The Power of Negative Thinking: A Model of Entrepreneurship, Aspirations, and Fear of Failure

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Abstract

Fear of failure can dominate the choices of individuals. We model its role in the decision to become an entrepreneur and subsequent investments made in pursuit of success using the framework of loss aversion. We show that when the threshold for success is sufficiently high, fear of failure motivates additional sacrifices by entrepreneurs. When the threshold for success stems from foregone outside options, on the other hand, fear of failure is always de-motivating. Finally, regardless of the yardstick used to measure success, fear of failure is negatively associated with entry into entrepreneurship. Our findings highlight the importance of the interaction between the degree of fear of failure and the aspirations of the would-be entrepreneur.

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1 Introduction

Jill Blashack Strahan calls it the fear factor—the moments of overpowering doubt when success seems impossible. For Strahan one such moment was when every aspect of life seemed to be collapsing around her. Three months behind on her mortgage and grieving over the death of her beloved brother, she was now, suddenly, a widow, confronting the scary prospect of raising her five year old son Zach alone. "The night after the funeral of my husband, I thought maybe I should give up, get a job and be a mom." said Strahan in an interview with Entrepreneur magazine.

No one would blame her for doing exactly that. Many in Strahan's position would put entrepreneurial dreams on hold, possibly permanently, even when confronted with less daunting circumstances. But not her. " I began living my life one minute at a time and living it with more intention," Strahan said in an interview. She threw her entire savings, and, indeed, her entire life into her venture, working 80 hour weeks without letup for more than a decade.

For Strahan, the fear factor became a source of strength and power—her secret weapon in the struggle to succeed. And succeed she did; her company, Tastefully Simple, grossed nearly $100 million in 2012 and continues to grow. Yet she acknowledges that not everyone responds as she did. People cope with fear of failure in many ways, some good, some bad. In Strahan's pithy phrase, "You always have a choice. You can get better or get bitter."[1]

Will Smith was a small-time Philadelphia rapper when his collaboration with DJ Jazzy Jeff, "Parents Just Don't Understand" became a breakout hit. For most artists this success represents their crowning moment, the pinnacle of their artistic career. It did for Jazzy Jeff, but not for Smith. He leapt from success to success, landing a lead role in a popular television series and then parlaying this into increasingly visible, and well paid, cinematic roles. By now, Smith ranks among the handful of A-list Hollywood actors who command tens of millions of dollars for a lead role.

When asked about his success, Smith turns the conversation to failure. "I've always had a horrible fear of not achieving . . . " he told Parade magazine. "All it takes is just one person telling me I can't do it, and I'll use the fear of failure as fuel."[2] Like Strahan, these negative thoughts are a source of power for Smith.

We all suffer from fear of failure to varying degrees. So pervasive is this problem that an enormous self-help industry has arisen to help people overcome these fears. Indeed, search the

[1] Sources: http://www.entrepreneur.com/article/230350 as well as http://directsellingnews.com/index.php/view/the_most_influential_women_in_direct_selling/P19#.VE_ArPnF868.

[2] Source: Parade, 11 July 2004, pg. 4-6, by: Dotson Rader, "My Fear Fuels Me".

terms entrepreneur and fear of failure and you will find thousands of sites and articles offering helpful advice. Yet, if Strahan and Smith are representative, such fears, perversely, represent a secret weapon for success.

We investigate the many facets of fear of failure–how it affects the decision to become an entrepreneur and the sacrifices made in pursuit of success. Of particular interest is whether the examples of Strahan and Smith are, in any way, representative. Put differently, we ask whether there is such a thing as "the power of negative thinking." Unfortunately, little data exists on this aspect of personality, so researchers have only anecdotes to go on. We add to this by offering a theory of entrepreneurship when individuals suffer from fear of failure.

By its very nature, fear of failure requires some internal yardstick for counting whether outcomes are successes or failures. In other words, outcomes are judged, not in their own right, but relative to some benchmark, which we call the aspiration level. The degree of fear of failure then represents the difference in the pain suffered from falling short of the mark versus the pleasure gained from exceeding it by the same amount. Modeled in this fashion, fear of failure can be seen as a type of loss aversion. We embed these payoffs in a setting where individuals choose between employment, which offers a safe return, and entrepreneurship, which is risky. Success in the latter depends upon investment in the venture and luck. It also depends on competition from other entrepreneurs.

While much of the extant literature focuses on the entry decision, the fateful moment when an individual takes the leap into entrepreneurship, at least as important are subsequent post-entry choices—the financial, physical, and emotional sacrifices made in pursuit of success. These are presumably less studied because they are much harder to measure. A key distinction in our model and results, is its focus on this aspect of entrepreneurship and, importantly, the connection between anticipated sacrifices made in pursuit of success and the decision to become an entrepreneur in the first place.That is, the impact of looking ahead to the obstacles and sacrifices on today's decision to make the leap.

Formally, we model entrepreneurship as a two stage game where individuals first make a career decision and, following this, decide upon how much to sacrifice in pursuit of success. Outcomes depend on the sacrifices made, the level of competition, and luck. Individuals measure outcomes (and expected outcomes) through the lens of their aspiration level together with the degree to which they suffer from fear of failure. Our main findings are the following:

1. Fear of failure can motivate additional sacrifices by entrepreneurs. Provided aspiration levels are high enough, greater fear of failure produces greater investment in the venture.

2. When aspiration levels are low or, alternatively, when they are calibrated to foregone employment earnings, fear of failure is de-motivating to entrepreneurs—greater fear of failure produces less investment.

3. Regardless of aspiration levels, fear of failure dissuades individuals from entrepreneurship: The greater is fear of failure, the less likely is an individual to choose entrepreneurship.

Our main result show that the impact of fear of failure varies enormously with context. When choosing a career, fear of failure is indeed debilitating and self-help articles offering ways to overcome this fear may indeed have value. Post-entry, however, fear of failure need not be an affliction to be "cured." Aspirations play a critical role. For those holding lofty aspiration, fear of failure can indeed be a "fuel" to success, as in Will Smith's phrase, but with more modest ambitions, Jill Strahan's fear factor merely produces sleepless nights and cautious days. Individuals hedge their bets, limiting the sacrifices owing to the worry that each dollar sacrificed will merely produce more pain and regret when things don't work out.

From an empirical perspective, we see these results as opening a new, and possibly quite fruitful, path of research. While fear of failure is much talked about, it has, so far, been little studied empirically. Perhaps the most important finding from our model is that these fears cannot be studied in isolation, but must be interacted with individual aspirations. Not doing so would produce a misleading (and misspecified) prediction.

The remainder of this section places the model in the context of the extant literature. In section 2, we formally lay out the model itself. Section 3 analyzes equilibrium behavior, ultimately endogenizing aspiration levels based on outside wages. Section 4 extends the basic model in various directions and shows that results 1-3 are not model specific, but rather general economic intuitions deriving from a context in which individuals measure outcomes relative to aspiration levels and suffer from fear of failure. Finally, section 5 concludes. An appendix contains proofs for all results.

Related Literature

Fear of failure is often seen as an impediment to starting a business. Several empirical studies investigate the strength of this effect using survey data and confirm a negative correlation between self-reported fear of failure and the likelihood of becoming an entrepreneur.[3] Interestingly, most studies view fear of failure as a form of risk aversion. We add to this debate by pointing out how fear of failure better represents a form of loss rather than risk aversion. The difference might seem to be mere semantics; however, the two models of risk evaluation differ in key respects, most notably

[3] E.g. Wagner and Sternberg, 2004; Arenius and Minniti, 2005; as well as Koellinger, Minniti and Schade, 2007.

the presence of a reference point under loss aversion, a threshold performance level for delineating between success and failure. We show that, holding fixed the degree of fear of failure, changes to the reference point produce sharp differences in equilibrium behavior. Since models of risk aversion have no such implications, this variation offers a sharp distinction between the two models. Along these lines, Devers, *et al.* (2007) offer suvey evidence that, in evaluating risky choices such as stock options, choices of (future) managers conform more to loss than risk aversion.

There are relatively few theory models of entrepreneurial entry and none which account for fear of failure and, more broadly, loss aversion. The previous models emphasize selection based on ability or risk preferences using conventional expected utility.[4] Besides our emphasis on fear of failure, a key contribution is to highlight how an entrepreneur's foregone outside options continue to drive post-entry decisions.

In assessing the usefulness of such a contribution, it is important to highlight recent evidence for selection based on loss aversion. Important findings in this regard come from Koudstaal, Sloof and van Praag (2014), who compare preferences of entrepreneurs to managers. Using non-incentivized survey data, they replicate the standard finding that entrepreneurs are less risk averse than managers. Using incentivized preference elicitation, these differences disappear. Instead, they find that entrepreneurs are less *loss averse* than are managers. Koudstaal, *et al.* explain this shift by noting that, when answering survey questions about risky choice, individuals tend to conflate risk and loss aversion.

