A Comment on "The Welfare Effects of Public Information" by Stephen Morris and Hyun Song Shin*

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Abstract

We re-examine the Morris and Shin (2002) model, but assume that players hold proper priors. This sharply alters equilibrium behavior, and welfare, particularly when coordination dominates individual payoffs. Whereas under improper priors the social value of public information is smallest, and possibly negative, in these situations, this is never true under proper priors. When coordination matters sufficiently, public information always has positive social value. With proper priors, the social value of public information always varies non-monotonically with coordination motives. The social value of public information can still be negative, but only when coordination motives are moderate. Despite these differences, the two models converge in the limit, though the rate of convergence varies markedly with the weight on coordination.

Keywords: Bayesian priors, social value of public information, coordination, aggregation games

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1 Introduction

In an important paper, Morris and Shin (2002) raise the provocative possibility that society can be harmed by additional public information. This overturned the canonical view that public information is, at worst, neutral, when individuals share the same objectives. After all, individuals could simply ignore the new information. In their model, individuals care about coordinating with one another, as well as matching the underlying state, but only the latter aspect of payoffs matters to welfare. The main point of the paper is to illustrate the dual role of public information. On the one hand, it adds to societal knowledge about the state and is helpful in that regard. On the other, it facilitates

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coordination, distorting equilibrium actions and reducing welfare. In the worst case scenario, where coordination motives dominate all other considerations, if public information is less precise than private, society is always worse off with its disclosure.

Important to this result is the benchmark against which public information is being weighed—a situation of purely private information. Morris and Shin assume that individuals have completely diffuse (improper) priors and hence, in the absence of public signals, choose first-best equilibrium actions regardless of the weight on coordination. In this paper, we examine the effect of instead imposing the more standard proper priors assumption. Such an exercise might seem to be mainly of technical interest rather than producing substantive new economic insights; however, in this case, proper priors fundamentally change the situation individuals face. This difference stems from the fact that, when individuals hold proper priors, however diffuse, the prior mean itself plays the role of a coordinating device. Moreover, much like with public information, the attractiveness of this device increases as coordination motives dominate, so much so that private information is completely ignored in the limit.

This reverses the welfare ranking of public information in the "worst-case scenario." Now, so long as the public signal contains any information whatsoever, it represents a welfare improvement. It also changes the nature of how the social value of public information varies with the intensity of coordination motives. With improper priors, a simple, monotonic relationship exists—the greater is the individual importance of coordination, the smaller is the social value of public information. With proper priors, however, the relationship is never monotonic. Initially, the social value of public information is declining, but, once coordination becomes sufficiently important to individuals, the relationship reverses and the social value of public information is *increasing* with the importance of coordination.

This suggests, perhaps, a discontinuity between the two sets of assumptions on prior beliefs. This, however, is not the case. As the variance of the state distribution becomes unbounded, equilibrium behavior in the proper priors model converges to that under improper priors; however the rate of convergence varies significantly depending on the importance of coordination. When coordination motives are relatively unimportant, convergence occurs quickly and the improper priors model reasonably approximates a proper priors model. As coordination motives become more important, ever larger variances are required for a reasonable approximation. Most importantly, when individuals place nearly all utility weight on coordination, convergence occurs only *at* the limit. In other words, for any finite variance of the state space, there exist coordination weights where the improper priors model poorly approximates behavior under proper priors. Morris and Shin envisage their exercise as one comparing a public signal to no public signal. Under proper priors, however, the effective comparison is between an uninformative public signal (the prior mean) and an informative public signal. Viewed in this light, our result is intuitive—an informative public signal is always better than an uninformative one. Thus, while equilibrium behavior in the models converges, when coordination motives dominate, welfare properties remain starkly different, except when evaluated *at* the limit.

Before placing our findings in the context of the growing literature spawned by Morris and Shin's insights and modeling approach, it is important to distinguish between the negative social value of public information and the role of coordination motives. Under improper priors, the two phenomena are perfectly correlated—the higher the weight on coordination, the lower the social value of public information. Our findings break this link but do not destroy the possibility that public information can have negative social value. As described above, regardless of prior beliefs, the value of public information falls as coordination motives increase, so long as these motives are not too strong. It is quite possible, as we demonstrate later by example, that public information has negative social value—but only for *intermediate* coordination weights, and only under more stringent conditions than the improper priors model. Thus, at the most basic level, we preserve the possibility that public information has negative social value, but offer a more nuanced description for the circumstances in which this occurs.

The rest of the paper proceeds as follows: The remainder of this section places our findings in the context of the literature succeeding the seminal Morris and Shin piece. Section 2 briefly reviews and slightly amends the canonical Morris-Shin model. Section 3 then shows how introducing proper priors reverses their result in three settings of increasing generality. Finally, section 4 concludes.

Related Literature

Morris and Shin's sharp result, and modeling technique, spawned a distinct literature devoted to examining problems of how large numbers of individuals process and respond to information, both public and private. Initially, the bulk of this literature dwelt on overturning the notion that disclosing public information can be welfare reducing. Most directly, Svensson (2006) argued that the parameter values required for this finding were unlikely to hold in practice, a point conceded by Morris, Shin and Tong (2006). Macro-models embedding the types of tradeoffs envisaged by Morris and Shin also produced results that public information was socially beneficial (see, e.g. Woodford, 2003; Hellwig, 2005; and Roca, 2010). Starting in 2004, Angeletos and Pavan offered a series of papers generalizing the Morris and Shin setting and comparing equilibrium and optimal use of information and coordinating actions. Among other things, they identify that the "beauty contest" nature of coordination in Morris and Shin, i.e. the idea that coordination has no social value, represents a necessary ingredient for their main result. Finally, James and Lawler (2011) support Morris and Shin's original point by highlighting that, if the policy maker is modeled as a strategic actor, one can recover the negative social value result, even in circumstances where coordination directly impacts welfare.

