

# Efficiency in Auctions: Theory and Practice\*

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August 25, 2000

## Abstract

In many policy contexts, efficiency is the primary consideration in structuring auctions. In this paper, we survey several sources of inefficiency arising in auctions. We first highlight the effects of demand reducing incentives, both in theory and in practice, in multi-unit auctions. Next, we study inefficiencies arising from interdependence in bidder valuations. Again, we highlight both theoretical insights as well as how these translate in practice. Finally, we present an impossibility theorem for attaining efficiency in sufficiently rich auction contexts. An auction form suggested by Klemperer is discussed as a means of ameliorating inefficiencies arising in practice.

JEL Classification Nos. D44, D82

Keywords: Auctions, Efficiency

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\*Updated copies of this paper can be found at [www.wws.princeton.edu/~rjmorgan](http://www.wws.princeton.edu/~rjmorgan).

<sup>†</sup>This paper was prepared for the conference, Central Bank Operations: Theory and Evidence. This conference was organized by the Center for European Integration Studies with the support of Bank of Spain, Deutsche Bundesbank, and European Central Bank. The financial assistance of the above entities is gratefully acknowledged. I also thank Michael Baye, Vijay Krishna, and Benny Moldovanu for useful comments. The editorial assistance of Heather Morgan was invaluable in enhancing the readability of this paper.

# 1 Introduction

In many contexts, efficiency is a primary consideration of policy makers in deciding how to allocate certain property rights. This is nicely illustrated in the opening letter of the information package for the US broadband PCS auctions. In this letter, Reed Hundt, Chairman of the US Federal Communications Commission, writes: “I am confident that the auction method we have chosen will put the spectrum in the hands of those who most highly value it and who have the best ideas for its use.”<sup>1</sup> Indeed, in the law authorizing the spectrum auctions, efficiency was explicitly stipulated by the US Congress to be the main goal. Likewise, efficiency was acknowledged to be the main criterion in the allocation of third generation (3G) spectrum licenses in the UK. (See Klemperer 2000a for details.)

From the perspective of the European Central Bank, efficiency in the provision of liquidity in European currency markets would likewise appear to be an essential objective in conducting effective monetary policy. In this paper, we survey a number of important findings, both theoretical and empirical, about the efficiency of various auction forms. Our hope is that, by providing a taxonomy of some of the causes of and solutions to efficiency problems, as well as assessing the effectiveness of these solutions in the field, this survey will serve as a useful guide to policy makers in the selection of an appropriate auction form for achieving desired outcomes.

In many respects, policy makers are in a unique position to “customize” their auction forms to address problems specific to the environment in which they operate. The dramatically declining cost of information technology now makes possible the creation of a virtual marketplace whose design can be strongly guided by efficiency concerns. Moreover, the ubiquitousness of electronic communication means that physical barriers to coordinating geographically disparate parties is much less an obstacle than in the past. Finally, information technology allows for the possibility of quick and transparent diffusion of information in auctions. Compelling examples of customized auction design are US and European spectrum auctions. In allocating spectrum licenses, policy makers, assisted by academicians, have developed new auction forms specifically designed to address problems unique to each market. For instance, the Federal Communications Commission turned to auction theorists to develop an auction design to best achieve their goal of allocat-

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<sup>1</sup>Quote taken from Ausubel and Cramton (1998), p. 2.

ing licenses efficiently. The auction form that was developed, known as the simultaneous-ascending auction, blended design elements of a number of different traditional auction forms into a unique hybrid. Other new auction forms continue to be developed in the areas of European spectrum, pollution rights, the regulation of greenhouse gases, and electric power transmission.

The remainder of the paper proceeds as follows: In Section 2, we introduce a model that is sufficiently flexible to be adaptable to all of the scenarios considered in the paper. We then highlight, in Section 3, the effects of demand reducing incentives, both in theory and in practice, in multi-unit auctions. The evidence suggests that the use of uniform-price auctions (Dutch auctions in the context of the European Central Bank) by policy makers may yield inferior results compared to an alternative sealed bid auction form. Next, we study, in Section 4, inefficiencies arising from interdependence in bidder valuations. Again, we highlight both theoretical insights as well as how these translate in practice. The policy implications of this section are that, when interdependent valuations are a concern, as is likely to be the case for European liquidity transactions, policy makers should be cognizant of the fact that open auction forms might be superior. Finally, we present, in Section 5, an impossibility theorem for attaining efficiency in sufficiently rich auction contexts. The Anglo-Dutch auction, an auction form suggested by Klemperer (2000a), which combines both open and sealed bid aspects is discussed as a means of overcoming a variety of possible inefficiencies arising in practice.

## 2 The Model

To study efficiency in the context of auctions, we begin by presenting a simple model that is nonetheless flexible enough to be employed in a wide variety of settings. An auctioneer is auctioning off a quantity of some (possibly divisible) good. We normalize the quantity available to be 1.<sup>2</sup> There are  $N \geq 2$  bidders, numbered  $i = 1, 2, \dots, N$  competing for the good. An allocation consists of a partition of the good among the bidders. A bidder's payoff only depends on the proportion of the good allocated to her. Let  $q_i$  denote the portion of the good allocated to bidder  $i$ .

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<sup>2</sup>This does not mean that we have restricted attention to single unit auctions. If one thinks of the good as partially or completely divisible, one obtains the multi-item case up to a continuum of items.

Each bidder  $i$  receives a (possibly multidimensional) signal  $s^i \in S^i$ . Signals have continuous density  $f_i(s^i) > 0$  and are independent across bidders. Bidders all have quasi-linear utilities of the following form:

$$V^i = \min(q_i, c_i) \sum_{j=1}^N a_i^j s^j + x_i.$$

The parameter  $c_i$  is the bidder’s “capacity.” That is, for a portion of the object above  $c_i$ , bidder  $i$  obtains zero marginal utility. The vector of coefficients,  $a_i^j \geq 0$ , reflect the impact of bidder  $j$ ’s signal on agent  $i$ ’s value for the object. Both  $c_i$  and the coefficients are assumed to be common knowledge. The variable  $x_i$  represents the money transfer to/from bidder  $i$ .

To get a flavor of the model, it is helpful to consider the canonical independent private-values case where there is a single, indivisible object being auctioned. In this case, each bidder receives a one dimensional signal,  $c_i = 1$ ,  $a_i^i = 1$ , and  $a_i^j = 0$  for all  $j \neq i$ .

A second-price sealed bid auction would have the following properties:  
Letting  $b_i$  be the bid of bidder  $i$ , then

$$\begin{aligned} \Pr(q_i = 1) &= 1 \text{ if } b_i > b_j \text{ for all } j \neq i \\ \Pr(q_i = 1) &= \frac{1}{m} \text{ in an } m\text{-way tie for highest} \\ \Pr(q_i = 1) &= 0 \text{ otherwise.} \end{aligned}$$

and  $x_i = -\max_{j \neq i}(b_j)$  if  $q_i = 1$  and  $x_i = 0$  otherwise.

### 3 Inefficiency in Multi-Unit Auctions

A fundamental question that policy makers face is devising procedures to allocate multiple, identical units of a good. The allocation of debt securities in the US is a classic example of this situation. Historically, US debt was auctioned using a pay-your-bid (also known as ‘discriminatory’) auction.<sup>3</sup> In this auction form, bidders submit a bid price for various quantities of securities. The Treasury then determines the market-clearing price and all bids exceeding this price are awarded their quantities demanded at the price

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<sup>3</sup>Also known as an ‘American’ auction in the context of the European Central Bank.

they offered. Friedman (1960) proposed that this mechanism be replaced by a ‘uniform-price auction.’ Under this auction, bidders once again submit bids for various quantities at a particular price. Once again, the Treasury determines the market clearing price. Bidders making offers above the market clearing price receive their quantities demanded; however, all winning bidders pay the *market clearing price*.