Our model adds to the growing literature incorporating behavioral factors to explain entrepreneurship. The extant literature has mainly emphasized overoptimism as a key selection force.[5] Nonetheless, there is increasing recognition of the importance of loss aversion. For instance, Langer and Waller (2003) observe that financial contracts produce differing effort incentives depending on the loss aversion of the entrepreneur entering into the contract. They derive optimal moral hazard contracts for financiers under these circumstances. Holmes, *et al.* (2008) offer a theoretical framework for how loss averse entrepreneurs evaluate opportunities.

While, to our knowledge, we are the first to theoretically study the effects of fear of failure/loss aversion on entrepreneurial entry and investment, our second stage investment model has appeared in Cornes and Hartley (2003) in the (mathematically isomorphic) form of a rent-seeking lottery

[4] Lucas (1978) and Ghatak, *et al.* (2007) emphasize selection on ability while Kihlstrom and Laffont (1979) emphasize selection on risk preferences.

[5] See for example Arabsheibani et. al., 2000; Cooper, *et al.*, 1988; Wright, *et al.*, 1997; Manove, 2000; and Landier and Thesmar, 2009. See Parker (2006) for a survey of entrepreneur optimism.

contest. A key distinction between our study and theirs, besides the obvious difference in application, concerns the role of the reference point. Unlike Cornes and Hartley, we allow the reference point to vary, sometimes endogenously, and many of our main findings relate to this variation. This permits us to examine how the yardstick used for determining success drives entry and subsequent investment, a lever entirely absent from the extant literature.

More broadly, Gill and Stone (2010) study loss aversion in two-player tournaments (a related game) where the reference point depends on relative investments. In a related paper, Gill and Prowse (2012) find experimental evidence of loss aversion in real effort tournaments. While the gist of these papers, that loss aversion is important in understanding risky choice, is similar to our study, the absence of entry, the structure of the game, and the formation of the reference point all differ substantially from our formulation.

2 The Model

Consider a situation where $N \geq 2$ individuals contemplate the choice between employment and entrepreneurship. Our main concern is to understand how fear of failure, i.e. the pain associated with falling short of some aspiration level, drives both entry into entrepreneurship as well as subsequent investment in the entrepreneurial venture. Fear of failure motives appear prominent both in surveys of entrepreneurs and in autobiographical descriptions entrepreneurs give about their motives. Our goal is to incorporate such motives formally in a game-theory model of choosing the entrepreneurship path. We describe precisely how we model these two features of preferences below.

In choosing employment, an individual enjoys a fixed wage w which is added to her initial wealth W_i. We assume that the cohort of potential entrepreneurs is small relative to the size of the (unmodeled) labor market, so each person takes the wage as given, and their choices have no effect on the market wage even if all N pursue employment.

Alternatively, an individual may choose entrepreneurship. Entrepreneurship differs from employment along many dimensions: Entrepreneurs do not answer to a boss, nor work on a set schedule of hours, but perhaps the most important difference concerns the connection between investment/effort and outcomes. Whereas a typical employee will have some small effect on the success or failure of the firm by her efforts, a typical entrepreneur's efforts and investment will have a profound effect on outcomes. Yet effort and investment alone do not guarantee success. Compe-

tition in the marketplace, including from other entrepreneurs, can seriously impede the chances of even the most hard-working. For instance, a restaurateur opening shop in a busy downtown area will likely not be alone—her success also depends, in part, on the investments of rival restaurants just down the street. Even apart from effort and competition, luck also has an important role to play. "Auntie" Anne Beiler ran one of the many small food stands at a farmer's market in Downingtown, Pennsylvania in 1988. She was trying to scrape by and hoped the stand would raise enough money for her real passion, community outreach. But her unique pretzel recipe, a chance discovery, propelled her to tremendous fortune with over 1,000 Auntie Anne's franchises operating currently.[6]

To account for the mix of effort, investment, strategic risk, and luck in determining outcomes, we model the entrepreneurship path as a winner-take-all market that awards a "prize" equal to $R > w$ to the winning individual. The prize, R, should be understood as the long-run value of becoming a dominant player in the market. That is, it represents the net present value of future cash flows resulting from success. Individuals not "winning" the market should be viewed as having achieved some small level of profit, but nothing that might be counted as true success. For instance, of all the farm stands at the Downingtown market, Auntie Anne's clearly "won," but this does not mean that the others received no customers whatever, simply that, compared to Auntie Anne's, their earnings were minimal.

Individuals choosing the entrepreneurship path simultaneously make investments e_i, which represent a combination of financial commitments as well as sweat equity in the form of long hours and intense effort. Investment represents efforts over and above those required under employment, which we normalize at zero. Furthermore, investment should be understood to be happening over time, as the business evolves from its embryonic stage. Typically, such investments are largely invisible to competitors and so may be safely modeled as a simultaneous game even though the investments themselves are not, literally, occurring at the same moment. Costs are assumed to be linear in investment, and there is no upper bound on the amount of investment an individual might choose to undertake.[7]

The combination of investment and luck determine a firm's "performance," i.e. a measure of its effectiveness in the market. Specifically, firm i's performance is equal to $y_i = e_i \times \varepsilon_i$ where

[6] Source: http://www.auntieannes.com/our-story/company-history

[7] Neither assumption is important. Nonlinear costs are readily handled simply by appropriately transforming the contest success function. Upper bounds on investment are likewise of no consequence so long as they are sufficiently generous. For instance, any upper bound on investment of R or higher leaves the analysis entirely unchanged.

e_i represents individual i's investment while ε_i represents luck, which is the realization from a Weibull extreme value distribution with mean equal to one.[8] Thus, luck is unbiased in that, on average, the highest investing firm will also be the highest performing. Moreover, the market is fair in the sense that the highest performing firm wins. The luck element, however, ensures that the highest investing firm is not necessarily the highest performing; surprises, like the unexpected success of Auntie Anne, are quite possible. Formally, the highest performing entrepreneur receives reward R while the others receive nothing, and everyone pays the cost of their investments. This implies that, when n individuals choose to become entrepreneurs, the probability that entrepreneur i, investing e_i, wins is simply $e_i / \sum_{j=1}^{n} e_j$ where, without loss of generality, we have assumed that the n individuals with the lowest indices choose to become entrepreneurs.

The structure of the success function, in particular, the pairwise independence in the probability of winning, the Luce property, means that the winner-take-all assumption is of little consequence. Were we instead to assume that there were k winners (and hence $n-k$ losers), all of whom received the same prize, nothing would change so long as we adapted the chance of winning so as to retain the Luce property. We show this formally in Section 4. From here, one can readily see that the same intuitions hold in a model where there are k, possibly differing, prizes, though the formal analysis of such a model would be fairly intractable. Thus, the winner-take-all assumption should be understood to be a mere convenience in performing analysis and gaining insights rather than a necessarily realistic modeling assumption for many entrepreneurial markets. For those inclined the use the model empirically, the extended, k prize model, is clearly more appropriate.

We now turn to entry. Here we assume that individuals choose their path (employment or entrepreneurship) in order of their indices. Thus, player 1 chooses first, followed by 2, and so on. At the moment of choice, each individual is aware of the current market size, i.e. the total number of entrepreneurs in the market up to that point. While highly stylized, the specifics of the entry model are of no great consequence. Essentially, we are after a model that yields the usual competitive equilibrium prediction that payoffs, both psychic and pecuniary, will be equalized across the two paths. Many extensive forms, including ours, produce this intuitive prediction. Thus, entry stage results will be robust to any alteration of the model retaining the competitive equilibrium property.

[8] Empirical readers will readily recognize this specification as being identical to the well-known McFadden random utility model for studying probabilitistic choice. (See, e.g. McFadden 1973, 1974.) The key advantage to this approach is that the likelihood of choosing option i versus j in a random utility context is *independent* of the other choices on offer, the so-called Luce property (Luce, 1959), which proves extremely useful and flexible econometrically. Indeed, it is the only error specification with this property. We adopt it for similar tractability and scalability reasons.

When determining entry and investment, individuals in the model care mainly about success and failure and, indeed, fear of failure plays a prominent role in their calculations. To model this, we assume that each individual i has a reference point, r_i, by which she judges success or failure. Pecuniary outcomes exceeding the reference point count as success while those falling below count as failure. Thus, her payoffs for a given monetary result, ω_i, are simply $\omega_i - r_i$. To capture the idea of fear of failure, we assume that these outcomes deal an additional psychic blow, with coefficient $\alpha_i - 1$, to an individual's self image, where $\alpha_i > 1$. Finally, we allow for the possibility of an ego boost, i.e. an additional psychic benefit $\beta_i - 1$ (where $\alpha_i > \beta_i \geq 1$) from success. Importantly, the psychic benefit from a given level of success is always smaller than the psychic harm inflicted by an equivalent level of failure, hence the additional inequality required of α_i. The ratio $\frac{\alpha_i}{\beta_i} \equiv \phi_i > 1$ measures the intensity of individual fear of failure, and is clearly increasing in α_i and decreasing in β_i. To summarize, when evaluating a given pecuniary outcome ω_i, individual i's payoffs are

$$U_i(\omega_i) = \begin{cases} \beta_i(\omega_i - r_i) & \text{if} \quad \omega_i > r_i \\ \alpha_i(\omega_i - r_i) & \text{if} \quad \omega_i \leq r_i \end{cases}$$

or equivalently

$$U_i(\omega_i) = \begin{cases} (\omega_i - r_i) & \text{if} \quad \omega_i > r_i \\ \phi_i(\omega_i - r_i) & \text{if} \quad \omega_i \leq r_i \end{cases}$$

The model is motivated by increasing interest in the effects of fear of failure on entrepreneurship; however, its essentials—the additional psychic cost of failure compared to success, and an aspiration level for counting outcomes as success—are analogous to the more generally observed phenomenon of loss aversion. Beginning with Kahneman and Tversky (1979), there is, by now, substantial evidence that individuals code events, including financial events, not as absolutes but rather in relation to some reference point. While the particular reference point differs depending on the setting and the individual, its presence has been fairly conclusively shown. Moreover, relative to the reference point, individuals tend to regard losses of a given magnitude as more painful than gains of the same magnitude are pleasurable. Many studies estimate that the difference between the coefficient on losses and gains is a factor approximately equal to two.[9] This is consistent with retrospective accounts of entrepreneurs being more affected by the costs of failure than with the benefits of success. While fear of failure motivates our analysis, the model is isomorphic to one of loss averse preferences, where the fear of failure motive represents a special case. In the

[9] See Kahneman and Tversky (1992) or Abdellaoui et al. (2007).

remainder of the paper, we sometimes use the term reference point, which should be understood to be synonymous with the aspiration level for success.