Less related is more recent work, such as Hellwig and Veldkamp (2009) and Myatt and Wallace (2012) who emphasize the endogeneity of private information acquisition and how public information provision affects these incentives.

Our contribution is twofold. First, we point out that, even within the exact framework of Morris and Shin without amendment to payoffs or information, the relationship between coordination motives and the social value of public information is nuanced and non-monotonic. When individuals hold proper priors, public information *never* has negative social value in circumstances where the divergence between private and social motives is most severe. Our second contribution is to highlight the heretofore unremarked upon role of the prior mean as a coordinating device analogous to a public signal. Embedding Morris and Shin's model in a context with proper priors reveals that the tradeoff is not between the presence and absence of a coordinating device, but rather between better and worse coordinating devices. In that sense, the envisaged tradeoff is inverted—compared to the prior mean, public information represents an unambiguous improvement in the quality of the coordinating device. Viewed in this light, our finding that public information can continue to have negative social value, albeit under differing circumstances from those offered in Morris and Shin, represents a somewhat counterintuitive result. Individuals sacrifice information for coordination when placing weight on either the prior mean or the public signal. Since the public signal is, itself, informative, individuals sacrifice much less when placing weight on it, and, for moderate coordination incentives, such distortions may be so severe that society is made worse off.

2 Model

For completeness, we briefly reprise the Morris-Shin model, but with the (necessary) addition of explicitly specifying the data generating process for the state variable. A unit mass of players, with representative player $i \in [0,1]$, each receive conditionally independent signals and then simultaneously choose actions. Payoffs for each individual depend on the proximity of the action to a state variable, whose realization is unknown to all, and on the proximity of the action to others' actions. The latter receives payoff weight r and the former $1-r$. Payoffs associated with matching others equal zero when summed over all individuals; thus, social payoffs consist of the sum of individual payoffs for matching the action to the state. The key variation in the model is the presence or absence of a conditionally independent public signal observed by all individuals, and its interaction with the weight on coordination, r.

Let the state θ be drawn from an atomless distribution F having positive density f over $(-\infty, \infty)$ with finite mean μ (normalized to zero without loss of generality) and finite variance σ_θ^2. Conditional on the state, each individual receives an independent private signal $x_i = \theta + \varepsilon_i$ where ε_i is normally distributed with mean zero and finite variance σ_ε^2. In some circumstances, all individuals receive an identical public signal $y = \theta + \delta$, where δ is also normally distributed with mean zero and finite variance σ_δ^2.

After receiving signals, each individual i simultaneously chooses an action $a_i \in \mathbb{R}$ and earns a payoff

$$u_i = -(1-r)(a_i - \theta)^2 - r\left(\int_t (a_i - a(t))^2\, dt - \int_j \int_t (a(j) - a(t))^2\, dt dj\right)$$

That is, payoffs depend on the proximity of i's action to the state, with weight

r, and the proximity of i's action to the actions taken by others, with weight $1-r$. The complexity of this latter aspect of payoffs arises due to a normalization ensuring that, in aggregate, this latter component has no effect on social welfare.

Straightforward algebra shows that these payoffs are equivalent to

$$u_i = -(1-r)(a_i - \theta)^2 - r(a_i - \overline{a})^2 + r\sigma_a^2$$

where $\bar{a} = \int_t a(t)\,dt$ and $\sigma_a^2 = \int_j (a(j) - \overline{a})^2\,dj$. That is, payoffs from actions depend on a weighted average of how well the action matches the state and how well it matches the aggregator $\bar{a}$, the mean action taken. Payoffs, though not best responses, also depend on the overall dispersion of actions, σ_a^2. Since the payoffs from coordination sum to zero in aggregate, social payoffs are simply

$$w = -(1-r)\int_i (a_i - \theta)^2\,di$$

The key tension in the model is that, while coordination plays no welfare role, it does affect individual choice. This may be readily seen in considering the best response of individual i given information I and some beliefs about the choices of others, which produce the aggregator $\bar{a}$.

$$a_i^* = (1-r)E[\theta|I] + rE[\bar{a}|I] \tag{1}$$

3 The Social Value of Public Information

We now re-analyze the Morris-Shin model, imposing proper priors, for three different data generating processes of the state variable: the uniform distribution, the normal distribution, and general distributions. Our main conclusion in all three situations is the same: for sufficiently large weight on coordination: (1) Social value of public information is increasing in r; and (2) public information *always* has positive social value. Both findings are the opposite of those under improper priors.

3.0.1 Uniform Distribution

A formal interpretation of Morris and Shin's improper priors assumption is that the state variable is uniform on $(-\infty, \infty)$. We compare their analysis with what one obtains by studying a sequence of games where the state is uniformly distributed on $[-l/2, l/2]$ when l takes on large, but finite value. Notice that, as $l \to \infty$, the model converges to Morris-Shin's original setting (but violates our assumption that the state has finite variance). We will show that the behavior of the model for (arbitrarily) large values of l differs qualitatively from the limit (improper priors) case.

We do, however, make two departures from a fully fledged analysis, both of which become negligible as l becomes large. First, we assume that individuals use a linear approximation of the conditional expectation function of θ, rather than the true (but highly nonlinear) expression. The true expression given a (private) signal $x_i \in [-l/2, l/2]$ is

$$
\begin{aligned}
E[\theta|x_i] &= \frac{\int_{-\frac{l}{2}}^{\frac{l}{2}} \theta f(x_i|\theta) \frac{1}{l} d\theta}{\int_{-\frac{l}{2}}^{\frac{l}{2}} f(x_i|t) \frac{1}{l} dt} \\
&= \frac{\int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{\theta}{l\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_i-\theta}{\sigma_x}\right)^2} d\theta}{\int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{1}{l\sigma_x\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_i-t}{\sigma_x}\right)^2} dt} \\
&= x_i + \frac{\frac{\sigma_x\sqrt{2\pi}}{\pi}\left(e^{-\frac{\left(x_i+\frac{l}{2}\right)^2}{2\sigma_x^2}} - e^{-\frac{\left(x_i-\frac{l}{2}\right)^2}{2\sigma_x^2}}\right)}{\operatorname{erf}\left(\frac{\sqrt{2}}{2\sigma_x}\left(x_i+\frac{l}{2}\right)\right) - \operatorname{erf}\left(\frac{\sqrt{2}}{2\sigma_x}\left(x_i-\frac{l}{2}\right)\right)} \qquad (2)
\end{aligned}
$$

where erf denotes the error function. Figure 1 plots the true function and a linear approximation when $l = 10,000$ and $\sigma_x^2 = 556$. The key thing to notice is that the true expression is simply the 45 degree line plus an adjustment factor, which, even for finite values of l, is small away from the endpoints of the state distribution.

