Contracting for Information under Imperfect Commitment*

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Abstract

We study optimal contracting under imperfect commitment in a model with an uninformed principal and an informed agent. The principal can commit to pay the agent for his advice but retains decision-making authority. Under an optimal contract, the principal should (a) never induce the agent to fully reveal what he knows—even though this is feasible—and (b) never pay the agent for imprecise information. We compare optimal contracts under imperfect commitment to those under full commitment as well as to delegation schemes. We find that gains from contracting are greatest when the divergence in the preferences of the principal and the agent is moderate.

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1 Introduction

There are many situations where those holding formal power to make decisions lack critical information or expertise. Often, this information or expertise lies in the hands of subordinates. Organizational theory suggests that a solution to this dilemma is to delegate authority—the power to make decisions should reside in the hands of those with the relevant information (see, for instance, Milgrom and Roberts, 1989 or Saloner et al. 2001).

This "delegation principle" implicitly assumes that the objectives of the person given authority do not differ from those of the firm. When such differences occur, it may be necessary to provide the right incentives to properly align the agent's objectives prior to delegation. Indeed, a second principle of organizational theory is the "alignment principle" which says that the alignment of incentives and the delegation of authority are complementary tools (see, for instance, Milgrom and Roberts, 1989, p. 17).

A second complication is that in practice, the authority given to subordinates is rarely absolute. That is, senior management may and, on occasion, does find it beneficial to intervene. In other words, there may be a commitment problem associated with the delegation of authority.

In view of these issues, how does the ability to commit, or the lack thereof, affect the delegation principle? With imperfect commitment, can a perfect alignment of objectives be attained? What is the optimal degree of alignment?

With the goal of shedding some light on these and related issues, we analyze the interaction between an uninformed principal and an agent who is informed about a payoff relevant state. Interest in the problem arises because the objectives of the agent may not coincide with those of the principal—a project that is optimal for one in a given state need not be optimal for the other. When the principal has no commitment power whatsoever, this interaction is captured in the cheap-talk model of Crawford and Sobel (1982). Several extensions of this model have studied the case where the principal can commit to give away decision making power entirely (i.e. delegate). Dessein (2002) studies this question in the context of delegation within firms while Gilligan and Krehbiel (1987) study it in the context of political institutions and rules.

In this paper, we enrich the Crawford-Sobel model by allowing for the possibility of contractual monetary transfers. Our goal is to study how the structure of optimal contracts is affected by the degree to which the principal can commit. We first examine the case of perfect commitment. Next, we relax this assumption and consider the case where the principal can commit to transfers but retains decision making authority. Finally, we compare these compensation contracts to delegation schemes.

We find that under perfect commitment, despite having the tools to fully align incentives, decisions are systematically distorted to favor the agent's preferences. That is, the principal economizes on transfers by agreeing to decisions that are never optimal for her given the realized states. Put simply, the alignment principle does not hold.

When the principal retains decision making authority, scope for contracting is, of course, more limited. Nevertheless, it is still feasible to fully align the agent's incentives solely through transfers while retaining decision-making authority. We show, however, that such contracts are never optimal. In a leading case of the model—the so-called uniform-quadratic case—optimal contracts can be explicitly characterized and involve no payment for imprecise information. In fact, an optimal contract is of the "bang-bang" variety—in one region of the state space, incentives are fully aligned, while in the other, no attempt is made to align incentives. Furthermore, when the objectives of the two parties diverge severely, contracting under imperfect commitment is of no value even though gains can still be achieved under perfect commitment.

Investigating optimal contracts in such environments is complicated by the fact that the standard "revelation principle," which allows one to restrict attention to direct contracts with truth-telling, cannot be invoked. Indeed, the standard revelation principle is known to fail when commitment is imperfect (Bester and Strausz, 2001). Without the revelation principle, there is no systematic way to determine the optimal contract—the class of contracts one may consider is necessarily *ad hoc*. A methodological contribution of this paper is to find a contract under this form of imperfect commitment that is optimal in the class of *all* feasible contracts. We do this by first establishing that a limited form of the revelation principle—sufficient for our needs—continues to hold even though commitment is imperfect.[1]

Related Literature

Our analysis builds on the classic "cheap talk" model of Crawford and Sobel (1982) which studies the interaction between an informed agent and an uninformed principal. In this paper we introduce the possibility that the principal compensates the agent for his advice. With this amendment, the Crawford and Sobel model can be thought of as a polar case where the principal has no commitment power. We study optimal contracting in this model under different degrees of commitment.

Baron (2000) studies the effect that "contracting" arrangements have in the uniform-quadratic "legislative rules" model of Gilligan and Krehbiel (1987). In the absence of a revelation principle type result, he exogenously restricts attention to a limited set of contracts. Furthermore, unlike our model, he allows for bi-directional transfers. Indeed the contract that is optimal in his class involves transfers from the agent to the principal. Finally, the principal is assumed to be able to commit to a transfer only when information is revealed.

Ottaviani (2000) also examines how the use of transfers can enhance the amount of information that the agent shares with the principal. For a uniform-quadratic specification, he shows the possibility of full revelation contracts (this is a special case of our Proposition 3) and that this contract is dominated by one that delegates authority to the agent directly but involves no transfers. He does not study optimal contracts.

[1] The positive result of Bester and Strausz (2001) cannot be applied to our model.

Dessein (2002) examines the benefits of delegation in a similar model, again in the uniform-quadratic case.[2] Unlike our setting, Dessein does not allow for the possibility of transfer payments by the principal. Further, the principal is assumed to be able to commit not to intervene in the project chosen by the agent; thus, issues associated with imperfect commitment are also absent.[3] In Section 5, we compare optimal contracts in our setting with delegation contracts along the lines of Dessein and also to the optimal delegation schemes considered by Holmström (1984), Melamud and Shibano (1991) and by Alonso and Matouschek (2008).[4]

A separate strand of the literature is concerned with solving the moral hazard problem of information gathering on the part of the agent or agents (see, for example Aghion and Tirole, 1997 and Dewatripont and Tirole, 1999). In contrast, our primary interest is in the role of contracts to elicit information from already informed agents. In these papers, incentive alignment for efficient information transmission, once the agent has gathered information, is a secondary consideration.

The remainder of the paper proceeds as follows: In Section 2, we sketch an amended version of the Crawford and Sobel model to allow for contracting and transfers. In Section 3, we characterize optimal contracts in this model under perfect commitment. In Section 4, we study optimal contracting when the principal can commit to transfers but not to actions. Section 5 compares the value of contracting with several alternative schemes. Section 6 concludes. Proofs are collected in the appendices.

2 A Model of Contracting for Information

The Crawford and Sobel framework studies the following situation: A *principal* has the authority to choose a project $y \in \mathbf{R}$, the payoff from which depends on some underlying state of nature $\theta \in \Theta \equiv [0, 1]$, which is distributed according to a continuous density function $f > 0$. While the realization of θ is unknown to the principal, there is an *agent* who observes it precisely.

The payoff functions of the players are of the form $U(y, \theta, b_i)$ where b_i is a bias parameter which differs between the two parties. The function $U(y, \theta, b_i)$ is twice-continuously differentiable and satisfies $U_{11} < 0$, $U_{12} > 0$, and $U_{13} > 0$. For each i, $U(y, \theta, b_i)$ is assumed to attain a maximum at some y. Since $U_{11} < 0$, then for each θ and b_i, there is a unique maximizing project.

The biases of the two parties are commonly known. The bias of the principal, b_0, is normalized to be 0 while the agent's bias is $b_1 = b > 0$. In what follows we write

[2] Dessein also looks at cases where the preferences are concave functions of the quadratic loss specification.

[3] Delegation is also considered in the context of open versus closed rules for legislative committees, where a closed rule is effectively a delegation scheme. See for instance, Gilligan and Krehbiel (1987, 1989).

[4] See also Krähmer (2006) for a model of message contingent delegation.

$U(y,\theta) \equiv U(y,\theta,0)$ as the principal's payoff function. Since $U_{13} > 0$, the parameter b measures how closely the agent's interests are aligned with those of the principal. Define $y^*(\theta) = \arg\max_y U(y,\theta)$ to be the *ideal* project for the principal when the state is θ. Similarly, define $y^*(\theta,b) = \arg\max_y U(y,\theta,b)$ be the ideal project for the agent. Since $U_{13} > 0$, $b > 0$ implies that $y^*(\theta,b) > y^*(\theta)$.When payoff functions are of the quadratic loss form (the leading case in the Crawford and Sobel framework), the principal's ideal project matches the true state exactly; that is, for all θ, $y^*(\theta) = \theta$. The ideal project for an agent with bias b is $y^*(\theta,b) = \theta + b$.

