Information Aggregation in Polls*

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Abstract

We study a model of information transmission via polling. A policy maker polls constituents to obtain information about a payoff relevant state variable and then chooses a policy that affects the welfare of all in the polity. Constituents, who differ in their ideologies, receive private signals about the state variable. We find that when relatively few constituents are polled, full revelation can be an equilibrium; however, full revelation is impossible as the poll size grows large. Considering equilibria in non-truth-telling strategies, when constituents and the policy maker have similar ideologies, full information aggregation can arise in an equilibrium where constituents endogenously sort themselves so that centrists answer truthfully while extremists bias their responses to the pollster; in contrast, when the policy maker is ideologically isolated, information aggregation is impossible. On a practical level, we show that ignoring strategic behavior gives rise to biased estimators and mischaracterization of the margin of error. Finally, we examine the properties of polls and elections and find that these two mechanisms give rise to different policy outcomes in equilibrium.

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1 Introduction

The use of polls for determining policy outcomes has become an increasingly important part of the political landscape. While polls are used for various purposes, one critically important use of polls is as a basis for determining what policy to undertake. Policy makers are often shy to admit that this purpose guides policy at all. President Kennedy, for instance, famously kept his polling numbers locked away in a safe in his brother's house rather than admit to using them. President Reagan, who is often famously viewed as making policy based mainly on his ideology, polled obsessively, taking polls "prior to his inauguration, while he was being inaugurated, and the day after he was inaugurated" (Green, 2002, 4). More recently, both the use of polls and those who conduct them has become more visible. The close relationship between President Clinton and his chief pollster, Richard Morris, is an acknowledgement of the importance of polls in determining policy outcomes in that administration. Why are policies influenced by polls? One possibility is that polls aggregate information dispersed among affected constituents. Another possibility is that poll results reflect ideological beliefs of constituents and thereby inform policy makers as to the political viability of policies. Most likely, poll responses reflect a combination of both information and ideology on the part of those polled.

The relationship between information and ideology in how constituents respond to polling leads to a fundamental question: if polls influence policy, and they undoubtedly do, and constituents being polled are aware of this fact, and they undoubtedly are, then might it be the case that those being polled respond *strategically* to reflect the fact that their responses shape the policy initiatives proposed by political leadership? In this paper, we study how strategic motives affect the information content of polls and, ultimately, policy outcomes.

To fix ideas, consider the situation of the Oregon Legislative Assembly in 1999.[1] That body was contemplating a change in the state's minimum wage law to lower the minimum wage for tipped workers and certain younger employees. The policy choice of the Assembly is continuous—policies might range from the status quo to a complete repeal of the minimum wage as well as everything in between. As economists at the Oregon Employment Department averred, predicting the economic impact of changes in minimum wage policy is extremely difficult. Thus, to obtain *information* about the economic effects of the proposed policy change, the Assembly sought comment from various interested parties. Many of these parties reported back polling data to provide this information. For instance, a poll conducted by the Oregon Restaurant Association offered evidence about the economic impact that the states' current minimum wage was having on restaurateurs. Other groups offered national polling data about both the desirability and impact of the minimum wage on workers and firms. Of course, in studying this polling data, the Assembly had to bear in mind that,

[1] The description of the policy process undertaken by the Oregon Legislative Assembly draws heavily from Thompson and Braun (1999).

while there was undoubtedly a factual component to the poll results, there was also an ideological component. For instance, the "data" provided by the Oregon restaurateurs in their poll responses differed (in a predictable way) with statistics available to the Assembly from other sources. It is also not hard to imagine that the reports of workers affected by the possible policy change were also influenced by their ideological views about the appropriate policy. In short, poll responses reflected a mix of information and ideology and, at least in the case of the restaurateurs, were clearly strategic.

The main results of the paper are as follows:

1. When the number of constituents polled is relatively small and the ideology of the populace is relatively homogeneous, then the informational component of the poll dominates—truthful information revelation is an equilibrium. Since the size of the poll is relatively small, however, the amount of information the policy maker obtains is limited.

2. As the number of constituents polled grows large, the amount of information obtainable by the policy maker increases. Unfortunately, it is precisely under these circumstances that ideological effects start to dominate poll responses. Indeed, we show there is a finite upper bound on the size of a poll where truthful information revelation is an equilibrium; put differently, information aggregation via truth-telling strategies is impossible in large polls.

3. Whether polls aggregate information under non truth-telling strategies depends on the distribution of ideologies in the polity. Under fairly mild conditions on the distribution of the ideologies of constituents, we show full information aggregation can arise in an equilibrium where those polled (endogenously) sort themselves into centrists, who answer truthfully, and extremists, who bias their answers based on their ideology. As the size of the poll grows large, the ideological bounds on centrism converge to the median ideology; that is, the fraction of centrists among those polled is vanishingly small. However, the number of centrists grows without bound, and full information aggregation occurs in the limit. (In an extension of the model, we show that information does not aggregate when there are no constituents whose ideologies lies within a neighborhood of the ideology of the policy maker.)

4. On a practical level, we study how the presence of strategic motives when answering poll questions affects statistical inference from the results. We show that ignoring strategic motives and using classical statistical inference leads to biased estimators of the state variable as well as a mischaracterization of confidence intervals for the value of the state variable. We offer estimators that correct for strategic effects in polls.

5. Finally, we show that policy outcomes arising from a poll typically differ from those obtained when policies are determined by voting, as in a referendum. Indeed, when the policy space is constrained to be binary (thus allowing for meaningful comparisons between the two mechanisms), voting in a referendum will typically convey some information held by constituents. In contrast, when policies are determined following a poll it may be *impossible* for constituents to credibly convey information in any equilibrium. Thus, the two mechanisms give rise to different policy outcomes in equilibrium.

While the motivating example discussed the use of polls in policy making in Oregon. Many other situations of decision making following polls share similar features. One possibility is that the influence of the poll on policy might be indirect. For instance, the Association for Children of New Jersey commissioned a poll to determine the appropriateness of levels of school funding across the state. The goal of the Association was to use the data to affect policy making regarding education funding in New Jersey. Obviously, there's a factual component of interest—the students' and parents' experiences in the schools—and an ideological component bound up in views about property taxes and state sponsored education. Similarly, the Wisconsin Association of Railroad Passengers and various other bodies in the Midwest conducted a series of polls to affect policy concerning regional rail service in the Midwest. A fundamental informational question of interest to policy makers are the likely usage patterns of any rail expansion, yet obviously ideological views about the importance of mass transit are reflected in poll responses. Outside the context of politics, consensus earnings forecasts of sell-side equity analysts can be viewed as the result of a poll. Clearly, there is an informational component—a firm's expected earnings per share—of interest to investors; however, in developing a forecast for inclusion in the poll, an individual analyst might choose to shade his or her views depending on the prospects of investment banking business with the client company or whether the analyst's firm has a long or short position in the firm's stock (see Michaely and Womack 1999). Thus, the key trade-off explored in the present paper—the interaction between information and "ideology"—is also present in the analyst setting.

While we focus on the informational aspect of polling, obviously polls are conducted for a variety of reasons. Political leaders use polls to test words, phrases, and ideas that they might use to generate public support for their policies. Pre-election polls serve to inform the electorate about the relative chances of the candidates and thereby coordinate voters on the candidates that are most likely to win in the election. In a political campaigns, polls can help candidates decide how to deploy their scarce advertising and human resources; and, political leaders gather information about the issues concerning the public so that they can respond with appropriate policies. Our model and results are appropriate for analyzing situations where the last reason suggested for polling is most salient.

The remainder of the paper proceeds as follows: Section 2 places this paper within the context of the broader literature. Section 3 describes the model. Section 4 studies equilibria involving truth-telling by all of those polled. Section 5 examines equilibria involving non truth-telling strategies and studies their limit properties. In section 6, we discuss how strategic motives affect classical statistical inferences drawn from poll data and how to correct for these. Section 7 extends the polling model. Among other things, in this section, we compare policies determined by polling outcomes with those obtained through referenda. Section 8 concludes. Proofs not appearing in the text are relegated to the Appendix.

2 Related literature

Our paper is related to the literature on (costless) strategic information transmission. Crawford and Sobel (1982) were the first to address this issue within the context of a communication game between an perfectly informed expert and an uninformed decision-maker. Their work has been extended in various directions: some work, for instance, has examined the effect of multiple experts with perfect information on the communication game (e.g., Gilligan and Krehbiel 1987 and 1989; Krishna and Morgan 2001a and 2001b; Ottaviani and Sorensen 2001; and Battaglini 2002); other work has considered the effect of multiple experts but with noisy information (e.g., Austen-Smith 1993; Li, Rosen, and Suen 2001; Wolinsky 2002; and Battaglini 2004). The main finding in this literature is that preference divergence between experts and the policy maker leads to information losses. Battaglini (2004) considers the case where there is an arbitrary number of experts with noisy multi-dimensional private information. He finds that although truth-telling is never an equilibrium, policy outcomes converge to the full information benchmark in the limit. In much of this literature, preferences of the experts and the policy maker are assumed to be common knowledge.[2]

Our paper is also related to the literature on strategic voting and information aggregation in elections.[3] Austen-Smith and Banks (1996) point out that truthful voting is not generally an equilibrium, while Feddersen and Pesendorfer (1998) show that information does not aggregate under various voting rules. In these models, all voters have identical policy preferences conditional on the underlying state.[4] Feddersen and Pesendorfer (1997) establish that truth telling is not an equilibrium but information does fully aggregate in a model of diverse preferences. Like our model, voters are privately informed about both the state and their ideology; however, unlike our model, the policy space is binary. All of the above models share the feature

[2] One exception is Morgan and Stocken (2003) who consider a single expert case where the preferences of the expert are private information.

[3] See Young (1988) for a statement of the classic early intuition on information aggregation in elections—the Condorcet Jury Theorem.

[4] See also Feddersen and Pesendorfer (1996), who allow for the quality of information of constituents to differ, as well as Persico (2004), who endogenizes both the policy rule and quality of information obtained by constituents.

that there is commitment to a fixed policy response as a function of the voting rule used. Razin (2003) relaxes this assumption in a model where voters have identical preferences by allowing an elected candidate to select policy following the vote. He shows that information does not fully aggregate. Somewhat related to our work is a literature studying polls prior to elections (see, for instance, Austen-Smith 1990; Fey 1997; and Coughlan 2000). Here, interest is in how the information derived from a poll affects voting. Finally, our work is somewhat related to Lohmann (1993), who studies a signaling model of political demonstrations.[5]

3 Model

When polls are conducted to gather information about the appropriate policy, those affected by the policy—the constituents—may have ideological differences about the appropriate policy choice. That is, given the same information, individuals may differ in their determination of the appropriate policy because an individual's ideology interacts with his information to produce policy choices. A polled constituent anticipates that his response to poll questions will have an effect, even if only a small one, on the chosen policy, and this might alter his response. Of course, the policy maker must recognize this possibility when interpreting poll results and translating them into policy. In this section, we introduce a model of polling that captures these features.

A polity consists of a continuum of individuals. Let the cumulative distribution function $F(b)$ having support $\left[\underline{b}, \overline{b}\right]$ denote the distribution of ideologies for this population of individuals. Without loss of generality, suppose the median of the ideological distribution occurs at $b = 0$. It is commonly known that the *policy maker* has the median ideology.[6] All others in the population are *constituents* of the policy maker.

The task of the policy maker is to select a policy, $y \in \mathbb{R}$. The selected policy affects the welfare of all of the constituents in the polity. The welfare effect of a given policy on a constituent also depends on the constituent's ideology and the realization of the state variable, θ, which is drawn from a Beta distribution with known parameters α, β. As we shall see, the Beta distribution has attractive properties for performing analysis of the polling problem while at the same time offering considerable modeling flexibility.