(See Figure 1 at end of this file)

Our linear approximation minimizes the sum of squared errors, so $E[\theta|x_i] \approx \gamma x_i$, where $\gamma = \frac{\sigma_\theta^2}{\sigma_x^2+\sigma_\theta^2}$. Notice that this expression is identical to the conditional expectation function when the state is normally distributed with the same parameter values, a fact that we will exploit later.

It may be readily verified that the nonlinear term in equation (2) goes to zero as $l \to \infty$; thus, our approximation becomes arbitrarily close to the true conditional expectation function as l gets large. The approximation serves the same purpose as improper priors did in the original model—to make equilibrium strategies linear and easy to compute. Later, we study other specifications without approximations and show that neither the approximation nor the linearity of equilibrium strategies affects our conclusions.

Our second departure is to neglect signal realizations lying outside the state space as well as the actions resulting therefrom. Again, as l becomes large, these signals occur with vanishingly small probability and may be safely neglected.

Private Signal Only

This situation requires virtually no analysis in the improper priors case. Private and social incentives are perfectly aligned and hence equilibrium actions are simply $a_i = x_i$ and social welfare is maximized given the available information.

We will show that this is not the case when individuals have proper priors—even in the limit. Equilibrium strategies, however, remain linear, as under improper priors, so long as conditional expectations are linear. Precisely,

Lemma 1 *When each individual receives a conditionally independent private signal, equilibrium choices are*

$$a_i = \frac{\gamma - \gamma r}{1 - \gamma r} x_i$$

Proof. Temporarily assume that all other individuals are following linear strategies of the form $a_j = \alpha_j x_j$, where α_j is a coefficient to be determined. This induces the expected aggregator

$$\begin{aligned} E[\bar{a}|x_i] &= \sum_j \alpha_j E(x_j|x_i) \\ &= \overline{\alpha} E(\theta|x_i) \end{aligned}$$

Using equation (1), i's best response is

$$\alpha_i x_i = (1 - r + r\overline{\alpha}) \gamma x_i$$

In equilibrium, $\alpha_i = \overline{\alpha} = \alpha$, so

$$\alpha = \frac{\gamma - \gamma r}{1 - \gamma r}$$

Finally, if all other individuals j choose strategies αx_j, then, substituting these expressions into the RHS of equation (1) and simplifying, it may be readily verified that $a_i = \alpha x_i$ is a best response. ■

For comparison purposes, it proves useful to write equilibrium actions as a function of $s = \frac{\sigma_\theta^2}{\sigma_x^2}$, the ratio of the variance of the state to the variance of the private signal, which implies that $\gamma = \frac{s}{s+1}$ and the equilibrium strategy becomes

$$a_i = \frac{(1-r)s}{(1-r)s+1} x_i$$

Notice that the limiting case where $s \to \infty$ recovers the equilibrium strategy under improper priors. Importantly, for finite values of s, coordination motives, expressed as r, distort equilibrium actions. This distortion can become arbitrarily severe—when $r \to 1$, equilibrium actions converge to the prior mean, zero.

Private/Public Signal

Next, we derive equilibrium choice behavior with the addition of a public signal under linear conditional expectations and interior signals, x_i and y (the realized public signal). Letting $p = \frac{\sigma_\theta^2}{\sigma_y^2}$, then the squared error minimizing linear approximation of the conditional expectation function is $E[\theta|x_i, y] \approx \gamma_1 x_i + \gamma_2 y$, where $\gamma_1 = \frac{s}{s+p+1}$ and $\gamma_2 = \frac{p}{s+p+1}$. Again, this produces linear equilibrium strategies, as the following lemma shows.

Lemma 2 *In the private/public signal game, equilibrium choices are*

$$a_i = \frac{(1-r)s}{p+1+(1-r)s} x_i + \frac{p}{p+1+(1-r)s} y$$

Proof. Conjecture that all others are playing linear strategies of the form $a_i = \alpha_{1i}x_i + \alpha_{2i}y$. This induces the expected aggregator

$$E_i\left(\bar{a}|x_i, y\right) = \bar{\alpha}_1 E_i\left(\theta|x_i, y\right) + \bar{\alpha}_2 y$$

Substituting this expression as well as that for $E\left[\theta|x_i, y\right]$, equation (1) becomes

$$a_i = (1-r)\left(\gamma_1 x_i + \gamma_2 y\right) + r\left(\bar{\alpha}_1\left(\gamma_1 x_i + \gamma_2 y\right) + \bar{\alpha}_2 y\right)$$

In equilibrium, $\bar{\alpha}_1 = \alpha_1$and $\bar{\alpha}_2 = \alpha_2$

$$\alpha_1 x + \alpha_2 y = (1-r)\left(\gamma_1 x_i + \gamma_2 y\right) + r\left(\alpha_1\left(\gamma_1 x_i + \gamma_2 y\right) + \alpha_2 y\right) \tag{3}$$

which may be readily solved for α_1 by evaluating the expression at $y = 0$ to obtain

$$\alpha_1 = \frac{(1-r)\,s}{1 + p + (1-r)\,s}$$

Substituting for α_1 and solving equation (3), we obtain

$$\alpha_2 = \frac{p}{1 + p + (1-r)\,s}$$

Substituting the resulting expressions into the RHS of equation (1), one can readily verify that $a_i = \alpha_i x_i + \alpha_2 y$ comprises the unique best response when all others play this same strategy. ■

As with the private information case, the equilibrium actions again converge to the derivation in Morris-Shin when the variance of the state becomes infinite. Coordination motives continue to distort equilibrium actions, as under improper priors, but the nature of the distortion changes. As $r \to 1$, $\alpha_1 = 0$ while $\alpha_2 = p/\left(1+p\right)$, i.e. individuals place no weight on the private signal but positive weight on the public signal and the prior mean.