After learning the state θ, the agent offers some "advice" to the decision maker in the form of a costless message m chosen from some fixed set M. Upon hearing the advice offered by the agent, the principal chooses the project y.

We augment the Crawford and Sobel framework by allowing the principal to contract with the agent and perhaps make transfer payments. We suppose that the preferences of the two parties are quasi-linear. Thus, if a payment $t \geq 0$ is made to the agent, then the payoff of the principal from project y in state θ is $U(y,\theta) - t$, while the payoff of the agent is $U(y,\theta,b) + t$. We assume that only nonnegative transfers $(t \geq 0)$ from the principal to the agent are feasible—in effect, the agent is protected by a "limited liability" clause and cannot be punished too severely. The principal is not subject to any budget constraint.

3 Contracts with Perfect Commitment

We begin by examining a somewhat standard problem in which the principal has the ability to commit perfectly—that is, she can write a contract that specifies both the project and the transfer as functions of the message sent by the agent. This will serve as a benchmark when we consider contracting environments where commitment is imperfect, i.e., where the principal can commit to transfers but retains decision making authority.

Under perfect commitment, the standard revelation principle applies, and it is sufficient to consider direct contracts—that is, those in which $M = [0,1]$—which satisfy incentive compatibility. A *direct* contract (y,t) specifies for each message $\theta \in [0,1]$, a project $y(\theta)$ and a transfer $t(\theta)$. A direct contract (y,t) is *incentive compatible* if for all θ, it is best for the agent to report the state truthfully, that is, if $\sigma = \theta$ maximizes $U(y(\sigma),\theta,b) + t(\sigma)$. Standard arguments show that, under perfect commitment, necessary and sufficient conditions for incentive compatibility are that: (i) $y(\cdot)$ is nondecreasing; and (ii) $t'(\theta) = -U_1(y(\theta),\theta,b)\, y'(\theta)$ at all points θ where $y(\cdot)$ is differentiable (see, for instance, Salanié, 1997).

There are several features of the model, however, that prevent the application of standard techniques to find an optimal contract. Specifically, a usual assumption about the agent's utility is that $U_2 > 0$; that is, a given project yields higher utility in higher states (see, for instance, Sappington, 1983). This guarantees that the agent's payoff in any incentive compatible contract is non-decreasing in the state; hence the limited liability constraint (or a participation constraint) is indeed met for all θ if it is met for the lowest type. In our model, however, U_2 changes sign depend on whether the project chosen is above or below the agent's ideal project. Hence, it is not enough to ensure that the limited liability constraint holds only for the lowest type. Nevertheless, an optimal contract may be found using control theory as we show below.

Optimal contracts The optimal contract is the solution to the following control problem

$$\max \int_0^1 (U(y,\theta) - t) f(\theta) \, d\theta$$

subject to the law of motion

$$t' = -U_1(y,\theta,b) \, u \tag{1}$$

and the constraints

$$\begin{aligned} y' &= u \\ t &\geq 0 \end{aligned}$$

where y and t are the state variables and u is the control variable. Notice that local incentive compatibility constraints are captured in the law of motion, which says that either: (i) y is locally strictly increasing, and in that case y and t are related according to (1); or (ii) y and t are both locally constant. It may be readily verified that for this problem, local incentive compatibility implies global incentive compatibility.

Some salient features of an optimal contract under perfect commitment can be inferred from these conditions. Appendix A contains the detailed analysis and shows:

Proposition 1 *Under perfect commitment, an optimal contract* (y,t) *has the following features:*

1. *projects* $y(\cdot)$ *are nondecreasing in* θ *and there is a* $z < 1$ *such that* $y(\cdot)$ *is constant over* $[z,1]$*;*

2. *transfers* $t(\cdot)$ *are nonincreasing in* θ *and* $t(\cdot)$ *is zero over* $[z,1]$*;*

3. $y(\theta) \leq y^*(\theta,b)$ *and if* $t(\theta) > 0$*, then* $y^*(\theta) < y(\theta)$*.*

While it is feasible for the principal to choose a contract that results in her ideal project always being chosen; in fact, an optimal contract under perfect commitment has precisely the opposite property. The principal's ideal project is *never* chosen. Nonetheless, since the interests of the two parties are partially aligned, an optimal contract specifies higher projects and lower transfers in higher states. In very high states, even this partial alignment of interests breaks down—the project chosen becomes unresponsive to the state and the agent receives no transfer.

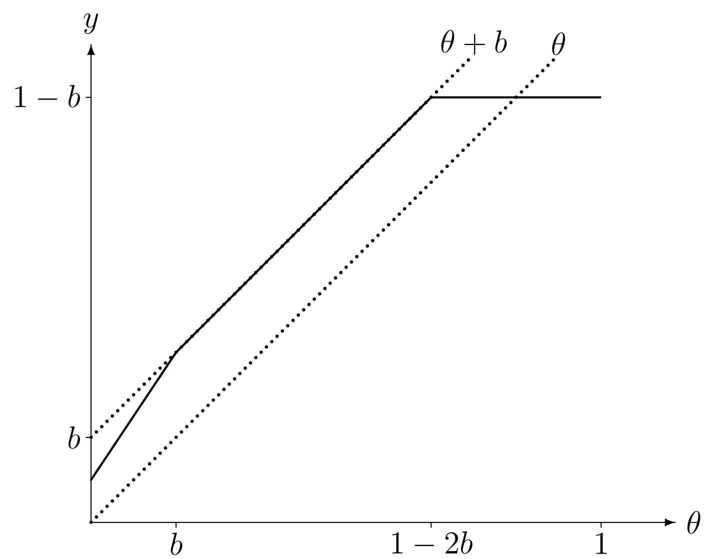

Figure 1: Optimal Contract with Perfect Commitment, $b \leq \frac{1}{3}$

Uniform-Quadratic case We conclude this section with an explicit characterization of an optimal contract for the *uniform-quadratic* case in which the distribution of states is uniform and the payoff function of the principal is:

$$U(y, \theta) = -(y - \theta)^2 \tag{2}$$

while that of the agent is

$$U(y, \theta, b_i) = -(y - (\theta + b))^2 \tag{3}$$

where $b > 0$.

The qualitative features of the contract when the bias is low differ somewhat from those when the bias is high. When the bias is low, that is, if $b \leq \frac{1}{3}$, an optimal contract has three separate pieces (see Figure 1). In low states, that is when $\theta \leq b$, the project $y(\theta) = \frac{3}{2}\theta + \frac{1}{2}b$ lies between that optimal for the principal $(y^*(\theta) = \theta)$ and that optimal for the agent $(y^*(\theta, b) = \theta + b)$. As θ increases, the project chosen tilts increasingly in favor of the agent, with a commensurate decrease in the transfer payments. For states between b and $1 - 2b$, the project that is best for the agent $(y^*(\theta, b) = \theta + b)$ is chosen and no transfers are made. It is as if the project choice were delegated to the agent. The set of feasible projects is "capped" at $\bar{y} = 1 - b$. For states above $1 - 2b$, the project is unresponsive to the state—that is, the agent always chooses project $\bar{y}$ and there is, effectively, pooling over this interval.

When the bias is high, that is, $\frac{1}{3} < b \leq 1$, an optimal contract consists of only two pieces (see Figure 2). In low states, the project again lies between the project ideal for the principal and that ideal for the agent. As in the case when the bias is

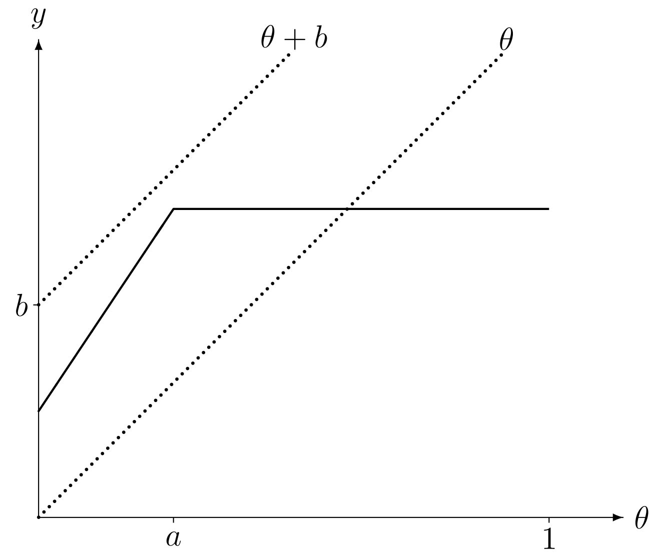

Figure 2: Optimal Contract with Perfect Commitment, $b > \frac{1}{3}$

low, the choice tilts in favor of the agent as the state increases with a corresponding decrease in the transfer payments. The set of feasible projects is again capped, but at a lower level. Indeed, as the agent becomes more biased, the cap decreases; that is, the agent becomes more constrained in his choice of projects. For high states, the agent always chooses the highest feasible project and there is, effectively, pooling over this interval. Unlike the case of low bias, there is no region in which the principal effectively delegates authority to the agent.