Like all models, ours is an incomplete description of all the features of entrepreneurship, employment, and the decision to switch between the two. For instance, the model contains nothing about the self-actualization benefits from being one's own boss or settting one's own hours. It contains nothing about an indivdual's life cycle and the possibility of repeatedly switching between the two paths. It says nothing about how access to capital bears on the entry decision or subsequent investment behavior. This is not because we think these factors are absent, but rather that they have no interplay with our main interest, fear of failure.

3 Equilibrium

In this section, we first study investments occurring post-entry. We pay particular attention to the effects of differing degrees of fear of failure and differing reference points for counting a result as a success. We then examine the entry stage, again with an eye toward how fear of failure affects the decision to become an entrepreneur in the first place.

The initial analysis treats the reference point as exogenous, yet, in many cases, the reference point is socially constructed. To allow for this, we close the model by imagining that individuals view the payoffs from "the road not taken"—employment—as a benchmark for success.

3.1 Entrepreneurial Investments

Assume that the first $i = 1, 2, ..., n \geq 2$ individuals choose the entrepreneurship path, and, for the moment, treat the reference point as exogenous. The location of the reference point plays a critical role in the analysis. The interesting case occurs when the following assumption is satisfied:

Assumption 1: $W_i < r_i < W_i + \frac{n+\phi_i(n-1)^2}{n+\phi_i n(n-1)}R$.

Assumption 1 says that an individual pursuing entrepreneurship aspires to a return exceeding her initial wealth level but less than the sum of initial wealth and a fractional value of the prize. The assumption merely ensures that winning the entrepreneurial market is coded as success while losing is coded as failure. Without Assumption 1, one can end up in situations where individuals code both winning and losing as success (or code both as failure) in which case preferences become, in effect, risk-neutral and the analysis straightforward.

The gain/loss utility to entrepreneur i who makes investment e_i is then

$$EU_i = \frac{e_i}{\sum_{j=1}^{n} e_j}\left(W_i + R - r_i - e_i\right) + \left(1 - \frac{e_i}{\sum_{j=1}^{n} e_j}\right)\phi_i\left(W_i - r_i - e_i\right) \tag{1}$$

Given beliefs as to the total investment $\sum_{k\neq i} e_k$, we may differentiate with respect to e_i to obtain a necessary condition for optimal (interior) investment:

$$\frac{\partial EU_i}{\partial e_i} = \frac{\sum_{k\neq i} e_k}{\left(\sum_{j=1}^{n} e_j\right)^2}R - \left\{\phi_i - \frac{e_i}{\sum_{j=1}^{n} e_j}\left(\phi_i - 1\right)\right\} + \frac{\sum_{k\neq i} e_k}{\left(\sum_{j=1}^{n} e_j\right)^2}\left[\left(\phi_i - 1\right)\left(r_i + e_i - W_i\right)\right] = 0 \tag{2}$$

The first term on the right-hand side represents the marginal pecuniary benefit of increased investment, i.e. the higher chance of winning the prize multiplied by its value. The second (bracketed) term represents the cost of increased investment (including psychic costs). The term ϕ_i represents the (normalized) psychic cost of a unit of investment conditional on the failure state. The remaining terms in this expression represent the portion of investment costs rebated back in the success state, multiplied by the chance of success, $e_i/\sum e_j$. Were entrepreneurs risk-neutral, these would be the only terms comprising the optimization condition.

Loss aversion (or fear of failure), however, creates an additional reason to investment: gains from coding a given outcome as a success rather than a failure. These gains are expressed by the term in square brackets, which merely says that a given gain loss outcome x, changes value from ϕ_i to 1 when it switches from being coded as a failure to being coded as a success. Finally, the summation expresssion is merely the increased chance of success from a unit of effort. A key feature of this investment motive is that, the higher are aspirations, the greater are the gains in moving from failure to success and hence the greater the incentive to invest. That is to say that lofty aspirations (even if borne of overconfidence) are self-motivating. A second key feature of this investment motive concerns fear of failure (ϕ_i). Notice that, the greater is the fear of failure, the greater is the incentive to invest in avoiding failed outcomes. Unlike the reference point, fear of failure also appears elsewhere in the optimality condition, as a determining factor in the cost of investment. There, it had an opposite effect, increasing the cost of investment and thus decreasing the incentive to invest. The overall effect of fear of failure on an entrepreneur's incentive to invest is thus unclear. There is, however, a clear interaction effect—the combination of loftier aspirations and greater fear of failure always produce greater incentives to invest, as a simple inspection of equation (2) reveals. We will return to this last point below, when we study equilibrium investment.

(Interior) equilibrium investments consist of a solution $\{e_1, e_2, ..., e_n\}$ to the n equation system whose representative equation is (2) . Since this system is highly nonlinear in e_i, conventional

methods are not usable to obtain closed form answers for equilibrium effort. To determine how the strategic interaction of entrepreneurs produces effort, we will suppose that entrepreneurs have similar characteristics to one another. Note, though, that these characteristics may be quite different than those of non-entrepreneurs. Formally, assume that for $i = 1, 2, ..., n$, all parameters with i subscripts (i.e. ϕ_i, r_i, and so on) are identical across entrepreneurs. In that case, we can look for a symmetric equilibrium, i.e. $e_i = e^*$ for all $i = 1, 2, ..., n$. Under this assumption, equation (2) simplifies to:

$$\frac{n-1}{n^2 e^*} R - \left\{ \phi - \frac{1}{n} (\phi - 1) \right\} + (\phi - 1) \frac{n-1}{n^2 e^*} (r + e^* - W) = 0 \tag{3}$$

and this may be rewritten in closed form as

$$e^* = \frac{n-1}{(2n-1) + \phi (n-1)^2} \times (R + (\phi - 1)(r - W)) . \tag{4}$$

Of course, this derivation is merely heuristic. Our next proposition formally establishes that a unique symmetric equilibrium exists. This follows from the form of equation (3) . Checking endpoint conditions and applying the intermediate value theorem yields existence, while linearity in e^* implies uniqueness, regardless of the reference point r or the number of entrants n. Formally,

Proposition 1 *Suppose that Assumption 1 holds. Then, for a given r and n, equilibrium investment in the unique symmetric equilibrium is given by equation* (4) .

Having established that there is a unique symmetric equilibrium, we can perform comparative analyses to see how investment responds to changes in the economic or psychic environment. First, we study the economic factors. From equation (4), it may be readily observed that investment falls when there are more competitors and rises as the stakes (R) increase. This is reassuring since all standard expected utility models make the same (sensible) prediction. Thus, the addition of fear of failure/loss aversion does not fundamentally overturn intuitions about response to rewards and competition.

Now, we study the psychic factors. As with our partial equilibrium study of equation (2) , when individuals have higher reference points, the model predicts that they will make more investments and generally work harder. Whereas this "motivation effect" gave an advantage to individual i when she was more motivated than her rivals, here, the reference points of *all* entrepreneurs increase. An important implication of this outcome is that, in entrepreneurial segments, such as technology start-ups, where aspiration levels are very high, participating entrepreneurs will work much harder

and invest much more than in segments, such as beauty salons, where aspiration levels are likely lower.

In normal conversation, we often associate hard work with greater chances of success. And, indeed, this is correct in a partial equilibrium sense even in our model. That is, fixing the efforts of all competitors save one, the harder working is this last competitor, the greater her chances to succeed. But, in comparing equilibria across settings, this is no longer true. In our setting, *all* entrepreneurs respond to higher aspiration levels by working harder, yet the net effect of all this additional effort is to leave entrepreneurs in precisely the same place (in terms of chances of winning), as the comparison group with lower aspirations. Put differently, if aspirations are a choice variable, i.e. entrepreneurs can "psych" themselves up by forming higher aspirations, then this will inevitably lead to a prisoner's dilemma for entrepreneurs. Each entrepreneur would wish to psych herself up with great expectations to commit her future selves to work hard, but the net result collectively is a disaster since, in the end, the benefits of each entrepreneur's increased effort is entirely cancelled out by other entrepreneurs, and failure is now more cataclysmic, given the resources poured into the venture.