The Social Value of Public Information

Using Lemmas 1 and 2, we now examine the social value of public information. Recall that the social loss function is proportional to the expected sum of the squared distance between the selected action and the state. In the private/public case, this may be readily shown to be

$$L_{Pub} \propto \sigma_\theta^2 \frac{(1-r)^2 s + p + 1}{\left(1 + p + (1-r)\,s\right)^2}$$

The loss function for the private signal case, L_{Pvt} is simply L_{Pub} evaluated at $p = 0$.

In their main result, Morris and Shin identify circumstances where public information has negative social value. To do this, they note that the loss function is independent of r in the private signals case and increasing in r when a public signal is added; thus, the larger is the coordination motive, the smaller is the social value of public information. Intuitively, the greater the weight on coordination, the greater the distortion in equilibrium actions in the presence of a public signal. The private signal case, however, has no such distortion. Hence, the larger is r, the more coordination motives interfere with information aggregation, and the more likely is the "CNBC Effect"—a negative social value of public information. If we express the social value of public information by the welfare ratio $W \equiv L_{Pvt}/L_{Pub}$, then one can readily show:

Remark 3 *Under improper priors, the social value of public information, measured by W, is strictly decreasing in r and is minimized when $r \to 1$.*

Furthermore, when $s > p$, adding a public signal always reduces social welfare when r is sufficiently large.

Our main finding is that incorporating proper priors fundamentally changes both the conclusion and the underlying intuition. First, as we noted above, with proper priors, the larger is the coordination motive, the smaller is the weight placed on the private signal. The same holds true of the private/public model. Thus, there is now a "race" to determine whether this distortion is exacerbated by the presence of public information. Our main finding is that this race always resolves unambiguously and, more importantly, in the opposite direction of the Morris-Shin prediction.

Proposition 4 *Suppose that the state is uniformly distributed on $[-l/2, l/2]$, l is large, and individuals use linear conditional expectations functions to form beliefs. Under proper priors, the social value of public information, W, is initially decreasing and then increasing in r, for r sufficiently large. It is maximized as $r \to 1$.*

Furthermore, for r sufficiently large, adding a public signal always improves expected welfare.

Proof. To establish that $r = 1$ maximizes W, notice that

$$W = \frac{\sigma_\theta^2 \frac{(1-r)^2 s+1}{(1+(1-r)s)^2}}{\sigma_\theta^2 \frac{(1-r)^2 s+p+1}{(1+p+(1-r)s)^2}} = \frac{\left((1-r)^2 s + 1\right)(1+p+(1-r)s)^2}{(1+(1-r)s)^2\left((1-r)^2 s + p + 1\right)}$$

The only extremum of this equation in the domain $(0,1)$ is $r = 1 - \frac{1}{s}\sqrt[3]{(1+p)s}$, which is a minimum. Therefore, for r sufficiently large, W is increasing in r. The two candidates for maxima are the endpoints $r = 0$ and $r = 1$. At $r = 0$, we have $W|_{r=0} = \frac{1+p+s}{1+s}$ while at $r = 1$, we obtain $W|_{r=1} = 1 + p$. Clearly $W|_{r=1} > W|_{r=0} > 1$, so the social value of public information achieves a global maximum as $r \to 1$. The last part of the proposition follows immediately since $W|_{r=1} > 1$ and is continuous. ■

Figure 2 illustrates how the social value of public information, measured by W, varies with the weight on coordination, r, under both sets of priors. Recall that the social value of public information is positive if and only if $W \geq 1$, so we plot the indifference line, $W = 1$, as well. Figure 2 uses values $s = 18$ and $p = 2$, but other combinations of parameters exhibit similar properties. For reasons explained below, with improper priors the social value of public information always decreases in r whereas it is first decreasing and then increasing with proper priors. In other words, Figure 2 illustrates the general features of variation in the social value of a public signal under both types of priors. Proper priors do not rule out the possibility that public information can reduce social welfare, as Figure 2 shows, but this can only occur for *intermediate* coordination weights, and not in all cases. Between $r = 0.64$ and $r = 0.87$, the public signal reduces welfare for the parameter values chosen.

(See Figure 2 at end of this file)

The key insight for the difference between the two plots is that, when individuals hold proper priors, the prior mean acts as a public signal. Thus, coordination motives distort equilibrium actions regardless of whether there is an explicit public signal. Under improper priors, this is not the case—without a public signal, equilibrium actions remain undistorted regardless of the weight of coordination motives.

To see the implications of this difference in equilibrium behavior on social welfare, consider the situation near $r = 0$. Public information always has positive social value at $r = 0$ since equilibrium actions are undistorted regardless of the formation of prior beliefs. The particular value of W, however, will differ as the conditional expectation of the state does depend on the prior. As r increases from zero, the social value of public information falls, though always at a faster rate under improper priors, which stems from the non-distortion of equilibrium actions absent a public signal.

Under proper priors, coordination motives distort actions with or without a public signal, hence one may wonder why W necessarily declines in r. The reason is that shifting weight from the private to the public signal is "cheaper" in terms of units of lost accuracy compared with shifting weight to the prior. Thus, for small values of r, distortions are always greater with a public signal, so W declines.