For very high biases, that is when $b > 1$, contracting is of no use—an optimal contract is no contract at all.

4 Compensation Contracts

We now consider situations in which the ability of the principal to commit is imperfect. The idea of imperfect commitment is captured by assuming that the principal can only contract on transfers and not on project choices—what we term *compensation contracts*.

A contract specifies the set of messages M that the agent may use to send information and a transfer scheme $T(\cdot)$ that determines the compensation $T(m) \geq 0$ that the agent will receive if he sends the message m. For instance, the message m sent by the agent could be advice that a specific project y be chosen and, in that case, the agent would be compensated on that basis. In other words, the set of messages M could be the same as the set of project choices.

A perfect Bayesian equilibrium (μ, Y, G) of the resulting game consists of (i) a strategy for the agent $\mu : \Theta \to \Delta(M)$ which assigns for every state θ, a probability distribution over M; (ii) a strategy for the principal $Y : M \rightarrow R$; and (iii) a belief function $G : M \rightarrow \Delta(\Theta)$ which assigns for every m a probability distribution over the states θ. It is required that, following any message m, the principal maximizes her expected payoffs given her beliefs; the beliefs G are derived from μ using Bayes' rule wherever possible; and the agent's strategy μ is optimal given Y.[5]

A modified Revelation Principle When the principal can perfectly commit—that is, to both a project $Y(m)$ and a transfer $T(m)$—then the classic revelation principle (see Myerson, 1991) may be invoked. Specifically, for any mechanism (M, Y, T) and any (Bayesian) equilibrium of this mechanism, there exists a direct mechanism—in which the agent reports his private information, so that $M = \Theta$—which results in the same projects and transfers and has the property that it is a optimal for the agent to tell the truth. The revelation principle is a powerful tool because in searching for optimal contracts, it allows the analyst to restrict attention to direct mechanisms that are incentive compatible.

As pointed out by Bester and Strausz (2001), with imperfect commitment, the revelation principle may fail—there may be equilibrium outcomes of an indirect mechanism that cannot be replicated by a direct mechanism. Bester and Strausz show, however, that with finite information any incentive-*efficient* payoff can be achieved using a direct mechanism although truth-telling is not guaranteed. In our contracting model, the state space is *continuous,* so we cannot simply appeal to the Bester and Strausz result. We can, however, obtain a stronger result: given an equilibrium of any mechanism, there exists an equilibrium of a direct mechanism which is *outcome equivalent* in the sense that it results in the same projects and transfers as in the original equilibrium for almost every state. Thus, any incentive-*feasible* payoff can be achieved as an equilibrium of a direct mechanism.[6] Formally,

Proposition 2 *In the contracting for information model, consider an indirect contract* (M, T) *with imperfect commitment and any equilibrium under this contract. Then there exists a pure strategy equilibrium under a direct contract* (Θ, t) *which is outcome equivalent.*

Proof. See Appendix B. ■

4.1 Full revelation contracts

Since the principal retains decision making power, her central concern remains acquiring information from the agent in such a way as to make good decisions. From the perspective of project choice, it would be ideal if the agent were to perfectly reveal the

[5] Because of the assumption that the principal's utility $U(\cdot, \theta)$ is strictly concave, it is unnecessary to allow for strategies in which the principal randomizes.

[6] By an "equilibrium" we always mean "perfect Bayesian equilibrium."

state. Under perfect commitment, it is feasible for the principal to induce the agent to fully reveal his information. At the other extreme, absent any commitment power whatsoever, Crawford and Sobel have shown that information loss is an inevitable consequence of the misalignment of incentives between the parties.

We examine two related questions: First, with imperfect commitment, is it even possible for the principal to design a contract that completely aligns the agent's interests with her own and gets him to fully reveal what he knows? Second, and more importantly, if it is possible, under what circumstances is this the best contract for the principal?

To address the first question, we devise a contract where the agent fully reveals his information. Under such a contract $\mu(\theta) = \theta$ and, furthermore, the principal anticipates that this will be the case; hence $y(\theta) = y^*(\theta)$. For truth-telling to be a best response requires that in every state θ

$$U(y^*(\theta), \theta, b) + t(\theta) \geq U(y^*(\theta'), \theta, b) + t(\theta')$$

for all $\theta' \neq \theta$. The first-order condition for the agent's maximization problem results in the differential equation

$$t^{*\prime}(\theta) = -U_1(y^*(\theta), \theta, b)\, y^{*\prime}(\theta)$$

Since $U_1(y^*(\theta), \theta, b) > 0$ and $y^{*\prime}(\theta) > 0$, a compensation schedule that induces full revelation is *downward sloping*. Thus, among all contracts that induce full revelation and satisfy limited liability, the least-cost one is:

$$t^*(\theta) = \int_\theta^1 U_1(y^*(\alpha), \alpha, b)\, y^{*\prime}(\alpha)\, d\alpha \tag{4}$$

Finally, it is routine to verify that nonlocal deviations to the above contract are not profitable for the agent either.[7] To summarize, we have shown:

Proposition 3 *Full revelation compensation contracts are always feasible.*

Now we turn to the second question: The following proposition establishes that the costs of aligning the agent's incentives always outweigh the benefits—full revelation is never cost effective.

Proposition 4 *Full revelation compensation contracts are* never *optimal.*

[7] An alternative derivation is as follows: A standard result in contract theory (see Salanié, 1997, p. 31) is that with full commitment every monotonic project choice can be implemented via a truthful direct mechanism with an appropriate transfer scheme. This implies that y^* can be so implemented. But since y^* is ex post optimal for the principal under truth-telling, no commitment is needed to ensure that the principal will in fact, choose $y^*(\theta)$ in state θ. Thus y^* can be implemented even without commitment.

Proof. See Appendix C. ■

To gain some intuition for the result, notice that, for very high states, the transfers required to induce truth-telling are quite modest ($t^* (\theta)$ is close to zero when θ is close to 1). The indirect cost of obtaining this revelation is, however, to raise the transfers for *all* lower states. Instead, the principal can *locally* give up a small amount of information by inducing pooling for the highest states, and save substantially through the *global* reduction in transfer payments for lower states. In the uniform-quadratic case, the cost of obtaining full revelation is so great that the principal is always better off simply offering no contract at all.

4.2 Optimal Compensation Contracts

Having shown that full revelation contracts are not optimal, it remains to identify the characteristics of an optimal contract. To obtain an exact characterization requires placing more structure on the distribution of states and the payoff functions of the actors. In this section, we offer an explicit characterization for the *uniform-quadratic* case. We begin by establishing some key properties of optimal compensation contracts.

4.2.1 No separation to the right of pooling

We first establish that inducing separation by fully aligning the interests of the agent with those of the principal is only cost-effective in low states. That is, once a contract calls for a pooling interval over a set of states, it never pays to induce separation for higher states. Specifically,

Proposition 5 *An optimal compensation contract involves separation in low states and pooling in high states.*

Proof. See Appendix D. ■

The property derived above implies that an optimal contract consists of separation for some set of low states, say for θ below some threshold a_0, followed by a number of pooling intervals $[a_{k-1}, a_k]$, $k = 1, 2, ...K$ that subdivide $[a_0, 1]$.

4.2.2 No payment for imprecise information

When the principal has no commitment power, the information an agent can credibly convey is necessarily coarse. As CS showed, the agent will reveal only that the state is in one of a finite number of subintervals. Contracts, however, enable the principal—at some cost, of course—to tailor incentives in a way that the agent is induced to reveal more than he would otherwise. Indeed, as we showed above, it is even possible to induce full revelation, but this is never cost-effective. This suggests that it is possible that an optimal contract would induce the agent to provide additional, but still not fully precise, information. Our next proposition shows that, in fact, this is not the case—the principal should never pay for *partial* revelation. The optimal contract is of the "bang-bang" variety—in low states, the principal pays the agent to fully reveal what he knows; in high states, the principal does not pay the agent at all, and, consequently, the agent reveals what he knows only imprecisely. Formally,

Proposition 6 *In an optimal compensation contract, the principal* ***never*** *pays for imprecise information.*

Proof. See Appendix D. ■

Why is it not optimal to pay for imprecise information? From Proposition 5, we know that an optimal contract involves full revelation in some interval $[0, a_0]$ and then a series of pooling intervals $[a_{k-1}, a_k]$, $k = 1, 2, ..., K$. Proposition 6 says that payments are made only when the reported state is low, that is, in $[0, a_0]$. Consider a change in the contract such that the principal still induces full revelation in $[0, a_0]$ but makes a small payment ε for states just above a_0. Doing this will distort the pooling intervals to some $\left[a'_{k-1}, a'_k\right]$, $k = 1, 2, ..., K$, thereby gaining more information in high states. Apart from the direct costs associated with this, there are also indirect costs. To maintain incentive compatibility, a payment of ε in the interval $[a_0, a'_1]$, raises the transfers in $[0, a_0]$ also—typically by at least ε. In other words, a payment in high states has an echo effect in *all* lower states. A *local* gain in information has a *global* cost.