One should, however, be a bit cautious in drawing this conclusion. The model assumes that the value of the entrepreneurial market is fixed and investments merely divide the market's (expected) value among the entrepreneurs. In many settings, entrepreneurial investment has two functions, both to capture surplus and to *create* surplus. Introducing this second motive into the model would mitigate, or possibly reverse, our finding that, in equilibrium, higher aspirations produce lower net surplus for all participants.

Fear of failure is more nuanced in its effects. In terms of the value of increasing the chance of winning, it acts in the same fashion as the reference point, increasing effort. This may be seen by examining the role of ϕ in the parenthetical expression to the right of the "$\times$" sign of equation (4). At the same time, the larger is ϕ, the smaller is the "rebate" of costs incurred to achieve this higher probability, which is reflected in the denominator term of equation (4). Which factor dominates depends critically on the reference point, as the following corollary shows.

Corollary 1 *Fix the number of competitors, $n \geq 2$, the reference point, r, and suppose that Assumption* 1 *holds. Then increased fear of failure (ϕ higher) raises equilibrium investment iff the reference point is sufficiently high, i.e. $r > W + R\left(\frac{n-1}{n}\right)^2$.*

Proving the corollary merely required routine differentiation of equation (4) with respect to

ϕ and then finding conditions where the derivative is positive. Notice that, by Assumption 1, $r < W + R$; thus, for large values of n, the condition given in the corollary will almost certainly not hold and fear of failure will be de-motivating. By contrast, when n is small, one can easily show that that the condition required in Corollary 1 is easily satisfied without violating Assumption 1. Thus, fear of failure can be highly motivating in settings with few competitors and sufficiently ambitious entrepreneurs, yet this same degree of fear can paralyze effort in more competitive settings, even when the level of ambition is unchanged.

Comparisons to Risk Neutrality

Corollary 1 also permits a ready comparison of efforts under loss aversion and those under risk neutrality since the risk-neutral case falls out when $\phi = 1$. Thus, we know that, if the reference point is sufficiently high, entrepreneurs governed by fear of failure will outwork a group of risk-neutral entrepreneurs. In low reference point situations, however, the reverse is true and risk-neutral entrepreneurs will work harder. Thus, the comparison would seem to come down to a question of what constitutes a reasonable reference point, an issue we shall return to in Section 3.3.

Is Fear of Failure Harmful?

There is, by now, a rich set of data concerning the actions, ambitions, and feelings of entrepreneurs. Our results suggest that the impact of fear of failure on outcomes such as level of investment or probability of success depend crucially on context. For instance, when there is a high degree of competition, fear of failure will be negatively associated with outcomes whereas, in situations with limited competition, the same level of fear may yield a positive association. Even for a fixed degree of competition, fear of failure can produce differential outcomes depending on an entrepreneur's ambitions. Fear of failure is motivating (i.e. yields higher outcomes) for a highly ambitious entrepreneur but de-motivating for someone less ambitious. The broader point is that no general statement can be made as to whether fear of failure represents a handicap to be overcome or a positive character trait to be embraced for an entrepreneur trying to succeed.

3.2 The Decision to Become an Entrepreneur

Before deciding on how much to invest in their business, an aspiring entrepreneur needs to decide whether to forego employment and enter into entrepreneurship. In this section, we analyze the entry decision of entrepreneurs. We assume that the entrepreneurship market is sufficiently large that we can ignore integer constraints. Entry, then, occurs up to the point where the gain/loss

utility from entrepreneurship equals the gain/loss utility from employment.[10] From Proposition 1, it is a simple matter to show that, for a given n and r, the equilibrium gain/loss utility from entrepreneurship is

$$EU(n,r) = \frac{R + (W-r)\left((n-1)\phi + 1\right)^2}{(2n-1) + \phi(n-1)^2} \tag{5}$$

As one would expect, competition reduces equilibrium utility from entrepreneurship: equation (5) may easily be seen to be decreasing in n. Of greater interest is how changes in the reference point affect equilibrium utility from entrepreneurship. That is, does a culture of high standards for success increase or decrease entrepreneurial involvement and happiness? On the one hand, a more ambitious reference point leads to more investment by entrepreneurs, but, collectively, produces lower gain/loss utility, as may be seen by inspection of equation (5).

The equilibrium number of entrepreneurs is determined by an entry condition equating the expected gain/loss utility from the two career paths. Denote the reference point delineating gains and losses under the outside option by ρ. Formally, for a given r and ρ, and treating n as continuous and interior, the equilibrium number of entrepreneurs, n^*, solves

$$EU(n^*,r) = \frac{R + (W-r)\left((n^*-1)\phi + 1\right)^2}{(2n^*-1) + \phi(n^*-1)^2} = \frac{\theta}{\beta}(W + w - \rho) = EU_0 \tag{6}$$

where EU_0 denotes the expected gain/loss utility from employment and $\theta \in \{\alpha, \beta\}$ represents the relevant gain or loss parameter depending on whether the payoffs from employment lie above or below the reference point. The interesting case is where there is *some* competition in the entrepreneurship market, i.e., $n^* \geq 2$. This requires a high enough payoff from winning the market. A sufficient condition, strengthening Assumption 1, is

Assumption 1a. Suppose that (1) a payoff equal to the endowment or less is always viewed as a loss, i.e. $r, \rho > W$, (2) Employment is not viewed as failure, i.e. $\rho \leq W + w$ and thus $\theta = \beta$; and (3) Entrepreneurship is sufficiently attractive compared to employment so that at least two entrepreneurs enter in equilibrium, i.e. $R > (\phi+1)^2(r-W) + (\phi+3)(w+W-\rho)$.

We are now in a position to characterize equilibrium entry:

Proposition 2 *Suppose that Assumption 1a holds, then there is a unique $n^* \geq 2$ consistent with a symmetric investment equilibrium, where*

$$n^* = \min\left(N, 1 - \frac{1}{\phi} + \frac{1}{\phi}\sqrt{\frac{R\phi + (\rho - w - W)(\phi - 1)}{(r-W)\phi + (w+W-\rho)}}\right) \tag{7}$$

[10] Accounting for integer constraints requires that we choose the integer floor of the value of n solving for this equality, making the analysis more cumbersome but changing nothing of substance.

With equation (7) in hand, we are in a position to examine how the various forces affect an individual's propensity to make the leap into entrepreneurship. Reassuringly, increases in the economic gains from entrepreneurship make individuals more likely to become entrepreneurs. Specifically, n^* may be readily seen to be increasing in the rewards from winning the entrepreneurial market, R, and (perhaps somewhat less readily seen to be) decreasing in the employment wage, w. Such results are, of course, standard when individuals are driven purely by pecuniary payoffs.

Rather more interesting, and unique to our setting, are the effects of various psychological factors on the decision to become an entrepreneur. Ambition, in the form of higher aspiration or reference points for success, dissuades individuals from pursuing entrepreneurship. The underlying motivation driving this outcome is that more ambitious individuals see the entrepreneurship path as potentially very costly. In the event they fail to succeed, failure is now more painful for having set such a high threshold. Moreover, success, even when attained, is less sweet since the outcome exceeds an individual's goal by only a small amount. A strategic effect is also present. Recall that ambition motivated individuals to invest much time and treasure in their ventures. While this motivation is helpful to success once an individual has made the jump into entrepreneurship, it has the opposite effect when individuals weigh their prospective futures under each path. Knowing that all will be risked once the commitment to entrepreneurship has been made, an ambitious individual pulls back from making the jump, wary of the stakes being risked. Formally, differentiating n^* with respect to r, routine calculations reveal a negative relationship.

By contrast, setting an unrealistically high standard for success in employment *increases* the chances of becoming an entrepreneur. The connection between ambition and career choice under this scenario is relatively straightforward. Recall that the pecuniary gains from employment in the model are fixed regardless of individual talent or effort. As a consequence, the higher the standard, the greater the disappointment will be experienced. Rather than pursuing such a frustrating path, an individual pursues entrepreneurship where, to some degree, she can control her destiny through effort and investment. Formally, it may be readily shown that n^* is increasing in ρ.

We now turn to fear of failure. The direct effect may be readily seen by examining the left-hand side of equation (6) , which computes expected gain/loss utility for a fixed n^*. Since the numerator is decreasing in ϕ and the denominator increasing, more fear of failure reduces expected utility. Since equilibrium entry equalizes this expression with an outside option payoff, which is independent of ϕ, it then immediately follows that greater fear of failure reduces the number of equilibrium entrants.

3.3 What Determines Aspirations?

Up until now, we have treated the measuring stick for determining success and failure merely as a given. Yet clearly, aspirations are not formed in a vacuum. A combination of social and economic factors contribute to affect the standard an individual applies to her own performance in deciding whether it counts as a success or not. One obvious factor influencing an individual's aspirations is her past experiences. For instance, MBAs at top universities are used to experiencing a high level of success in school, at work ,and in leadership activities outside of both spheres. It is often a rude shock to experience "failure," perhaps obtaining a B grade in a core class, after a lifetime of academic success. The same, no doubt, informs the expectations of entrepreneurs. Those that experienced significant financial and leadership success in their previous employment will tend to have similar aspirations in the entrepreneurial sphere whereas those coming from more modest circumstances might count any improvement as a success.