While the constituents are uninformed about the realized state, each constituent i receives a conditionally independent private signal $s_i \in \{0, 1\}$ that is correlated with the state: Specifically, conditional on state θ, a constituent i receives the signal $s_i = 1$ with probability θ and the signal $s_i = 0$ with complementary probability.

[5] Grossman and Helpman (2001) offer an excellent summary of some of the key insights from both strands of research on the role of special interest groups in the policy making process.

[6] While we have assumed this exogenously, one could readily derive it as a consequence of a standard voting model where votes are cast prior to any information about the state being realized.

Unfortunately, the policy maker does not observe the state nor receive a signal about it. Instead, the policy maker must rely on a poll to obtain information about the state.[7] Before choosing a policy, the policy maker conducts a poll consisting of a commonly known (finite) sample size of n of the constituents numbered $i = 1, 2, ..., n$.[8] Each polled constituent i simultaneously sends a binary message $m_i \in \{0, 1\}$. The message sent does not directly enter any constituent's payoff function nor is the message in any way constrained by constituent i's signal or ideology—the message is pure cheap talk. The policy maker learns the count of the total number k of constituents sending the message $m = 1$; we shall refer to k as the *outcome* of the poll.

Since individuals obtain conditionally independent binary signals about the state, if each constituent truthfully reveals his signal (i.e., reports $m_i = s_i$), the poll captures all information relevant to the policy maker's choice. Moreover, as the size of the poll becomes arbitrarily large, the outcome of the poll divided by its size (i.e., k/n) provides an arbitrarily good estimate of θ.

After learning the results of the poll, the policy maker selects a policy, and payoffs are realized. An individual's payoff function is given by $U(y, \theta, b)$; that is, an individual's payoffs depends on the policy, the state, and the individual's ideology. To conserve on notation, we simply omit the b argument from the payoff function for the policy maker. We assume that $U_{11} < 0$ and that there exists a unique finite policy, y, that maximizes this expression for each θ and b. We also assume that in higher states, higher policies are preferred by all individuals, that is $U_{12} > 0$. Further, higher policies are likewise preferred by individuals with higher values of the ideology parameter, b, that is $U_{13} > 0$.[9] Individuals with ideologies $b < 0$ are thought of as "left-biased" relative to the policy maker while those with ideologies $b > 0$ are thought of as "right-biased". For a given state, a left-biased individual prefers a lower policy than the policy maker while a right-biased individual prefers a higher policy.

The posterior beliefs of the policy maker after learning that a poll of n constituents had outcome k is given by the cumulative distribution function $G(\theta| \langle n, k\rangle)$ .Where appropriate, we shall use $g(\theta| \langle n, k\rangle)$ to denote the probability density function associated with $G(\theta| \langle n, k\rangle)$. The notation $\langle n, k\rangle$ denotes a poll of size n receiving support k.Given these beliefs, the necessary and sufficient condition for the policy maker to

[7] It is easier to analyze the model if we assume that the policy maker does not receive a signal about the state. Since we are interested in information aggregation properties of large polls, the informational effect of this assumption is negligible. The model, however, generalizes to the case where the policy maker obtains a signal.

[8] The common knowledge assumption is for simplicity. Alternatively, one could endogenize the size of the poll by assuming a constant marginal cost, c, to the policy maker per person contacted in a poll. The analysis below would be identical in a sequential equilibrium where the equilibrium number of individuals polled, n, is a pure strategy for the policy maker.

[9] These assumptions are standard in the costless information transmission literature (see Crawford and Sobel 1982).

choose an optimal policy, $y(k)$, is

$$\int_0^1 U_1(y,\theta)\, dG(\theta|\langle n,k\rangle) = 0. \tag{1}$$

We shall sometimes consider a special case of payoffs that occurs when individuals suffer quadratic losses associated with the deviation of the chosen policy from an individual's ideal policy.[10] Specifically,

$$U(y,\theta,b) = -(y-(\theta+b))^2$$

In this quadratic loss specification, the solution to equation (1) becomes $y(k) = E_G[\theta|\langle n,k\rangle]$; that is, the chosen policy is set equal to the policy maker's point estimate of the ideal policy given his information.[11]

The time line of the game and notation is summarized in Figure 1.

We study perfect Bayesian equilibria of this game. In such equilibria: (i) the policy maker uses Bayes' rule where possible in determining his posterior beliefs about the state; (ii) given beliefs, the policy is chosen optimally; (iii) given beliefs about the strategies of the other players, each constituent chooses a message to maximize his expected payoff.[12]

4 Truthful Revelation in Polls

In this section, we study conditions in which truth-telling—honest reporting of one's signal—is an equilibrium. Recall that when constituents are non-strategic in their polling responses, then, as the size of the poll grows arbitrarily large, the policy maker's estimate of the state becomes arbitrarily precise. In other words, the information distributed across the polity is fully aggregated in a poll. In contrast, when constituents are strategic and their ideological preferences are not identical to those of the policy maker, we find a constituent might seek to influence policy choices by misrepresenting his signal in the poll. In particular, we establish that, regardless of the ideological distribution of the constituents, it is never the case that truth-telling by *all* polled constituents comprises an equilibrium once the number of constituents polled

[10] This specification is also standard in the costless information transmission literature (see Crawford and Sobel 1982) Typically, it is coupled with the assumption that the state, θ, is uniformly distributed; thus giving rise to the so-called "uniform-quadratic" case, which is widely used in applications (e.g., Gilligan and Krehbiel 1987, 1989; Krehbiel 1990; Grossman and Helpman 2001; and Krishna and Morgan 2001a, b).

[11] Following Dessein (2002), it is straightforward to generalize the quadratic specification as follows: If one lets payoffs be $U(y,\theta,b) = \psi\left(-(y-(\theta+b))^2\right)$ where $\psi(\cdot)$ is an arbitrary increasing function, then all of our results obtained for the quadratic loss specification straightforwardly generalize to this case as well.

[12] This solution concept corresponds exactly to the notion of "legislative equilibrium" used in Gilligan and Krehbiel (1987, 1989).

becomes sufficiently large. Thus, the interaction between information and ideology makes information aggregation in polls (through truthful revelation) an impossibility.

To establish the result, suppose the policy maker conducts a poll of size n and believes (correctly in equilibrium) that all of those polled are telling the truth. The posterior beliefs of the policy maker for a poll with outcome k are Beta distributed with parameters $k+\alpha$ and $n-k+\beta$. The next two lemmas establish structural properties of any truth-telling equilibrium; the proofs of these lemmas are contained in the Appendix. Lemma 1 establishes the intuitive property that higher outcomes, k, of the poll lead to higher equilibrium policies, $y(k)$. Lemma 2 shows that as the size of the poll increases, the effect of any polled constituent on the policy decreases.

Lemma 1 *If all polled constituents reveal truthfully, then* $y(k+1) > y(k)$ *.*

Lemma 2 *Fix* $\varepsilon > 0$*. If all polled constituents reveal truthfully, then for* n *large enough,* $y(k+1) - y(k) < \varepsilon$*.*

We now use these structural properties an equilibrium with truthful revelation to prove the main result in this section:

Proposition 1 *For a sufficiently large sample of constituents, truthful reporting is never an equilibrium.*

Proof. Suppose to the contrary that truth-telling is an equilibrium. Then it must be the case that for any constituent i:

$$E_{s_{-i,\theta}}\left[U\left(y\left(\sum_{j\neq i} s_j + s_i\right), \theta, b_i\right)|s_i\right] \geq E_{s_{-i,\theta}}\left[U\left(y\left(\sum_{j\neq i} s_j + m_i'\right), \theta, b_i\right)|s_i\right]$$

where $m_i' \neq s_i$ and where $E_{s_{-i,\theta}}[\cdot]$ is the expectation over all other polled constituents' signals, s_{-i}, and over the distribution of the state, θ.

In particular, suppose without loss of generality that $b_i > 0$ and $s_i = 0$. Then the incentive compatibility condition becomes

$$\sum_{k=0}^{n-1} \Pr\left(\sum_{j\neq i} s_j = k|s_i = 0\right)\int_0^1 \left(U(y(k), \theta, b_i) - U(y(k+1), \theta, b_i)\right) g(\theta|\langle n,k\rangle)\, d\theta \geq 0.$$

Since $U_{13} > 0$, it follows that for every k

$$\int_0^1 \left(U(y(k), \theta, b_i) - U(y(k)+\varepsilon, \theta, b_i)\right) g(\theta|\langle n,k\rangle)\, d\theta < 0$$

for $\varepsilon > 0$ small enough.

By Lemma 1, $y(k) < y(k+1)$, and by Lemma 2, for any $\varepsilon > 0$, $y(k+1) \leq y(k) + \varepsilon$ for n sufficiently large. Hence, in large polls

$$\sum_{k=0}^{n-1} \Pr\left(\sum_{j \neq i} s_j = k | s_i = 0\right) \int_0^1 \left(U\left(y\left(k\right), \theta, b_i\right) - U\left(y\left(k+1\right), \theta, b_i\right)\right) g\left(\theta | \langle n, k \rangle\right) d\theta < 0,$$

which contradicts the claim that truth-telling is an equilibrium. An analogous argument shows that a polled constituent with preference parameter $b_j < 0$ has a profitable deviation when receiving a signal $s_j = 1$. ■

Proposition 1 highlights that, in general, information aggregation via truth-telling in polls is inconsistent with equilibrium behavior. Indeed, there is a finite upper bound to the size of a poll where truth-telling is incentive compatible. The key observation is that in large polls, since an individual constituent's influence on policy is relatively small, his incentive to misrepresent his signal in the direction of his ideological preference is large.

In determining how to respond to a poll, a constituent lacks considerable information about the policy effect of such misrepresentation. In particular, an individual constituent knows only his own signal; the signals of all others polled are unknown. Thus, a constituent contemplating the deviation is uncertain about the exact policy effect of misrepresenting his signal. Indeed, one possibility is that, by deviating, the effect on policy could be to "overshoot" the constituent's ideal policy. That is, a constituent might prefer a more right leaning policy than the policy maker, but policies that are too far to the right relative to the state are not desirable for either the constituent or the policy maker. Proposition 1 illustrates the equilibrium effects of the trade-off between influencing policy choice and possibly "overshooting" the desired policy.

It is important to contrast this strategic effect with those typically arising in voting models. In voting models, the strategic dimension centers on the chances that a voter will be pivotal and thus change the chosen candidate/policy/platform discretely. Of course, the odds of this event occurring are small in large elections, but, since an individual has no effect on policy in any other circumstance, the pivotal event looms large strategically. In contrast, a constituent's response to a poll always affects the policy. When the poll is relatively small, a constituent's effect on policy is relatively large and "overshooting" the optimal policy is a key concern. As the size of the poll grows large, an individual's effect on policy shrinks and overshooting ceases to be a serious concern. Without this discipline, incentives to reveal truthfully break down.

Proposition 1 highlights how the interaction between ideology and information precludes the possibility of truth-telling equilibria in large polls. To get a sense of the potential informational significance of the effect, it is useful to study a version of the quadratic loss specification of the model which will enable us to explicitly characterize the largest size truth-telling poll. We find that the size of polls need not be very large before truth-telling equilibria are ruled out. Furthermore, the loss of information associated with limits on the size of the largest poll where truth-telling is an equilibrium increase *linearly* with the standard deviation of the distribution of ideologies in the polity.

Consider the following special case of the model, which we shall refer to as the *quadratic loss with binary ideology* specification. Suppose that payoffs are quadratic and that those polled have bias b_i, which is equally likely to be $-b$ or b where $b > 0$; that is, the degree of bias is identical for all constituents but the direction of the bias varies. This specification is especially useful in studying ideological heterogeneity in that the bias parameter, b, is equal to the standard deviation of the ideological distribution.