The "echo effect" of a local change on global payment schemes is analogous to the intuition offered for why full revelation over the entire state space is never optimal. As Proposition 4 shows, the informational gains in high states are always outweighed by the increased costs of aligning incentives in low states. One might expect that a similar resolution of the costs and benefits will occur more generally in an optimal contract; however, to make this calculation requires a precise characterization of an optimal contract. This can be done in the uniform-quadratic setting.

Proposition 6 also sheds light on an important aspect of organizational theory, which stresses that incentives and delegation are complements. That is, if the principal is going to effectively push decision making authority downstream, then she must provide incentives to the agent to act in a manner consistent with the organizational objectives. Of course, this is problematic in the case of imperfect commitment since the principal cannot irreversibly transfer decision making power. Thus, a key contracting question is how the principal should resolve this tension. Proposition 6 illustrates that "compromise" in the form of incentives that somewhat align the agent's preferences with those of the principal are never optimal. Depending on the realized state, the contract either aligns interests perfectly or dispenses with monetary incentives altogether.

4.2.3 Characterization

Propositions 5 and 6 together imply that in an optimal contract, the agent is induced to reveal up to some state a_0 and not compensated thereafter. Further, for any value of a_0, it can be shown that the number of pooling intervals, K, is uniquely determined—it is the no contracting outcome that maximizes the principal's expected payoffs. (For a formal statement, see Lemma 5 in Appendix D.)

Thus, an optimal contract can be completely characterized as the solution to the problem of choosing a_0 to maximize

$$EV = -\int_0^{a_0} \left(2b\left(a_0 - \theta\right) + t\left(a_0\right)\right) d\theta - \sum_{k=1}^{K} \int_{a_{k-1}}^{a_k} \left(y\left(\left[a_{k-1}, a_k\right]\right) - \theta\right)^2 d\theta$$

where K and a_k are determined as in Lemma 5 in Appendix D.

Finally, we show that the interval over which separation takes place and contractual payments are made is "relatively small." In particular, an optimal contract never involves paying for information more than one-fourth of the time.

Proposition 7 *An optimal compensation contract involves: (i) positive payments and separation over an interval* $[0, a_0]$ *where* $0 < a_0 \leq \frac{1}{4}$*; (ii) no payments and a division of* $[a_0, 1]$ *into a finite number of pooling intervals.*

Proof. See Appendix D. ■

A "Taxation Principle" A common objection to direct mechanisms in the perfect commitment setting is that they are unrealistic—one never sees "direct message games" played between principals and agents to determine economic outcomes. A standard rejoinder to this criticism is the so-called "taxation principle" which points out a variety of realistic indirect mechanisms which are equivalent. For instance, a direct mechanism in a monopolistic pricing setting is equivalent to a nonlinear tariff schedule.

The taxation principle also operates in settings where commitment is imperfect. To see this, recall that an optimal direct contract in Proposition 7 involves full revelation in the interval $[0, a_0]$ and then a division of $[a_0, 1]$ into K intervals $[a_{k-1}, a_k]$, $k = 1, 2, ..., K$ where $a_K = 1$. An equivalent indirect mechanism is the following: First, the principal offers the following transfer schedule associated with the various projects

$$t(y) = \begin{cases} t_0 - 2by & \text{if} \quad y \leq a_0 \\ 0 & \text{if} \quad y > a_0 \end{cases}$$

where t_0 is the payment associated with a project choice of $y = a_0$. The agent then selects his preferred project, and the principal engages in a "project review" phase. Any project $y \leq a_0$ is automatically approved while projects $y > a_0$ require

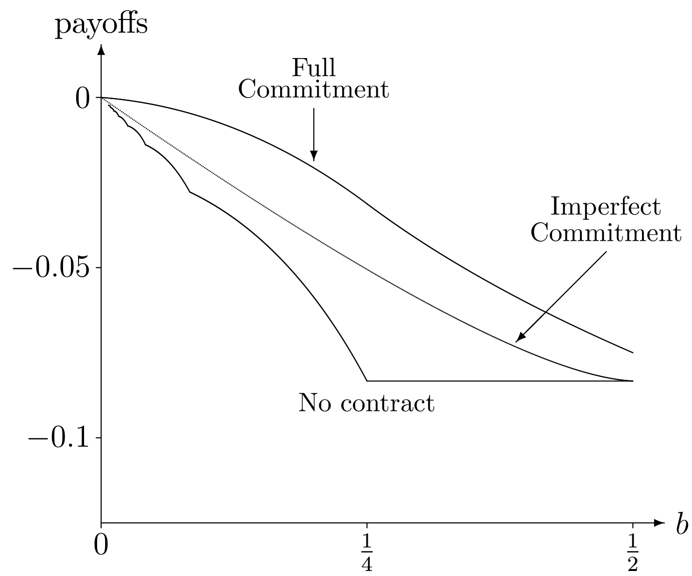

Figure 3: Value of Contracting

additional scrutiny. Specifically, if the agent chooses any project $y \in [a_{k-1}, a_k]$ other than $y_k = \frac{1}{2}(a_{k-1} + a_k)$, the principal overrules the agent and implements y_k.

5 Comparing Contracts

In this section, we compare the key features of optimal compensation contracts with three benchmarks: optimal contracting under perfect commitment, no contracting at all, and "delegation," where the principal can commit to projects but cannot compensate the agent.

Comparison to perfect commitment Figure 3 displays the payoffs to the principal under the optimal compensation contract (with imperfect commitment) compared to an optimal contract with perfect commitment. Obviously, the payoffs are higher under perfect commitment. Moreover, once the agent is sufficiently biased ($1 > b > \frac{1}{2}$), the ability to compensate the agent for his advice without the ability to commit to project is of no value whatsoever. In contrast, contracting continues to be valuable under full commitment.

The key difference between the two contracts is that, under perfect commitment, the project chosen is frequently a compromise—it lies between the principal's ideal choice and that of the agent. Under imperfect commitment, the project chosen is (almost) never a compromise. Compromising is valuable because it lets the principal save on transfer payments to the agent while still obtaining project choices that are responsive to the underlying state. In short, compromise requires commitment.

A key feature shared by both contracts is that the principal never pays the agent for imprecise information. Under imperfect commitment, this results in project choices that jump discontinuously with the underlying state. Under perfect commitment, this feature manifests itself in the form of delegation with caps on project choice. That is, under perfect commitment, the selected project is responsive to the underlying state (up to the cap), but reflects the optimal choice from the perspective of the agent rather than the principal.

Comparison to no contracting Williamson (1975), among others, has argued that formal contracting arrangements between principals and agents are inherently costly; thus, it may be the case that the principal is better served by eschewing contracting entirely. Figure 3 also displays the expected payoffs to the principal under imperfect commitment compared to no contracting as a function of the agent's bias.

Notice that the gains from contracting are *non-monotonic* in the degree of bias. Clearly, when the preferences of the agent and the principal are closely aligned, the latter's payoff is close to her first-best level and the gains from contracting are small. As the bias increases, informational losses to the principal become more severe, and there is more scope for contracting to "fix" the incentive problem. Indeed, when $b \geq \frac{1}{4}$, the agent can credibly reveal no information absent contracting. Resorting to contracts improves the situation, but the cost of aligning the agent's preferences increases until, at $b \geq \frac{1}{2}$, it becomes prohibitive. Thus, when the agent's preferences are extreme, the gains from contracting are also limited.

This suggests that if there were some costs associated with "formalizing" the exchange of information between principals and agents by writing contracts, one would expect to see contracts in cases of intermediate bias, but not when incentives are relatively closely aligned nor when the agent being consulted is an extremist.

Comparison to delegation Strategic management texts often suggest that, for businesses faced with decentralized information, delegation (or a flat organizational structure) is the appropriate response. For example, Saloner, Shepard, and Podolny (2001, pp. 79-80) write: "One basic principle of organization design is to assign authority to those who have information."

In the context of our model, the validity of the delegation principle may be examined by comparing *full delegation*—the unconditional assignment of authority to the person with information—to an optimal contracting arrangement with imperfect commitment. By "full" delegation we mean that the principal commits not to exercise any discretionary authority and so no longer has the freedom to intervene ex post. Specifically, there are no "caps" on what project the agent may choose. In that case, the agent will, of course, choose his favorite project $y^*(\theta, b) = \theta + b$ in each state, and the payoff of the principal is simply $-b^2$.