To capture this idea, we imagine that all individuals begin on the employment path, but, at some point, they face the decision of whether to become an entrepreneur. This could be a decision they themselves initiate or a chance water cooler conversation with a colleague looking for a partner in a startup. In deciding whether to make the jump, individuals look back at their past earnings, effort, and responsibilities, as well as projecting forward, to some degree, imagining raises and promotions that might arise in the near-term were they to continue with their current employer. They might also contemplate the prospects from entrepreneurship as well though, having never been an entrepreneur and faced with the uncertainties of a new project, these expectations are likely to be rather more fuzzy.

Thus, we suppose that individuals anchor on the known and the foreseeable in making their determination of whether to switch or remain in employment. Specifically, they use the sam yardstick, recent employment earnings, to measure success and failure, regardless of which path they choose. For those that make the jump to entrepreneurship, this amounts to measuring success by "the road not taken" in the words of Robert Frost. Formally, the reference point along both paths becomes

$$r^* = \rho^* = W + w$$

With this embodiment of r^* and ρ^*, it is straightforward to determine equilibrium investment, e^{**}, and number of entrants n^{**}. Substituting the relevant reference points into equation (7) yields a unique solution to the equilibrium number of entrepreneurs (ignoring integer constraints), n^{**}. The

two star notation indicates an equilibrium value with an endogenous reference point. Substituting the value of n^{**} into equation (4) then yields the equilibrium investment per entrepreneur. Formally, we have shown that:

Proposition 3 *Suppose that Assumption 1a holds and that the reference points, r^* and ρ^* are determined by employment earnings. Then there is a unique symmetric equilibrium of the model where*

$$n^{**} = 1 + \frac{1}{\phi}\left(\sqrt{\frac{R}{w}} - 1\right) \tag{8}$$

individuals become entrepreneurs and choose investment

$$e^{**} = \sqrt{wR} - w \tag{9}$$

Proposition 3 offers a prediction of entrepreneurship choices based on the workplace earnings foregone. The behavior of entry and investment choices is intuitive and sensible. Individuals in high wage positions are loath to give these up to become an entrepreneur. On the other hand, ideas with fabulous upside, expressed as a higher value of R, are enticing. The better the prospects from winning the market, the more likely it is that an individual will make the jump. Once in the entrepreneurship arena, economic forces continue to matter both directly, through R, and indirectly, through w. As the value of the entrepreneurial market increases so too do investments. This is not as obvious as it might seem, at first blush, since there are two competing forces. The direct effect is that, since the market is now worth more, making extra efforts to win now have higher payoffs. But, as we discussed above, with increased value comes increased entry, which dilutes the value of each dollar invested. All else equal, more competition produces less investment in equilibrium. In equilibrium, however, the direct effect always dominates the competitive effect. By contrast, the effects of increased employment wages are easily seen and require no such trade-off. Higher wages (prevailing at the time), create a higher aspiration for success, which in turn promotes effort. Also, higher wages act as a barrier to entry, depressing the amount of competition. Since success is now more attainable, the rewards for each additional hour spent or dollar sent are now higher. Both effects push up investment.

But what about our main concern, fear of failure? Unsurprisingly, fear of failure is a powerful deterrent to entry. Intuitively, greater fear of failure pushes individuals toward the safer employment path and so the equilibrium number of entrepreneurs falls. Once individuals make the leap into entrepreneurship, we emphasized that, depending on aspirations, fear of failure could be motivating

or de-motivating. When success is measured by the yardstick of the employment wage, remarkably, fear of failure has no direct effect on investments. As we saw, depending on aspirations, fear of failure could be motivating or de-motivating. When aspirations are retrospective, and based on wage, we end up in the knife edge case where aspirations are entirely neutral. On the one hand, the culling effect of fear of failure on the number of rivals encourages those making the leap to invest more. But there is a countervailing direct effect—fixing the number of competitors, higher fear of failure *always* reduces investment when we endogenize the reference point in this fashion. Essentially, aspirations borne of foregone opportunities are never sufficiently ambitious for fear of failure to be motivating. Strikingly, the two effects *exactly* offset one another. Thus, one implication of this explanation of the origin of aspirations is that, the greater the fear of failure, the smaller the total investment made by entrepreneurs. The following result formalizes these observations. We eschew a formal proof since signing the associated derivatives of equations (8) and (9) is routine and straightforward.

Proposition 4 *Suppose that Assumption 1a holds and that the aspiration levels, r^* and ρ^* are determined by employment earnings.*

The higher the value to the entrepreneurial market, the more entrepreneurs and the greater the investment per entrepreneur undertaken.

The higher the employment wage, the fewer the number of entrepreneurs and the greater the investment per entrepreneur undertaken.

More fear of failure produces fewer entrepreneurs but no less investment. Nonetheless, total investment in entrepreneurship activities always falls as fear of failure becomes more severe.

So what does this view of the origins of aspirations tell us about entrepreneurship? First, notice that this view of reference points is either retrospective or centered on the very immediate future. Individuals judge success by projecting their recent past and near future and looking to exceed these expectations. The model highlights that such a yardstick is, inherently, conservative. Against such a yardstick, fear of failure can never be motivating, either in entry or in investment. As best, it can be neutral (with investment) and at worst, depressing (as with entry). Under this view, the widespread advice to conquer one's fears and banish negative thoughts is, indeed, on the mark. But while such short-run yardsticks of success and failure might dominate the thoughts of many if not most entrepreneurs, a key distinction with the most successful entrepreneurs seems to be a yardstick based much more on the long-run. Admittedly, such a view is merely an impression

based on interview and other non-empirical data; nonetheless, it is striking how the entrepreneurs one thinks of as singularly successful—Bill Gates, Steve Jobs, Sam Walton—measured success not in terms of beating the part of the near future, but rather in terms of more distant and, at the time, hopelessly optimistic goals. Ironically, such dreamy goals, when paired with fear of failure, do not produce paralysis or failure, but rather singular efforts toward success.

The 2008-2009 recession produced vast number of such "entrepreneurs of necessity," who enterered entrepreneurship for lack of credible employment options. Our model suggests that these individuals will have low aspirations and, consequently undertake little investment and minimize their exposure to risk. Remarkably, there is evidence that this behavior carries over throughout an entrepreneur's career, which is entirely consistent with decisions borne of fear of failure and, especially, aspirations built around the context of the entry decision. For instance, Schoar and Zuo (2011) find evidence that not only are entrepreneurs who began their ventures during a recesion more conservative in their investment choices than their counterparts entering during a boom, but these differences persist even as economic times change. In short, the values and strategies an entrepreneur hold at the time of entry seem to "imprint" on that person and affect choices throughout her life.

Many governments pursue policies to spur entrepreneurship including subsidized credit, mentoring, and education about entrepreneurship. At the same time, liberal governments often pursue policies such as minimum wage hikes or greater social safety nets that would seem to undercut the incentives for entrepreneurship. Indeed, these policies do have the expected (adverse) effect if one merely counts the number of entrepreneurs entering into the market, but the model suggests that the situation is more nuanced. Proposition 4 highlights a tradeoff between quantity and quality of ventures—higher wages lead to fewer, but higher quality ventures (assuming quality is correlated with investment). Thus, a policy maker primarily concerned with the overall quality (investment) of the entrepreneurial market, would view subsidized credit and increased social safety nets as complementary activities toward attaining this goal.

3.4 Empirical Implications of Fear of Failure

In this section, we examine the predictions of the model that might be tested empirically. To the best of our knowledge, none of these implications have been tested for the simple reason that survey data identifying aspirations and the degree of fear of failure do not yet exist. While data exist with one or the other of these factors, as we show below, a key result concerns the *interaction* of the

two. To recap, our model offers predictions concerning both the propensity to enter as well as the intensity with which an entering entrepreneur will compete along these two dimensions. In terms of entry, the analysis supports the conventional view that fear of failure acts as a deterrent. Less intuively, the analysis also indicates that high aspirations also act as a deterrent to entry. The intuition, as we stated above is that, with high aspirations and any degree of fear of failure, a would-be entrepreneur anticipates that failure, if it comes will be severe owing to how far below aspirations failure lies. Thus, the would-be entrepreneur is repelled by this option more than, say, another individual with the same fear of failure and lower aspirations. We summarize these predictions of the model in hypothesis 1:

Hypothesis 1: *In predicting entrepreneurship among a pool of individuals, entrepreneurship is negatively associated with fear of failure and the level of aspirations for the venture to count as a success.*

A key feature of our model is to study not just who becomes an entrepreneur but what entrepreneurs do after entering. While higher aspirations deter entrepreneurs from entering, post entry, higher aspirations produce larger investments and hence, all else equal, a greater chance at success. The effects of fear of failure on post-entry choices is more nuanced. When paired with low aspirations, fear of failure is paralyzing and so produces low levels of investment and hence worse chances at success. By contrast, the combination of fear of failure and high aspirations strongly motivates individuals to work hard. While this motivation is entirely negative—working hard to avoid failure—it still contributes to success. Thus, conditional on being a successful entrepreneur, the analysis predicts both high aspirations and considerable fear of failure. Certainly, this seems a reasonable characterization of Jill Blashack Strahan, referred to in the Introduction. This leads to Hypothesis 2.