Much of the debate over the merits of these competing auction forms has been guided by observations in the case where bidders have private valuations for a single object being auctioned. Some of the earliest work in this area is due to Vickrey (1961) who made the observation that by having bidders submit sealed bids, awarding the object to the highest bidder but having that bidder pay the second highest bid, it would then be in the interests of the bidders simply to reveal their values truthfully in the form of their bids. As a consequence, the object would be efficiently allocated to the bidder valuing it most highly. This auction form, known as the Vickrey auction, represented a successful first attempt to guide the design of auctions through considerations of efficiency. Indeed, professional economists widely believed that the uniform-price auction was the clear, multi-unit analog of that proposed by Vickrey. For instance, in an address to the Treasury, economist Robert Weber (1992) notes: “The second-price [Vickrey] auction naturally generalizes to a uniform-price auction, where all bidders pay an equal price corresponding to the highest rejected bid.”<sup>4</sup>

It has been widely argued that the merit of the uniform price auction over the discriminatory auction is a reduction in bid-shading. For instance, Merton Miller offers the following argument against the discriminatory auction, “People will shave their bids downward. All of that is eliminated if you use the [uniform-price] auction. You just bid what you think it’s worth.”<sup>5</sup> In a similar vein, Friedman suggests: “[The uniform price auction] has the major consequence that no one is deterred from bidding by being stuck with an excessively high price. You do not have to be a specialist. You need only know the maximum amount you are willing to pay for different quantities.”<sup>6</sup> Taken together, these arguments suggest two advantages for the uniform price over the discriminatory auction. The first is that, by reducing distortions in bidding, the uniform price auction improves the allocative efficiency

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<sup>4</sup>Taken from the text of Weber’s June 3, 1992 opening address at the US Federal Reserve/Treasury’s “Forum on Change.”

<sup>5</sup>As quoted in Ausubel and Cramton (1998), p 3.

<sup>6</sup>*Ibid.*

of the auction directly. Second, by simplifying the bidding strategies needed to compete successfully, the uniform price auction might encourage entry – especially among non-specialists. This latter effect has been little studied by auction theorists to date.

The intuition academics rely upon for the absence of bid-shading in the uniform-price auction appears to derive from a special case of this auction form where each bidder wants at most one unit of the object being auctioned. In this case, it is a simple matter to show that in the uniform-price auction, bidding one’s value is a weakly dominant strategy. That is, regardless of the bidding strategies being employed by competitors, a bidder can do no better than to simply bid his or her value for a single unit of the good. In contrast, equilibrium in the discriminatory auction entails bid shading. (Indeed, bidding one’s value in this auction form is a weakly dominated strategy.) Moreover, the appropriate bid shading strategy for a bidder in this auction depends crucially on his or her conjectures about how aggressive the bids of her competitors will be. Thus, computations as to the appropriate amount to bid in this auction are arguably more complicated than in the uniform-price auction. Interestingly, in the canonical case where bidders are symmetric; valuations are independent and private; and there is a single indivisible object being auctioned; there is nothing to distinguish between the two auction forms. Both have equilibria that are fully efficient and both raise exactly the same amount in revenues.<sup>7</sup>

### 3.1 Theoretical Results on Efficiency

#### The Inefficiency of the Uniform-Price Auction

When bidders demand more than one unit, simply bidding one’s value is no longer optimal in the uniform-price auction. This can be illustrated by an extremely simple example.

**Example 1** Suppose that valuations are common knowledge. There are two identical objects being auctioned. If bidder 1 receives one of the objects, she values it at \$10. If she receives both objects, then, she values the second object at an additional \$9. Bidder 2, on the other hand, only desires one object, which he values at \$8. Bidder 2 derives no additional value for the

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<sup>7</sup>This follows from the “Revenue Equivalence Theorem.” See Klemperer (2000b) for a particularly clear statement.

second object. It is clear that from an efficiency standpoint both objects should go to bidder 1.

Suppose that the auctioneer holds a uniform-price auction. If both bidders simply bid their values, then bidder 1 will receive both objects and pay the market clearing price. In this case, that price is bidder 2's bid of \$8 for each object. In that case, bidder 1's surplus is \$3 (\$10 minus \$8 for the first object and \$9 minus \$8 for the second object.) The auctioneer earns revenues of \$16.

Clearly, bidder 2 can do no better than to bid his value, but bidder 1 can improve her surplus by deviating. Suppose that instead of bidding her value (\$9) for the second object, bidder 1 reduces her demand for the second object by bidding \$0 for it. In this case, bidder 1 receives only one object, bidder 2 receives one object, and the market clearing price drops to \$0. This allocation is inefficient and raises no revenues for the seller; however, it does dramatically increase the surplus to bidder 1 (and to bidder 2 as well). Indeed, it is straightforward to verify that this strategy is an equilibrium for both bidders; whereas bidding one's value is not.

The example starkly illustrates the key difference between the single and multiple object cases. In the single object case, one's bid never affects the final price paid by a bidder. This is always determined by a competitor's bid. This is not the case when there are multiple objects. As the example showed, bidder 1's bid for the second object indeed determined the price she paid for one of the objects. Thus, for exactly the reasons in the discriminatory auction, bid-shading on subsequent objects beyond the first is an appropriate strategy for a bidder in a uniform-price auction.

Of course, the inefficiency in this auction arises not from bid shading *per se*, but rather from differential bid shading. Neither bidder shades his or her bid for the first object, but does shade for subsequent objects. This creates the possibility that bidder 1's shaded bid for the second object will fall below bidder 2's unshaded bid, thus splitting the allocation of the objects between the bidders instead of efficiently awarding them to bidder 1.

An important result of Ausubel and Cramton (1998) shows that the uniform price auction is generally inefficient. To illustrate this, we now consider a version of the model presented in Section 2. Suppose that the object is infinitely divisible and that bidders have identical capacities  $c_i = c'$  for all  $i$ .<sup>8</sup> Further, suppose that there are no interdependencies in the valuations of

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<sup>8</sup>The arguments here are presented for the simple case where all the capacities are equal.

the bidders, so the coefficients are  $a_i^i = 1$  and  $a_i^j = 0$  for all  $i, j \neq i$ .

In this model, bidders submit bid functions  $b_i(q)$  that are right continuous and weakly decreasing in  $q$ . It is equivalent, however, for the bidders to submit demand functions  $q_i(b)$  that are left continuous and weakly decreasing in  $b$ . Let the aggregate demand curve be given by  $Q(b) = \sum_{i=1}^N q_i(b)$ . A market clearing price,  $p$ , is such that  $\lim_{b \downarrow p} Q(b) \leq 1$  and  $\lim_{b \uparrow p} Q(b) \geq 1$ . All inframarginal agents then receive their demand at price  $p$ . If  $\lim_{b \uparrow p} Q(b) > 1$ , then the marginal agents are proportionally rationed. Let marginal agent  $i$ 's incremental demand at price  $p$  be

$$\Delta_i(p) = \lim_{b \downarrow p} q_i(b) - q_i(p).$$

Such an agent receives

$$q_i = \left(1 - \lim_{b \downarrow p} Q(b)\right) \frac{\Delta_i(p)}{\sum_{j=1}^N \Delta_j(p)}.$$

As usual, an outcome of an auction consists of an allocation  $(q_1, q_2, \dots, q_N)$  and a set of transfers  $(x_1, x_2, \dots, x_N)$ . An outcome is efficient if the good goes to bidders with the highest valuations up to their capacity constraint. Let  $m$  denote the largest integer such that  $mc' < 1$ . Further, suppose that  $c' \times (m + 1) > 1$ . Thus, an ex post efficient allocation gives  $c'$  of the good to the  $m$  bidders with the highest valuations and gives a rationed share  $1 - mc'$  to the bidder with the  $m + 1$ st highest valuation.