Alternatively, one can think of delegation schemes along the lines of Holmström

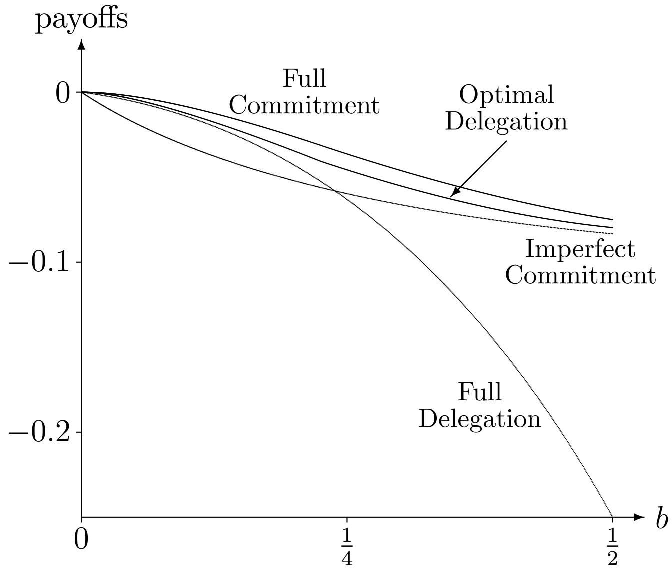

Figure 4: Contracts vs. Delegation

(1984), Melamud and Shibano (1991) and Alonso and Matouschek (2007) where the principal gives authority to the agent to select projects but places constraints on his choice of projects. It can be shown that, under the *optimal delegation* scheme, the principal restricts project choices of the agent to be from 0 up to a maximum of $1-b$. The payoff to the principal under this scheme is $-b^2+\frac{4}{3}b^3$, and clearly this is superior (by the amount $\frac{4}{3}b^3$) to the full delegation scheme. By precluding the agent from taking extremely high projects (including those that are not optimal in any state), the principal is able to enjoy the benefits of delegation in low states while suffering less from incentive misalignment in the high states. This improves payoffs.

How does the ability to commit to projects but not transfers (i.e., the delegation regimes) compare to the reverse—the ability to commit to transfers but not to projects? In other words, which form of commitment is more valuable? To examine this question, Figure 4 compares the principal's expected payoffs from an optimal contract under imperfect commitment with those from full delegation and optimal delegation.[8] As a benchmark, payoffs under the full commitment contract (commitment both to projects and transfers) are also shown.

As the figure shows, contracting under imperfect commitment is superior to full delegation only when the bias of the agent is high, $b > 0.244$. Recall that an optimal contract lies between the principal's favorite project and that of the agent. This arises because it is more cost-effective for the principal to economize on transfers by compromising on projects. Full delegation is an extreme version of this idea—the principal pays no transfers but instead of a compromise, in effect concedes to the

[8]In an important paper, Dessein (2002) has shown, again for the uniform-quadratic case, that delegation is superior to no contracting when the bias of the agent is not too extreme.

agent, giving him the freedom to choose his preferred project. When the preferences of the two parties are relatively closely aligned, the complete transfer of authority is more cost-effective for the principal than aligning incentives via transfers and retaining authority. As the bias increases, the transfer of authority becomes increasingly costly for the principal and transfers start to become more cost-effective. If the principal has the authority to restrict the space of projects that an agent might select from, the project represents a compromise between the principal and the agent's optimal choices. As a result, optimal delegation contracts dominate commitment only to transfers *regardless of the bias of the agent.* In short, the ability to commit to projects is more valuable than the ability to commit to transfers.

6 Conclusions

Absent any contracting tools, a key lesson from the strategic information literature is that differences in preferences lead to loss of information and, consequently, poor decision making. In this paper, we examine how the situation improves when the decision maker (principal) can write contracts. In particular, we study the case where the principal retains decision making authority but can compensate the agent for his advice.

Even with this limited contracting capability, the principal can avoid information loss entirely—regardless of the differences in preferences, it is always feasible to induce the agent to fully reveal his information. However, despite the apparent power of such contracts for solving the information loss problem, we show that they are *never* optimal for the principal. Instead, an optimal contract has the following property: in some states, the agent is compensated in a way that induces him to fully convey what he knows, while in other states, no payment is made and the agent conveys noisy, but still informative, messages. In other words, an optimal contract *never* involves any payment for imprecise information.

Finally, we studied the gains from contracting under imperfect commitment as well as perfect commitment and compared the payoffs under these schemes to the case where no contracts are possible as well as to the case where the principal (optimally) delegates the decision to the agent. In general, gains from contracting are greatest when the bias of the agent is moderate.

We have focused on the role of contracts in improving information transmission and abstracted away from their role in providing the right incentives for information acquisition. In many instances, the two problems—information transmission and information acquisition—can be effectively decomposed and our analysis is directly relevant. In other cases the problems cannot be considered separately. It remains for future research to study how our conclusions about the nature of optimal contracts change in cases where effort incentives are also important.

A Appendix

This appendix derives properties of an optimal contract under perfect commitment. The optimal contract is the solution to the following control problem

$$\max \int_0^1 \left(U\left(y,\theta\right) - t\right) f\left(\theta\right) d\theta$$

subject to the law of motion

$$t' = -U_1\left(y,\theta,b\right) u \tag{5}$$

and the constraints

$$\begin{aligned} y' &= u \\ t &\geq 0 \end{aligned}$$

where y and t are the state variables and u is the control variable.

If we write the generalized Hamiltonian

$$L = \left(U\left(y,\theta\right) - t\right) f\left(\theta\right) - \lambda_1 U_1\left(y,\theta,b\right) u + \lambda_2 u + \mu t$$

the resulting Pontryagin conditions are: there exist non-negative costate variables λ_1, λ_2 and a nonnegative multiplier μ that satisfy:

$$\begin{aligned} \lambda_1' &= -\frac{\partial L}{\partial t} = f\left(\theta\right) - \mu && (6) \\ \lambda_2' &= -\frac{\partial L}{\partial y} = -U_1\left(y,\theta\right) f\left(\theta\right) + \lambda_1 U_{11}\left(y,\theta,b\right) u && (7) \\ 0 &= \frac{\partial L}{\partial u} = -\lambda_1 U_1\left(y,\theta,b\right) + \lambda_2 && (8) \\ 0 &= \mu t && (9) \end{aligned}$$

and the transversality conditions are:

$$\lambda_1\left(1\right) = 0 \text{ and } \lambda_2\left(1\right) = 0 \tag{10}$$

Lemma 1 *For all $\theta \in \left(0,1\right)$, $y\left(\theta\right) \leq y^*\left(\theta,b\right)$.*

Proof. Suppose that the contrary is true, that is, there exists a θ such that $y\left(\theta\right) > y^*\left(\theta,b\right)$. Recall that in any optimal contract

$$-\lambda_1 U_1\left(y,\theta,b\right) + \lambda_2 = 0$$

and since $\lambda_1\left(\theta\right) \geq 0$ and $\lambda_2\left(\theta\right) \geq 0$. If $\lambda_1\left(\theta\right) > 0$, then the contradiction is immediate since $U_1\left(y,\theta,b\right) < 0$. Suppose that $\lambda_1\left(\theta\right) = 0$ then $\lambda_2'\left(\theta\right) = -U_1\left(y,\theta\right) f\left(\theta\right) > 0$ and hence $\lambda_2\left(\theta\right) > 0$ and again there is a contradiction. ■

An immediate implication of the previous lemma is that the transfers are nonincreasing in the state.

Lemma 2 $t(\cdot)$ *is nonincreasing.*

Proof. The law of motion (5), is

$$t' = -U_1(y, \theta, b)\, u$$

and from the fact that any incentive compatible $y(\cdot)$ is nondecreasing, we know that $u = y' \geq 0$. Now Lemma 1 implies that $U_1(y, \theta, b) \geq 0$ and so $t' \leq 0$. ■

Lemma 3 *If* $t(\theta) > 0$*, then* $y^*(\theta) < y(\theta)$.

Proof. If $t(\theta) > 0$, then from Lemma 2, for all $\sigma < \theta$, $t(\sigma) > 0$. This means that $\mu(\sigma) = 0$ for all $\sigma \in [0, \theta]$. Now (6) implies that

$$\lambda_1(\theta) = F(\theta) + \lambda_1(0)$$

where F is the cumulative distribution function associated with f and from (7)

$$\lambda_2(\theta) = (F(\theta) + \lambda_1(0))\, U_1(y, \theta, b)$$

and differentiating this results in

$$\lambda_2'(\theta) = f(\theta)\, U_1(y, \theta, b) + (F(\theta) + \lambda_1(0))\,(U_{11}(y, \theta, b)\, u + U_{12}(y, \theta, b))$$

Equating this with the expression in (7), we get

$$U_1(y, \theta, b) + U_1(y, \theta) = -\frac{F(\theta) + \lambda_1(0)}{f(\theta)} U_{12}(y, \theta, b) < 0$$

since $U_{12} > 0$. But since $y \leq y^*(\theta, b)$ this implies that $y > y^*(\theta)$. ■

Finally, an optimal contract must involve some pooling in high states. Thus, even though the principal has the option of full revelation, this is too expensive and never optimal.

