \tau_H)$. To the left of this cutoff the geometry of the portfolio policies are reversed to that of the right of this cutoff. Namely, for the portfolio policy $\pi_{A, B}^{A, -A}$, to the left of the cutoff, it is concave-like in the on-equilibrium wealth $w^{(\theta_H, \tau_H)}$, whereas to the right of the cutoff, it is convex-like; the similar can be said for the two performance fee policies. In this case, $w^{(\theta_L, \tau_L)}$ suffered a negative shock, and the deviant strategy pair $(\theta_L, \tau_H)$ is the case when Manager $A$ had deviated while Manager $B$ was compliant. Given this, to ensure incentive compatibility for Manager $A$, the Principal must thus boost the wealth allocations to Manager $A$. Hence, as the compliant wealth $w^{(\theta_H, \tau_H)}$ increases, the Principal will reduce portfolio allocations $\pi_{A, B}^{A, -A}$ to Manager $B$ and redirect them to Manager $A$. Likewise, the Principal will increase the performance fees $y_{A, B}^{A, -A}$ to Manager $A$, while decreasing the performance fees $y_{B, A}^{A, -A}$ to Manager $B$. Note also that there are indeed ranges of compliant wealths $w^{(\theta_H, \tau_H)}$ where no performance fees in $[0, 1]$ will exist, and these are the regions where a contract will not exist for decentralized delegation, and this is especially true if the realized wealths $w^{(\theta_H, \tau_H)}$ are not sufficiently high.
8.1.4 Numerical illustrations of joint tail probabilities via copulas

As discussed in Section 8, the incentive compatibility constraints in dynamic decentralized delegation are akin to VaR constraints on the $t = 0$ portfolio and performance fee policies. Furthermore, as discussed and even further illustrated in Figures 8, 9, 10 and 11, we see that extreme downside risks are a particular concern. This motivates us to investigate how exactly do joint tail probabilities affect the contracting environment. For this purpose, we will need a method to model the joint tail probabilities, of which a natural tool is via copulas. Two numerical illustrations are given in Figure 12 and Figure 13. Please see details in Section H.

To focus on the risk channel, we will assume that the deviant strategies $\theta_L$ and $\tau_L$ now have identical means. That is, for the numerical parameters in the following illustrations, we use the parameters of Table H but further amended with that of Table A.
| Mean return on deviant strategy $\theta_L$ | $\mu_{\theta_L}$ | 0.08 |
| Mean return on deviant strategy $\tau_L$ | $\mu_{\tau_L}$ | 0.08 |

Table 2: The base parameters are amended to that of Table 1.

Figure 12: Numerical illustration of the Principal’s $t = 0$ value function along with the $t = 0$ optimal policies against various parameters of the Gumbel-Hougaard copula. The base parameters are the same as Table 1 along with the amendment Table 2. We assume the marginal distributions of the strategy returns $(R_{\theta_L}, R_{\tau_L}, R_{\theta_H}, R_{\tau_H})$ follow by a discrete approximation to the normal distribution. The joint distribution is modeled by the Gumbel-Hougaard copula, in which it has a parameter $\delta_{\text{Gumbel}} \geq 1$. See Example 1 of Section 11 for details. A low $\delta_{\text{Gumbel}}$ parameter implies the joint distribution is nearly independent, whereas a higher value implies increasing upper tail dependence of the joint distribution. As the joint returns have increasing upper tail dependence, the Principal’s value function decreases.

Figure 13: Numerical illustration of the Principal’s $t = 0$ value function along with the $t = 0$ optimal policies against various parameters of the Clayton copula. The base parameters are the same as Table 1 along with the amendment Table 2. We assume the marginal distributions of the strategy returns $(R_{\theta_L}, R_{\tau_L}, R_{\theta_H}, R_{\tau_H})$ follow by a discrete approximation to the normal distribution. The joint distribution is modeled by the Clayton copula, in which it has a parameter $\delta_{\text{Clayton}} > 0$. See Example 1 of Section 11 for details. A low $\delta_{\text{Clayton}}$ parameter implies the joint distribution has high lower tail dependence, whereas a higher value implies the joint distribution is almost independent. As the joint returns have increasing lower tail dependence, the Principal’s value function decreases.
9 Conclusion

We study a problem of centralized delegation versus decentralized delegation, where there is moral hazard risk over the investment strategy choice within each asset class. In the static model under first best, it is simply a matter of which form of delegation that offers better risk sharing with respect to the investment strategies.