Ausubel and Cramton (1998) show that bidding in ex post efficient auctions must have the following properties:

**Lemma 1** *In the model above, ex post efficiency implies symmetric, flat bidding functions for almost every  $s^i$ . Moreover, bid functions are increasing in  $s^i$ .*

We first offer some intuition for these results: For any  $s > s'$ , it must be the case that  $b_i(c'; s) \geq b_i(0; s')$ . Otherwise, for some realizations it will be efficient for  $i$  to win  $c'$  with valuation  $s$ , and  $q_i = 0$  with valuation  $s'$ . But if  $b_i(c'; s) < b_i(0; s')$ , then for some realizations of valuations, the auction will allocate the objects inefficiently. Thus, strictly downward sloping bid functions are not consistent with efficiency. For similar reasons, strictly upward

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The economic intuitions for this case readily generalize.

sloping bidding functions are also not consistent with efficiency. Thus, an efficient auction must entail flat bidding functions.

Since bidders in ex post efficient auctions have flat bidding functions, it is useful to define  $\phi_i(s)$  to be agent  $i$ 's cutoff price for demanding  $q'$ . That is, for all prices below  $\phi_i(s)$ , bidder  $i$  demands  $c'$  units and, above this, she demands none. Symmetric bidding strategies imply that, for all agents,  $\phi_i(s) = \phi(s)$ .

To see that this is required for efficiency, suppose instead that two bidders,  $i$  and  $j$ , use differing cutoff strategies. Then for some realizations  $s^i > s^j$ , it will be the case that  $\phi_i(s^i) < \phi_j(s^j)$ . Denote by  $s_{(k)}$  the realized valuation of the agent with the  $k$ th highest valuation. Suppose that  $s^i > s_{(m+1)} > s^j$ . Under these conditions, efficiency requires that  $i$  receive  $c'$  units of the good and  $j$  receive none. However, since  $\phi_i(s^i) < \phi_j(s^j)$ , bidder  $j$  will be allocated a positive amount of the good. Thus, differing cutoff strategies can lead to inefficient outcomes. The same argument also implies that cutoffs must be increasing in  $s$ .

The main result of Ausubel and Cramton (1998) is to show that no equilibrium in the uniform price auction entails bidding functions satisfying Lemma 1.

First, recall that in the uniform price auction, each bidder simply pays the market clearing price for each object that he or she obtains. That is,  $x_i = pq_i$ .

Proving the result consists of showing two steps:

1 In an efficient equilibrium, bidders must bid their values.

Bidder  $i$  is bidding against  $N - 1$  flat bidding functions. Thus, the aggregate demand function she faces consists of steps at  $c'$ ,  $2c'$ , ...,  $(N - 1)c'$ . For any share  $q < mc'$ , bidder  $i$ 's own bid does not affect her price. Thus, it is a weakly dominant strategy to bid one's value for these units. Combining this observation with Lemma 1 implies that bidders must bid their values.

2. Bidding one's value is not a best response to "sincere" bidding by all other bidders.

Consider the following deviation:

$$q_i(b, v) = \begin{cases} c' & \text{if } b \in [0, b') \\ 1 - mc' & \text{if } b \in [b', v] \\ 0 & \text{otherwise} \end{cases}$$

where  $b' \leq v$ . Obviously, sincere bidding is a degenerate case of the above

strategy. The key is to show that expected payoffs are decreasing in  $b'$  when evaluated at  $b' = v$ .

There are three regions to consider:

1. Suppose that the  $m$ th highest valuation of the other bidders is less than  $b'$ . In this case, bidder  $i$  gets  $c'$  and a slight decrease in  $b'$  does nothing to expected payoff.
2. Suppose that the  $(m + 1)$ st highest valuation exceeds  $b'$ . In this case, bidder  $i$  gets nothing and a slight decrease in  $b'$  does nothing to expected payoff.
3. Suppose that  $b'$  is between the  $m$ th and  $(m + 1)$ st valuations of the other bidders (this happens with positive probability). Then, bidder  $i$  obtains quantity  $1 - mc'$  of the good and lowers her price by decreasing  $b'$  below  $v$ . This improves her expected profits.

Thus, efficiency is impossible in the uniform-price auction.

The simple intuition is captured in the model; however the result may be generalized to the case of different capacities, correlated values, and downward sloping demands. In all of these cases, the general intuition that bid shading leads to inefficiency plays a crucial role.

To summarize:

**Theorem 1** *There is no equilibrium in the uniform price auction that is efficient.*

### The Vickrey Auction

A key insight in Vickrey's (1961) work on auction design was the observation that by having a winning bidder pay an amount unaffected by her own bid, sincere bidding becomes incentive compatible. Moreover, by making the amount paid by the winning bidder equal to the "externality" imposed on the other bidders; that is, by having the winning bidder pay an amount equal to the surplus in the absence of that bidder, private and social incentives are aligned and efficiency results. While an auction achieving these goals is quite straightforward in the unit demand case, the most obvious generalization in the multi-unit world, the uniform-price auction, does not share its desirable properties.

To see the appropriate generalization, it is useful to return to Example 1. Recall that bidder 1 had values of \$10 and \$9 respectively for the 1st and 2nd units of the good; whereas bidder 2 had values of \$8 and \$0. Suppose that bidders bid sincerely and we order the bids from highest to lowest. In this case, the order is \$10, \$9, \$8, \$0. The highest two bids are awarded units – in this case both units go to bidder 1.

For sincere bidding to be in the interests of the bidders, it is necessary that what you pay should be unaffected by what you bid. Hence, bidder 1's payments will only be affected by bidder 2's bids and vice-versa. Next, to achieve efficient allocations, we would like each bidder to pay an amount equal to the surplus that would have been achieved had that bidder been absent. For instance, absent bidder 1, both objects would have been allocated to bidder 2. Bidder 2 valued these objects at \$8 and \$0; hence his surplus is \$8. Equivalently, since bidder 1 displaced the bids \$8 and \$0 by bidder 2, she pays \$8 for the two units.

More generally, consider a  $k$ -object auction with the following rules: Bidders each submit bids for both objects.. Bids are arranged highest to lowest. The  $k$  highest bids are awarded the object. Each bidder pays an amount equal to the bids *that she did not submit* that were displaced from winning.

Clearly, bidder 2 can do no better than to bid his values since bidding above bidder 1 simply results in losses. Likewise, since 1's bids do not affect her payments, she can do no better than to bid sincerely as well.

While this design admirably solves the efficiency problem. It can have rather unfortunate equity consequences which might make it less desirable from a policy standpoint.

**Example 2** Suppose that bidder 1 has valuations of \$10 and \$7, respectively. Bidder 2 still has valuations of \$8 and \$0. Under sincere bidding, the Vickrey auction correctly allocates one object to each of the bidders; however bidder 2 pays \$7 for his object while bidder 1 pays nothing for hers. Arguably, it is unfair to have a bidder who values the object less end up paying more for it.

Even more unpleasant constructions are possible. Suppose that there are 10 objects being auctioned. Bidder 1 values the first 9 at \$10 each and the last one at \$7. Bidder 2 values the first at \$8 and the remainder at zero. Again, the Vickrey auction will efficiently give 9 objects to bidder 1 and one

object to bidder 2. Again, bidder 2 will pay \$7 for his object, while bidder 1 will get all nine of the objects for free. Such an outcome would seem to be fairly unpalatable from a political standpoint.

We now return to the model and offer a formal definition for allocation and payment rules in a Vickrey auction.

Define the aggregate demand of all bidders except  $i$  to be

$$Q_{-i}(b) = \sum_{j \neq i} q_j(b).$$

Define  $p^0$  to be the market clearing price as above and define  $p^{-i} = \inf \{p : Q_{-i}(p) \geq 1\}$ . That is,  $p^{-i}$  is the market clearing price in the absence of bidder  $i$ . Ignoring rationing, the Vickrey auction awards a quantity  $q_i(p^0)$  to bidder  $i$  and requires  $i$  to pay

$$x_i = q_i(p^0) p^0 - \int_{p^{-i}}^{p^0} (1 - Q_{-i}(t)) dt.$$

This is illustrated in the following diagram.