Lemma 4 *There exists a* $z < 1$, *such that* y *is constant over* $[z, 1]$.

Proof. We claim that there exists a $z < 1$, such that $t(z) = 0$. If $t(\theta) > 0$ for all $\theta \in (0, 1)$, then we have that for all $\theta \in (0, 1)$, $\mu(\theta) = 0$. Now (6) together with the transversality condition implies that $\lambda_1(\theta) = F(\theta) - 1$, which is impossible since $\lambda_1(\theta) \geq 0$. ■

The uniform-quadratic case. In the uniform-quadratic case, the Pontryagin conditions (6) to (9) are also sufficient since the relevant convexity conditions are satisfied (see for instance, Seierstad and Sydsæter, 1987). Some qualitative features of the solution differ depending on whether the bias b is less than or exceeds $\frac{1}{3}$. These are depicted in Figures 1 and 2, respectively.[9]

B Appendix

Proof of Proposition 2

Suppose that (μ, Y, G) is a perfect Bayesian equilibrium under the contract (M, T). Recall that since the principal's payoff function is strictly concave in y, she never randomizes. Given any state θ, define $\overline{Y}(\theta) = \sup \{Y(m) : m \in \operatorname{supp} \mu(\cdot \mid \theta)\}$ and $\underline{Y}(\theta) = \inf \{Y(m) : m \in \operatorname{supp} \mu(\cdot \mid \theta)\}$ be the "largest" and "smallest" actions induced in state θ, respectively.

Consider two states $\theta_1 < \theta_2$. Then we claim that $\overline{Y}(\theta_1) \leq \underline{Y}(\theta_2)$. Suppose to the contrary that $\overline{Y}(\theta_1) > \underline{Y}(\theta_2)$. Let Y_1^n be a sequence in the set $\{Y(m) : m \in \operatorname{supp} \mu(\cdot \mid \theta_1)\}$ that converges to $\overline{Y}(\theta_1)$. Similarly, let Y_2^n be a sequence in $\{Y(m) : m \in \operatorname{supp} \mu(\cdot \mid \theta_2)\}$ converging to $\underline{Y}(\theta_2)$. For large n, $Y_1^n > Y_2^n$. If T_1^n and T_2^n are the transfers associated with Y_1^n and Y_2^n, respectively, then by revealed preference of Y_1^n in state θ_1 we have that $U(Y_1^n, \theta_1, b) - U(Y_2^n, \theta_1, b) \geq T_2^n - T_1^n$. Since $U_{12} > 0$, we have that $U(Y_1^n, \theta_2, b) - U(Y_2^n, \theta_2, b) > T_2^n - T_1^n$ which is a contradiction since this means that it is better to induce action Y_1^n and transfer T_1^n in state θ_2. Thus, $\overline{Y}(\theta_1) \leq \underline{Y}(\theta_2)$ and so in equilibrium, any two states have at most one project in common. Moreover, this also implies that the function $\overline{Y}(\cdot)$ is monotone.

Next, suppose that θ is such that $\underline{Y}(\theta) < \overline{Y}(\theta)$. Then from the previous paragraph, for all $\theta' < \theta$, we have $\overline{Y}(\theta') \leq \underline{Y}(\theta) < \overline{Y}(\theta)$ and so the function $\overline{Y}(\cdot)$ is discontinuous at θ. But a monotonic function can be discontinuous only on a countable set and this implies that $\underline{Y}(\theta) < \overline{Y}(\theta)$ for at most a countable number of points θ. To summarize, we have so far shown that, in any equilibrium of any indirect mechanism, the agent induces a *unique* project $\overline{Y}(\theta)$, and hence a unique corresponding transfer $t(\theta)$, in almost every state.

We will construct an equilibrium under a direct contract that is outcome equivalent to the original contract in the sense that, for almost every θ, the induced project and the resulting transfer is the same. Consider the direct contract (Θ, t).[10] Define $Z(\theta) = \left\{\sigma : \overline{Y}(\sigma) = \overline{Y}(\theta)\right\}$ to be the set of states in which the project induced is the same as that induced in state θ. By the monotonicity of $\overline{Y}$, $Z(\theta)$ is an interval, possibly degenerate.

To complete the proof, let the *pure* strategy of the agent in the direct contract be as follows: for all $\sigma \in Z(\theta)$ send message $z(\theta) = E_F[\sigma \mid \sigma \in Z(\theta)]$. This strategy leads

[9] The exact solutions in the uniform-quadratic case may be obtained from the authors.

[10] Let $t(\theta) = 0$ at points of discontinuity of $\overline{Y}$.

the principal to hold posterior beliefs identical to those in the original equilibrium of the indirect contract, and so the project chosen by the principal in state θ will be the same in the two equilibria. Thus, this pure strategy equilibrium of the direct contract (Θ, t) is outcome equivalent to the original, possibly mixed, equilibrium. This completes the proof.

C Appendix

Proof of Proposition 4. We exhibit a contract that is superior to the best full revelation contract. Consider a contract $t(\cdot)$ that induces the following: the agent reveals any state $\theta \in [0, z]$ where $z < 1$ and pools thereafter. No payment is made if the reported state $m > z$. At $\theta = z$, the agent must be indifferent between reporting that the state is z and reporting that it is above z. If we denote by t_z the payment in state z, then we must have

$$U\left(y^*\left(z\right), z, b\right) + t_z = U\left(y\left(\left[z, 1\right]\right), z, b\right) \tag{11}$$

where $y([z, 1]) = \arg\max E\left[U\left(y, \theta\right) \mid \theta \in [z, 1]\right]$ is the optimal project conditional on knowing that $\theta \in [z, 1]$. Since for z close to 1, $U\left(y^*\left(z\right), z, b\right) < U\left(y\left(\left[z, 1\right]\right), z, b\right)$, it follows that $t_z > 0$.

It is routine to verify that

$$\left.\frac{dt_z}{dz}\right|_{z=1} = U_1\left(y^*\left(1\right), 1, b\right)\left(\left.\frac{d}{dz} y\left[z, 1\right]\right|_{z=1} - y^{*\prime}\left(1\right)\right)$$

Incentive compatibility over the interval $[0, z]$ requires that

$$t\left(\theta\right) = t_z + \int_\theta^z U_1\left(y^*\left(\alpha\right), \alpha, b\right) y^{*\prime}\left(\alpha\right) d\alpha$$

which is again always greater than zero, so this alternative contract is also feasible.

It is useful to note that:

$$\frac{dt\left(\theta\right)}{dz} = \frac{dt_z}{dz} + U_1\left(y^*\left(z\right), z, b\right) y^{*\prime}\left(z\right)$$

That is, on the interval $[0, z]$, the new contract t is parallel to the full revelation contract t^*. Indeed, for all $\theta \le z$ we have,

$$t\left(\theta\right) - t^*\left(\theta\right) = t_z - t^*\left(z\right)$$

The expected utility of the principal resulting from the new contract is

$$V = \int_0^z \left(U\left(y^*\left(\theta\right),\theta\right) - t\left(\theta\right)\right) f\left(\theta\right) d\theta + \int_z^1 U\left(y\left[z,1\right],\theta\right) f\left(\theta\right) d\theta$$

Differentiating with respect to z, we obtain

$$\begin{aligned}
\frac{dV}{dz} &= \left(U\left(y^*\left(z\right),z\right) - t_z\right) f\left(z\right) - U\left(y\left[z,1\right],z\right) f\left(z\right) \\
&\quad - \int_0^z \left(\frac{dt\left(\theta\right)}{dz}\right) f\left(\theta\right) d\theta \\
&= \left(U\left(y^*\left(z\right),z\right) - t_z\right) f\left(z\right) - U\left(y\left[z,1\right],z\right) f\left(z\right) \\
&\quad - \int_0^z \left(\frac{dt_z}{dz} + U_1\left(y^*\left(z\right),z,b\right) y^{*\prime}\left(z\right)\right) f\left(\theta\right) d\theta
\end{aligned}$$

When $z = 1$, we have

$$\begin{aligned}
\left.\frac{dV}{dz}\right|_{z=1} &= -\left.\frac{dt_z}{dz}\right|_{z=1} - U_1\left(y^*\left(1\right),1,b\right) y^{*\prime}\left(1\right) \\
&= -\left(U_1\left(y^*\left(1\right),1,b\right)\left(\left.\frac{d}{dz}y\left[z,1\right]\right|_{z=1} - y^{*\prime}\left(1\right)\right)\right) - U_1\left(y^*\left(1\right),1,b\right) y^{*\prime}\left(1\right) \\
&= -U_1\left(y^*\left(1\right),1,b\right) \left.\frac{d}{dz}y\left[z,1\right]\right|_{z=1} \\
&< 0
\end{aligned}$$

where the inequality follows from the fact that $U_1\left(y^*\left(1\right),1,b\right) > 0$ and $\frac{d}{dz}y\left[z,1\right] > 0$. Thus we have shown that for z close enough to 1, the alternative contract $t\left(\cdot\right)$ yields a higher expected utility for the principal than the full revelation contract $t^*\left(\cdot\right)$.