With an improper prior, the social value of public information declines monotonically in W for the same reasons as with small r. In contrast, the decline in the social value of public information stops, and eventually reverses completely, under proper priors. In all cases, the weight on the private signal goes to zero as r goes to one, but the distribution of this weight differs. Absent a public signal, the prior mean is the sole coordinating device, so all weight is transferred there. With a public signal, both it and the prior mean receive positive weight. As a consequence, the social value of public information is always positive for r sufficiently large, and the relationship between the value of public information and the weight on coordination is necessarily *non-monotonic*.

3.0.2 Normal Distribution

We now reproduce the analysis above when the state is normally distributed with mean zero and (finite) variance σ_θ^2. The expressions given for the approximate conditional expectations functions above now become exact expressions. Moreover, signals can no longer lie outside the range of states. As a consequence, the equilibrium strategies derived above for the uniform case correspond to exact equilibrium expressions for a normally distributed state. The rest of the analysis then immediately follows, but now requires no caveats. To summarize:

Proposition 5 *Suppose that the state is normally distributed. Under proper priors, the social value of public information, W, is initially decreasing and then increasing in r, for r sufficiently large. It is maximized as $r \to 1$.*

Furthermore, for r sufficiently large, adding a public signal always improves expected welfare.

3.0.3 General State Distributions

Notice that the intuition accompanying Figure 2 in no way depended on the particulars of the state distribution. We now show formally that our main results hold for arbitrary state distributions. We do this in two steps. Lemma 6 demonstrates that, for any symmetric equilibrium $\alpha(x)$ in a situation absent a public signal, as coordination weights goes to one, equilibrium strategies converge to placing all weight on the prior mean. Lemma 7 shows something similar when a public signal is present—equilibrium strategies converge to the conditional expectation function conditional on the public signal only. Thus, limiting equilibrium strategies place weight on both the prior mean and the public signal. Since the public signal is informative, it then follows that public information must have positive social value for r sufficiently large.

Lemma 6 *For any data generating process for θ, in the private signal case, any symmetric equilibrium $\alpha(x) = E[\theta]$ as $r \to 1$.*

Given the equilibrium strategy $\alpha(x)$, define $\bar{\alpha}(\theta)$ to be the expectation of the aggregator function conditional on θ. Formally,

$$\begin{aligned} \bar{\alpha}(\theta) &= E_z[\alpha(z)|\theta] \\ &= \int_z \alpha(z)\phi(z-\theta)\,dz \end{aligned}$$

We claim that

$$E_z[\alpha(z)|x] = E_\theta[\bar{\alpha}(\theta)|x]$$

To see this, note that, by the law of total probability, we may write:

$$\begin{aligned} E_z[\alpha(z)|x] &= E_\theta[E_z[\alpha(z)|\theta,x]|x] \\ &= E_\theta[E_z[\alpha(z)|\theta]|x] \\ &= E_\theta[\bar{\alpha}(\theta)|x] \end{aligned}$$

Thus, the equilibrium condition becomes

$$\alpha(x) = (1-r)E[\theta|x] + rE_\theta[\bar{\alpha}(\theta)|x]$$

In the limit, the equilibrium condition satisfies $\alpha(x) = E_\theta[\bar{\alpha}(\theta)|x]$, where

$$E_\theta[\bar{\alpha}(\theta)|x] = \frac{\int_\theta\left[\int_z \alpha(z)\phi(z-\theta)\,dz\right]\phi(x-\theta)f(\theta)\,d\theta}{\int_t \phi(x-t)f(t)\,dt}$$

Cross-multiplying, we obtain

$$\alpha(x)\int_\theta \phi(x-\theta)f(\theta)\,d\theta = \int_\theta\left[\int_z \alpha(z)\phi(z-\theta)\,dz\right]\phi(x-\theta)f(\theta)\,d\theta$$

or, equivalently,

$$\begin{aligned} & \int_\theta\left\{\alpha(x) - \int_z \alpha(z)\phi(z-\theta)\,dz\right\}\phi(x-\theta)f(\theta)\,d\theta \\ = & \int_\theta\{\alpha(x)-\bar{\alpha}(\theta)\}\phi(x-\theta)f(\theta)\,d\theta \\ = & \; 0 \end{aligned}$$

but since $\bar{\alpha}(\theta)$ is independent of x, it follows that $\alpha(x) = \bar{\alpha}$ for this condition to be satisfied.

Lemma 7 *For any data generating process for θ, in the private/public case, any symmetric equilibrium $\alpha(x,y) = E[\theta|y]$ as $r \to 1$.*

Proof. Recall that the equilibrium equation may be written as:

$$\alpha(x,y) = (1-r)E[\theta|x,y] + rE_z[\bar{\alpha}(z,y)|x,y]$$

Integrating both sides over x weighted by $g(x|y)$, we obtain:

$$E_x[\alpha(x,y)|y] = (1-r)E_x[E[\theta|x,y]|y] + rE_x[E_z[\bar{\alpha}(z,y)|x,y]|y]$$

Simplifying,

$$\bar{\alpha}(y) = (1-r)E[\theta|y] + r\bar{\alpha}(y)$$

And hence, along the equilibrium path, we have

$$\bar{\alpha}(y) = E[\theta|y]$$

Finally, using arguments identical to Lemma 6, it may be shown that, $a(x,y)$ is independent of x in the limit as $r \to 1$. Hence, in the limit, $\bar{\alpha}(y) = \alpha(y) = \alpha(x,y)$. Finally, since y is correlated with θ, it follows immediately that $\bar{\alpha}(y)$ depends on y. ■

Together, these two lemmas imply that public information always has positive social value for r sufficiently large. Absent any public signal, actions become completely unresponsive to the realized state in the limit; whereas this is not the case in the presence of a public signal. Hence public information improves welfare.