With the presence of moral hazard in centralized delegation, the Principal needs to compensate the single Manager $C$ for the private costs of taking the Principal’s desired strategy pair but also the opportunity cost for any foregone long-short trading profits from deviant strategy pairs the Manager could have enjoyed. This implies if the Manager’s investment opportunity set is too wide, in that the mean return differences of the asset classes under management by Manager $C$ are large, or that the Manager $C$ is nearly risk neutral, no centralized contract will exist to implement the Principal’s desired strategy pair. In decentralized delegation, the restricted investment opportunity sets of the respective Manager $A$ and Manager $B$ confine their deviations to their own asset classes. Thus, the aforementioned opportunity cost in centralization simply does not exist in decentralization. But when the decentralized Manager $A$ and Manager $B$ consider a strategy deviation within their own respective asset classes, they only care for the mean and volatility differences between the compliant and deviant strategies, and do not take into account the correlation of their joint strategies. Hence, if the strategies within the asset classes have similar mean returns, it may be impossible for the Principal to induce his desired strategy pair with the decentralized Managers because only the Principal can capture the diversification benefits of the strategies’ correlations, and not Manager $A$ and Manager $B$ themselves.

In a dynamic decentralization model with committed reinvestments, Manager $A$ and Manager $B$ will have a motive to intertemporally hedge their future wealths with that of the Principal. Using the Principal’s intertemporal wealth as a bridge between multiple Managers’ payoffs, the Principal can thus incentivize the Managers to implement his desired investment strategy pair. The dynamic model shows that the incentive compatibility constraints can be viewed as value-at-risk constraints on the Principal’s portfolio and performance fee policies. Thus, even though individuals have mean-variance preferences and only linear contracts are considered, the Principal endogenously requires the knowledge of the full joint probability distribution of the compliant and deviant strategy pair returns, beyond just the first and second moments. The analogous dynamic centralized delegation model is studied in Section D. Via copulas, we numerically investigate how tail dependence of strategy returns affect the dynamic contracting environment.

It would be interesting to further study this problem of centralized versus decentralized delegation with a more general contract space, even in a static model. This paper focuses the contract space to be linear
in returns for both centralization and decentralization. But as emphasized in the paper, the restriction
versus relaxation in the investment opportunity set in centralization versus decentralization, respectively,
drives most of the differences between the two delegation forms. This highly suggests that a “one size
fits all” contract space for both delegation forms is inappropriate. In particular, this is to say that the
problem suggests that the optimal contract in a more general contract space should look rather different
in centralization versus decentralization. However, as also noted in the paper, when Managers can actively
access the financial markets to modify the incentive effects, the determination of an optimal contract is really
a joint problem of optimal contract design and optimal financial market restriction. We leave this interesting
problem to future research.

Appendix

A Proofs for Section 4

Proof of Proposition 4.1. (a) Using first order sufficient and necessary conditions, we can see that the value to (3.2d)
will be given by (4.1).
(b) Substitute in the optimal portfolio found above into the mean and variance expressions.
(c) Since the fixed fee \( x_C \) is linear respect to the Principal’s objective function, it implies that in equilibrium,
the individual rationality constraint (3.2f) constraint binds. This implies the Principal’s objective function can
be rewritten as,

\[
E[W_y(C)] - \frac{\eta}{2} \text{Var}(W_y(C)) = 1 + E[\hat{R}_{\theta, \tau}] - (c(\theta) + c(\tau))
- \frac{\eta^2}{2} y_C \text{Var}(\hat{R}_{\theta, \tau}) - \frac{\eta^2}{2} (1 - y_C^2) \text{Var}(\hat{R}_{\theta, \tau}).
\]

Now, by first order conditions on \( y_C \), we see that the above becomes a fourth order polynomial (i.e. quartic)
equation, and has the following roots,

\[
y_C \in \left\{ \frac{\eta^2}{\eta^2 \sigma^4 (1 - \rho^2_{\theta, \tau})} \right\}.
\]

The first root is clearly in \((0, 1)\); the second root is negative and hence not in \((0, 1)\); the third and fourth roots
(with \( \pm \)) are not in \( \mathbb{R} \) since \((-1)^{2/3} \notin \mathbb{C} \). Thus, an interior solution exists and is uniquely given by the first root.

(d) Simply substitute in the optimal fixed and optimal fees found earlier.
(e) Analogous to the above.