Given the posterior beliefs of the policy maker and equation (1), the policy chosen under truth-telling in a poll of n constituents with outcome k is

$$\begin{aligned} y(k) &= E_G[\theta| \langle n, k\rangle] \\ &= \frac{k+\alpha}{n+\alpha+\beta}. \end{aligned}$$

With this structure, we can then find a closed-form solution, the proof of which is given in the Appendix, for the largest size poll where truth-telling is an equilibrium. Formally,

Proposition 2 *In the quadratic loss with binary ideology specification, the largest poll where truth-telling is an equilibrium is*

$$\bar{n} = \max\left\{ \left\lfloor \frac{1}{2b} - \alpha - \beta \right\rfloor, 0 \right\}$$

where $\lfloor x \rfloor$ *denotes the integer component of* x.

The importance of strategic effects on the size of a poll where truth-telling is an equilibrium may be readily seen in Figure 2. This figure defines the upper bound poll size under truth-telling when the state is distributed uniformly. As Figure 2 reveals, when the standard deviation of the ideological distribution, b, exceeds 0.016, then the largest poll where truth-telling is an equilibrium is at most 30.

5 Centrist-Extremist Equilibria in Polls

Having shown that truthful revelation is not an equilibrium when large numbers of constituents are polled, we now study a class of equilibria that we refer to these equilibria as *centrist-extremist equilibria.* In a centrist-extremist equilibrium, which is characterized by a pair (b_l, b_r) where $b_l \leq b_r$, all constituents with ideology $b < b_l$ report $m = 0$ regardless of their signal, all constituents with ideology $b > b_r$ report $m = 1$ regardless of their signal, and constituents with ideology $b \in [b_l, b_r]$ report truthfully. That is, constituents with ideologies lying outside the bounds $[b_l, b_r]$ are extremists in the sense that they always report a signal favoring their ideology; in contrast, those constituents with ideologies lying inside the bounds $[b_l, b_r]$ are centrists who report truthfully. This class of equilibria have a number of useful and intuitive properties: this class nests both truthful revelation (i.e. $b_l = \underline{b}$, $b_r = \bar{b}$) and babbling (i.e., $b_l = b_r = 0$); they are also monotonic in ideology which is consistent with the $U_{13} > 0$ assumption. In election contexts, the study of centrist-extremist equilibria is fairly standard.[13]

To analyze the informational and statistical properties of centrist-exptremist equilibria requires that we place additional structure on the model: specifically, we assume that preferences are of the quadratic loss form and that the ideological distribution has a positive density on $[-1, 1]$ and is symmetric around $b = 0$. The assumption on the support of the ideological distribution implies that truth-telling is not an equilibrium regardless of the size of the poll.

5.1 Polls with Stratified Sampling

First, we consider a special case of the model—what we refer to as a "stratified poll". This special case is useful in that it permits easy construction of closed form solutions to centrist-extremist equilibria in polls. Moreover, as we shall see, all of the informational and statistical properties of this model are true more generally but more easily understood in the present context.

Typically, polls are conducted in a fashion that "stratifies" the sample to reflect the ideological distribution of the polity. We model a stratified poll as follows: Suppose that when (an odd number) of n individuals are polled, one can arrange the ideological preferences of the constituents in increasing order such that the ideological preferences of polled constituent i are

$$b_i = F^{-1}\left(\frac{i-1}{n-1}\right).$$

Notice that the pollster does not know the ideological preference of a polled constituent but does know the exact ideological distribution of the set of n polled constituents. There are a number of practical ways such stratification is achieved (see Voter Contact Services 1994). For instance, one can poll on the basis of zip codes. Since zip code information combined with past voting records is a reliable indicator of ideology for those living in that zip code, one can then create a stratified poll with an ideological distribution mirroring that of the polity. As the number of polled individuals grows arbitrarily large, i.e., $n \to \infty$, the ideological distribution of the polled individuals will converge to the ideological distribution of the polity under a stratified poll (as well as under an unstratified poll).

[13] See, for instance, Feddersen and Pesendorfer (1997).

We also suppose that the constituent ideologies are uniformly distributed on $[-1,1]$ .Thus, the ideology of polled constituent i is

$$b_i = \frac{2(i-1)-(n-1)}{(n-1)}$$

for $i = 1, \ldots, n$.[14] Finally, we assume that the parameters (α, β) of the underlying state distribution are natural numbers.

We construct a symmetric centrist-extremist equilibrium where centrists reveal information truthfully whereas extremists answer the poll in a direction that is consistent with their ideology; that is, a left-wing extremist constituent with preference parameter $b_i < 0$ always reports $m_i = 0$ and a right-wing extremist with preferences $b_j > 0$ always reports $m_j = 1$. The equilibrium construction then consists of characterizing the preference bounds on left and right-wing extremism given the policy maker behaves optimally.

Before proceeding, it is useful to observe that in a poll of n constituents where c constituents are known to report truthfully, while ρ always report $m = 1$, and the remaining $\lambda = n - c - \rho$ constituents always report $m = 0$, the policy maker's beliefs in this case is Beta distributed with parameters $k - \rho + \alpha$ and $c - (k - \rho) + \beta$.

In terms of centrist-extremist equilibria in stratified polls, suppose constituents with indices 1 to $l-1$ are left-wing extremists, those with indices l to r are centrists, and those with indices $r+1$ to n are right-wing extremists. Symmetry requires $l = n - r + 1$. In such an equilibrium, the policy maker correctly interprets a size n poll having outcome k as one receiving $k - (n-r)$ reports of $s_i = 1$ from $n - 2(n-r)$ centrist constituents.

In such an equilibrium, the optimal policy for the policy maker is:

$$y(k) = \frac{k - (n-r) + \alpha}{n - 2(n-r) + \alpha + \beta}.$$

Notice that

$$y(k+1) - y(k) = \frac{1}{n - 2(n-r) + \alpha + \beta}.$$

which is independent of the poll outcome k. This proves extremely useful for evaluating incentive constraints and allows simple construction of symmetric centrist extremist equilibria. First, define $I \equiv \alpha + \beta + 1$ and let n_j be the sequence

$$n_j = 2\left(j^2 - jI\right) + 1$$

for $j = 1, 2, .., \infty$.

[14]The formulation of the distribution of ideologies for the model of polls with stratified sampling is identical to the preference specification used in Lohmann (1993)

Proposition 3 *In a stratified poll of size n_j, there exists a symmetric centrist-extremist equilibrium where the index of the right-most centrist is given by*

$$r_j = \frac{1}{2}(j - I) + j(j - I) + 1$$

The proof is contained in the Appendix. Next, we examine the limiting properties of this sequence. Notice that, the number of centrists associated with a poll of size n_j and the associated-centrist extremist equilibrium is

$$\begin{aligned} c_j &= n_j - 2(n_j - r_j) \\ &= j - I + 1. \end{aligned}$$

Notice that, the fraction of centrists in the population vanishes in the limit

$$\begin{aligned} \lim_{j\to\infty} \frac{c_j}{n_j} &= \lim_{j\to\infty} \frac{j - I + 1}{2(j^2 - jI) + 1} \\ &= 0, \end{aligned}$$

while the *number* of centrists becomes arbitrarily large ($\lim_{j\to\infty} c_j \to \infty$). This last property implies that information fully aggregates. To summarize, we have shown that

Proposition 4 *In a stratified poll, there exists a sequence of poll sizes n_j, and an associated symmetric centrist-extremist equilibrium, in which full information aggregation results in the limit even though the fraction of centrists in the poll goes to zero.*

To gain some intuition for the result, it is useful to examine the nature of the endogenous bounds on extremism. For the jth element of the sequence, the index number of the right-most centrist is $r_j = \frac{1}{2}(j - I) + j(j - I) + 1$. The percentile of the ideology distribution of this constituent is given by $\frac{r_j}{n_j}$, which yields

$$\frac{r_j}{n_j} = \frac{\frac{1}{2}(j - I) + j(j - I) + 1}{2(j^2 - jI) + 1}.$$

In the limit, $\lim_{j\to\infty} \frac{r_j}{n_j} = \frac{1}{2}$. Thus, the ideologies of the centrists collapse to that of the median individual as the size of the poll grows large. This collapse occurs because the policy change from reporting $m_i = 1$ versus $m_i = 0$ becomes arbitrarily small as the poll size grows large, and consequently, the incentive constraints require correspondingly smaller values of bias. While the ideological bounds on the centrists are collapsing in the limit, the number of constituents polled between the interval of ideologies, $[l, r]$ is becoming large as the poll increases in size. Thus, there is a "race" between the speed at which the ideological bounds collapse and the increase in the number of centrists between the bounds.[15] Proposition 4 shows that, regardless of the parameters of the state distribution, the ideological bounds on centrism collapse at a slower rate than the number of centrists increases. Accordingly, while full information aggregation results in the limit, convergence occurs relatively slowly. In particular:

Remark 1 *In the sequence of centrist-extremist equilibria, the number of centrists increases at rate* $\sqrt{n}$.

5.2 Polls with Random Sampling

In the previous section, we made a number of assumptions about the polling technology to obtain the precise equilibrium construction in Proposition 3. In this section, we relax many of these assumptions and show that the informational features highlighted above continue to hold; of course, the cost of relaxing these assumptions is that a construction of an equilibrium sequence is no longer possible. Specifically, we assume that polled constituents are drawn randomly from the underlying population, and we drop the restrictions that ideology be uniformly distributed and that α and β be natural numbers. Throughout the remainder of the paper, we shall refer to this framework as *the model of polls with random sampling.*

Before proceeding it is useful to introduce some additional notation. Consider a poll of size n and an associated centrist-extremist equilibrium (b_l, b_r). We may equivalently represent this equilibrium by a pair (q_c, q_r) where

$$\begin{aligned} q_r &= 1 - F(b_r) \\ q_c &= F(b_r) - F(b_l) \end{aligned}$$

the notation q_t describes the quantile measure of constituents of type $t \in \{c, r\}$ in the population. Second, we say that a centrist-extremist equilibrium is *non-degenerate* when $q_c > 0$. We first establish that a non-degenerate centrist-extremist equilibrium always exists.

As usual, to show that a non-degenerate centrist-extremist exists requires that the policy maker choose policies optimally after learning the outcome of the poll, and, given optimal policy making, that constituents prefer to report according to their type (centrist, left-wing extremist, or right-wing extremist) rather than deviate.

[15] A similar "race" occurs in a recent paper by Taylor and Yildirim (2005). In a model of costly voting, they show that full information aggregation occurs even though the fraction of voters casting votes goes to zero in the limit.

Given quadratic preferences, the optimal policy following poll outcome k is

$$
\begin{aligned}
y(k) &= E[\theta|\langle n,k\rangle] \\
&= \frac{\sum_{c_1=0}^{k}\sum_{c_0=0}^{n-k}\frac{n!}{(k-c_1)!c_1!c_0!(n-k-c_0)!}(q_c)^{c_1+c_0}(q_r)^{k-c_1}(1-q_c-q_r)^{n-k-c_0}\frac{\Gamma(\beta+c_0)\Gamma(\alpha+c_1+1)}{\Gamma(\alpha+\beta+c_0+c_1+1)}}{\sum_{c_1=0}^{k}\sum_{c_0=0}^{n-k}\frac{n!}{(k-c_1)!c_1!c_0!(n-k-c_0)!}(q_c)^{c_1+c_0}(q_r)^{k-c_1}(1-q_c-q_r)^{n-k-c_0}\frac{\Gamma(\alpha+c_1)\Gamma(\beta+c_0)}{\Gamma(\alpha+\beta+c_0+c_1)}}
\end{aligned}
$$

Given the optimal policy rule, a non-degenerate centrist-extremist equilibrium is a solution to the following pair of incentive compatibility constraints:

1. A constituent with ideology b_r and a signal $s=0$ must be indifferent between reporting honestly and lying:

$$
\begin{aligned}
&\sum_{k=0}^{n-1}\left(\Pr[\langle n-1,k\rangle|s=0]\left(E[\theta|\langle n,k+1\rangle]-E[\theta|\langle n,k\rangle]\right)\right. \\
&\left.\times\left(E[\theta|\langle n,k+1\rangle]+E[\theta|\langle n,k\rangle]-2E[\theta|\langle n-1,k\rangle,s=0]\right)\right) \\
&-2F^{-1}(1-q_r) \\
=\ & 0 \qquad (2)
\end{aligned}
$$

2. A constituent with ideology b_l and a signal $s=1$ must be indifferent between reporting honestly and lying:

$$
\begin{aligned}
&\sum_{k=0}^{n-1}\left(\Pr[\langle n-1,k\rangle|s=1]\left(E[\theta|\langle n,k+1\rangle]-E[\theta|\langle n,k\rangle]\right)\right. \\
&\left.\times\left(-E[\theta|\langle n,k+1\rangle]-E[\theta|\langle n,k\rangle]+2E[\theta|\langle n-1,k\rangle,s=1]\right)\right) \\
&+2F^{-1}(1-q_r-q_c) \\
=\ & 0 \qquad (3)
\end{aligned}
$$

We are now in a position to prove that a non-degenerate centrist-extremist equilibrium always exists in our model.