Turning to the other polar case, it is a simple matter to show that, at $r = 0$, public information is beneficial. To see this, recall that individual best responses in that case are simply to choose $a = E[\theta|I]$, where I represents the information available to a given player. Since the public signal is correlated with the state, it follows that it will receive positive weight in computing the conditional expectation and hence actions will correlate more closely with the state in the presence of the public signal than in its absence.

This implies that public information can only have negative social value for intermediate weights on coordination, as with our earlier analysis. Thus, we have shown:

Theorem 8 *For all data generating processes for the state variable, under proper priors, the social value of public information, W, is positive for r sufficiently small and r sufficiently large.*

Furthermore, if public information has negative social value then r must be interior and welfare must be non-monotonic in r.

4 Convergence of Proper and Improper Priors Models

Our previous results might suggest that there is a discontinuity in welfare between the two models of priors. This is not the case for a specific order of limits as may be readily seen when the state space is normally distributed. If one first takes the limit as the variance of the state distribution becomes unbounded, i.e. $\sigma_\theta \to \infty$, equilibrium actions converge to those of the improper priors model. In particular, actions in the absence of a public signal no longer depend upon r whereas those with a public signal do. It then immediately follows that the welfare results of the original Morris-Shin model also obtain: The social value of public information is decreasing in r and is negative as $r \to 1$ when $s > p$. By contrast, our analysis above fixed the variance at some finite amount and then examined the social value of public information as r varied.

Formally, recall that, for finite variance,

$$\lim_{r \to 1} W = 1 + p$$

Now, taking limits as $\sigma_\theta \to \infty$,

$$\lim_{\sigma_\theta \to \infty} \lim_{r \to 1} W \to \infty$$

which is intuitive since, as $r \to 1$, actions absent a public signal converge to the prior mean while those taken under a public signal converge to a convex combination of the prior and the public signal. In the limit as $\sigma_\theta \to \infty$, actions correlating with the state at all will be infinitely better than those totally uncorrelated with the state.

In contrast, if we take limits in the opposite order, we first obtain:

$$\lim_{\sigma_\theta \to \infty} W = \frac{1}{\sigma_y^2} \frac{\left(\sigma_x^2 + (1-r)\,\sigma_y^2\right)^2}{\sigma_x^2 + (1-r)^2\,\sigma_y^2}$$

Now, taking limits as $r \to 1$, we have

$$\lim_{r \to 1} \lim_{\sigma_\theta \to \infty} W = \frac{\sigma_x^2}{\sigma_y^2}$$

which yields the finding that the social value of public information is negative if and only if the public signal is less precise than is the private signal, i.e. $s > p$.

Taking limits in this order shows that, unsurprisingly, the Morris and Shin model is simply the limit of a proper priors model as the variance of the state grows arbitrarily large. Thus, it would seem to be a reasonable approximation of the situation where the variance of the state is large but finite. How then can we reconcile the apparently different qualitative features of the two models?

Figure 3 graphically illustrates convergence behavior. It depicts the welfare ratio under improper priors, which is independent of σ_θ, and that under proper priors for varying values of σ_θ. Notice that the two curves approximately coincide for low values of r before diverging, with the proper priors curve turning upward and the improper priors curve continuing downward. Notice too that, as σ_θ increases, so too does the interval over which the two curves approximately coincide. In the limit, the two curves coincide over the entire interval of r. The figure shows that, as the state variance increases, the improper priors model well-approximates proper priors for ever larger intervals of coordination weights. For large coordination weights, however, it is always a poor approximation. The reason this is problematic is that the region of interest, where the social value of public information is most likely to be negative, occurs when coordination weights are large. In the neighborhood of $r = 1$, the two models appear discontinuous despite converging in the limit.

(See Figure 3 at end of this file)

5 Conclusions

While at a broad level, the main observation of the seminal Morris-Shin paper remains valid—public information can have negative social value—the intuition is more nuanced than what is revealed by the improper priors model. In that model, decisions made under a public signal become increasingly distorted as individuals care more about coordinating with one another, and the worst-case scenario occurs when these motives have the largest possible weight. Here, the social value of public information is most likely to be negative. This is never the case under proper priors—quite the contrary, we showed that public information has the *highest* social value under these same circumstances.

The key insight is that the prior mean itself is a type of public signal, and this fundamentally changes equilibrium behavior. The prior mean's role can be seen starkly in situations absent public information. With improper priors, coordination motives are irrelevant to equilibrium actions, since there is no shared information upon which to coordinate. With proper priors, coordination motives distort actions, just as they do with a public signal. Indeed, as coordination motives become paramount, equilibrium actions in the two models differ radically—actions make optimal use of information under improper priors and make *no* use of information under proper priors; instead, individuals simply coordinate on the prior mean.

Conclusions about welfare also change. Unlike the case with improper priors, the social value of public information always varies non-monotonically with coordination weights, first falling, then rising, and ultimately exceeding, the social welfare absent public information. When coordination weights are high, the positive contribution of the public signal to welfare becomes obvious. Without such information, individuals simply coordinate on the prior mean; with this information, they coordinate on the somewhat informative public signal as well.

Despite these differences, the improper priors model does not represent a singularity or a knife-edge case. The two models converge as the variance of the state variable becomes unbounded, but the nature of convergence varies with the weight on coordination motives. With low coordination weights, behavior is similar in the two models; whereas with high weights, it is not. As prior beliefs become more diffuse, the boundary between low and high moves inexorably upward, finally reaching the top only in the limit. The key issue is that this poorly approximated region contains the most important results from the improper priors model and hence produces qualitatively different welfare patterns.