Proof of Proposition 4.2. (a) By binding the (IR) constraints (3.2f), we obtain the optimal fixed fee form, and we

\(28\) It should be noted that in general, quartic equations (and naturally arising here because of first order conditions) are
notoriously difficult to obtain simple and explicit solutions for. It is conjectured that if one extends to consider more than two
risky investment strategies, or that we extend to more general non-linear contracts, it would be difficult to obtain a closed form
contract for even first best centralized delegation. Indeed, the most difficult step in the proof of this Proposition is this step,
as everything else is straightforward. It was actually somewhat surprising to this author that despite a rather complicated first
order condition, an economically sensible and intuitive solution for the performance fee arises.
can rewrite the Principal’s objective function as,

$$E[W^p_{\tau}(\theta, \tau)] - \frac{\eta}{2} \text{Var}(W^p_{\tau}(\theta, \tau)) = -(c(\theta) + c(\tau)) + 1 + \pi(\mu_\tau - \mu_\theta) + \mu_\theta - \frac{\eta}{2} \sigma^2(1 - \pi)^2$$

$$- \frac{\eta}{2} \left[ \pi^2 ((1 - y_\theta)^2 \sigma^2 + (1 - y_\lambda)^2 \sigma^2 - 2(1 - y_B)(1 - y_A)\rho_{\theta, \tau} \sigma^2) + 2\pi(1 - y_\lambda) ((1 - y_B)\rho_{\theta, \tau} \sigma^2 - (1 - y_\lambda)\sigma^2) + (1 - y_\lambda)^2 \sigma^2 \right].$$

(A.1)

(b) By first order conditions applied to (A.1), we arrive at three different stationary points of \((\pi, y_\lambda, y_B)\),

$$\begin{align*}
(\pi, y_\lambda, y_B) \in & \left\{ \left( 0, \frac{\eta \pi}{\eta \eta + \eta \rho_{\theta, \tau}}, 1 + \frac{(\eta \eta + \eta \rho_{\theta, \tau})(\mu_\tau - \mu_\theta) - \eta \eta \rho_{\theta, \tau}\sigma^2}{\eta \eta \rho_{\theta, \tau}\sigma^2}, \frac{\eta \pi}{\eta \eta + \eta \rho_{\theta, \tau}} \right) \right. \\
& \left( 1 + \frac{(\eta \eta + \eta \rho_{\theta, \tau})(\mu_\tau - \mu_\theta) - \eta \eta \rho_{\theta, \tau}\sigma^2}{\eta \eta \rho_{\theta, \tau}\sigma^2}, \frac{\eta \eta + \eta \rho_{\theta, \tau} \sigma^2}{\eta \eta + \eta \rho_{\theta, \tau}} \right), \\
& \left( \pi^*, y_\lambda^*, y_B^* \right) \right\}
\end{align*}$$

The first and second stationary points, which would imply zero wealth invested into either of the agents, will violate the individual rationality constraint \((A.1)\). Thus, only the third stationary point is a candidate for an interior solution.

(c) This is simply applying Assumption \((B.1)\). The value function computation is straightforward.

Proof of Proposition \((B.1)\). Use Proposition \((A.1)\) and Proposition \((A.2)\).

B Additional Results and Proofs for Section 5

Proof of Proposition \((B.1)\). (a) This is the same proof as that of Proposition \((A.1)\).

(b) This is evident since the arguments in Proposition \((A.1)\) for deriving Manager C’s optimal portfolio choice holds true for any arbitrary contract.

(c) By Assumption \((B.2)\) and Proposition \((A.1)\), if the Principal wants to implement and induce the investment strategy pair \((\theta_H, \tau_H)\), then the Principal needs to write a contract that prevents Manager C from taking on the deviant strategies \((\theta’, \tau’) \in S_{-(\theta_H, \tau_H)}\). These are captured by the incentive compatibility constraints in \((A.1)\). One should note that these three constraints can be collapsed to a single one by equivalently writing,

$$-2c + \frac{1}{2}(\mu_{\theta_H} + \mu_{\tau_H})y - \frac{1}{4} \eta \eta \sigma^2(1 + \rho_{\theta_H, \tau_H})y^2 \geq \max_{(\theta’, \tau’)} \left\{ -(c(\theta’) + c(\tau’)) + \frac{1}{4} \frac{(\mu_{\theta'} - \mu_{\tau'})^2}{\eta \eta \sigma^2(1 - \rho_{\theta', \tau'})} + \frac{1}{2} (\mu_{\theta'} + \mu_{\tau'})y - \frac{1}{4} \eta \eta \sigma^2(1 + \rho_{\theta', \tau'})y^2 \right\},$$

(B.1)

where we take the maximum on the right hand side over \((\theta’, \tau’) \in S_{-(\theta_H, \tau_H)}\), which is clearly then equivalent to \((B.2)\).