*Figure 1 here.*

It is straightforward to show that sincere bidding is an equilibrium in this auction mechanism. To summarize:

**Proposition 1** *Sincere bidding is an efficient equilibrium in the Vickrey auction.*

### 3.2 Efficiency in the Field

Recall that demand reduction was the key factor leading to inefficiency in the uniform-price auction. Demand reduction was, of course, a strategic response to the possibility that one's own bids for succeeding units would affect the price paid for earlier units. While theoretically correct, from a policy perspective one may well wonder how important this factor is in affecting actual bidding behavior. Indeed, it seems plausible that once there are sufficiently many bidders in the market, the possibility of one's own bid affecting the price becomes negligible and hence the efficiency gains from the Vickrey auction become quite small. Further, even these small benefits might be more than offset by the perceived inequities that can arise using the Vickrey auction.

List and Lucking-Reiley (2000) and Engelbrecht-Wiggans, List, and Lucking-Reiley (2000) examined these issues in a series of field experiments. In the US, there is an existing active market in the trading of sports memorabilia, such as sports cards, autographs, and so on. A common way in which buyers and sellers meet in this market is through sports card trading shows. Card dealers typically set up stalls displaying their cards at these shows and negotiate sales prices with buyers. The details of how these negotiations come to pass are variable.

The field experiments take advantage of this existing market and compare allocations under a uniform-price auction and a Vickrey auction as the number of bidders varies. Specifically, two-unit auctions consisting of 2, 3, and 5 bidders were run using each of the auction formats. The “subjects” of the experiment consisted of either only trading card dealers or only non-dealers. The objects being auctioned consisted (mainly) of three different versions of a Cal Ripken, Jr. rookie card.<sup>9</sup> Three companies manufactured Ripken rookie cards: *Topps*, whose card is considered the most valuable, and *Fleer* and *Donruss*, whose cards are less valuable than *Topps*, but equal to one another. The *Topps* card has a “book” value of \$70, while the others have a book value of \$40.

In each of the experiments, bidders were taken through the following four step experimental procedure:

1. A subject was invited to participate, told that the auction would take about five minutes, and given an opportunity to examine the cards being auctioned.
2. The subject was given the instructions for the auction and asked to compute an example to demonstrate knowledge of the auction rules. The subject was also told that she would be randomly matched with a set number of other bidders of her type (dealers if she was a dealer and non-dealers if she was a non-dealer).
3. Subjects submitted bids.
4. The monitor explained the ending rules for the auction.

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<sup>9</sup>In one of the treatments, a *Score* Barry Sanders rookie card was auctioned. Its estimated value is comparable to the *Topps* Ripken card.

Notice that winning bidders were required to make payments in accordance with the auction rules and received the card or cards. Bidders were segregated into dealer and non-dealer groups since the experimenters believed that the motivation for demand by dealers (i.e. the possibility of resale) caused their valuations to differ substantially from the non-dealers. Participation in the experiments was, of course, entirely voluntary.

Table 1 below summarizes the treatments:

*Insert Table 1 here.*

Since the valuations that bidders place on each of the cards is unobservable, it is not possible to directly estimate whether bidders are employing sincere bidding strategies; however, by comparing across auction types, it is possible to test for the existence of demand reduction effects. Further, by comparing allocations under the two auction forms, it is again possible to infer some rough measure of the impact of demand reduction on efficiency in allocations. Note, however, that all of these measures take as given that bidders are employing something close to sincere bidding in the Vickrey auction. We shall return to this assumption later.

Table 2 below presents summary statistics of the results of the auctions.

*Insert Table 2 here.*

We now highlight the key aspects of Table 2. First, in the case of the two bidders treatments, there is strong evidence consistent with demand reduction. More formally, one can reject the null hypothesis that the unit 2 bids equal one another in favor of the one-sided alternative that the Vickrey bids lie above the Uniform bids at the 1% significance level. Further evidence of this occurs in the form of zero bids, which are a much higher percentage of the bids in the uniform-price auction as compared to the Vickrey auction. Finally, demand reduction appears to have a strong impact on allocations. There are far more split allocations under the uniform price auction as compared to the Vickrey auction. Thus, for the case of two bidders, it appears that strategic bidding does occur in the uniform-price auction and that this has an effect on economic outcomes.

Turning to the three and five bidder treatments, it appears that the introduction of even a small number of additional bidders substantially reduces demand reduction effects. In one treatment, ND5, mean second unit bids in the Vickrey auction are in fact lower than in the uniform-price auction. In the remaining treatments, the results are consistent with demand reduction; however, none of these differences is statistically significant at conventional levels. The proportion of zero bids still tends to be higher in uniform auc-

tions; however it is difficult to see an economic impact from the small amount of demand reduction. Split allocations are largely the same in both auction forms. This proxy for efficiency, however, becomes extremely coarse as the number of bidders increases. To sum up, the field experiments at least suggest that demand reduction becomes less of an issue with even modest increases in the number of bidders.

A clear limitation in this experimental design is that, to make sensible comparisons between the auction forms, one must assume that equilibrium strategies are a good description of bidder behavior. In the case of the Vickrey auction, this may appear to be an unproblematic assumption in that it is a weakly dominant strategy to bid sincerely in private values settings. However, there may be good reason to doubt this characterization of behavior. Returning to the two bidder treatments in Table 2, notice that mean bids for the first unit are much lower than those under the uniform-price auction despite the fact that, in a private values setting, it is a weakly dominant strategy in both auctions to bid sincerely. The theoretical prediction of no difference between first unit bids across auction forms is rejected at conventional significance levels. In the three and five bidder treatments, there are no significant differences in first unit bids. Thus, one is left to wonder whether the differences in the allocations in the two bidder treatment derive from demand reduction or from anomalous bidding behavior in one or both auction forms.

One can get a clearer sense of bidding behavior in Vickrey auctions through laboratory experiments. In these experiments, it is possible to observe a bidder's valuation for each unit as well as to ensure that the conditions specified in the model are satisfied. The drawback of these experiments relative to field experiments is that the environment is somewhat less natural and thus behavior may not be reflective of what one sees in practice. Nonetheless, it is useful to highlight two sets of relevant laboratory experiments.

Kagel and Levin (1999) use an innovative laboratory design to make comparisons between uniform price and a dynamic analog to the Vickrey auction, called the Ausubel auction (see Ausubel (1997)), that is also theoretically efficient.<sup>10</sup> In their design, a single human subject with induced flat demand for 2 units competes against a variable number of computerized bidders with

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<sup>10</sup>To be more precise, they compared a number of variations of the uniform-price auction with the dynamic Vickrey mechanism proposed by Ausubel. In describing their results, we focus on standard sealed bid versions of the uniform price auction.

single unit demand in a two unit auction. The computerized bidders bid sincerely for their single unit demand and the human bidder is made aware of the fact that the computerized bidders are employing this strategy. This design has the advantage of eliminating strategic uncertainty over the actions of rival bidders as an explanation of observed bidding behavior. In the uniform-price auction, it is optimal to bid sincerely for the first unit and to bid zero for the second unit. In the Ausubel auction, sincere bidding for both units is optimal.

The results for the auction forms were quite interesting. In the uniform price auction, the modal bid for the first unit was equal to value; however, a substantial proportion of bidders selected bids in excess of valuations. When competing against three computer rivals, bids exceeded valuations more than one-fourth of the time. When competing against five computer rivals, bids exceeded values more than 40% of the time. As predicted by the theory, bids on second units exhibited substantial demand reduction (more than 61% of bids are below value), but the extreme prediction of zero bids was borne out relatively infrequently. Thus, while demand reduction is observed, there are substantial departures from theoretical predictions in bidding in this auction.

In the Ausubel auction, behavior was closer to equilibrium predictions. In both three and five rival treatments, over 85% of all first unit bids were within 5 cents of a bidder's valuation. Likewise, there was considerably less demand reduction in second unit bids under the Ausubel auction.