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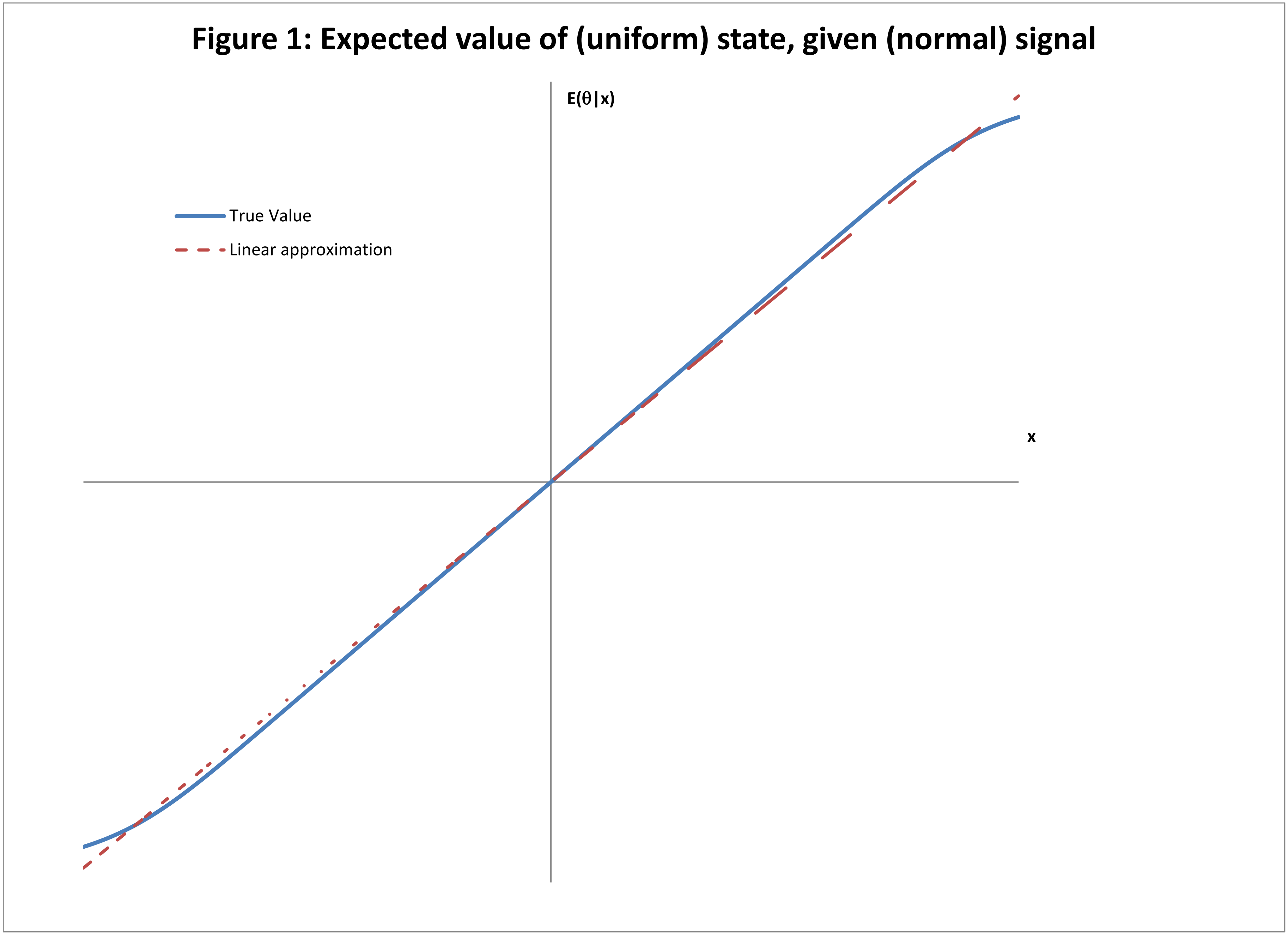
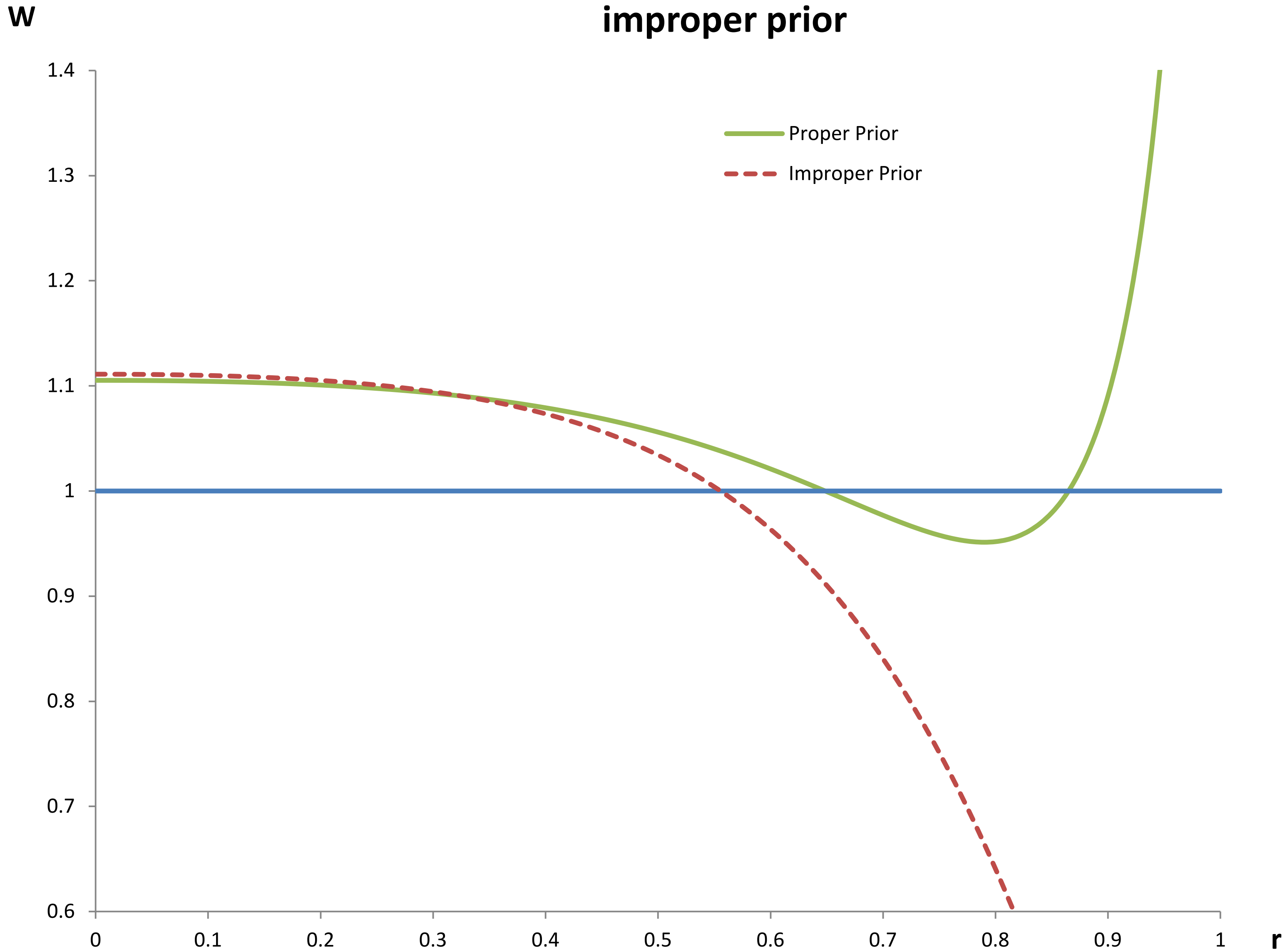