Note that by Assumption \((B.2)\), we have that \(\mu_{\theta'} \leq \mu_{\theta_H} \) and \(\mu_{\tau'} \leq \mu_{\tau_H} \), and where at least one of these two inequalities are strict, and hence \(\mu_{\theta'} - \mu_{\theta_H} + \mu_{\tau'} - \mu_{\tau_H} < 0 \). Likewise, \(c(\theta’) + c(\tau’) - 2c < 0 \). However, since we only assume that the correlations \(\rho_{\theta, \tau} \) for all investment strategy pairs \((\theta, \tau)\) are different, and in particular no special sign and order restrictions, so we have that if \(\rho_{\theta', \tau'} - \rho_{\theta_H, \tau_H} > 0 \), then the component is concave in \(y \), and if \(\rho_{\theta', \tau'} - \rho_{\theta_H, \tau_H} < 0 \), it is convex in \(y \). Thus, we have a pointwise maximum of convex and/or concave functions, and in general, one has no particular geometric form of this.

(d) From the condition \((B.2)\), we substitute in the first best solution to check the condition under which none of the incentive compatibility constraints will bind. This is condition \((B.3)\).

(e) Suppose the conditions on the private costs \((B.3)\) are such that a first best solution will not be attained in second best. While we could indeed proceed to use Kuhn-Tucker conditions (with say three Kuhn-Tucker multipliers) to solve for the optimal solution, we can proceed with a much more geometric proof here. Firstly, by \((B.1)\) or equivalently \((B.2)\), it is clear that when a binding solution (that is in \([0, 1]\)) exists, only one of the constraints
will bind. Suppose that \((θ^h, τ^h) \in S_{\ell} (θ^h, τ^h)\) is the pair of deviant investment strategies for which its associated incentive compatibility constraint binds.

Given the quadratic form of constraints, we are motivated to define the discriminant for the binding deviant pairs \((θ^e, τ^e)\). Notice that the sign of the discriminant is heavily dependent on the sign of \(ρ_{θ^e, τ^e} - ρ_{θ^h, τ^h}\). Provided that \(D(θ^h, τ^h) \geq 0\), so that roots will exist for the quadratic associated with the binding incentive compatibility constraint \((θ^h, τ^h)\), we compute the roots as

\[
\tilde{y}_{\pm}(θ^h, τ^h) = \frac{1}{2} \left( -\frac{μ_{θ^h} - μ_{θ^h} + μ_{θ^h} - μ_{θ^h}}{2} \pm \sqrt{D(θ^h, τ^h)} \right)
\times \left[ -(c(θ^h) + c(τ^h) - 2c) + \frac{1}{4} \frac{(μ_{θ^h} - μ_{θ^h})^2}{η^2σ^2(1 - ρ_{θ^h, τ^h})} \right]^{-1}.
\]

With our current assumptions, it is not difficult to show that the negative root \(\tilde{y}_{-}(θ^h, τ^h) < 0\). Thus, let’s focus on the positive root \(\tilde{y}_{+}(θ^h, τ^h)\) of \((\tilde{y}, θ^h, τ^h)\). We must now recall that our solution must be confined in \([0, 1]\). Hence, a second best solution will exist only if \(\tilde{y}_{+}(θ^h, τ^h) \in [0, 1]\), and likewise, if \(\tilde{y}_{-}(θ^h, τ^h) \notin [0, 1]\), then no second best solution will exist.

**Corollary B.1.** Consider the second best centralized delegation setup in Proposition B.2, and suppose the conditions (i.e. conditions (i) and (ii) of part (d)) for the existence of a second best contract holds. In particular, recall Corollary B.1. 5.5. This is simply by binding the (IR) constraints (5.2). Recall the setup of Proposition B.2. The optimal performance fee is \(\tilde{y}_{+}(θ^h, τ^h)\) of (5.1). Consider the second best centralized delegation setup in Proposition B.2. The optimal performance fee is \(\tilde{y}_{+}(θ^h, τ^h)\). With our current assumptions, it is not difficult to show that the negative root \(\tilde{y}_{-}(θ^h, τ^h) < 0\). Thus, let’s focus on the positive root \(\tilde{y}_{+}(θ^h, τ^h)\) of (5.1). We must now recall that our solution must be confined in \([0, 1]\). Hence, a second best solution will exist only if \(\tilde{y}_{+}(θ^h, τ^h) \in [0, 1]\), and likewise, if \(\tilde{y}_{-}(θ^h, τ^h) \notin [0, 1]\), then no second best solution will exist.

**Proof of Corollary B.1.** This is simply rewriting out the condition (5.2) more explicitly.

**Proof of Proposition B.2.** (a) This is simply by binding the (IR) constraints (5.2).
(b) This is simply rewriting the (IC) constraints (5.2).
(c) This will be seen as a special case of Proposition B.2.