Hypothesis 2: *Among a pool of entrepreneurs, higher aspirations lead to higher investment and more success. Conditional on low aspirations, higher fear of failure leads to lower investment and success; however, just the opposite is true conditional on high aspirations.*

To perform equilibrium analysis while retaining tractability, we symmetrized the model; however, the predictions of Hypothesis 2 apply with equal force to an asymmetric model under the following conditions: Fix the investment of all other entrepreneurs and differentiate equation (2) , the first-order condition for investment. The sign of this derivative explains the direction of the change. A similar method establishes Hypothesis 1.

4 Extensions

The goal of this section is to show that the intuitions derived from our model are not merely artifacts of specific assumptions, but rather represent forces present in any model wherein individuals suffer from fear of failure. We highlight two potential fragilities of the model: the winner-take-all assumption and the determination of firm performance. With regard to the first, we extend the model to allow k winners and show that this changes our intuitions about fear of failure not a whit. Next, we show that the same is true of our specification of firm performance. With regard to the second, we generalize the perfomance process to a parametric class of performance functions that cover the gamut in their mix of luck and investment, ranging from all luck to all investment and all cases in between. We show that this too has no effect on our findings. One technical note: Throughout the analysis, we relied on the fact that first-order conditions characterize optimal choices. It is routine to extend standard results to show that second-order conditions automatically hold with the distributional assumption we selected for the error term. More generally, this need not be the case. In an appendix, we establish the second-order conditions for these extensions.

Multiple Prizes

Here, we show that the winner-take-all aspect of the market plays no important role in driving entry and investment behavior of would-be entrepreneurs in the face of fear of failure. Using a framework proposed by Vesperoni (2013), we can extend the model to allow for $k \geq 1$ winners. The key is amending the success function to accommodate multiple winners while still maintaining tractability. Vesperoni proposes a success function where, when there are 3 entrepreneurs and 2 winners, each of whom earns the same reward, the chance that entrepreneur 1 wins a prize is given by

$$\Pr\left[1 \text{ wins}\right] = \frac{e_1e_2 + e_1e_3}{e_1e_2 + e_1e_3 + e_2e_3}$$

More generally, entrepreneur i's probability of winning when there are n entrepreneurs and k winners is the sum of the product of all permutations consisting of investments of k entrepreneurs that include e_i divided by the sum of the product of all permutations consisting of investments of k entrepreneurs. This formula is complicated in general, but simplifies dramatically if we assume that all entrepreneurs $j \neq i$ choose identical investments e while entrepreneur i chooses investment e_i, which suffices for symmetric equilibrium analysis. In that case, the chance of success becomes

$$\Pr\left[i \text{ wins}\right] = \frac{e_i\binom{n-1}{k-1}e^{k-1}}{e_i\binom{n-1}{k-1}e^{k-1} + \binom{n-1}{k}e^{k}}$$

The point of such a complicated formulation is to retain the pairwise independence or Luce property of relative success while allowing for multiple prizes.[11] Vesperoni shows that his formulation is the unique solution (up to a class of monotone transformations of e) where the Luce property holds. The basic idea of the formulation is this. Fixing the effort levels of all n contestants, consider a situation where any two contestants, i and j, are removed from the general competition and pitted directly against one another. Using our earlier formulation, we can readily compute the chance that i wins or, equivalently, that i is ranked higher than j. Vesperoni's axiom requires that this same chance prevails in the n person competition and thus the relative ranking probability be independent of the number of other competitors. Thus, if i is 60% likely to defeat j in a two person competition, then i is also, with 60% probablility, likely to rank higher than j in n player competition. Notice that, when $k = 1$, this formulation reduces to the success function used earlier.

Utilizing this structure, we may now easily extend the model to allow for k winners in the entrepreneurship market. To maintain comparability with our early, winner-take-all structure, we normalize the overall value of winning the market to be R and hence each individual prize becomes R/k when there are k winners. Thus, when n entrepreneurs enter the market and compete for k prizes, entrepreneur i chooses investment e_i to maximize:[12]

$$\begin{aligned} EU_i &= \frac{e_i\binom{n-1}{k-1}e^{k-1}}{e_i\binom{n-1}{k-1}e^{k-1}+\binom{n-1}{k}e^{k}}\left(W+\frac{R}{k}-r-e_i\right) \\ &\quad +\left(1-\frac{e_i\binom{n-1}{k-1}e^{k-1}}{e_i\binom{n-1}{k-1}e^{k-1}+\binom{n-1}{k}e^{k}}\right)\phi\left(W-r-e_i\right) \end{aligned} \tag{10}$$

Following steps exactly analogous to the $k = 1$ analysis and amending Assumption 1a appropriately we obtain

Assumption 1a′. Suppose that (1) a payoff equal to the endowment or less is always viewed as a loss, i.e. $r, \rho > W$, (2) Employment is not viewed as failure, i.e. $\rho \leq W + w$ and thus $\theta = \beta$; and (3) Entrepreneurship is sufficiently attractive compared to employment so that at least two entrepreneurs enter in equilibrium, i.e. $R > \frac{((2-k)\phi+k)^2}{k}(r-W)+\left(\frac{((2-k)\phi+k)^2}{k\phi}+\frac{(\phi-1)k}{\phi}\right)(w+W-\rho)$.

Proposition 5 *In a market with n entrepreneurs consisting of k winners each receiving value R/k,*

[11] An analogous complication arises in the consumer choice context if consumers can select multiple options for a given observation rather than only one option.

[12] While the formula above is valid only for integer numbers of players, it easily extends to the continuous case by substituting $\frac{\Gamma[n]}{\Gamma[k]}$ for $\binom{n-1}{k-1}$ and $\frac{\Gamma[n]}{\Gamma[k+1]}$ for $\binom{n-1}{k}$ in equation (10).

the unique equilibrium investment equals

$$e^* = \frac{(n-k)\left(R + k\left(r - W\right)\left(\phi - 1\right)\right)}{(k(2n-k) + \phi(n-k)^2))}$$

Furthermore, given Assumption 1a', if entry is unrestricted,

$$n^* = \min\left(N, k\left(1 - \frac{1}{\phi}\right) + \frac{1}{\phi}\sqrt{\frac{k\left(R\phi + (\rho - w - W)\left(\phi - 1\right)k\right)}{\left((w + W - \rho) + (r - W)\,\phi\right)}}\right)$$

entrepreneurs are entering the market.

The formulation of equilibrium effort, e^*, and the number of entrants, n^*, in Proposition 5 is extremely familiar, which is hardly surprising since it nests the winner-take-all case as $k = 1$. Notice, however, that k enters in a straightforward way in both equations and hence our earlier analysis about the impact of ambitions, fear of failure, and their interaction is entirely unaltered (qualitatively) by this extension of the model.

Luck, Skill, and Success

Previously we assumed that performance was given by the product of investment and luck, which we modeled as $y_i = e_i \times \varepsilon_i$. Since we assumed that ε_i was extreme value Weibull distributed, this imposes a specific mix of investment and luck on the chances of success. Suppose instead, we slightly amended the model to the more flexible form: $y_i = e_i^{\lambda} \times \varepsilon_i$, where λ is a parameter common to all contestants. As with our multiple prize analysis, this extension of the model also nests our original formulation as a special case where $\lambda = 1$. If we perform a change of variable, letting $\eta = \lambda/(1+\lambda)$, then it is readily apparent that the parameter η represents the weight on investment in the chances of success. The complement, $1 - \eta$, represents the weight placed on luck. When $\eta = 0$, which corresponds to $\lambda = 0$, only luck matters, whereas, when $\eta = 1$, which corresponds to $\lambda = \infty$, only investment matters. The main model places equal weight on investment and skill, since $\eta = 1/2$ when $\lambda = 1$.

It may be readily shown that this specification produces a success function given by:

$$\Pr\left[i \text{ wins}\right] = \frac{e_i^{\lambda}}{\sum_{j=1}^{n} e_j^{\lambda}}$$

or, equivalently, using weighting notation

$$\Pr\left[i \text{ wins}\right] = \frac{e_i^{\frac{\eta}{1-\eta}}}{\sum_{j=1}^{n} e_j^{\frac{\eta}{1-\eta}}}$$

We will extend our results, characterizing equilibrium levels of investment and entry with exogenous reference points while varying the weight placed on luck.[13] First, we show that the main effects of fear of failure identified in the main model also hold for its extension. Second, we highlight the intuitive role played by the weight on luck, both on investment and entry.