D Appendix

This appendix contains proofs of some results pertaining to the structure of optimal contracts under imperfect commitment in the uniform-quadratic case.

Proof of Proposition 5. Suppose there is pooling in the interval $[w,s]$ and revelation in the interval $[s,z]$. In the interval $[s,z]$ the contract must satisfy

$$t\left(\theta\right) = 2b\left(z-\theta\right) + t\left(z\right) \tag{12}$$

Then the indifference condition at s is

$$-\left(\frac{w+s}{2}-(s+b)\right)^2+t_{ws}=-b^2+t(s) \tag{13}$$

Notice that $t_{ws}>0$. Otherwise, at s, both the projects $\frac{w+s}{2}$ and s are too low for the agent.

At w, the agent must be indifferent between some equilibrium project y together with some transfer t_y ,and the project $\frac{w+s}{2}$ together with the transfer t_{ws}. Hence, we have

$$\begin{aligned} t_y &= (y-(w+b))^2-\left(\frac{w+s}{2}-(w+b)\right)^2+t_{ws} \\ &= w^2+2zb+y^2-2yw-2yb+t(z) \end{aligned}$$

using (13) to substitute for t_{ws}. It is important to note that the transfer t_y does not depend on s.

Hence, the principal's utility in this interval

$$\begin{aligned} EV &= \int_w^s \left(-\left(\frac{w+s}{2}-\theta\right)^2-t_{ws}\right)d\theta-\int_s^z (2b(z-\theta)+t(z))\,d\theta \\ &= ws^2-sw^2+t(z)\,w-w^2b-\frac{1}{3}s^3+\frac{1}{3}w^3+2bzw-bz^2-t(z)\,z \end{aligned}$$

Now consider a small change in s, keeping fixed all projects and transfers not in the interval $[w,s]$. As noted above, this does not affect the transfer t_y associated with the project y to the left of w. Moreover, since $t_{ws}>0$, a small change in s is feasible. The change in expected utility from an increase in s is:

$$\frac{dEV}{ds}=-(w-s)^2$$

and this is negative provided $s>w$. This means that no contract in which there is pooling over some nondegenerate interval $[w,s]$ followed by separation over some interval $[s,z]$ can be optimal.

The following lemma is a first step in establishing Proposition 6.

Lemma 5 *Suppose that a contract calls for revelation on* $[0,a]$ *and pooling with no payment thereafter. Such a contract is feasible if and only if the no-contract equilibrium that subdivides* $[a,1]$ *into the maximum number of pooling intervals is played.*

Proof. First, suppose that with no contracts, a size K partition of $[a,1]$ is possible, then the "break-points" of the partition are

$$a_j = \frac{j}{K} + \frac{K-j}{K}a - 2bj\,(K-j)$$

for $j = 1, 2, ..., K$.

For a size K partition to be feasible $(a_1 > a)$ and a size $K+1$ partition to be infeasible $(a_1 \leq a)$ together requires that:

$$\frac{1-a}{2K\,(K+1)} \leq b < \frac{1-a}{2K\,(K-1)} \tag{14}$$

In state a, incentive compatibility implies that the agent is indifferent between the project a and the project $\frac{1}{2}\,(a + a_1)$,

$$-b^2 + t_0 = -\left(\frac{a+a_1}{2} - (a+b)\right)^2$$

where t_0 is the transfer associated with a report $\theta = a$. Substituting for a_1 yields

$$t_0 = \frac{1}{4}\frac{(1-a-2K\,(K-1)\,b)\,(2bK\,(K+1)-(1-a))}{K^2} \tag{15}$$

The condition that $t_0 \geq 0$ in any feasible contract is the same as (14), the condition that there be at most K partition elements in the interval $[a, 1]$. ■

Proof of Proposition 6. Proposition 5 implies that an optimal contract must have separation over some interval $[0, z_0]$ (possibly degenerate) and then a number of pooling intervals (say n^*). Suppose that the total expected transfer in this contract is B^*. Since the contract is optimal it must also maximize the principal's expected payoffs among all contracts in which the expected expenditure is B^*, which one may think of as the "budget" of the principal. We will argue that every solution to a budget constrained problem—and an optimal contract must be a solution to such a problem—has the "no payment for pooling" property.

Choose $n \geq \max(n^*, N(b))$ where $N(b)$ is the maximum number of partition elements of $[0, 1]$ with no transfers. Further, let the budget B be arbitrary. Given a budget B, we want to construct the equilibrium maximizing the principal's expected utility among those that consist of revealing over the interval $[0, z_0]$ followed by at most n intervals of pooling in a way that the expected transfers add up to exactly B. Let the revealing interval be $[0, z_0]$ and let the cut points be denoted by $z_1, z_2, ..., z_{n-1}$ with payments t_i over the interval $[z_{i-1}, z_i]$. Payments for any θ in the revealing interval $[0, z_0]$ are $t_0 + 2b\,(z_0 - \theta)$. For notational convenience, we adopt the convention that $z_n = 1$.

For $i = 1, 2, ..., n-1$, incentive compatibility on the part of the agent implies that, in state z_i,

$$-\left(\frac{z_i + z_{i-1}}{2} - (z_i + b)\right)^2 + t_i = -\left(\frac{z_i + z_{i+1}}{2} - (z_i + b)\right)^2 + t_{i+1}$$

and solving this recursively, we obtain

$$t_i = \frac{1}{4}\left(z_i - z_{i-1}\right)^2 - \left(z_i + z_{i-1}\right) b - \frac{1}{4}\left(1 - z_{n-1}\right)^2 + \left(1 + z_{n-1}\right) b + t_n \tag{16}$$

Incentive compatibility also implies that, in state z_0,

$$-b^2 + t_0 = -\left(\frac{z_0 + z_1}{2} - (z_0 + b)\right)^2 + t_1$$

and, using the solution for t_1 obtained in (16) we get

$$t_0 = -2z_0 b - \frac{1}{4}\left(1 - z_{n-1}\right)^2 + \left(1 + z_{n-1}\right) b + t_n \tag{17}$$

Given a budget B, an optimal contract under imperfect commitment is the solution to the following:

Problem 1 *Choose $z_0, z_1, ..., z_{n-1}$ and t_n to maximize*

$$EU = -\frac{1}{12}\sum_{i=1}^{n}\left(z_i - z_{i-1}\right)^3$$

subject to the constraints that (i) the total expected transfers

$$z_0\left(bz_0 + t_0\right) + \sum_{i=1}^{n} t_i\left(z_i - z_{i-1}\right) \leq B$$

and (ii) for $i = 0, 1, ..., n-1$,

$$t_i \geq 0$$

where t_i are given by (16) and (17).

The Lagrangian associated with Problem 1 is

$$L = U + \lambda\left(B - z_0\left(bz_0 + t_0\right) - \sum_{i=1}^{n} t_i\left(z_i - z_{i-1}\right)\right) + \sum_{i=0}^{n-1}\mu_i t_i$$

where λ and μ_i are multipliers. The first-order necessary conditions require that the following expressions equal zero:

$$\frac{\partial L}{\partial z_0} = \frac{1+3\lambda}{4}\left(z_1 - z_0\right)^2 - 2\mu_0 b - \frac{1}{2}\mu_1\left(z_1 - z_0 + 2b\right) \tag{18}$$

for $i = 1, 2, ..., n-2$

$$\frac{\partial L}{\partial z_i} = \frac{1+3\lambda}{4}\left(\left(z_{i+1} - z_i\right)^2 - \left(z_i - z_{i-1}\right)^2\right) + \frac{1}{2}\mu_i\left(z_i - z_{i-1} - 2b\right) - \frac{1}{2}\mu_{i+1}\left(z_{i+1} - z_i + 2b\right) \tag{19}$$

$$\begin{aligned}\frac{\partial L}{\partial z_{n-1}} &= \frac{1+3\lambda}{4}\left(\left(1 - z_{n-1}\right)^2 - \left(z_{n-1} - z_{n-2}\right)^2\right) - \frac{1}{2}\lambda\left(1 - z_{n-1} + 2b\right) \\ &\quad + \frac{1}{2}\left(1 - z_{n-1} + 2b\right)\left(\sum_{i=0}^{n-2}\mu_i\right) + \frac{1}{2}\mu_{n-1}\left(1 - z_{n-2}\right)\end{aligned} \tag{20}$$

$$\begin{aligned}\frac{\partial L}{\partial t_n} &= -\lambda\left(z_0\frac{\partial t_0}{\partial t_n} + \sum_{i=1}^{n}\frac{\partial t_i}{\partial t_n}\left(z_i - z_{i-1}\right)\right) + \sum_{i=0}^{n-1}\mu_i\frac{\partial t_i}{\partial t_n} \\ &= -\lambda + \sum_{i=0}^{n-1}\mu_i\end{aligned} \tag{21}$$

Notice that the expected cost of full revelation is b. Thus, when the budget is large enough, that is, $B \geq b$, then full revelation is feasible and clearly solves the budget constrained problem.