**Proposition B.2.** Recall the setup of Proposition B.2.

(a) Consider the following conditions on the private cost \(c\) imply the optimal second best decentralized delegation optimal portfolio and performance fee policies (5.1, \(\tilde{y}, \tilde{y}_{A}, \tilde{y}_{B}\) have the following form:
(i) If,
\[ 0 < c \leq \eta \Delta \mu \Delta \mu_r (1 + \rho_{h, r_i}) \min \left\{ \frac{1}{\eta \Delta \mu \Delta \mu_r (1 + \rho_{h, r_i}) + \Delta \mu_r (2\eta M + \eta p (1 + \rho_{h, r_i}))} \right\} \]
then,
\[ (\hat{x}, \hat{y}_A, \hat{y}_B) = (\hat{x}^{FB}, \hat{y}_A^{FB}, \hat{y}_B^{FB}) = \left( \frac{1}{2}, \frac{\eta p (1 + \rho_{h, r_i})}{\eta M + \eta p (1 + \rho_{h, r_i})}, \frac{\eta p (1 + \rho_{h, r_i})}{\eta M + \eta p (1 + \rho_{h, r_i})} \right). \]

(ii) If,
\[ \frac{\Delta \mu \Delta \mu_r \eta p (1 + \rho_{h, r_i})}{\eta \Delta \mu \Delta \mu_r (1 + \rho_{h, r_i}) + \Delta \mu_r (2\eta M + \eta p (1 + \rho_{h, r_i}))} < c \leq \frac{\Delta \mu \Delta \mu_r \eta p (1 + \rho_{h, r_i})}{\eta \Delta \mu \Delta \mu_r (1 + \rho_{h, r_i}) + \Delta \mu_r (2\eta M + \eta p (1 + \rho_{h, r_i}))} \]
then,
\[ (\hat{x}, \hat{y}_A, \hat{y}_B) = \left( \frac{(\Delta \mu_r - c) (\eta M + \eta p (1 + \rho_{h, r_i}))}{\Delta \mu_r (2\eta M + \eta p (1 + \rho_{h, r_i}))}, \frac{c (2\eta M + \eta p (1 + \rho_{h, r_i}))}{\eta M \Delta \mu_r + c (\eta M + \eta p (1 + \rho_{h, r_i})}, \frac{\eta p (1 + \rho_{h, r_i})}{\eta M + \eta p (1 + \rho_{h, r_i})} \right). \]

(iii) If,
\[ \frac{\Delta \mu \Delta \mu_r \eta p (1 + \rho_{h, r_i})}{\eta \Delta \mu \Delta \mu_r (1 + \rho_{h, r_i}) + \Delta \mu_r (2\eta M + \eta p (1 + \rho_{h, r_i}))} < c \leq \frac{\Delta \mu \Delta \mu_r \eta p (1 + \rho_{h, r_i})}{\eta \Delta \mu \Delta \mu_r (1 + \rho_{h, r_i}) + \Delta \mu_r (2\eta M + \eta p (1 + \rho_{h, r_i}))} \]
then,
\[ (\hat{x}, \hat{y}_A, \hat{y}_B) = \left( \frac{\eta M \Delta \mu_r + c [\eta M + \eta p (1 + \rho_{h, r_i})]}{\Delta \mu_r [2\eta M + \eta p (1 + \rho_{h, r_i})]}, \frac{\eta p (1 + \rho_{h, r_i})}{\eta M + \eta p (1 + \rho_{h, r_i})}, \frac{c (2\eta M + \eta p (1 + \rho_{h, r_i}))}{\eta M \Delta \mu_r + c (\eta M + \eta p (1 + \rho_{h, r_i})} \right). \]

(iv) If,
\[ \eta \Delta \mu \Delta \mu_r (1 + \rho_{h, r_i}) \max \left\{ \frac{1}{\eta \Delta \mu \Delta \mu_r (1 + \rho_{h, r_i}) + \Delta \mu_r (2\eta M + \eta p (1 + \rho_{h, r_i}))} \right\} \]
then,
\[ (\hat{x}, \hat{y}_A, \hat{y}_B) = \left( \frac{1}{2}, \frac{2\Delta \mu_c}{c (\Delta \mu_r - \Delta \mu_r) + \Delta \mu_r \Delta \mu_r}, \frac{2\Delta \mu_c}{c (\Delta \mu_r - \Delta \mu_r) + \Delta \mu_r \Delta \mu_r} \right). \]

(v) Else if none of the conditions above are satisfied, then there does not exist an optimal second best decentralized delegation contract.