In Table 3, we compare the efficiency of both types of auctions. Efficiency in Table 3 is measured by comparing the realized surplus with the maximum potential surplus in the auction. As the table makes clear, both auction forms achieved high levels of efficiency – in every session, over 95% of the available surplus was realized. Consistent with the theoretical prediction, the Ausubel auction consistently achieved higher surplus than the uniform price auction.

*Insert Table 3 here*

Manelli, Sefton, and Wilner (2000) compare bidding behavior in multi-unit sealed bid Vickrey auctions with the Ausubel auction. In one treatment of interest, they induced private valuations for bidders and had them compete in a Vickrey-type auction. Specifically, three bidders competed for three units of a good. Each bidder had a flat demand for up to two units and valued the third unit at zero.

*Insert Table 4 Here*

As Table 4 shows, more than a third of all bids made for the first two units exceed the bidders' values. Thus, in both the sealed bid uniform price

auctions studied in Kagel and Levin (2000) and the sealed bid Vickrey auctions studied in Manelli, Sefton, and Wilner, there is substantial overbidding on the first unit despite the fact that sincere bidding is a weakly dominant strategy. As Kagel and Levin (2000) highlight by considering alternative “frames” for the uniform price auction, the sealed bid frame seems to be in large part responsible for inducing this type of overbidding behavior.

Unlike the Kagel and Levin studies, overbidding in the Manelli, Sefton, and Wilner experiments translates into significant welfare losses: The Vickrey auction achieves on 88% efficiency as compared to the theoretical prediction of 100%. Indeed, almost 11% of the time, a single bidder receives all three units even though the third unit is completely worthless to her.

### 3.3 Summary

From a theoretical perspective, the Vickrey auction is preferred to the uniform-price auction on efficiency grounds. The reason for the inefficiency in the uniform-price auction is that, since sometimes a bidder’s payments for objects won are affected by her bids for “later” units, there is a strategic incentive to engage in bid shading or demand reduction in multi-unit auctions. As a consequence of this differential bid shading – more shading for more units – there is no efficient equilibrium in the uniform-price auction. The Vickrey auction remedies this defect in two ways. First, by making a bidder’s payments independent of her bids, sincere bidding becomes weakly incentive compatible. Second, by linking payments to the “externality” each bidder exerts on the other bidders through her participation in the auction, private and social incentives are aligned. As a consequence, full efficiency is obtained in weakly dominant strategies.

There are, however, a number of drawbacks to the Vickrey scheme. First, it is possible that the socially efficient outcome may not be particularly equitable. It could well be that a bidder who values the object more pays less for it. It is also possible that a bidder who obtains multiple units will end up paying less than a bidder obtaining one unit. It may also not be palatable to have two bidders, each of whom received exactly one unit, pay different prices. Second, in laboratory settings, the optimal strategy in the Vickrey auction is far from transparent – even to experienced bidders. As a result, there may be a wide gap between the theoretical and actual efficiency properties of this auction. Finally, as the number of bidders grows larger, the possibility of affecting the price with one’s own bids diminishes. This

ameliorates demand reduction incentives and may lead to smaller efficiency losses from the uniform-price auction. In field experiments with three or more bidders, there is little to distinguish bids for multiple units in Vickrey versus uniform-price auctions.

## 4 Interdependent Valuations

As European economies become increasingly integrated, it is important to recognize that the valuations of bidders participating in ECB auctions may be interdependent. For instance, demand for securities by a bank in the Netherlands may well be tied to economic variables in France, Belgium, and Germany, but is perhaps less affected by Italian economic variables. On the other hand, a German bank might be affected by Belgian, Dutch and Italian variables. Put differently, there is a common value component in the valuations of the Dutch and German banks. In this section, we examine how efficient allocation in auctions is affected by the presence of interdependencies among bidders. In many circumstances, a sealed bid auction form will not yield an efficient allocation. However, we illustrate how efficiency can be restored by using a dynamic auction form. Finally, we explore the practical implications of these observations by comparing the performance of various mechanisms in laboratory experiments.

Of course, there are numerous other policy contexts in which bidders have interdependent valuations. One prominent example is in the area of spectrum auctions. In this context, there is considerable uncertainty about the uses to which the spectrum might be most profitably put, as well as the ultimate size of wireless markets. Obviously, bidders' valuations were also affected quite directly by the allocation of related licenses. Likewise, interdependencies appear prominently in auctions for oil tracts and lumber. The results of this section apply also to these other settings.

### 4.1 Theoretical Results on Interdependent Valuations

We begin by studying a version of the model presented in Section 2 where valuations are interdependent in a simple fashion. Suppose that there are three bidders, each of whom receive uni-dimensional signals distributed uniformly. There is a single indivisible unit being auctioned. Bidder  $i$ 's utility

is

$$V_i = q_i (s^i + \alpha s^{i+1}) + x_i$$

where we use the convention that if  $i + 1 = 4$ , then  $i + 1 = 1$ . Suppose that for all  $i$ , the coefficients  $0 < \alpha < 1$ . It is helpful to think of this situation as one where bidders are seated in a circle. A bidder's valuation is most affected by her own signal, but somewhat affected by the signal of the bidder on her right, and this is true for all bidders.

Ex post efficiency requires that we award the object to the bidder who values it most highly. Notice, however, that this bidder does not necessarily have the highest signal. For instance, suppose that  $\alpha = \frac{1}{2}$  and that the realized signals are  $(\frac{3}{4}, 1, 0)$ . In this case, bidder 1 values the object at 1.25, bidder 2 values it at 1, and bidder 3 values it at  $\frac{3}{8}$ . Thus, efficiency requires that the object go to bidder 1 even though her signal is only second-highest.

Suppose that we run a Vickrey auction to allocate the object. Consider a symmetric equilibrium where each bidder employs the bidding strategy  $\beta(s)$  and where  $\beta$  is increasing and differentiable. Then when bidder  $i$  obtains signal  $s_i$ , her optimization becomes:

$$\max_b \int_0^{\beta^{-1}(b)} \left( \int_0^{s^{i+1}} (s^i + \alpha s^{i+1} - \beta(s^{i+1})) ds^{i-1} + \int_{s^{i+1}}^{\beta^{-1}(b)} (s^i + \alpha s^{i+1} - \beta(s^{i-1})) ds^{i-1} \right) ds^{i+1}.$$

where we use the convention that if  $s^{i-1} = 0$ , then  $s^{i-1} = 3$ .

Differentiating with respect to  $b$  and setting equal to zero:

$$\frac{\int_0^{\beta^{-1}(b)} (s^i + \alpha \beta^{-1}(b) - \beta(\beta^{-1}(b))) ds^{i-1}}{\beta'(\beta^{-1}(b))} + \frac{\int_0^{\beta^{-1}(b)} (s^i + \alpha s^{i+1} - \beta(\beta^{-1}(b))) ds^{i+1}}{\beta'(\beta^{-1}(b))} = 0$$

In a symmetric equilibrium,  $b = \beta(s_i)$ . Hence, a candidate equilibrium needs to satisfy:

$$\begin{aligned} \int_0^{s^i} (s^i (1 + \alpha) - \beta(s^i)) dt + \int_0^{s^i} (s^i + \alpha t - \beta(s^i)) dt &= 0 \\ \left( (s^i (1 + \alpha) - \beta(s^i)) + \left( s^i \left( 1 + \frac{\alpha}{2} \right) - \beta(s^i) \right) \right) s^i &= 0 \end{aligned}$$

This yields

$$\beta(s) = s \left( 1 + \frac{3}{4}\alpha \right)$$

which may be verified to be an symmetric equilibrium bidding strategy. Note, however, that since this strategy is increasing in the bidder's own signal, it will have the property that the object is misallocated in circumstances similar to the one given above. Note also that, with interdependent values, it is no longer the case that simply bidding one's signal is a weakly dominant strategy of the Vickrey auction. The key difficulty is that, in determining her bid, a bidder is missing important information – namely the signal of her neighbor. As a consequence, inefficient allocations are possible.