Symmetrizing preferences and repeating our earlier analysis with this new specification when there are n entrepreneurs, we obtain the equilibrium investment amount

$$e^* = \frac{(n-1)\lambda\left(R + (\phi - 1)(r - W)\right)}{n + (n-1)\lambda + \phi(n-\lambda)(n-1)}$$

As usual, whether fear of failure is motivating or de-motivating depends on aspirations. In this case, it may be readily shown that e^* is increasing in fear of failure, ϕ, if and only if

$$r > W + \frac{(n-1)(n-\lambda)}{n^2}R$$

The effects of placing greater weight on luck is, intuitively, to reduce effort, which may be readily seen by differentiating e^* with respect to λ to obtain the expression

$$\frac{\partial e^*}{\partial \lambda} = \frac{(n-1)\left(R + (\phi-1)(r-W)\right)\left(n + \phi n(n-1)\right)}{\left(n + (n-1)\lambda + \phi(n-\lambda)(n-1)\right)^2} > 0$$

and similarly, using backward induction, we can find the equilibrium number of entrepreneurs as

$$\begin{aligned} n^* \quad = \quad & \min\left(N, \frac{(\phi-1)\left((1+\lambda)(w+W-\rho) + (r-W)2\phi\right) - R(\lambda-1)\phi}{2\phi\left(w+W-\rho+(r-W)\phi\right)}\right. \\ & \left. + \frac{\sqrt{\left((w+W-\rho)(\phi-1) - R\phi\right)\varkappa}}{2\phi\left(w+W-\rho+(r-W)\phi\right)}\right) \end{aligned}$$

where

$$\varkappa = -R\phi(\lambda-1)^2 + (w-\rho)\left((\lambda-1)^2\phi - (1+\lambda)^2\right) + W(\phi-1) - 4r\lambda\phi$$

Assumption 1a provided conditions for situations where the reference point was not so high that winning the entrepreneurial competition was counted as a loss, and the prize was sufficiently attractive that at least two individuals would become entrepreneurs. The analogous condition for the more flexible success function is:

Assumption 1a″. Suppose that (1) a payoff equal to the endowment or less is always viewed as a loss, i.e. $r, \rho > W$, (2) Employment is not viewed as failure, i.e. $\rho \leq W + w$ and thus $\theta = \beta$;

[13] We are, to some degree, limited in studying the full range of weighting schemes by our use of first-order conditions to characterize investment. This approach is valid if and only if equilibrium is in pure strategies, which is known to be the case only if the luck component is not too small in the risk-neutral model. (See Baye, et al. 1990). We require an analogous condition in the fear of failure model.

and (3) Entrepreneurship is sufficiently attractive compared to employment so that at least two entrepreneurs enter in equilibrium, i.e. $R > \frac{(W-r)(1+\phi)^2+(\phi(\lambda-2)-(2+\lambda))(W+w-\rho)}{\phi(\lambda-1)-1}$.

The key thing to notice is that implications about the level of ambitions and fear of failure are entirely unchanged by this new formulation; thus, Hypotheses 1 and 2 remain valid regardless of the weight given to luck versus investment (so long as equilibrium is pure strategies).

This version of the model also permits us to explore how weights placed on luck affect entry and subsequent competition among entrepreneurs. One might think that individuals would be more attracted to markets where they are more in control of their fate, i.e. where investment dominates; however this is not the case. To see this, first notice that, since no individual has an advantage in investment generation over her rivals, in equilibium, the chance of winning remains the same, $1/n$, regardless of the weight placed on luck. Hence, greater control offers no greater chance of winning. Moreover, as the weight placed on luck increases, the marginal returns to investment fall and hence competition becomes less fierce. Thus, intuition suggests, and formal analysis confirms that the greater the weight placed on luck, the *more* attractive the market, i.e. n^* is strictly decreasing in λ. We prove this claim formally in the appendix.

5 Conclusions

The decision to pursue entrepreneurship—founding an internet startup, pursuing an invention, or opening a restaurant—is momentous, and often life-changing. Fundamentally, this choice is about perceived gains and losses: the payoffs from entrepreneurship may be large relative to the safer employment path, but the sacrifices required and the potential losses experienced are also great. In part, these hazards are inherent to the entrepreneurship environment. But the dice of fate are, in effect, loaded depending on the level of competition. Success is more likely the fewer the number of competitors pursuing the same idea.

The conventional view is that gains and losses represent purely pecuniary outcomes (or money metric equivalents in terms of the sacrifice of time and effort). Yet individuals rarely use such language to describe their situation or their choices. Rather, individuals think of themselves as either succeeding or not, a contextual view based on some internal yardstick for coding results as success or failure. Moreover, many individals cite fear of failure as a hurdle to pursuing entrepreneurship in the first place. The practical literature on becoming an entrepreneur often confers advice about how one can overcome such fears, mainly by revising the yardstick itself.

In this paper, we take this notion seriously, defining payoffs, which represent a combination of the pecuniary and the psychic, as gains or losses relative to a benchmark for success. Fear of failure, then, can be understood as the degree to which falling short of the benchmark, i.e. failure, is more painful than success, to the same degree, is pleasurable. Expressed in this way, the model becomes isomorphic to one in which individuals experience loss aversion, where the success benchmark represents the reference point.

We then study how changes in the benchmark, i.e. differing aspirations, interact with fear of failure to affect behavior. While many studies focus purely on the decision to become an entrepreneur, we also study the choices made post-entry—how large a sacrifice will entrepreneurs make? How do these sacrifices change with aspirations and fear of failure? In part, we are motivated by the many stories of successful entrepreneurs viewing fear of failure not as something debilitating, but rather as providing additional motivation. Is this mere post-hoc rationalization meant to portray success and sacrifice in a more flattering light, or is there something to the idea of fear of failure as motivation? We find that, whether fear of failure is motivating or de-motivating depends crucially on aspirations. Those with high aspirations derive strength to make additional sacrifices and undertake extremes of effort from fear of failure. By contrast, those with more moderate aspirations find fear of failure profoundly depressing and de-motivating in the model. Importantly, this suggests a new channel for understanding why some entrepreneurs succeed and others do not. Even more importantly, it suggests that simply adding fear of failure as an explanatory variable in an empirical study is unlikely to yield significant results—its interaction with aspirations is essential. While our formal equilibrium characterization relies on competitors being similar, we strove to show which implications are general, and would survive in an asymmetric model.

From the perspective of entry, however, the advice books for entrepreneurs are on the mark. The model suggests that fear of failure reduces the chance of entry, regardless of aspiration level. Moreover, this stems not from naive behavior where individuals fail to anticipate its motivational effect, but from sophisticated behavior where motivational aspects are anticipated. Simply put, from an *ex ante* perspective, a rational individual, albeit one deriving payoffs relative to a personal benchmark for success, will see the gains in motivation outweighed by the costs associated with possible failure, costs which will be amplified by the enormous sacrifices undertaken in pursuit of success. Thus, by studying entry alone, the model predicts a deceptively negative view of fear of failure. Indeed, viewed solely from that perspective, overcoming such fears seems a useful exercise.

All of this leaves open the question of just where the benchmark for success derives. Clearly,

aspirations derive, to some degree, from an individual's experiences and opportunities. Thus, an immigrant facing poverty in his home country and a minimum wage job in his adopted country will obviously have a very different idea of success than a young engineer in Palo Alto having recently graduated from Stanford. We try to capture this idea by assuming that aspirations derive from the payoffs from wage work, i.e. the foregone earnings from becoming an entrepreneur. Such aspirations resolve the ambiguity as to whether fear of failure motivates or paralyzes entrepreneurial effort—individuals holding such views about success merely suffer from fear of failure rather than finding it motivational.

This places our earlier, more positive, result about fear of failure in a new context: It is widely held that individuals entering entrepreneurship are overconfident, though what exactly that they are overconfident about remains in dispute. If overconfidence might reasonably produce higher aspirations—overconfident individuals expect to succeed more often and at higher levels. The above result shows that, when overconfidence is missing, fear of failure cannot be motivating. By contrast, individuals suffering from significant overconfidence can, in fact, benefit from fear of failure. We do not pursue the connection between overconfidence and fear of failure formally, but it strikes us as a useful avenue for the future.

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6 Appendix

This appendix contains proofs of the propositions offered in the text.

Proposition 1 *Suppose that Assumption 1 holds. Then, for a given r and n, equilibrium investment in the unique symmetric equilibrium is given by equation* (4) .

Proof. Temporarily assume that winning the market is coded as a gain and losing is coded as a loss. (We will verify that this is the case given Assumption 1 later.) Differentiating equation (1) with respect to e_i yields the first-order condition:

$$\frac{\sum_{k\neq i} e_k}{\left(\sum_{j=1}^{n} e_j\right)^2}\left(\left(W+R-e_i-r\right)+\phi\left(e_i+r-W\right)\right)+\frac{e_i}{\sum_{j=1}^{n} e_j}\left(\phi-1\right)-\phi=0$$

It is routine to verify that equation (1) is strictly concave in e_i; hence the first-order condition is both necessary and sufficient.

Solving for a symmetric equilibrium, we have

$$\frac{(n-1)}{n^2 e}\left(\left(W+R-e-r\right)\right)-\frac{1}{n}-\left(\frac{n-1}{n}\right)\phi+\frac{(n-1)}{n^2 e}\phi\left(e+r-W\right)=0$$

and, taking a common denominator, the condition reduces to

$$(n-1)\left(\left(W+R-e-r\right)\right)-ne-(n-1)\,\phi ne+(n-1)\,\phi\left(e+r-W\right)=0$$

which yields equation (4).