Proof of Proposition 6.2. (a) After binding the (IR) constraints (15.3) into the Principal’s objective function, it remains that the portfolio and performance fee policy \( (\pi, y_A, y_B) \) have to respect the (IC) constraints (15.2), and the box constraints \( (\pi, y_B, y_A) \in \mathbb{R} \times [0,1]^2 \). However, we observe that \( (\pi, y_A, y_B) \) being on the boundary of \([0,1]^3\) would immediately violate either the (IR) constraints, the (IC) constraints, or both. Hence, for a solution to exist, \( (\pi, y_A, y_B) \) must be in the interior of \([0,1]^3\), that being \( (0,1)^3 \). Hence, given a feasible solution \( (\pi, y_A, y_B) \in (0,1)^3 \), we must then cycle through the \( 2 \times 2 = 4 \) cases where either the (IC) constraint (15.3) of Manager A bind or not, and whether (IC) constraint (15.4) of Manager B bind or not.

(i) This is the case when we obtain an interior solution and neither (IC) of Manager A nor (IC) of Manager B bind. Substitute in the first best solution from Proposition 6.4, under Assumption 6.2, into (6.6) and replace \( \geq \) with \( > \) to get the conditions on the private costs \( c \).

(ii) This is the case when only (IC) of Manager A binds and when that of Manager B does not bind. This
happens when, after substituting the first best solution into (5.6), and we obtain,

\[ \Delta \mu_A \eta (1 + \rho_{i_1, i_2}) > 2c(\eta_1 + \eta_2 (1 + \rho_{i_1, i_2})), \]

\[ \Delta \mu_B \eta (1 + \rho_{i_1, i_2}) \leq 2c(\eta_1 + \eta_2 (1 + \rho_{i_1, i_2})). \]

The binding condition also allows for us to get the portfolio policy \( \pi \) as a function of \( y_A \). Via first order conditions on the objective function, substitute back and then we solve for \((\pi, y_A, y_B)\). However, we still need to satisfy the interior box constraints \((\pi, y_A, y_B) \in (0, 1)^3\). We have \( y_A \in (0, 1) \) holding. Here, \( y_B > 0 \) and to have \( y_B < 1 \), we need,

\[ c < \Delta \mu_A. \]

Under such condition, we would also have \( \pi \in (0, 1) \). Putting those three conditions on the private cost \( c \) together yields the displayed condition.

(iii) This is the case when (IC) of Manager A does not bind, but that of Manager B does bind. The argument is completely analogous to the previous one.

(iv) This is the case when both (IC)'s of Manager A and Manager B bind. Here, we need to differentiate between two sub-cases — when \( \Delta \mu_A = \Delta \mu_B \) and when \( \Delta \mu_A \neq \Delta \mu_B \).

If \( \Delta \mu_A = \Delta \mu_B \equiv \Delta \mu \), then we immediately have that \((\pi, y_A, y_B) = (1/2, 2c/\Delta \mu, 2c/\Delta \mu)\). So, the condition to ensure that \((\pi, y_A, y_B) \in (0, 1)^3\) is clearly when,

\[ c < \frac{\Delta \mu}{2}. \]

Suppose \( \Delta \mu_A \neq \Delta \mu_B \), and without loss of generality, suppose \( \Delta \mu_A > \Delta \mu_B \). To have \( y_A > 0 \), we would need,

\[ \frac{\Delta \mu_A \Delta \mu_B}{\Delta \mu_A - \Delta \mu_B} > c, \]

and to have \( y_A < 1 \), one would need,

\[ c < \frac{\Delta \mu_A \Delta \mu_B}{\Delta \mu_A + \Delta \mu_B}. \]

Finally, to have \( \pi > 0 \), we would need,

\[ c < \frac{\Delta \mu_A \Delta \mu_B}{\Delta \mu_A - \Delta \mu_B}. \]

Putting these conditions together implies we need,

\[ \frac{\eta (1 + \rho_{i_1, i_2})}{2 (\eta_1 + \eta_2 (1 + \rho_{i_1, i_2}))} \Delta \mu_A \leq c < \min \left\{ \frac{\Delta \mu_A \Delta \mu_B}{\Delta \mu_A - \Delta \mu_B}, \frac{\Delta \mu_A \Delta \mu_B}{\Delta \mu_A + \Delta \mu_B}, \right\}. \]

Simplifying and generalizing to the case when \( \Delta \mu_A < \Delta \mu_B \), we have the displayed condition.

\[ \Box \]