Figure 1: Expected value of (uniform) state, given (normal) signal
E(θ|x)
True Value
Linear approximation
x

Figure 2: Welfare Effect of Public Information, under proper and improper prior



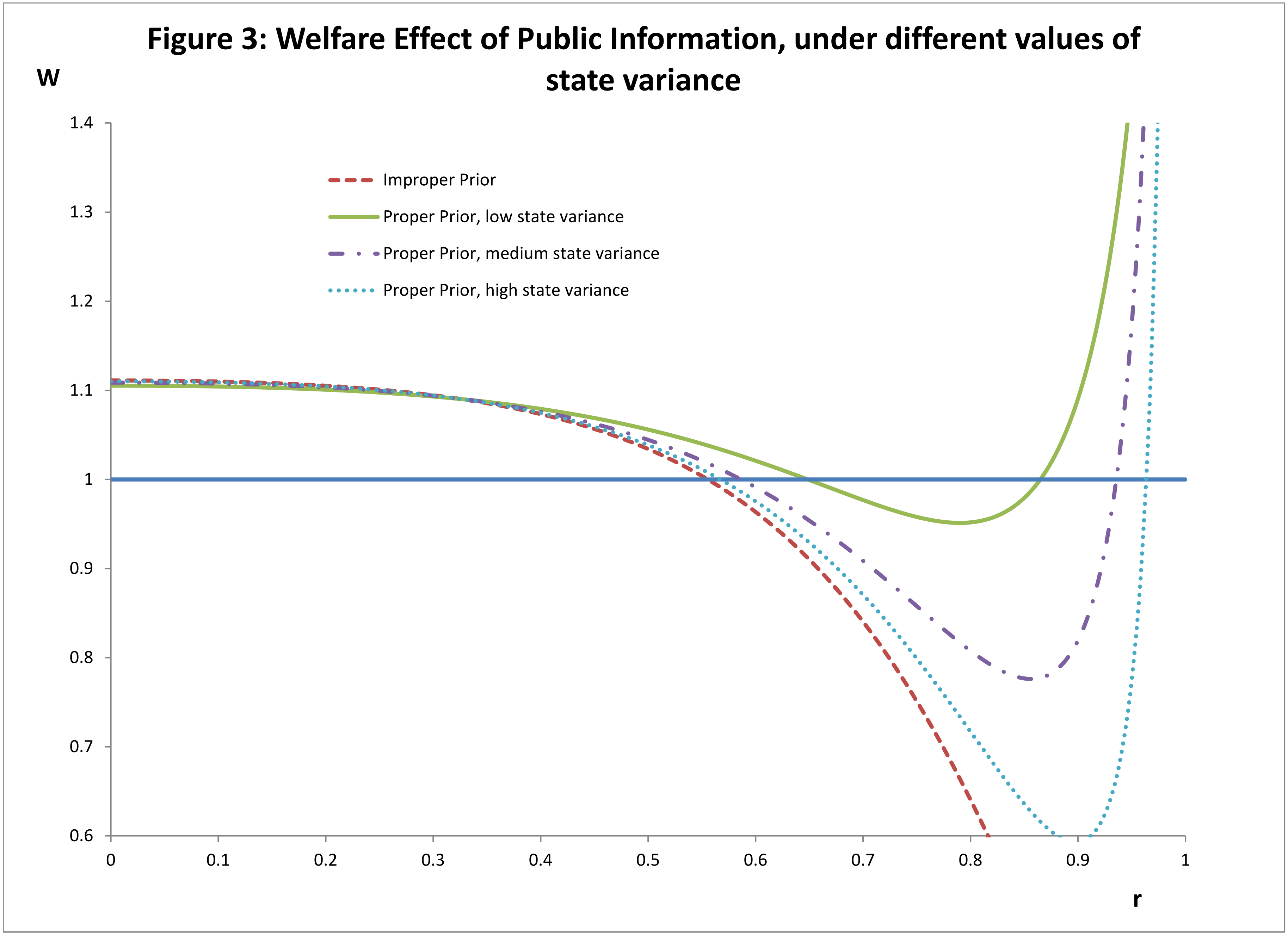


Figure 3: Welfare Effect of Public Information, under different values of state variance
W
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
Improper Prior
Proper Prior, low state variance
Proper Prior, medium state variance
Proper Prior, high state variance
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
r