Finally, we need to verify that, under the equilibrium investment, winning the market constitutes a gain; that is $W+R-e>r$ is satisfied. Substituting for the equilibrium value of e, the required inequality becomes:

$$W+R-(n-1)\,\frac{R+(r-W)\,(\phi-1)}{(2n-1)+\phi\,(n-1)^2}>r$$

Cross-multiplying

$$\begin{aligned}\left(W+R-r\right)\left((2n-1)+\phi\,(n-1)^2\right)-(n-1)\left(R+(r-W)\,(\phi-1)\right) &> 0\\ R\left(n+\phi\,(n-1)^2\right)-(r-W)\left(n+\phi n\,(n-1)\right) &> 0\end{aligned}$$

And this condition is satisfied when:

$$r-W<\frac{n+\phi\,(n-1)^2}{n+\phi n\,(n-1)}R$$

which is identical to Assumption 1. ■

Corollary 1 *Fix the number of competitors $n \geq 2$, the reference point r, and suppose that Assumption 1 holds. Then increased fear of failure (ϕ higher) raises equilibrium investment iff the reference point is sufficiently high, i.e. $r > W + R\left(\frac{n-1}{n}\right)^2$.*

Proof. We measure the intensity of loss aversion by ϕ; thus differentiating equation (4) with respect to this measure yields:

$$\frac{\partial e^*}{\partial \phi} = (n-1)\frac{n^2(r-W) - R(n-1)^2}{\left(2n-1+\phi(n-1)^2\right)^2}$$

The sign of this expression turns on

$$n^2(r-W) - R(n-1)^2$$

and hence $\frac{\partial e^*}{\partial \phi} > 0$ iff

$$r > W + R\left(\frac{n-1}{n}\right)^2$$

We will show that, depending on the location of the reference point, investment may increase or decrease with ϕ. It is straightforward to identify the region where investment decreases. Simply notice that $W + R\left(\frac{n-1}{n}\right)^2 > W$ and hence, for all $r \in [W, W + R\left(\frac{n-1}{n}\right)^2]$, equilibrium investment (weakly) falls with an increase in the intensity of fear of failure. On the other hand, for all $r \in [W + R\left(\frac{n-1}{n}\right)^2, W + R\frac{n+\phi(n-1)^2}{n+\phi n(n-1)}]$ equilibrium investment increases in the intensity of loss-aversion. Lastly we need to check whether, given Assumption 1, the latter case can arise. To see that this is possible, notice that $W + R\left(\frac{n-1}{n}\right)^2 < W + R\frac{n-1}{n}$ since $n > 1$. We will now show that the upper bound on r given in Assumption 1 is greater than the RHS of this inequality. That is, we will show that

$$W + R\frac{n-1}{n} < W + \frac{n+\phi(n-1)^2}{n+\phi n(n-1)}R.$$

Simplifying, we require only that

$$\frac{n+\phi(n-1)^2}{n+\phi n(n-1)} > \frac{n-1}{n}$$

And cross-multiplying yields the condition

$$n^2 > n(n-1)$$

which always holds since $n > 1$. Thus, when the reference point is high enough, or

$$r \in \left(W + R\left(\frac{n-1}{n}\right)^2, W + R\frac{n+\phi(n-1)^2}{n+\phi n(n-1)}\right)$$

then $\frac{\partial e^*}{\partial \phi} > 0$. ■

Proposition 2 Suppose that Assumption 1a holds, then there is a unique $n^* \geq 2$ consistent with a symmetric investment equilibrium, where

$$n^* = \min\left(N, 1 - \frac{1}{\phi} + \frac{1}{\phi}\sqrt{\frac{R\phi + (\rho - w - W)(\phi - 1)}{(r - W)\phi + (w + W - \rho)}}\right)$$

Proof. To establish uniqueness of n^*, we will show that $EU(n, r)$ is strictly decreasing in n. To see this, recall that

$$EU(n, r) = \frac{R + (W - r)((n-1)\phi + 1)^2}{(2n-1) + \phi(n-1)^2}$$

and differentiating this expression with respect to n, we obtain

$$\frac{\partial EU(n, r)}{\partial n} = -2(\phi(n-1) + 1)\frac{R + (r - W)(\phi - 1)}{\left(2n - 1 + \phi(n-1)^2\right)^2} < 0$$

where the inequality follows from the fact that $n > 2$ and $r > W$. Since $EU(n, r)$ is strictly decreasing in n while U_0 is constant in n, there is a unique number of entrants in equilibrium.

Next, we solve for n^* in closed form. Clearly, if $EU(N, r) \geq U_0$, then $n^* = N$. If $EU(N, r) < U_0$, then n^* solves $EU(n, r) = U_0$, which yields

$$n^* = 1 - \frac{1}{\phi} + \frac{1}{\phi}\sqrt{\frac{R\phi + (\rho - w - W)(\phi - 1)}{(r - W)\phi + (w + W - \rho)}}$$

To see that this expression has the property that $n^* \geq 2$, note that if we solve

$$2 \leq 1 - \frac{1}{\phi} + \frac{1}{\phi}\sqrt{\frac{R\phi + (\rho - w - W)(\phi - 1)}{(r - W)\phi + (w + W - \rho)}}$$

for R, we obtain Assumption 1a; i.e. $R > (\phi + 1)^2(r - W) + (\phi + 3)(w + W - \rho)$. Thus, we may conclude that there is a unique $n^* \geq 2$ consistent with a symmetric equilibrium where

$$n^* = \min\left(N, 1 - \frac{1}{\phi} + \frac{1}{\phi}\sqrt{\frac{R\phi + (\rho - w - W)(\phi - 1)}{(r - W)\phi + (w + W - \rho)}}\right)$$

■

Extension: Multiple Prizes

Proof. $n^*$and e^* are obtained using analogous methods as in the baseline model and have the same comparative statics as the baseline model. It remains to verify the second-order conditions hold locally and that global deviations are not profitable. Taking the second derivative of equation 10 with respect to e_i, using symmetry and simplifying we have

$$\left.\frac{\partial^2 EU_i}{\partial e_i^2}\right|_{e_i=e} = -\frac{2k\,(n-k)\,(R+(k\,(r-W+e)-ne)\,(\phi-1))}{n^3e^2}$$

Inserting e^* and simplifying we get

$$\left.\frac{\partial^2 EU_i}{\partial e_i^2}\right|_{e_i=e=e^*} = -\frac{2k\left(\phi\,(n-k)^2+k\,(2n-k)\right)}{(n-k)\,n\,(R+k\,(r-W)\,(\phi-1))} < 0$$

and thus the second order conditions always hold.

Furthermore, a deviation to $e_i = 0$ given all other entrepreneurs invest e^* is not profitable as

$$U\,(0) = \phi\,(W-r) < (W+w-\rho)$$

■

Extension: The Luck Component of Entrepreneurial Success

Proof. While deriving the equilibrium from the first-order conditions is straightforward and parallel to our earlier analysis, we still need to verify that the first-order conditions are necessary and sufficient. Here, we establish under which conditions the second-order conditions hold.

Expected gain-loss utility from entering the entrepreneurship market is equal to

$$\frac{e_i^\lambda}{\sum_{j=1}^n e_j^\lambda}\,(W+R-e_i-r)+\left(1-\frac{e_i^\lambda}{\sum_{j=1}^n e_j^\lambda}\right)\phi\,(W-e_i-r)$$

n^* and e^* are found following the analogous procedures as in the proofs of the baseline model. Since there is a unique symmetric solution to these equations, ensuring the sufficiency of the first-order approach requires that we locally check the second-order conditions and verify that the payoffs under the putative symmetric equilibrium exceed those from corner solutions.

To establish local concavity, we twice differentiate e_i and evaluate at the equilibrium, which, using e^*, yields

$$\frac{(n^*+\lambda\,(n^*-1)+(n^*-1)\,\phi\,(n^*-\lambda))\,((\lambda-1)\,(n^*-1)\,\phi-1-\lambda)}{\lambda\,(n^*-1)\,n^*\,((r-W)\,(\phi-1)+R)}$$

Note that this expression is always negative for $\lambda \leq 1$. Thus for these cases our analysis always applies. For $1 < \lambda \leq 2$, we require for the second order conditions to hold that

$$\begin{aligned}(\lambda-1)\,(n^*-1)\,\phi-1-\lambda &< 0\\ \lambda &< \frac{(n^*-1)\,\phi+1}{(n^*-1)\,\phi-1}\end{aligned}$$

This need not always hold. Notice that the right hand side is decreasing in ϕ. Thus, for high levels of fear of failure, the second order conditions may be violated for $1 < \lambda \leq 2$.

It remains to show that a player cannot profitably deviate by switching to a corner solution where $e_i = 0$. The expected gain/loss utility under such a deviation is

$$U(0) = \phi(W - r) < (W + w - \rho)$$

and thus this deviation is not profitable. ■