**B.1 When there is only moral hazard over mean returns**

An interesting special case that neither neither potentially favors nor biases centralized delegation is when there is no moral hazard over correlations, \( \rho \equiv \rho_{i_1} \) for all \((\theta, \tau)\), and the potential mean return losses between the two investment strategies are identical, \( \Delta \mu \equiv \Delta \mu_A = \Delta \mu_B \). In this case, the incentive compatibility constraints (5.6) of decentralized delegation have the form,

\[ 0 \geq c - (1 - \pi) y_A \Delta \mu, \]
\[ 0 \geq c - \pi y_A \Delta \mu, \]

which is effectively the same form as before, but the incentive compatibility constraint (5.7) for centralized delegation reduces to,

\[ 0 \geq \max_{(\theta', \tau')} \left\{ 2c - (c(\theta') + c(\tau')) + \Delta \mu y_C \right\} = 2c + \Delta \mu y_C \]  \hspace{1cm} (B.3)

Thus, in this special case for centralized delegation, the centralized Manager C has incentives that are very much aligned with the Principal, as the alternative investment strategies \((\theta_L, \tau_L)\) have the same mean \( \mu_{\theta_L} = \mu_{\tau_L} = \mu - \Delta \mu \), same volatility and same correlations, this implies that a long-short strategy is not profitable.

**Corollary B.3.** Assume that there is no moral hazard over correlations \( \rho \equiv \rho_{i_1} \) for all strategy pairs \((\theta, \tau) \in S\), and the mean return differences between the two strategies are identical, \( \Delta \mu \equiv \Delta \mu_A = \Delta \mu_B > 0 \).
Consider the second best decentralized delegation problem.

(i) The optimal performance fee is,

\[ \hat{y}_C = \begin{cases} \frac{\eta}{2}, & 0 < c < \frac{\eta}{2} + \frac{\Delta \mu}{2}, \\ \frac{2c}{\Delta \mu}, & 1 \leq c < \frac{\Delta \mu}{2}, \\ \emptyset, & \text{otherwise}. \end{cases} \]

(ii) The associated Principal’s value function in second best decentralized delegation is,

\[ \mathbb{E}[W_{c,p}] - \frac{\eta}{2} \text{Var}(W_{c,p}) \bigg|_{SB,(\theta_H, \gamma_H)} = \begin{cases} -2c + \mu - \frac{\eta \theta_H}{4(\theta_H + \eta_H)} \sigma^2 (1 + \rho), & 0 < c < \frac{\eta}{2} + \frac{\Delta \mu}{2}, \\ -2c + \mu - \frac{4(\eta_H c^2 + \eta_H (\Delta \mu - 2c^2) (1 + \rho) \sigma^2)}{4(\Delta \mu)^2}, & 1 \leq c < \frac{\Delta \mu}{2}, \\ -\infty, & \text{otherwise}. \end{cases} \]

Consider the second best decentralized delegation problem.

(i) The optimal portfolio and performance fee policies are,

\[ (\hat{x}, \hat{y}_{A}, \hat{y}_{B}) = \begin{cases} \left( \frac{1}{2}, \frac{\eta_H(1 + \rho)}{\eta_H + \eta_H(1 + \rho)}, \frac{\eta_H(1 + \rho)}{\eta_H + \eta_H(1 + \rho)} \right), & 0 < c < \frac{\eta_H(1 + \rho)}{2(\eta_H + \eta_H(1 + \rho))} \Delta \mu, \\ \frac{2c}{\Delta \mu}, & 1 \leq c < \frac{\Delta \mu}{2}, \\ \emptyset, & \text{otherwise}. \end{cases} \]

(ii) The associated Principal’s value function in second best decentralized delegation is,

\[ \mathbb{E}[W_{p}] - \frac{\eta}{2} \text{Var}(W_{p}) \bigg|_{SB,(\theta_H, \gamma_H)} = \begin{cases} -2c + \mu - \frac{\eta \theta_H(1 + \rho)}{4(\theta_H + \eta_H(1 + \rho))} \sigma^2, & 0 < c < \frac{\eta_H(1 + \rho)}{2(\eta_H + \eta_H(1 + \rho))} \Delta \mu, \\ -2c + \mu - \frac{\sigma^2 [4(\theta_H + \eta_H(1 + \rho)) \sigma^2 - \eta_H(1 + \rho) (\Delta \mu - 4c^2)]}{4(\Delta \mu)^2}, & 1 \leq c < \frac{\Delta \mu}{2}, \\ -\infty, & \text{otherwise}. \end{cases} \]

Let’s compute the difference between the Principal’s value function under second best decentralized delegation and that of second best centralized delegation.