In this case, however, the solution to the efficiency problem is straightforward. Suppose that instead of running a sealed bid Vickrey auction, we instead ran an ascending (English) auction. That is, bidders publicly decide whether to stay in or drop out of an auction as increasing prices are announced. When all bidders but one have decided to drop out, the auction ends, and the object is awarded to the remaining bidder at the last drop out price.

Before studying this auction, the following property of ex post efficient allocations is useful.

**Lemma 2** *An efficient allocation never awards the object to the bidder with the lowest signal.*

Suppose that bidder  $i$  had the lowest signal,  $s$ . Compare  $i$ 's valuation to bidder  $i + 1$  with signal  $s'$ . In this case,  $i$ 's valuation is  $s + \alpha s'$  whereas  $i + 1$ 's valuation is  $s' + \alpha s''$  where  $s'' \geq s'$ . Since  $s', s'' \geq s$  then  $i + 1$ 's valuation exceeds  $i$ 's.

Suppose that bidders initially follow a drop out strategy that is increasing in own signals. In this case, the bidder with the lowest signal is the first to drop out. Suppose, without loss of generality that bidder 3 drops out. In that case, bidder 2 knows exactly his valuation for the object and so it is a weakly dominant strategy for him to drop out once this value has been reached. That is, the cutoff price for bidder 2 is

$$p_2 = s^2 + \alpha s^3.$$

At each price,  $p$ , bidder 1 performs the following thought exercise: Knowing bidder 3's signal and 2's strategy, 1 can infer 2's signal if he drops out of the auction at the next instant. Thus, conditional on bidder 2 dropping out, 1's value is  $s^1 + \alpha (p - \alpha s^3)$ . Thus, bidder 1 should drop out when

$$s^1 + \alpha (p - \alpha s^3) = p.$$

Solving yields the cutoff strategy for bidder 1 of:

$$p_1 = \frac{s^1 - \alpha^2 s^3}{1 - \alpha}.$$

Using these strategies, bidder 1 wins the auction whenever

$$\begin{aligned} p_1 &\geq p_2 \\ \frac{s^1 - \alpha^2 s^3}{1 - \alpha} &\geq s^2 + \alpha s^3 \\ s^1 + \alpha s^2 &\geq s^2 + \alpha s^3. \end{aligned}$$

But this is exactly the condition for ex post efficiency. Notice that the distribution of signals was not important to generating this result. These observations were made in Maskin (1992) and Kirchkamp and Moldovanu (2000). Krishna (2000) generalizes these results as follows:

**Theorem 2** *Suppose that there is a single indivisible object being auctioned among  $N$  bidders. Suppose that for all  $i, l < N$ ,  $a_{i+l \bmod N}^i$  is strictly decreasing in  $l$ . Then, there exists an efficient equilibrium in an English auction.*

The lesson here is clear:

*A dynamic Vickrey (English) auction is efficient in many circumstances where a sealed-bid Vickrey auction is not.*

To recap, by making bidding strategies dynamic, it becomes possible for a bidder to incorporate relevant information into her bid in a way which is impossible under a sealed bid auction form. This can have beneficial consequences in terms of efficiency.

## 4.2 Laboratory Results on Interdependent Valuations

Until recently, the costs associated with holding oral auctions on a large scale effectively precluded the use of these auction forms in many contexts. However, with the rise of information technology, electronic bidding in real-time is no longer a serious difficulty, nor is it especially costly to implement. Thus, some of the practical advantages of running a sealed-bid auction as opposed to an oral auction have disappeared. Still, before advocating a change to the more efficient, but perhaps more cumbersome English auction over a sealed bid form, it is useful to assess whether the theoretical efficiency gains translate into actual bidder behavior.

A set of experiments by Kirchkamp and Moldovanu (2000) explore bidding in the three bidder, single object context of the example above. Specifically, they compare bidding in a second price sealed bid auction and an ascending auction using a variety of parameter values for  $\alpha$ .<sup>11</sup> In assessing behavior in English auctions, it is useful to distinguish the strategic problems facing the three bidders. The first bidder to drop out needs to be playing a monotonic bidding strategy for other bidders to correctly infer her signal. Suppose bidder  $i$  drops out first. Then, bidder  $i - 1$ 's problem of determining what to bid is considerably easier than that faced by  $i + 1$ . In equilibrium,  $i - 1$  can perfectly infer  $i$ 's signal from her dropout bid, so it is a weakly dominant strategy for  $i - 1$  to bid up to her value. Equilibrium bidding for bidder  $i + 1$ , on the other hand, requires that he infer his value based on the possibility that  $i - 1$  will drop out at the next moment. This is obviously a more cognitively difficult strategy.

Kirchkamp and Moldovanu find that the low bidder does indeed follow a monotonic strategy that is relatively close to the equilibrium prediction. The low bidder tends to stay in longer than the equilibrium prediction for low signals and relatively shorter for high signals. Bidders to the “left” of the low bidder likewise play strategies that are close to equilibrium strategies. On the other hand, bidders to the “right” of the low bidder tend to be far from equilibrium strategies. In particular, their bids are relatively insensitive to their *own* signal as well as to the implied signal of the low bidder (which affects them only indirectly). Bids in the sealed bid auction are also relatively close to theoretical predictions.

We now turn to efficiency in the two auctions. In analyzing efficiency, it is useful to divide the analysis into two cases: “Easy” cases are those in which the bidder with the highest signal also has the highest value. “Hard” cases are where the bidder with the highest signal does not have the highest value. The theoretical prediction is that there should be no difference in the efficiency of the competing auction forms in easy cases, but that the English auction should dominate the sealed bid auction in the hard cases. Figure 2 illustrates the efficiency outcomes of the Kirchkamp and Moldovanu

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<sup>11</sup>In these experiments, the implementation of the second price sealed bid auction is non-standard. Kirchkamp and Moldovanu implement this mechanism by running an ascending clock auction where the time at which the first bidder drops out is unobservable. All clocks stop when there is only a single active bidder remaining. As a consequence, the drop out strategy of the high bidder is censored, and the frame in which the auction is conducted differs from most other implementations of the second price sealed bid auction.

experiments. In this figure, the circles indicate the mean efficiency for the English auction under various  $\alpha$  parameters while the crosses indicate mean efficiency for the sealed bid auction. In all cases for the English auction, the theoretical prediction is 100% efficiency. The figure highlights the fact that the English auction does represent an efficiency improvement over the sealed bid auction; however, because of deviations from equilibrium bidding behavior on the part of the “right” bidder, the efficiency of the English auction is below 100%. Dividing this into easy versus hard cases, one can see that there is little to distinguish the two auction forms in easy cases, but that efficiency in hard cases is much higher with the English auction. Interestingly, deviations in equilibrium bidding behavior in the sealed bid auction result in a much higher frequency of efficient outcomes (about 30% of the time) as opposed to the theory prediction (0% of the time).

*Figure 2 here.*

As was mentioned in Section 3.2 above, Manelli, Sefton, and Wilner (2000) also compare oral and sealed bid auction forms. In addition to the private values treatment already considered, Manelli, Sefton, and Wilner also study the case where bidders have interdependent valuations. As in the Kirchkamp and Moldovanu experiments, the sealed bid auction form they use is not theoretically efficient; whereas the oral auction is. In the Manelli, Sefton, and Wilner experiments, three bidders compete for three units; however, each bidder only demands two units. A bidder’s valuation for the first two units is:  $\frac{1}{2}s^i + \frac{1}{4}(s^{i+1} + s^{i-1})$  where we use the usual numbering convention. Notice that in the unit demand case, this interdependence does not affect the efficiency properties of the Vickrey auction; however, with multi-unit demand, interdependencies create demand reduction incentives, which harm efficiency.