Proposition 5 *In a poll with random sampling, a non-degenerate centrist-extremist equilibrium exists.*

Next, we study the information aggregation properties of non-degenerate centrist-extremist equilibria when sampling is random. As we shall see, the intuition and the main properties of polls as the sample grows large are the same under random sampling as under stratified polling. In some sense, this might not be surprising since the distribution of ideologies under random and stratified sampling both converge in distribution to the underlying ideological distribution of the populace.

For a fixed poll size n, define $q_{c,n}$ to be the supremum of the set of values of q_c arising in a centrist-extremist equilibrium; let $q_{r,n}$ be the value of q_r associated with q_c. Notice that if $q_{c,n}$ remained fixed as the size of the poll grew large, then a constituent would correctly realize that his reporting choice would have little effect on the policy. Moreover, since the information contained in the outcome of the poll allows the policy maker to better infer the state, there is little chance that by lying about his signal, a constituent would induce an action that does not enhance his payoff. As a result, constituents whose ideologies lie near the boundary of centrism (i.e. constituents with ideologies close to b_l or b_r) will be increasingly tempted to lie. As a consequence, for reasons analogous to the stratified sampling case, the bounds b_l and b_r must tend toward zero as the size of the poll increases; that is, an increasingly small fraction of the populace will act as centrists.

We formalize this intuition in the following proposition:

Proposition 6 *In a poll with random sampling, as the size of the poll grows arbitrarily large, the fraction of constituents telling the truth goes to zero. Formally,* $\lim_{n\to\infty} \{q_{c,n}\} = 0$.

Notice that, while we examined the sequence of equilibria containing the largest fraction of centrists, the limit result above implies that all non-degenerate centrist-extremist equilibria converge to $q^* = 0$ in the limit.

What happens to information aggregation? As we show in the next proposition, information fully aggregates even under a poll with a random sample. To gain some intuition for why this is the case, consider the converse: suppose information did not fully aggregate. In that case, a constituent's own signal, even after having observed the outcome of the poll, leads to a shift in his posterior beliefs about the state. That is, there is a difference in the posterior beliefs of the constituent and those of the policy maker when both observe the poll's outcome. Constituents with ideologies close to those of the policy maker prefer to tell the truth and shift the policy in the direction of the state rather than to lie and shift the policy away from it. As the sample size increases with out bound, there are an infinite number of these constituents, which implies information must, in fact, fully aggregate to avoid this contradiction. In short, while there might appear to be a "race" between the speed at which the bounds on centrism collapse and the number of centrists lying inside these bounds, the above intuition shows that the race is always decided in favor of full information aggregation so long as there is a positive density of constituent ideologies in an epsilon neighborhood around the ideology of the policy maker. We return to this intuition in section 5 and show that, absent this condition, information aggregation need not result.

The following proposition formalizes the above intuition.

Proposition 7 *As the size of the poll grows arbitrarily large, the number of constituents telling the truth becomes arbitrarily large. Formally,* $\lim_{n\to\infty} n\{q_{c,n}\} = \infty$.

Corollary 1 *Information fully aggregates.*

6 Statistical Properties of Polls

Suppose a policy maker were to analyze poll results using classical statistics while ignoring strategic effects. For instance, in a poll to assess the appropriateness of school funding in New Jersey, the Monmouth University Polling Institute, who were commissioned to conduct the poll, surveyed 803 New Jersey residents that were 18 years or older in October 2006. To analyze the results of their polls, the Institute relied on classical statistics to derive confidence intervals. Specifically, they report a margin of error attributable to sampling of plus or minus three and a half percentage points based on a underlying Bernoulli model with a parameter equal to one half.[16]

Pollsters typically recognize that in addition to sampling error, bias might be introduced thorough selection issues—the survey was based on telephone interviews necessitating respondents have telephones—and framing effects—bias attributable to question wording. That being said, we know of no cases where polling organizations have attempted to correct confidence intervals to account for the possibility that respondents might answer strategically. As we highlighted above, it is precisely in large polls, where sampling error is small and confidence bounds apparently "tight," where strategic motives play the largest role. Thus, a policy maker needs to recognize these strategic motives to draw correct inferences from polling data.

6.1 Polls with Stratified Sampling

To illustrate how the presence of strategic behavior affects the inferences drawn from a poll, consider Figure 3. This figure illustrates the most informative symmetric centrist-extremist equilibrium when a stratified poll consisting of 261 constituents is conducted. In the figure, the parameter of interest, θ, is uniformly distributed. Using Proposition 4, one can show there is a symmetric centrist-extremist equilibrium where 11 of those polled are centrists and the remainder are extremists; thus, the outcome of the poll, k, can range from 125 to 136. This range is illustrated on the x-axis of the figure. The conventional sample mean, $\hat{\theta}$, is simply the outcome of the poll divided by the size of the poll, $\hat{\theta} = \frac{k}{n}$. As the graph shows, the sample mean is increasing, but given the large number of extremists in the polling populace, not surprisingly $\hat{\theta}$ always lies in the neighborhood of 1/2. Since the results of a poll are approximately normally distributed when the sample size n is large, the confidence interval then

[16] The margin of error associated with a 95 percent confidence level and a (large) random sample of size n is $1.96 \times \sqrt{\theta(1-\theta)/n}$; the margin of error that is typically reported assumes $\theta = 1/2$.

consists of adding and subtracting $z_{\gamma/2}$ times the standard error to and from $\hat{\theta}$; the figure depicts a $\gamma = .95$ confidence interval. Notice that, because the size of the sample is large, the confidence interval fairly tightly brackets the sample mean.

The constituents do not all truthfully reveal their information. Indeed, there are only eleven constituents—the centrists—telling the truth in the poll. This has two implications: First, it implies that a small variation in the outcomes of the poll lead to large differences in the sample mean once one recognizes that the effective sample size is 11 rather than 261. This difference can be readily seen in Figure 3 by comparing the conventional sample mean to the adjusted sample mean $\hat{\theta}' = (k - (n - r))/c$, where the term *adjusted* refers to the fact that strategic motives are recognized. Notice that the adjusted sample mean, which is the minimum variance unbiased estimator for θ, is much more steeply sloped as a function of the outcome of the poll than is the conventional sample mean. Second, since the effective sample size is 11 as opposed to 261, when the conventional and adjusted sample means are relatively close to one another, such as when $k = 130$, the standard error and hence the confidence interval is much wider once one accounts for the extremists in the sample population. On the other hand, for extreme outcomes, such as $k \leq 127$ or $k \geq 134$, the adjusted standard error becomes extremely small and hence the confidence interval for the adjusted statistics is tighter than when one does not account for strategic motives. Indeed, for these values, the conventional and adjusted confidence intervals do not overlap at all. Further, the presence of strategic behavior dramatically effects the poll's margin of error: when the pollster ignores the fact that respondents behave strategically, the error attributable to the sample is plus or minus six percentage points with 95 percent confidence; in contrast, when the pollster recognizes that respondents behave strategically, the error is plus or minus thirty percentage points with 95 percent confidence!

In the remainder of the section, we show that for the model of polls with stratified sampling, the conventional sample mean is biased toward moderate values of θ compared to the adjusted sample mean. We also establish that the conventional confidence bands are too narrow for moderate values of k and too wide for extreme values of k.

Bias

We begin by studying the relationship between the conventional sample mean and the unknown parameter, θ. Recall that the sample mean of a poll consisting of n constituents having outcome k is $\hat{\theta} = k/n$. The sample mean is biased if $E\left[\hat{\theta}\right] \neq \theta$. To see that it is biased, fix a value of θ and using the definitions of n_j, r_j, and c_j

contained in Proposition 4, observe that

$$\begin{aligned} E\left[\hat{\theta}\right] &= \frac{E\left[k|\theta\right]}{n_j} \\ &= \frac{1}{n_j} \times \left[(n_j - r_j) + \sum_{i=0}^{c_j} i \binom{c_j}{i} \theta^i (1-\theta)^{c_j - i}\right] \\ &= \theta + \frac{(2j-1)(j-I)}{4j(j-I)+2}\left(\frac{1}{2} - \theta\right). \end{aligned}$$

When $j > I$, notice that the estimator $E\left[\hat{\theta}\right] > \theta$ when $\theta < \frac{1}{2}$, and vice-versa when $\theta > \frac{1}{2}$; thus, the sample mean has positive bias when the state is low and negative bias when the state if high. The intuition for this observation is that the sample mean $E\left[\hat{\theta}\right]$ is determined in a fashion that fails to recognize that only a portion of the constituents surveyed truthfully reveal their signal to the pollster (i.e., the centrists). Overstating the sample size moderates $E\left[\hat{\theta}\right]$ as an estimate of θ.

We have shown:

Proposition 8 *For large polls, the sample mean $\hat{\theta}$ is biased.*

Confidence Intervals

We now consider the accuracy of the sample mean as an estimator of the unknown parameter θ by constructing confidence intervals. When one ignores strategic effects, the approximate confidence intervals for θ can be constructed using the statistic

$$Z = \frac{\hat{\theta} - \theta}{\sqrt{\frac{\hat{\theta}\left(1-\hat{\theta}\right)}{n}}},$$

which is approximately standard normal for n sufficiently large. The γ-percentage confidence interval is $\hat{\theta} \pm z_{\gamma/2}\sqrt{\frac{\hat{\theta}\left(1-\hat{\theta}\right)}{n}}$ where $z_{\gamma/2}$ is a standard normal z statistic. The width of this confidence interval is

$$w = 2z_{\gamma/2}\sqrt{\frac{\hat{\theta}\left(1-\hat{\theta}\right)}{n}}.$$

Because $\hat{\theta}$ is biased and the sample size n is effectively overstated in a centrist-extremist equilibrium, the test statistic Z yields an incorrect confidence interval. If one adjusts the statistic to account for the strategic motives of the constituents, the adjusted statistic is

$$Z' = \frac{\frac{k-(n-r)}{c} - \theta}{\sqrt{\frac{\frac{k-(n-r)}{c}\left(1-\frac{k-(n-r)}{c}\right)}{c}}},$$

and the width of the adjusted confidence interval is

$$w' = 2z_{\gamma/2}\sqrt{\frac{\frac{k-(n-r)}{c}\left(1-\frac{k-(n-r)}{c}\right)}{c}}.$$

Observe that when $r = n$, that is, when all of those polled reveal truthfully, the conventional and adjusted confidence intervals are identical for all poll outcomes. However, when $r < n$; that is, when some constituents do not answer truthfully, the confidence intervals do not coincide. In this case, the difference between confidence intervals w and w' for a poll having outcome k is proportional to an expression we denote by $\phi(k)$. That is,

$$w - w' \propto \left(\left(2n^2k + 4k^2r\right)\left(n - 2(n-r)\right) + 2n^2k^2 + n^3r - 8knr^2\right) \equiv \phi(k).$$

Solving for the roots of $\phi(k)$ yields:

$$\begin{aligned} k_0 &= \frac{1}{2}n\left(1 - \frac{2r-n}{\sqrt{(n-r)^2 + 3r^2}}\right) \\ k_1 &= \frac{1}{2}n\left(1 + \frac{2r-n}{\sqrt{(n-r)^2 + 3r^2}}\right); \end{aligned}$$

notice that $k_0 < \frac{n}{2} < k_1$.