For any $B < b$, we will show that a solution to the budget constrained problem is characterized as follows:

First, for any point $\theta = a$ define K to be the integer satisfying

$$\frac{1-a}{2K\left(K+1\right)} \leq b < \frac{1-a}{2K\left(K-1\right)}$$

We know from CS that there is a partition equilibrium of $[a, 1]$ into K intervals with cut points

$$a_j = \frac{j}{K} + \frac{K-j}{K}a - 2bj\left(K-j\right)$$

for $j = 0, 1, 2, ..., K$ and no transfers. Clearly since $a \leq 1$, it follows immediately that $K \leq N\left(b\right)$ and from Lemma 5, $t_0 \geq 0$..

Second, let a be the solution to:

$$a\left(ba-\left(\frac{a+a_1}{2}-(a+b)\right)^2+b^2\right)=B$$

that is, a is such that the entire budget is exhausted in getting the agent to reveal all states $\theta\in[0,a]$.

Case 1: $n=K$. It is useful to begin with the case in which $n=K$.
The solution in this case is: for $n=0,1,2,...,n-1$,

$$z_j=a_j \tag{22}$$

where $a_0\equiv a$. In addition,

$$t_n=0 \tag{23}$$

We also need to specify the values for the various multipliers. These are:

$$\lambda=-\frac{\frac{4}{3}K^2\left(K^2-1\right)b^2+(1-a)^2}{(2K(K+1)b-1)(2K(K-1)b-1)-4a+3a^2} \tag{24}$$

which is positive.

$$\mu_0=0 \text{ and } \mu_1=\frac{1+3\lambda}{2}\frac{r_1^2}{f(0)} \tag{25}$$

and for $i=2,...,n-1$

$$\mu_i=\frac{(1+3\lambda)}{g(i-2)\,g(i-1)}\left(4b\sum_{j=0}^{i-2}g(j)^2+\frac{1}{2}r_1^2g(-1)\right) \tag{26}$$

where $r_1=\frac{1-a}{K}-2b(K-1)$ and $g(j)=r_1+4jb+2b$.

It may be verified that the values for z_i,t_n together with the multipliers λ and μ_i solve the necessary first-order conditions for Problem 1.

Case 2: $n>K$. When $n>K$, a solution to the first-order conditions can be obtained by setting $z_0=z_1=...=z_{n-K}=a$ and for $i=1,2,...,K$, $z_{n-K+i}=a_i$. The indices of the remaining variables are also displaced by $n-K$.

This completes the argument that the solution specified in (22) to (26) satisfies the necessary first-order conditions (18) to (21) associated with Problem 1. We now show that in fact this is an optimal solution. We do this by showing that it satisfies both the necessary and sufficient conditions for an equivalent problem.

Consider the following alternative specification of the budget constrained problem in which the choice variables are the lengths of the intervals $r_i=z_i-z_{i-1}$ rather than their end points z_i.

Problem 2 *Choose* $z_0, r_1, ..., r_n$ *and* t_n *to maximize*

$$EU = -\frac{1}{12}\sum_{i=1}^{n} r_i^3$$

subject to the constraints that: (i) the total expected transfers

$$z_0 (bz_0 + t_0) + \sum_{i=1}^{n} t_i r_i \leq B$$

(ii) for $i = 0, 1, ..., n-1,$

$$t_i \geq 0$$

and (iii)

$$z_0 + \sum_{i=1}^{n} r_i = 1$$

where t_i *are given by (16) and (17).*

Problem 2 is the same as Problem 1 except for a change of variables. Since they share all local extrema, for every solution to the first-order conditions for Problem 1 there exists a corresponding solution to the first-order conditions for Problem 2. But in Problem 2, the objective function is concave in the choice variables and the constraints are all convex functions, the first-order conditions for Problem 2 are also sufficient. Thus any solution to the first-order conditions for Problem 1 constitutes a global optimum.

We have thus shown that an optimal solution to the budget constrained problem entails that except for t_0, all other $t_i = 0$. In other words, in an optimal contract, the principal never pays for pooling. This completes the proof of Proposition 6.

Proof of Proposition 7. We claim that the optimal value of a is

$$a_0 = \frac{3}{4} - \frac{1}{4}\sqrt{4 + \frac{1}{3}\left(3 - 8bK\left(K-1\right)\right)\left(8bK\left(K+1\right) - 3\right)} \tag{27}$$

where K is the unique integer such that

$$\frac{3}{8K\left(K+1\right)} \leq b < \frac{3}{8K\left(K-1\right)} \tag{28}$$

It is routine to verify that $a_0 \leq \frac{1}{4}$.

First, we show that for all b, the payoff to the principal from choosing $a > a_0$ is worse than her payoff from choosing $a = a_0$. At $a = \frac{1}{4}$, the most informative partition has K elements where K is the unique integer satisfying (28). For any $a > a_0$,

$$\frac{\partial EV}{\partial a} = \frac{1}{6}\frac{8b^2K^4 - 8b^2K^2 - 6bK^2 + 3(1-2a)(1-a)}{K^2} < 0$$

using (27). This shows that all $a > a_0$ are suboptimal since for any such a the most informative partition of $[a, 1]$ can have at most K elements. In particular, $\frac{dU}{da} < 0$ at $a = \frac{1}{4}$.

Next, we show that for all b, the payoff to the principal from choosing $a < a_0$ is worse than her payoff from choosing $a = a_0$. For $a < a_0$ and fixed K, one may readily verify that

$$\frac{\partial EV}{\partial a} > 0$$

The only thing left to verify is that for $a < a_0$, the utility is lower than at a_0 even if the number of elements in the most informative partition of $[a, 1]$ is greater than K.

Suppose that when $a = 0$, the maximal size of the partition of $[a, 1]$ is N (as in CS).

For $L = N-1, N-2, ...K+1, K$ define a_L to be the smallest a for which it is *not* possible to make a size $L+1$ partition. That is,

$$-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{2(1-a_L)}{b}} = L$$

The principal's expected payoff function is not differentiable at the points a_L since there is a "regime change" from $L+1$ to L element partitions. We can however, find the right and left derivatives of EV at a_L and a_{L-1}, respectively.

The right derivative of EV at $a = a_L = 1 - 2bL(L+1)$ is,

$$\left.\frac{\partial EV}{\partial a}\right|_{a=a_L}^{+} = \frac{1}{3}8b(2L+1)(L+1)\left(b - \frac{3}{8L(L+1)}\right) \tag{29}$$

But since for all $a \in [a_L, a_{L-1})$, there does *not* exist a partition of $[a, 1]$ with $L+1$ elements and $a < \frac{1}{4}$, we have

$$b \geq \frac{(1-a)}{2L(L+1)} > \frac{3}{8L(L+1)}$$

and so (29) is positive.

Similarly, the left derivative of U at $a = a_{L-1} = 1 - 2bL(L-1)$

$$\left.\frac{\partial EV}{\partial a}\right|_{a=a_{L-1}}^{-} = \frac{1}{3}8b(2L-1)(L-1)\left(b - \frac{3}{8L(L-1)}\right) \tag{30}$$

But since at a_{L-1} , there does not exist a partition of $[a_{L-1}, 1]$ with L elements and

$a_{L-1} < \frac{1}{4}$

$$b \geq \frac{(1-a_L)}{2L\,(L-1)} > \frac{3}{8L\,(L-1)}$$

and so we have that (30) is also positive.

The proof is completed by noting that when $L = K$, we have

$$\left.\frac{\partial EV}{\partial a}\right|_{a=a_K}^{+} > 0 \text{ and } \left.\frac{\partial EV}{\partial a}\right|_{a=a_{K-1}}^{-} < 0$$

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