Table 5 summarizes some of the main findings of these experiments. First, notice that unlike the sealed bid auction in the Kirchkamp and Moldovanu experiments, the Manelli, Sefton, and Wilner experiments show significant overbidding. This is also true, but less so, for the oral auction. Thus, in both auction forms, there are significant departures from equilibrium bidding. These departures directly translate into efficiency losses. This is may be easily seen by looking at the frequency by which all three units were allocated to a single bidder. In the case of the oral auction, the theoretical prediction is 0%. In contrast, this happened slightly more than 7% of the time in the experiments. Moreover, despite the fact that the oral mechanism is predicted to yield 100% efficiency, in fact, its efficiency of around 84% was slightly lower

than for the sealed bid auction.

*Table 5 here.*

To recap, the theoretical prediction that oral auctions have efficiency advantages over sealed bid auctions receives mixed support in laboratory settings when bidders have interdependent valuations. In the single object case, a dynamic Vickrey (English) auction outperforms a sealed bid Vickrey auction. In multi-unit settings with interdependent valuations, the dynamic Vickrey (Ausubel) auction does slightly worse than a sealed bid Vickrey auction in terms of efficiency.

## 5 Further Obstacles to Efficiency

In this section, we highlight a number of additional obstacles to efficiency. From a theoretical perspective, we begin by showing that once the determinations of bidder valuations become sufficiently rich, theoretical efficiency becomes impossible. We present a theorem formalizing what “sufficiently rich” means. Next, we consider some practical obstacles to efficiency raised by Klemperer (2000a) and others. Finally, we highlight a “hybrid” auction form suggested by Klemperer as a means of overcoming some of these obstacles.

### 5.1 Theoretical Obstacles to Efficiency

We previously showed that, with multi-unit demand, efficiency in the uniform-price auction is lost. Further, with certain types of interdependencies among bidder valuations, efficiency in sealed bid auction forms is lost. In each case, we were able to offer alternative auction forms that restored theoretical efficiency. We now show that, in many circumstances, auctions that yield efficient outcomes simply do not exist.

To see this, consider the case of the model where there are  $K < \infty$  feasible allocations of the object(s) being auctioned. At its most general, an auction consists of an *allocation rule*,  $\pi$ , and a *payment rule*,  $x$ . Let the space of all signals received by bidders be  $S$ . Under these circumstances, an allocation rule is a function mapping realized signals into probabilities of each of the feasible allocations. That is  $\pi : S \rightarrow \mathfrak{R}^K$  where for all  $k$ ,  $\pi_k(s) \in [0, 1]$  and where for all  $s$ ,  $\sum_{k=1}^K \pi_k(s) = 1$ . An allocation rule is efficient if it allocates the objects such that the weighted sum of bidders’ valuations is

maximized. A direct revelation mechanism (DRM) consists of a pair  $(\pi, x)$  mapping reported signals into allocations and transfers. A DRM is *incentive compatible* if it is an equilibrium to truthfully report one's signals. From the revelation principle, it is known that any indirect mechanism capable of achieving an efficient outcome has a DRM analog.

Perhaps the most transparent intuition for the impossibility result can be obtained from the following simple example.

**Example 3** Suppose that there is a single indivisible object being auctioned between two bidders. Bidder 1 receives two signals,  $s_i = (s_1^1, s_1^2)$ . The first signal tells bidder 1's valuation for the good, the second signal bidder 2's valuation. Bidder 2 is uninformed about values. Suppose that the utilities of each bidder are:

$$\begin{aligned} V^1 &= q_1 s_1^1 + x_1 \\ V^2 &= q_2 s_1^2 + x_2. \end{aligned}$$

Obviously, it is efficient to give the object to bidder 1 when  $s_1^1 \geq s_1^2$ .

Consider transfer rules when  $s_1^1 > s_1^2$ . Incentive compatibility requires that the transfer rule must be constant in all of these cases. If not, bidder 1 will economize by choosing the pair  $s_1$  that maximizes  $x_1$  subject to the constraint the  $s_1^1 > s_1^2$ . The same reasoning implies that in the case where  $s_1^1 < s_1^2$ , the transfer rule must likewise be constant. Let  $x_1^1$  denote the constant transfer for reports  $s_1^1 > s_1^2$ , and let  $x_1^2$  be likewise defined.

Thus, for a given realization of  $(s_1^1, s_1^2)$ , bidder 1 will choose the larger of  $s_1 + x_1^1$  and  $x_1^2$ . Thus, bidder 1 will report  $s_1^1 > s_1^2$  when  $s_1^1 > x_1^2 - x_1^1$  and will report  $s_1^1 < s_1^2$  when the opposite inequality holds. Notice, however, that these reporting strategies are independent of  $s_1^2$  – in other words, incentive compatibility requires an allocation strategy that is independent of  $s_1^2$ . However, the relationship between  $s_1^2$  and  $s_1^1$  is absolutely crucial to allocate the objects efficiently. As a result, even in this simple setting, constructing an efficient auction is impossible.

A more complicated example (from Jehiel and Moldovanu (2000)) illustrates how this intuition generalizes. Suppose that a pair of bidders are competing for a single, indivisible object. Each bidder receives a pair of signals  $(s_i^1, s_i^2)$ . The signal  $s_i^1$  indicates the impact of  $i$ 's signal on bidder 1 and  $s_i^2$  should be interpreted likewise. Suppose bidders have utility

$$V^1 = q_1 (s_1^1 + \alpha s_2^1) + x_1$$

$$V^2 = q_2 (s_1^2 + \alpha s_2^2) + x_2.$$

where  $\alpha < 1$ . Again, efficiency requires that the object be allocated to bidder 1 when

$$s_1^1 + \alpha s_2^1 > s_1^2 + \alpha s_2^2.$$

As in the previous example, each bidder has a signal  $s_i^j$  that does not affect her own utility in the event that she receives the object. By reasoning similar to the example above, incentive compatibility implies that the probability that  $i$  is allocated the object must be independent of  $s_i^j$ . As a result, incentive compatibility and efficiency are mutually incompatible in this setting.

Jehiel and Moldovanu (2000) offer the following result generalizing this intuition.

**Theorem 3** *Let  $(\pi, x)$  be an efficient DRM. Assume that the following are satisfied: 1. For some allocation  $k$ , there exist  $i \neq j$  such that  $q_i > 0$ ,  $a_i^i \neq 0$  and  $a_j^i \neq 0$ . 2. There exist open neighborhoods  $\Theta^i \in S^i$ ,  $\Theta_1^{-i}, \Theta_2^{-i} \in S^{-i}$  such that  $\pi_k(s^i, s^{-i}) = 1$  for all  $(s^i, s^{-i}) \in \Theta^i \times \Theta_1^{-i}$  and  $\pi_k(s^i, s^{-i}) = 0$  for all  $(s^i, s^{-i}) \in \Theta^i \times \Theta_2^{-i}$ . Then  $(\pi, x)$  cannot be incentive compatible.*

In words, when bidders have interdependent valuations, as is likely to be the case in ECB contexts, full efficiency may be impossible. To summarize:

*When one bidder possesses information about another bidder's valuation that is central in determining an efficient allocation, designing an efficient auction is impossible.*

## 5.2 Practical Obstacles to Efficiency

Klemperer (2000a) highlights the fact that collusive behavior among bidders may be an important practical obstacle to achieving efficient allocations. Moreover, he goes on to argue that open auctions may be more susceptible to collusive behavior than sealed bid auctions. The idea is the following: in an open, ascending auction early bids on items may be used to signal an implicitly collusive arrangement for the division of the objects. In the US broadband auctions, Cramton and Schwartz (1999) find evidence of this type of signaling. In these auctions, the last three digits of (multi-million dollar) early bids corresponded to area codes of particular regions for which bandwidth was being auctioned. Klemperer also highlights observations made by

Jehiel and Moldovanu (2000a and 2000b) regarding bidding behavior in German spectrum auctions. In this case, there were ten licenses available. In the initial round of bidding, Mannesman, a key player in this market, made five high bids on licenses and five much lower bids. This was viewed by a rival as an offer to split the licenses between them a five apiece. The result was non-aggressive bidding on the part of both bidders and an even split of the licenses.