As we show in Proposition 9, the realized outcome of the poll determines whether the conventional confidence interval overstates or understates the precision of poll information. In particular, as Figure 3 illustrates, when relatively few (or relatively many) centrists report $m_i = 1$, the conventional confidence interval is wider than the adjusted confidence interval—the precision of the poll information is understated. The converse is true when a moderate number of centrists report $m_i = 1$. The proof of the following proposition is contained in the Appendix.

Proposition 9 *For large polls of size n, the conventional confidence interval is strictly wider than the adjusted confidence interval for outcomes $k < k_0$ or $k > k_1$ and conversely for outcomes $k \in [k_0, k_1]$.*

The intuition underlying the results of Proposition 9 is straightforward. Suppose the pollster observes extreme realizations of k. The pollster is more likely to observe extreme values for k when the sample size is large than the same number of extreme values for k when the sample size is small. The sample size is overstated when the pollster does not recognize the effects of strategic polling behavior. Therefore, when the pollster observes extreme realizations of k, the pollster can less confidently draw inferences about the unknown population parameter θ. Consequently, we observe

$w - w' > 0$. Alternatively, suppose the pollster observes a moderate value for k. In this case, the pollster can more confidently draw inferences about the unknown population parameter θ when the sample size is large than when it is small. Because the sample size is overstated when the pollster does not recognize the effects of strategic polling behavior, we find $w - w' < 0$.

6.2 Polls with Random Sampling

Next, we turn to polls with random sampling. As we shall see below, the statistical properties of bias and misstated confidence intervals arise in a similar fashion under this framework as they did under a poll with stratified sampling. In other words, the distortions to the statistical properties of polls are a general consequence of the presence of strategic incentives and not merely a by-product of the particular modeling assumptions used earlier.

Bias

The sample mean of a poll consisting of n constituents having outcome k is $\hat{\theta} = k/n$. Recall that in equilibrium a polled constituent i's message m_i is a Bernoulli random variable with parameter $\theta q_c + q_r$. The sample mean of such a distribution is

$$E\left[\hat{\theta}\right] = \frac{1}{n}\sum_{i=1}^{n} E\left[m_i\right] = \left(q_c\theta + q_r\right),$$

and this is a biased estimator of θ provided $q_c \neq 1$.Thus, we have shown

Proposition 10 *For large polls, the sample mean $\hat{\theta}$ is biased.*

By recognizing that constituents' behave strategically, it is possible to obtain an unbiased statistic for θ. Specifically, consider the statistic

$$\tilde{\theta} = \frac{k - nq_r}{nq_c}.$$

Notice that

$$\begin{aligned} E\left[\tilde{\theta}\right] &= E\left[\frac{k - nq_r}{nq_c}\right] \\ &= \frac{1}{nq_c}\left(E\left[k\right] - nq_r\right) \\ &= \frac{1}{nq_c}\left(\left(q_c\theta + q_r\right)n - nq_r\right) \\ &= \theta. \end{aligned}$$

Hence, to obtain a statistic $\tilde{\theta}$ that is an unbiased estimator of θ, two adjustments to the conventional sample mean $\hat{\theta}$ of the poll are needed: first, one needs to reduce the outcome of the poll by the expected number of right-wing constituents; second, the sample size needs to be adjusted to account for the expected fraction of those polled who are centrists.

Confidence intervals.

A poll under a centrist-extremist equilibrium (q_c, q_r) will produce reported results that are Bernoulli distributed with parameter τ, where $\tau = q_c\theta + q_r$. When the sample size n is large, the sample mean of the poll, $\hat{\theta}$, is approximately distributed $N\left(\tau, \frac{\tau(1-\tau)}{n}\right)$. We are interested not in the parameter τ, however, but in the parameter θ. Accordingly, define the function

$$g(x) = \frac{x - q_r}{q_c}.$$

Notice that, $g(\tau) = \theta$ and $g\left(\hat{\theta}\right) = \tilde{\theta}$ as defined above. Using the delta method (see Greene, 2003), it follows that

$$\sqrt{n}\left(g\left(\hat{\theta}\right) - g(\tau)\right) \rightarrow N\left(0, \left(g'(\tau)\right)^2 \tau(1-\tau)\right)$$

for large n. Therefore, the estimator $\tilde{\theta}$ converges in distribution to $N\left(\theta, \frac{\tau(1-\tau)}{n(q_c)^2}\right)$ for large n.

Comparing the statistical properties of the estimator for θ taking account of strategic effects to the usual classical estimator, two points are worth noting: First, the classical estimator produces biased estimates of θ. Second, the variance of the classical estimator $\hat{\theta}$ understates the variance of the estimator $\tilde{\theta}$. Under the classical model, the variance of the underlying (Bernoulli) distribution is scaled down by n; in contrast, accounting for strategic effects means that a poll of size n exhibits greater variance than it would in the absence of strategic effects; this effect is reflected in the correction of the variance by dividing by $(q_c)^2$. Notice that when truth-telling is an equilibrium (i.e., $q_c = 1$), the properties of the two estimators coincide. At the other extreme, when q_c approaches zero,the variance of the $\tilde{\theta}$ estimator becomes infinite because the poll results reveal little about the state, θ, in this case. By correcting the estimator and adjusting the variance, one can use poll results from a centrist-extremist equilibrium to construct appropriate confidence intervals—or margin of error—for the parameter of interest, θ.

7 Extensions

Up until now, we have shown that, even though constituents have incentives to respond to polling questions strategically once the poll size is sufficiently large, the poll still performs well in the sense of aggregating information. To arrive at this conclusion, we made several key assumptions. First, we assumed that there was sufficient flexibility on the part of the policy maker that the set of policies that might be chosen following the poll could be represented by a continuum. Second, we assumed that the ideological distribution of the constituents could also be represented by a continuum with the policy maker representing the median ideology. In this section, we relax these two assumptions and explore the properties of the model.

7.1 Polls and Elections

The first variation is a comparison between policy making via an elected representative who uses polls to glean information versus direct democracy such as the use of a referendum to make policy. A recent famous example of this was when Governor Arnold Schwarzenegger "went to the people" to implement policies related to the resolution of budget impasses rather than working with the California state legislature. Of course, the policy outcomes associated with this type strategy are often uncertain—all four of the ballot measures placed by the governor were defeated by voters in the 2004 referenda. A natural question is how the policy performance of polls differs from that of elections, which might be thought of as referenda.

To allow for a meaningful comparison between policy making using polls and referenda, we amend the earlier model such that the policy maker is restricted to choosing one of two possible policies. When then compare the policy response under circumstances where the policy is chosen by the policy maker following a poll versus where the policy is selected by "the people" via majority rule in a referendum. Before proceeding with the analysis, it is useful to reflect on the difference between the two mechanisms. Like elections, polls have the feature that they provide a mechanism for aggregating private information. Indeed, a poll with a policy maker who has the median ideology and whose policy choice space is binary seems similar to a two-candidate election with majority rule. An important difference, however, is that in a poll the policy maker chooses the optimal action *after* observing the poll outcome whereas in an election the rule for choosing the winning candidate is fixed. In other words, the policy maker cannot *commit* to a policy rule in advance of the polling outcome while in election, the policy maker, in effect, can commit to a policy rule as a function of the response of constituents. As we shall see below, the absence of commitment has important effects on the informational properties and equilibrium policy response of a poll compared to an elections.

We amend the model of polls with random sampling as follows: suppose the policy space is given by $y \in \{y_0, y_1\}$ where $0 \leq y_0 < y_1 \leq 1$. Further, to ensure the policy choice is non-trivial under truth-telling, suppose $E[\theta|s=0] \leq \frac{y_1+y_0}{2} < E[\theta|s=1]$ .The following definition proves useful:

Definition 1 *The policy maker is said to follow a k* ***rule*** *if policy y_0 is selected when the support of the poll is less than or equal to k and policy y_1 otherwise.*

In a referendum, the k rule is committed to in advance. for instance, under majority rule where y_0 represents the status quo policy, the vote amounts to committing to a $k = \lfloor \frac{n}{2} \rfloor$ rule. It is straightforward to show (and also follows from Austen-Smith and Banks, 1996) that truth-telling is not an equilibrium in the referendum model; thus, we study the analog of a centrist-extremist equilibrium for the case of a referendum. The referendum model is closely related to Feddersen and Pesendorfer (1997) and shares many of the same properties.[17] In particular, using a proof along the same lines as their existence proof, we can show that

Proposition 11 *A centrist-extremist voting equilibrium exists.*

Next, we contrast the results from a referendum with those of polls. We first show that a key feature of the model of polls with random sampling—that truth-telling is never an equilibrium for a sufficiently large poll—continues to hold in the amended model. The intuition is as follows: The endogenous k rule followed by the policy maker occurs where she is almost indifferent between policy y_0 and policy y_1 conditional on the information in the poll. When the poll becomes sufficiently large, this means that it is likely that the true value of the state is such that the policy maker is indeed indifferent between the two possible policy choices. This, in turn, implies that constituents with ideologies even slightly to the right of the policy maker will strictly prefer y_1 even given the signal $s = 0$ and hence distort their response to the poll. The incentives are similar for constituents with ideologies just to the left of the policy maker. It then follows that truth-telling is not an equilibrium. Formally,

Proposition 12 *Truth-telling is never an equilibrium for a sufficiently large poll.*

A common intuition might suggest that a symmetric centrist-extremist equilibrium in a poll leads to policy outcomes similar to a referendum under majority rule when the policy space is binary. Intuitively, if there are, on average, the same number of left-wing extremists as right-wing extremists, then when the majority of centrists indicate having received $s = 1$,then one might expect this would lead to a change in the policy choice of the policy maker. That is, one might imagine that a symmetric centrist-extremist equilibrium endogenously leads the policy maker to adopt a majority k rule. As we show below, this, intuition fails: when the prior beliefs of the policy maker sufficiently favor one policy ex ante, it might require a supermajority of evidence to lead to a policy change.

Remark 2 *An equilibrium in a referendum under a given k rule is generally not an equilibrium in a poll.*

[17] Our assumptions are not identical to the Feddersen and Pesendorfer (1997) model. Hence, we cannot immediately invoke their results for the referendum model. The differences in the assumptions, however, do not appear to qualitatively change the results.

An equilibrium in an election consists of a k rule and a resulting pair of equilibrium cutpoints (b_l, b_r) .For this to be an equilibrium in a poll, this further requires that

$$E[v(\theta,0) \mid \langle n,k\rangle] \leq 0 \leq E[v(\theta,0) \mid \langle n,k+1\rangle]$$

In general, there is no reason for this additional condition to hold.

In particular, suppose that $y_0 = 0$ and $y_1 = 1$; further, let $n = 5$, $\alpha = 2$, $\beta = 1$, and suppose the constituents' ideology is uniformly distributed on $[-1, 1]$. In this case, the prior beliefs of the policy maker favor policy y_1. While there is a non-degenerate centrist-extremist equilibrium in a referendum under the majority rule (and indeed under any k rule), There is no non-degenerate centrist extremist equilibrium in a poll where the policy maker endogenously chooses the majority rule.

One might suspect, however, that given the prior beliefs of the decision maker, one would need to modify the k rule such that there is sufficient evidence of a low state to induce a policy change. That is, one might imagine that, while the example fails for a majority k rule, an equilibrium might arise under a supermajority k rule.[18] This turns out not to be the case either. Indeed, in the example above, an endogenous k rule precludes the possibility of meaningful information revelation on the part of *any constituents in a poll.*

In particular, it is straightforward to verify that, in the example above, there is does not exist a non-degenerate centrist-extremist equilibrium in a poll with binary policies. In other words, the unique centrist-extremist equilibrium for the above parameters consists of babbling on the part of all constituents together with a totally unresponsive policy rule.