(i) Suppose \( \rho \in (-1, 0) \). Then \( \frac{\eta_H}{\eta_H + \eta_H} > \frac{\eta_H(1 + \rho)}{\eta_H + \eta_H(1 + \rho)} \), and,

\[ \left( \mathbb{E}[W_{p}] - \frac{\eta}{2} \text{Var}(W_{p}) \right) \bigg|_{SB,(\theta_H, \gamma_H)} - \left( \mathbb{E}[W_{c,p}] - \frac{\eta}{2} \text{Var}(W_{c,p}) \right) \bigg|_{SB,(\theta_H, \gamma_H)} = \begin{cases} \frac{\eta \theta_H \rho(1 + \rho)^2}{4(\theta_H + \eta_H)(\theta_H + \eta_H(1 + \rho))}, & 0 < c < \frac{\eta_H(1 + \rho)}{\eta_H + \eta_H(1 + \rho)}, \\ \frac{\sigma^2 [4c \Delta \mu \eta_H(1 + \rho)] - 4c^2 (\eta_H + \eta_H)(\theta_H + \eta_H(1 + \rho)) - (\Delta \mu)^2 (1 + \rho)}{4(\theta_H + \eta_H)(\Delta \mu)^2}, & \frac{\eta_H(1 + \rho)}{\eta_H + \eta_H(1 + \rho)} \leq c < \frac{\eta_H(1 + \rho)}{\eta_H + \eta_H}, \\ \frac{\eta_H c^2 \sigma^2}{(\Delta \mu)^2 \rho}, & \eta_H + \eta_H \leq c < \frac{\Delta \mu}{2}, \\ \text{undefined}, & \text{otherwise}. \end{cases} \]

In particular, for all \( c \in (0, \Delta \mu/2) \),

\[ \left( \mathbb{E}[W_{p}] - \frac{\eta}{2} \text{Var}(W_{p}) \right) \bigg|_{SB,(\theta_H, \gamma_H)} - \left( \mathbb{E}[W_{c,p}] - \frac{\eta}{2} \text{Var}(W_{c,p}) \right) \bigg|_{SB,(\theta_H, \gamma_H)} < 0. \]
(ii) Suppose \(\rho \in (0, 1)\). Then 
\[
\frac{\eta \rho (1 + \rho)}{\eta M + \eta \rho (1 + \rho)} > \frac{\eta M}{\eta M + \rho},
\]
and,
\[
\left( \mathbb{E}[W_P] - \frac{\eta M}{2} \operatorname{Var}(W_P) \right)_{SB, (\theta M, \tau M)} - \left( \mathbb{E}[W_c.P] - \frac{\eta M}{2} \operatorname{Var}(W_c.P) \right)_{SB, (\theta M, \tau M)} > 0.
\]

\[
\begin{aligned}
\left( \mathbb{E}[W_P] - \frac{\eta M}{2} \operatorname{Var}(W_P) \right)_{SB, (\theta M, \tau M)} - \left( \mathbb{E}[W_c.P] - \frac{\eta M}{2} \operatorname{Var}(W_c.P) \right)_{SB, (\theta M, \tau M)} > 0.
\end{aligned}
\]

In particular, for all \(c \in (0, \Delta \mu / 2)\),
\[
\left( \mathbb{E}[W_P] - \frac{\eta M}{2} \operatorname{Var}(W_P) \right)_{SB, (\theta M, \tau M)} - \left( \mathbb{E}[W_c.P] - \frac{\eta M}{2} \operatorname{Var}(W_c.P) \right)_{SB, (\theta M, \tau M)} > 0.
\]

(iii) If \(\rho = 0\), then for all \(c \in (0, \Delta \mu / 2)\),
\[
\left( \mathbb{E}[W_P] - \frac{\eta M}{2} \operatorname{Var}(W_P) \right)_{SB, (\theta M, \tau M)} - \left( \mathbb{E}[W_c.P] - \frac{\eta M}{2} \operatorname{Var}(W_c.P) \right)_{SB, (\theta M, \tau M)} = 0.
\]

Proof of Corollary B.3. This is a special case of Proposition 4. and Proposition 5. ■

Corollary B.3 illustrates that when there is no moral hazard over correlations, \(\rho \equiv \rho_{\theta M, \tau M} = \rho_{\theta L, \tau L}\), and that the mean return losses due to moral hazard are equal, \(\Delta \mu \equiv \Delta \mu_0 = \Delta \mu_\rho\), then this substantially aligns the interests of the centrally delegated single Manager \(C\). And as a result, in this special case, our results are essentially identical to the first best case of Proposition 4. that we had studied earlier. In particular, centralized delegation is favored when the correlations are negative \(\rho < 0\), decentralized delegation is favored when the correlations are positive \(\rho > 0\), and both forms of delegation are equal when the investment strategies are uncorrelated \(\rho = 0\).
References