In circumstances where many of the efficiency problems highlighted in this paper are paramount, Klemperer (2000a) suggests employing an auction form he calls the “Anglo-Dutch” auction. In the simple single object case, this auction consists of an ascending auction which proceeds until only two bidders remain, followed by a sealed bid auction where bids can be no lower than the price level reached during the ascending phase of the auction. First, as we showed in Section 4, interdependencies in bidder valuations require the information aggregation properties of the ascending auction to facilitate efficient allocations. By correctly selecting the form of the sealed bid auction, demand reducing incentives (emphasized in Section 3) may be avoided. Finally, the sealed bid phase of the auction guards against collusive possibilities. It remains to assess, both theoretically and in the lab, the effectiveness of this hybrid auction form in multiple-unit auction settings.

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Figure 1: Payment Rule in the Vickrey Auction

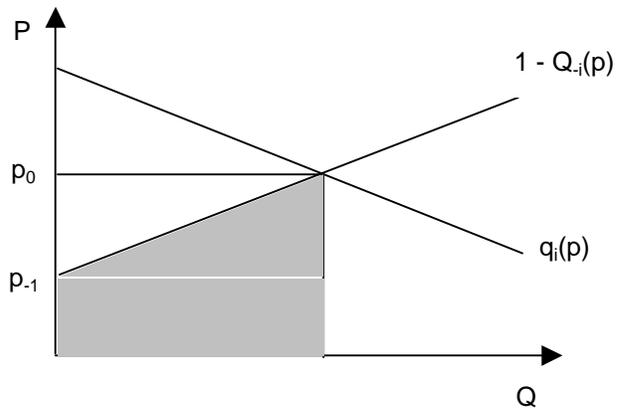
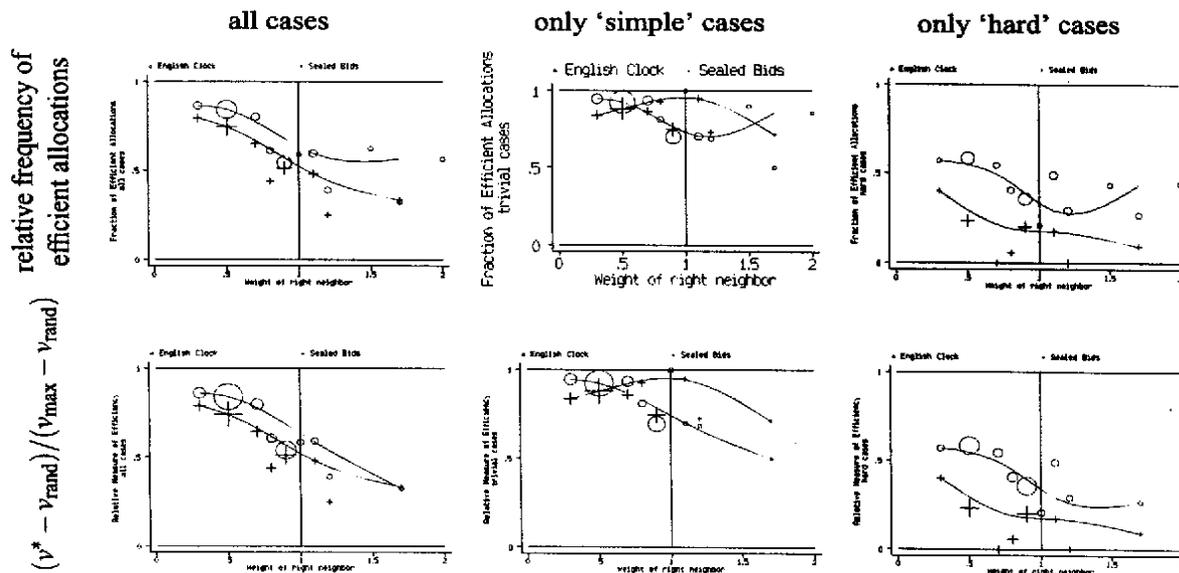


Figure 2: Efficiency in Interdependent Value Auctions



Sizes of the symbols are proportional to the number of observations. Splines connect four median bands. The figure shows that the higher efficiency of the English auction is obtained primarily in 'hard' cases.

Source: Kirchkamp and Moldovanu (2000), Figure 7.

Table 1: Summary of Experiments

<b>Treatment</b>	<b>Card Auctioned</b>	<b>Bidder Type</b>	<b>Vickrey Auctions</b>	<b>Uniform Auctions</b>
D2	Ripken Topps	2 Dealers	15	15
ND2	Sanders Score	2 Non-dealers	17	17
D3	Ripken Topps	3 Dealers	9	9
ND3	Ripken Donruss	3 Non-dealers	12	12
D5	Ripken Fleer	5 Dealers	6	6
ND5	Ripken Fleer	5 Non-dealers	6	6

Source: Engelbrecht-Wiggans, List, and Lucking-Reiley (2000) Table 1.

Table 2: Summary Statistics

Treatment	Bid on 1st Unit		Bid on 2nd Unit		Split Allocations		Zero Bids	
	Vickrey	Uniform	Vickrey	Uniform	Vickrey	Uniform	Vickrey	Uniform
D2	49.60 (15.19)	62.67 (15.28)	41.77 (14.46)	30.60 (13.43)	11.7%	43.3%	0.0%	3.3%
ND2	51.82 (15.19)	62.21 (15.28)	28.82 (14.46)	16.62 (13.43)	27.9%	42.6%	5.9%	20.6%
D3	39.74 (26.87)	38.17 (26.44)	29.26 (25.05)	22.44 (17.81)	86.1%	91.7%	11.1%	14.8%
ND3	20.03 (13.60)	20.68 (13.46)	9.69 (10.46)	8.63 (9.94)	74.1%	77.8%	30.6%	41.7%
D5	31.12 (22.81)	31.45 (15.91)	20.68 (14.82)	18.63 (15.00)	90.0%	93.3%	6.7%	20.0%
ND5	20.77 (14.20)	19.44 (15.93)	9.77 (11.03)	10.48 (11.74)	95.0%	93.3%	26.7%	40.0%

Terms in parentheses are standard deviations.

Source: Engelbrecht-Wiggans, List, and Lucking-Reiley (2000) Tables 3 and 4.

Table 3: Summary of Uniform and Ausubel Experiments

Session	Number of		Efficiency	
	Rivals	Auction	Actual	Predicted
1	3.00	Uniform	98.29 (0.72)	97.30 (0.25)
2	3.00	Uniform	95.36 (0.92)	96.72 (0.29)
3	5.00	Uniform	98.19 (0.83)	98.46 (0.16)
9	5.00	Ausubel	99.90 (0.07)	100.00
10	3.00	Ausubel	98.60 (0.38)	100.00

Source: Kagel and Levin (2000) Tables 4 and 10.

## Table 4: Summary of Vickrey Experiments

Overbid	2.075 (1.085)
Three objects	0.108 (0.167)
Efficiency	0.871 (0.070)

*Overbid* denotes the number of bids for the 1st or 2nd unit that exceed a bidder's valuation.

*Three objects* denotes the proportion of the time a single bidder obtained all three objects

Parentheses denote standard deviations.

Source: Manelli, Sefton, and Wilner (2000) Table 2.

Table 5: Summary of Interdependent Values Experiments

	Sealed Bid (Vickrey)	Oral (Ausubel)
Overbid	1.092 (0.689)	0.433 (0.407)
Three objects	0.017 (0.083)	0.071 (0.039)
Efficiency	0.853 (0.065)	0.844 (0.082)

*Overbid* denotes the number of bids for the 1st or 2nd unit that exceed a bidder's valuation.

*Three objects* denotes the proportion of the time a single bidder obtained all three objects

Parentheses denote standard deviations.

Source: Manelli, Sefton, and Wilner (2000) Table 2.