We have shown that,

Proposition 13 *Under binary policies, a referendum has a non-degenerate centrist-extremist equilibrium for all k rules while meaningful information revelation need not arise in a poll under any k rule.*

An immediate implication of the proposition is that the distribution of policy outcomes occurring under a given state when a referendum is held to determine policy will typically differ from those of a poll. For the example above, in all states, the policy outcome under a poll is policy y_1 whereas under a referendum, policy y_0 is chosen some fraction of the time. From the point of view of the policy maker, therefore, it would be better to *commit* to a k rule in advance (i.e., to hold a referendum) rather than to retain the power to choose policy after learning the results of a poll. In short, when the policy space is binary, the absence of commitment implied by choosing policies using polls is welfare reducing from the perspective of the policy maker compared to a referendum.

[18] In this case, a supermajority amounts to a k rule where $k < 2$ since evidence in favor of a low state is needed to overturn the priors of the decision maker.

7.2 Binary ideologies

Our next variation amends the model of polls with random sampling to include the possibility that there are no constituents with ideologies near those of the policy maker. Recall that, when the ideology of the constituents is continuously distributed around the policy maker, who has the median ideology, information aggregates because the pollster is able to infer the underlying state from the constituents with ideologies close to that of the policy maker. To highlight the sensitivity of the equilibrium outcome to the ideological distribution of the constituents and how this distribution determines whether information aggregations occurs, we focus on a binary ideology specification. In this specification, constituents either have left-wing preferences ($-b$) or right-wing preferences (b) while the policy maker is stuck between the two ideological camps. This situation might arise for a particular issue where the polity is ideologically polarized compared to the policy maker, i.e., where no constituent has an ideology close to that of the policy maker. We showed that, when the ideological distribution was binary, that is $b_i = \{-b, b\}$ with equal probability, the largest size poll supporting truth telling was given by $n = \bar{n}$ as defined in Proposition 2. Further, we continue to suppose that individuals suffer quadratic losses associated with the deviation of the chosen policy from an individual's ideal policy.

In the next proposition, which is proved in the appendix, we show that $\bar{n}$ represents an upper bound on the number of constituents reporting truthfully in *any* centrist-extremist equilibrium. In other words, the interaction of information and ideology leads to a situation where information does not fully aggregate in polls regardless of poll size. Formally,

Proposition 14 *In the quadratic loss with binary ideology specification, there does not exist a centrist-extremist equilibrium where information fully aggregates. Specifically, at most* $\bar{n} = \max\left\{\left\lfloor \frac{1}{2b} - \alpha - \beta \right\rfloor, 0\right\}$ *constituents truthfully reveal their signals in any centrist-extremist equilibrium.*

Although this analysis pertains to the case where ideology of constituents is binary, one could readily generalize Proposition 14 to a situation where there is a difference of at least $b > 0$ between the ideology of the policy maker and the ideology of any of the constituents. In other words, for any ideological distribution with the property that there is zero mass on the interval $(-b, b)$, full information aggregation will not occur. This finding contrasts with our earlier finding that information aggregates.

While this extension of the model is interesting in highlighting the sensitivity of the information aggregation results to alternative assumptions, the binary ideology model would seem to be a rather poor description of reality. After all, why should it be that the policy maker holds ideological views that are "far" from those of other constituents in the polity? How did such a policy maker get elected in the first place? Why are her views so different? This extension is useful mainly on a theoretical basis for understanding models of polling, but probably does not reflect the institutional reality within which polling occurs.

8 Conclusions

The influence of poll results on policy outcomes has a long history. For instance, President Richard Nixon decided to ban oil drilling off the California coast largely on the basis of poll information (Green 2002). Polled constituents are aware of this connection between their poll responses and policy outcomes, and hence, will be *strategic* in their responses. In determining their responses to a poll, constituents are influenced both by their underlying ideology as well as specific information they have about the desirability of various policies, such as the ecological effects of oil drilling off the California coast. Consequently, poll answers will reflect the interaction of ideology and information in the sense that respondents might "shade" their answers in the direction of their ideology, thereby distorting their information.

We study equilibrium responses to polls in the presence of constituents' informational and ideological motives. We show that when constituents' ideologies are relatively homogeneous and the poll is sufficiently small, poll respondents reveal their information truthfully. In contrast, as the size of the poll increases, we find the gains to respondents from "shading" their information in the direction of their ideology dominate. As a consequence, constituents have an incentive to distort their information even in polls with modest sample sizes.

Given that truth-telling is not an equilibrium once the sample size of the poll is sufficiently large, we examine whether polls aggregate information under non truth-telling strategies. On one hand, when the ideology of constituents is similar to that of the policy-maker, we show full information aggregation can arise in an equilibrium where those polled endogenously sort themselves into centrists, who answer truthfully, and extremists, who do not. As the sample size increases, the fraction of centrists in the poll declines and indeed vanishes in the limit. Nevertheless, the *number* of centrists increases with the sample size of the poll and then information aggregates in the limit. On the other hand, information aggregation breaks down when the policy maker is ideologically isolated from her constituents.

An important practical implication of our results is that the use of classical statistics to derive conclusion from poll data can lead to seriously flawed inference about the underlying environment. Indeed,when poll respondents are strategic the sample mean is typically not an unbiased estimator of the parameter of interest and the margin of error is always understated. We propose amendments to the standard classical statistics to correct for strategic effects and therefore allow meaningful inference.

Finally, we compare polls to elections. While votes in an two candidate election will typically convey some of the information held by constituents, this need not be the case in a poll. In contrast, when policies are determined following a poll and the policy space is constrained to be binary, constituents may be unable to convey information in any equilibrium. Accordingly, we conclude these two mechanisms yield different policy outcomes.

A Appendix

Proof of Lemma 1:

Suppose to the contrary that $y(k+1) \leq y(k)$. Since $y(k+1)$ is optimal when a poll has outcome $k+1$, it follows from equation (1) that:

$$\int_0^1 U_1(y(k+1), \theta) \frac{g(\theta|\langle n, k+1\rangle)}{g(\theta|\langle n, k\rangle)} g(\theta|\langle n, k\rangle) d\theta = 0. \tag{4}$$

Furthermore, there exists a unique state, $\theta = \theta_{k+1}$, where $U_1(y(k+1), \theta) = 0$. Using this fact, we can rewrite the left-hand side of equation (4) as

$$\begin{aligned}
& \int_0^{\theta_{k+1}} U_1(y(k+1), \theta) \frac{g(\theta|\langle n, k+1\rangle)}{g(\theta|\langle n, k\rangle)} g(\theta|\langle n, k\rangle) d\theta \\
& + \int_{\theta_{k+1}}^1 U_1(y(k+1), \theta) \frac{g(\theta|\langle n, k+1\rangle)}{g(\theta|\langle n, k\rangle)} g(\theta|\langle n, k\rangle) d\theta \\
> & \int_0^{\theta_{k+1}} U_1(y(k+1), \theta) \frac{g(\theta_{k+1}|\langle n, k+1\rangle)}{g(\theta_{k+1}|\langle n, k\rangle)} g(\theta|\langle n, k\rangle) d\theta \\
& + \int_{\theta_{k+1}}^1 U_1(y(k+1), \theta) \frac{g(\theta_{k+1}|\langle n, k+1\rangle)}{g(\theta_{k+1}|\langle n, k\rangle)} g(\theta|\langle n, k\rangle) d\theta \\
= & \frac{g(\theta_{k+1}|\langle n, k+1\rangle)}{g(\theta_{k+1}|\langle n, k\rangle)} \int_0^1 U_1(y(k+1), \theta) g(\theta|\langle n, k\rangle) d\theta,
\end{aligned}$$

where the strict inequality follows from two facts: First, when $\theta < \theta_{k+1}$, $U_1(y(k+1), \theta) < 0$ and when $\theta > \theta_{k+1}, U_1(y(k+1), \theta) > 0$. Second, since the posterior beliefs of the policy maker are Beta distributed, the family of posterior densities $\{g(\cdot|\langle n, k\rangle)\}$ satisfies the strict monotone likelihood ratio property. Thus, when $\theta < \theta_{k+1}$, $\frac{g(\theta_{k+1}|\langle n,k+1\rangle)}{g(\theta_{k+1}|\langle n,k\rangle)} > \frac{g(\theta|\langle n,k+1\rangle)}{g(\theta|\langle n,k\rangle)}$ and when $\theta > \theta_{k+1}$, $\frac{g(\theta_{k+1}|\langle n,k+1\rangle)}{g(\theta_{k+1}|\langle n,k\rangle)} < \frac{g(\theta|\langle n,k+1\rangle)}{g(\theta|\langle n,k\rangle)}$.

Finally, we claim that

$$\frac{g(\theta_{k+1}|\langle n, k+1\rangle)}{g(\theta_{k+1}|\langle n, k\rangle)} \int_0^1 U_1(y(k+1), \theta) g(\theta|\langle n, k\rangle) d\theta \geq 0.$$

The likelihood ratio $\frac{g(\theta_{k+1}|\langle n,k+1\rangle)}{g(\theta_{k+1}|\langle n,k\rangle)}$ is positive. Further, the strict concavity of the policy maker's payoff function implies that for all $y \leq y(k)$,

$$\int_0^1 U_1(y, \theta) g(\theta|\langle n, k\rangle) d\theta \geq 0.$$

Since, by assumption, $y(k+1) \leq y(k)$, then $\int_0^1 U_1(y(k+1), \theta) g(\theta|\langle n, k\rangle) \geq 0$. Therefore,

$$\int_0^1 U_1(y(k+1), \theta) \frac{g(\theta|\langle n, k+1\rangle)}{g(\theta|\langle n, k\rangle)} g(\theta|\langle n, k\rangle) d\theta > 0,$$

which contradicts equation (4).■

Proof of Lemma 2:

To establish the result, we will show that for any truthful sequence, k_n,

$$\lim_{n\to\infty}\frac{g\left(\theta|\left\langle n,k_n\right\rangle\right)}{g\left(\theta|\left\langle n,k_n+1\right\rangle\right)}=1$$

Therefore

$$\begin{aligned}
\lim_{n\to\infty}\frac{g\left(\theta|\left\langle n,k_n\right\rangle\right)}{g\left(\theta|\left\langle n,k_n+1\right\rangle\right)} &= \left(\frac{1-\theta}{\theta}\right)\lim_{n\to\infty}\left(\frac{\int t^{(k_n+1)+\alpha-1}\left(1-t\right)^{n-(k_n+1)+\beta-1}dt}{\int t^{k_n+\alpha-1}\left(1-t\right)^{n-k_n+\beta-1}dt}\right)\\
&= \left(\frac{1-\theta}{\theta}\right)\lim_{n\to\infty}\left(\frac{\frac{1}{\Gamma(n+\alpha+\beta)}\Gamma\left(\alpha+k_n+1\right)\Gamma\left(n+\beta-k_n-1\right)}{\frac{1}{\Gamma(n+\alpha+\beta)}\Gamma\left(\alpha+k_n\right)\Gamma\left(n+\beta-k_n\right)}\right)\\
&= \left(\frac{1-\theta}{\theta}\right)\lim_{n\to\infty}\left(\frac{\alpha+k_n}{\beta+n-\left(k_n+1\right)}\right)\\
&= \left(\frac{1-\theta}{\theta}\right)\lim_{n\to\infty}\left(\frac{\frac{\alpha}{n}+\frac{k_n}{n}}{\frac{\beta}{n}+1-\left(\frac{k_n}{n}+\frac{1}{n}\right)}\right)\\
&= 1,
\end{aligned}$$

where we use the fact that $\lim_{n\to\infty}\frac{k_n}{n}=\theta$. Since $\lim_{n\to\infty}\frac{g(\theta|\langle n,k_n\rangle)}{g(\theta|\langle n,k_n+1\rangle)}=1$ for all θ, the optimal policy following a poll having outcome k_n converges to that of the optimal policy following a poll having outcome k_n+1. Thus, for any $\varepsilon>0$, there exists a sufficiently large n such that

$$y\left(k+1\right)-y\left(k\right)<\varepsilon.$$

This completes the proof. ■

Proof of Proposition 2:

Consider a right-biased constituent with ideology $b_i=b$. Clearly, when this constituent receives a signal $s_i=1$, he can do no better than to report truthfully. For truth-telling to be incentive compatible, a constituent i receiving a signal $s_i=0$ must prefer to report truthfully than to dissemble, that is,

$$\begin{aligned}
&\sum_{k=0}^{n-1}\Pr\left(\sum_{j\neq i}s_j=k|s_i=0\right)\int_0^1\left(U\left(y\left(k\right),\theta,b\right)-U\left(y\left(k+1\right),\theta,b\right)\right)g\left(\theta|\left\langle n,k\right\rangle\right)d\theta\\
=&\sum_{k=0}^{n-1}\Pr\left(\sum_{j\neq i}s_j=k|s_i=0\right)\left(\left(y\left(k+1\right)-y\left(k\right)-2b\right)\left(y\left(k+1\right)-y\left(k\right)\right)\right)\\
=&\sum_{k=0}^{n-1}\Pr\left(\sum_{j\neq i}s_j=k|s_i=0\right)\left(\frac{1}{n+\alpha+\beta}-2b\right)\left(\frac{1}{n+\alpha+\beta}\right)\\
\geq&\ 0.
\end{aligned}$$

Therefore, for incentive compatibility, we must have

$$\frac{1}{n+\alpha+\beta} \geq 2b.$$

An identical inequality occurs for left-based constituents to tell the truth when $s_i = 1$. Therefore,

$$n \leq \frac{1}{2b} - \alpha - \beta. \tag{5}$$

If the right-hand side of equation (5) is negative, no poll (or equivalently an $\bar{n} = 0$ size poll) is consistent with a truth-telling equilibrium. If the right-hand side of equation (5) is positive, then, since the size of the poll must be integer valued, the largest poll where truth-telling is an equilibrium is

$$\bar{n} = \left\lfloor \frac{1}{2b} - \alpha - \beta \right\rfloor.$$

This completes the proof. ■

Proof of Proposition 3

Given the above construction, we need to establish that no polled constituent can profitably deviate. First, consider the incentive constraint of a centrist with ideology $b > 0$. For truth-telling to be optimal requires that, having received the signal $s = 0$,the payoff to that constituent is greater by reporting $m = 0$ than from reporting $m = 1$. That is,

$$\begin{aligned} & \sum \Pr\left[\langle n-1, k\rangle \,|s=0\right] \left(y\left(k+1\right) - y\left(k\right)\right) \\ & \times \left(y\left(k+1\right) + y\left(k\right) - 2E\left[\theta|\, \langle n-1, k\rangle, s=0\right] - 2b\right) \\ \geq\ & 0 \end{aligned}$$

Recall that optimality on the part of the policy maker requires that $y\left(k\right) = E\left[\theta|\, \langle n, k\rangle\right]$ for all k.

We make the following claim:

Claim: In a centrist-extremist equilibrium under stratified polling, for any constituent who is a centrist with signal $s = 0$, it is the case that $E\left[\theta|\, \langle n-1, k\rangle, s=0\right] = E\left[\theta|\, \langle n, k\rangle\right]$.

To establish the claim, notice that

$$\begin{aligned} & E\left[\theta|\, \langle n-1, k\rangle, s=0\right] \\ = & \frac{\int_0^1 \theta \binom{c-1}{k-\rho} \theta^{k-\rho} \left(1-\theta\right)^{c-1-(k-\rho)} \left(1-\theta\right) \theta^{\alpha-1} \left(1-\theta\right)^{\beta-1} d\theta}{\int_0^1 \binom{c-1}{k-\rho} t^{k-\rho} \left(1-t\right)^{c-1-(k-\rho)} \left(1-t\right) t^{\alpha-1} \left(1-t\right)^{\beta-1} dt} \\ = & \frac{k-\rho+\alpha}{c+\alpha+\beta} \\ = & E\left[\theta|\, \langle n, k\rangle\right] \end{aligned}$$

Hence, the incentive constraint of a centrist with signal $s = 0$ reduces to

$$\sum \Pr\left[\langle n-1, k\rangle | s=0\right] \left(y\left(k+1\right) - y\left(k\right)\right) \left(y\left(k+1\right) - y\left(k\right) - 2b\right) \geq 0$$

Clearly, this constraint is most binding for the right-most centrist. Substituting for b using the index r_j,

$$\begin{aligned} & \sum \Pr\left[\langle n_j - 1, k\rangle | s=0\right] \left(\frac{1}{n_j - 2\left(n_j - r_j\right) + \alpha + \beta}\right) \\ & \times \left(\frac{1}{n_j - 2\left(n_j - r_j\right) + \alpha + \beta} - 2\left(\frac{2\left(r_j - 1\right) - \left(n_j - 1\right)}{\left(n_j - 1\right)}\right)\right) \\ = & \sum \Pr\left[\langle n_j - 1, k\rangle | s=0\right] \left(\frac{1}{n_j - 2\left(n_j - r_j\right) + \alpha + \beta}\right) \\ & \times \left(\frac{1}{n_j - 2\left(n_j - r_j\right) + \alpha + \beta} - 2\left(\frac{2\left(r_j - 1\right) - \left(n_j - 1\right)}{\left(n_j - 1\right)}\right)\right) \end{aligned}$$

Notice that the sign of this expression depends solely on the sign of

$$\left(\frac{1}{n_j - 2\left(n_j - r_j\right) + \alpha + \beta} - 2\left(\frac{2\left(r_j - 1\right) - \left(n_j - 1\right)}{\left(n_j - 1\right)}\right)\right).$$

Substituting for n_j and r_j this expression reduces to zero. Hence, no centrist with signal $s = 0$ can profitably deviate.

Next, consider a centrist with signal $s = 1$ and ideology $b < 0$. The incentive constraint is

$$\begin{aligned} & \sum_k \Pr\left[\langle n-1, k\rangle | s=1\right] \left(y\left(k+1\right) - y\left(k\right)\right) \\ & \times \left(2E\left[\theta | \langle n-1, k\rangle, s=1\right] - y\left(k+1\right) - y\left(k\right) + 2b\right) \\ \geq & \; 0 \end{aligned}$$

Claim: In a centrist-extremist equilibrium under stratified polling, for any constituent who is a centrist with signal $s = 1$, it is the case that $E\left[\theta | \langle n-1, k\rangle, s=1\right] = E\left[\theta | \langle n, k+1\rangle\right].$

To establish the claim, notice that

$$\begin{aligned} E\left[\theta | \langle n-1, k\rangle, s=1\right] &= \frac{\int_0^1 \theta \binom{c-1}{k-\rho} \theta^{k-\rho} \left(1-\theta\right)^{c-1-(k-\rho)} \theta \times \theta^{\alpha-1} \left(1-\theta\right)^{\beta-1} d\theta}{\int_0^1 \binom{c-1}{k-\rho} t^{k-\rho} \left(1-t\right)^{c-1-(k-\rho)} t \times t^{\alpha-1} \left(1-t\right)^{\beta-1} dt} \\ &= \frac{k+1-\rho+\alpha}{c+\alpha+\beta} \\ &= E\left[\theta | \langle n, k+1\rangle\right] \end{aligned}$$

Hence, the incentive constraint of a centrist with signal $s = 1$ reduces to

$$\sum_k \Pr\left[\langle n-1, k\rangle | s=1\right] \left(y\left(k+1\right) - y\left(k\right)\right) \left(y\left(k+1\right) - y\left(k\right) + 2b\right) \geq 0$$

and, by symmetry, this constraint is also satisfied.

Now consider a right-wing extremist signal $s = 0$. His incentive constraint is

$$\begin{aligned} & \sum \Pr\left[\langle n-1, k\rangle | s=0\right] \left(y\left(k+1\right) - y\left(k\right)\right) \\ & \times \left(y\left(k+1\right) + y\left(k\right) - 2E\left[\theta | \langle n-1, k\rangle, s=0\right] - 2b\right) \\ < \; & 0 \end{aligned}$$

Claim: In a centrist-extremist equilibrium under stratified polling, for any constituent who is a right-wing extremist with signal $s = 0$, it is the case that

$$E\left[\theta | \langle n-1, k\rangle, s=0\right] < E\left[\theta | \langle n, k\rangle\right].$$

To establish the claim, notice that

$$\begin{aligned} E\left[\theta | \langle n-1, k\rangle, s=0\right] &= \frac{\int_0^1 \theta^{k-(\rho-1)} \left(1-\theta\right)^{c-(k-(\rho-1))} \left(1-\theta\right) \theta^{\alpha} \left(1-\theta\right)^{\beta-1} d\theta}{\int_0^1 t^{k-(\rho-1)} \left(1-t\right)^{c-(k-(\rho-1))} \left(1-t\right) t^{\alpha-1} \left(1-t\right)^{\beta-1} dt} \\ &= \frac{k-\rho+\alpha+1}{c+\alpha+\beta+1} \end{aligned}$$

Define the function

$$\psi\left(z\right) = \frac{k-\rho+\alpha+z}{c+\alpha+\beta+z}$$

and notice that this is increasing in z. Then

$$\begin{aligned} E\left[\theta | \langle n, k\rangle\right] &= \psi\left(0\right) \\ &< \psi\left(1\right) \\ &= E\left[\theta | \langle n-1, k\rangle, s=0\right] \end{aligned}$$

Hence, it follows that

$$\begin{aligned} & \sum \Pr\left[\langle n-1, k\rangle | s=0\right] \left(y\left(k+1\right) - y\left(k\right)\right) \\ & \times \left(y\left(k+1\right) + y\left(k\right) - 2E\left[\theta | \langle n-1, k\rangle, s=0\right] - 2b\right) \\ < \; & \sum \Pr\left[\langle n-1, k\rangle | s=0\right] \left(y\left(k+1\right) - y\left(k\right)\right) \left(y\left(k+1\right) - y\left(k\right) - 2b\right) \\ < \; & 0 \end{aligned}$$

where the last inequality follows from the fact that right-wing extremists have higher values of b than do centrists. Hence, there is no profitable deviation.

Finally, consider a left-wing extremist with signal $s = 1$. His incentive constraint is

$$\begin{aligned} & \sum_k \Pr\left[\langle n-1,k\rangle \,|s=1\right]\left(y\left(k+1\right)-y\left(k\right)\right) \\ & \times\left(2E\left[\theta|\left\langle n-1,k\right\rangle ,s=1\right]-y\left(k+1\right)-y\left(k\right)+2b\right) \\ < & \ 0 \end{aligned}$$

Claim: In a centrist-extremist equilibrium under stratified polling, for any constituent who is a left-wing extremist with signal $s = 1$, it is the case that $E\left[\theta|\left\langle n-1,k\right\rangle ,s=1\right] < E\left[\theta|\left\langle n,k+1\right\rangle\right]$.

Substituting

$$\begin{aligned} E\left[\theta|\left\langle n-1,k\right\rangle ,s=1\right] &= \frac{\int_0^1 \theta \binom{c}{k-\rho}\theta^{k-\rho}\left(1-\theta\right)^{c-(k-\rho)}\theta\times\theta^{\alpha-1}\left(1-\theta\right)^{\beta-1}d\theta}{\int_0^1 \binom{c}{k-\rho}t^{k-\rho}\left(1-t\right)^{c-(k-\rho)}t\times t^{\alpha-1}\left(1-t\right)^{\beta-1}dt} \\ &= \frac{k-\rho+\alpha+1}{c+\alpha+\beta+1} \end{aligned}$$

Next, notice that

$$\begin{aligned} E\left[\theta|\left\langle n-1,k\right\rangle ,s=1\right] &= \frac{k-\rho+\alpha+1}{c+\alpha+\beta+1} \\ &< \frac{k-\rho+\alpha+1}{c+\alpha+\beta} \\ &= E\left[\theta|\left\langle n,k+1\right\rangle\right] \end{aligned}$$

Hence, it follows that

$$\begin{aligned} & \sum_k \Pr\left[\langle n-1,k\rangle \,|s=1\right]\left(y\left(k+1\right)-y\left(k\right)\right) \\ & \times\left(2E\left[\theta|\left\langle n-1,k\right\rangle ,s=1\right]-y\left(k+1\right)-y\left(k\right)+2b\right) \\ < & \sum_k \Pr\left[\langle n-1,k\rangle \,|s=1\right]\left(y\left(k+1\right)-y\left(k\right)\right)\left(y\left(k+1\right)-y\left(k\right)+2b\right) \\ < & \ 0 \end{aligned}$$

where the last inequality follows from the fact that left-wing extremists have lower values of b than do centrists. Hence, there is no profitable deviation.■

Proof of Proposition 5:

The proof relies on the following series of Lemmas.

Lemma 3 $E\left[\theta|\left\langle n,k+1\right\rangle\right] < E\left[\theta|\left\langle n-1,k\right\rangle ,s=1\right]$

Proof of Lemma 3:

Fix the strategy of all individuals at (q_c, q_r). Suppose that the policy maker conducts two polls, one consisting of a single individual and one consisting of $n-1$ individuals and receives 1 support in the first poll and k support in the second poll. Clearly, this is equivalent to holding a single poll consisting of n individuals and receiving $k+1$ support. Hence

$$E\left[\theta|\left\langle n,k+1\right\rangle\right]=E\left[\theta|\left\langle n-1,k\right\rangle,\left\langle 1,1\right\rangle\right]$$

Next, suppose that the policy maker learned that, in conducting the poll with a single individual, that individual was telling the truth. Then the change to the policy maker's posterior would be higher than if she were uncertain about the truthfulness of the individual. Hence

$$E\left[\theta|\left\langle n-1,k\right\rangle,s=1\right]>E\left[\theta|\left\langle n-1,k\right\rangle,\left\langle 1,1\right\rangle\right]$$

This proves the claim. ■

Lemma 4 $E\left[\theta|\left\langle n,k\right\rangle\right]>E\left[\theta|\left\langle n-1,k\right\rangle,s=0\right]$

Proof of Lemma 4:

Fix the strategy of all individuals at (q_c, q_r). Suppose that the policy maker conducts two polls, one consisting of a single individual and one consisting of $n-1$ individuals and receives 0 support in the first poll and k support in the second poll. Clearly, this is equivalent to holding a single poll consisting of n individuals and receiving k support. Hence

$$E\left[\theta|\left\langle n,k\right\rangle\right]=E\left[\theta|\left\langle n-1,k\right\rangle,\left\langle 1,0\right\rangle\right]$$

Next, suppose that the policy maker learned that, in conducting the poll with a single individual, that individual was telling the truth. Then the change to the policy maker's posterior would be greater than if she were uncertain about the truthfulness of the individual. Hence

$$E\left[\theta|\left\langle n-1,k\right\rangle,s=0\right]<E\left[\theta|\left\langle n-1,k\right\rangle,\left\langle 1,0\right\rangle\right]$$

This proves the claim. ■

Lemma 5 *In any centrist-extremist equilibrium, a constituent with index b_i is a right-wing extremist only if $b_i > 0$. Likewise, a constituent with index b_i is a left-wing extremist only if $b_i < 0$.*

Proof of Lemma 5:

Suppose not. Consider the case of a constituent with index $b_i > 0$ who is supposed to be a left-wing extremist. Then, when $s_i = 1$,it must be the case that

$$\sum_{k=0}^{n-1} \Pr\left[\langle n-1,k\rangle | s_i = 1\right]\left(E\left[\theta | \langle n,k+1\rangle\right] - E\left[\theta | \langle n,k\rangle\right]\right)$$
$$\times\left(-E\left[\theta | \langle n,k+1\rangle\right] - E\left[\theta | \langle n,k\rangle\right] + 2E\left[\theta | \langle n-1,k\rangle, s=1\right] + 2b_i\right) < 0$$

However, since

$$E\left[\theta | \langle n-1,k\rangle, s=1\right] \geq E\left[\theta | \langle n,k+1\rangle\right] \geq E\left[\theta | \langle n,k\rangle\right]$$

it then follows that, for all k,

$$-E\left[\theta | \langle n,k+1\rangle\right] - E\left[\theta | \langle n,k\rangle\right] + 2E\left[\theta | \langle n-1,k\rangle, s=1\right] + 2b_i \geq 0$$

and hence

$$\sum_{k=0}^{n-1} \Pr\left[\langle n-1,k\rangle | s_i = 1\right]\left(E\left[\theta | \langle n,k+1\rangle\right] - E\left[\theta | \langle n,k\rangle\right]\right)$$
$$\times\left(-E\left[\theta | \langle n,k+1\rangle\right] - E\left[\theta | \langle n,k\rangle\right] + 2E\left[\theta | \langle n-1,k\rangle, s=1\right] + 2b_i\right) \geq 0$$

which is a contradiction. The proof for the reverse case is identical. ■

From this lemma, we may then conclude that as $q_c \to 0, q_r \to \frac{1}{2}$. Clearly, as $q_c \to 1, q_r \to 0$.

We are now in a position to prove the main result.

First, fix b_r. Suppose that $b_l = -1$. Substituting this value for b_l yields

$$\begin{aligned} & \left(-y(k+1) - y(k) + 2E\left[\theta | \langle n-1,k\rangle, s=1\right] + 2b_l\right) \\ = & \left(-y(k+1) - y(k) + 2E\left[\theta | \langle n-1,k\rangle, s=1\right] - 2\right) < 0 \end{aligned}$$

because $y(k+1), y(k), E\left[\theta | \langle n-1,k\rangle, s=1\right] > 0$. Because $(y(k+1) - y(k)) > 0$, it follows that the left-hand side of (3) is negative. Now suppose $b_l = 0$. Then substituting this value for b_l yields

$$\begin{aligned} & \left(-y(k+1) - y(k) + 2E\left[\theta | \langle n-1,k\rangle, s=1\right] + 2b_l\right) \\ = & -E\left[\theta | \langle n,k+1\rangle\right] - E\left[\theta | \langle n,k\rangle\right] + 2E\left[\theta | \langle n-1,k\rangle, s=1\right] \\ > & \ 0 \end{aligned}$$

where the inequality follows from the fact that $E\left[\theta | \langle n,k\rangle\right] < E\left[\theta | \langle n,k+1\rangle\right] < E\left[\theta | \langle n-1,k\rangle, s=1\right]$ established in Lemma 3. Thus, the left-hand side of (3) is positive. Finally, since the left-hand side of (3) is continuous in b_l it follows from the intermediate value theorem that there exists a value for b_l such that

$$\begin{aligned} &\sum_{k=0}^{n-1} (\Pr[\langle n-1,k\rangle | s=1] (E[\theta| \langle n,k+1\rangle] - E[\theta| \langle n,k\rangle]) \\ &\times (-E[\theta| \langle n,k+1\rangle] - E[\theta| \langle n,k\rangle] + 2E[\theta| \langle n-1,k\rangle, s=1] + 2b_l)) \\ = \;& 0 \end{aligned}$$

Second, fix b_l. When $b_r = 0$, then

$$\begin{aligned} &(y(k+1) + y(k) - 2E[\theta| \langle n-1,k\rangle, s=0] - 2b_r) \\ = \;& y(k+1) + y(k) - 2E[\theta| \langle n-1,k\rangle, s=0] \\ > \;& 0 \end{aligned}$$

because $E[\theta| \langle n,k+1\rangle] > E[\theta| \langle n,k\rangle] > E[\theta| \langle n-1,k\rangle, s=0]$, which was established in Lemma 4. It then follows that the left-hand side of (2) is positive. When $b_r = 1$, then

$$\begin{aligned} &(y(k+1) + y(k) - 2E[\theta| \langle n-1,k\rangle, s=0] - 2b_r) \\ = \;& y(k+1) + y(k) - 2E[\theta| \langle n-1,k\rangle, s=0] - 2 \\ < \;& 0 \end{aligned}$$

and the left-hand side of (2) is negative. Finally, since the IC constraint is continuous in b_r it follows from the intermediate value theorem that there exists a value for b_r such that

$$\begin{aligned} &\sum_{k=0}^{n-1} (\Pr[\langle n-1,k\rangle | s=0] (E[\theta| \langle n,k+1\rangle] - E[\theta| \langle n,k\rangle]) \\ &\times \left(E[\theta| \langle n,k+1\rangle] + E[\theta| \langle n,k\rangle] - 2E[\theta| \langle n-1,k\rangle, s=0] - 2F^{-1}(1-q_r)\right)) \\ = \;& 0 \end{aligned}$$

Combining the first and second arguments together with Kakutani's fixed point theorem implies the existence of a centrist-extremist equilibrium. ■

Proof of Proposition 6:

Consider a convergent subsequence of $\{q_{c,n}\}$ (note that since $\{q_{c,n}\}$ is bounded, such a subsequence exists). Call this subsequence $\{q_{c,n_i}\}$ with subsequence limit point L. We claim that for such subsequences, the subsequence limit equals $L = 0$. To see this, suppose to the contrary that the limit of a convergent subsequence was

$$\lim_{n\to\infty} \{q_{c,n_i}\} = L > 0$$

In that case, either

$$\lim_{n\to\infty} \{q_{c,n_i}\} < \frac{1}{2}$$

or

$$\lim_{n\to\infty} (1 - \{q_{r,n_i}\} - \{q_{c,n_i}\}) < \frac{1}{2}$$

Suppose, without loss of generality that $\lim_{n\to\infty} \{q_{r,n_i}\} < \frac{1}{2}$. It then follows that the associated bound on the ideology of the right-most centrist satisfies:

$$\lim_{n\to\infty} \{b_{r,n_i}\} = \bar{b}_r > 0$$

By the strong law of large numbers, one can show that

$$\Pr\left(\lim_{n\to\infty} (E[\theta| \langle n,k\rangle] - \theta) = 0\right) = \Pr\left(\lim_{n\to\infty} (E[\theta| \langle n-1,k\rangle, s=0] - \theta) = 0\right) = 1$$

Therefore, $\Pr(\lim_{n\to\infty} (E[\theta| \langle n,k\rangle] - E[\theta| \langle n-1,k\rangle, s=0]) = 0) = 1$. The relevant incentive constraint thus simplifies in the limit, and it follows *almost surely* that

$$\begin{aligned} & \lim_{n\to\infty} \frac{\sum_{k=0}^{n-1} \Pr[\langle n-1,k\rangle |s=0] (E[\theta| \langle n,k+1\rangle] - E[\theta| \langle n,k\rangle])^2}{2\sum_{k=0}^{n-1} \Pr[\langle n-1,k\rangle |s=0] (E[\theta| \langle n,k+1\rangle] - E[\theta| \langle n,k\rangle])} \\ = \;& 0, \end{aligned}$$

which is a contradicts the fact that $\bar{b}_r > 0$.

Thus, we have shown that every convergent subsequence of $\{q_{c,n}\}$ converges to zero. It then follows from Rudin (1976, 51) that if every subsequence of $\{q_{c,n}\}$ converges to q, then $\{q_{c,n}\}$ converges to q. This result implies $\{q_{c,n}\}$ converges to zero. ■

Proof of Proposition 7:

Suppose not. In that case

$$\lim_{n\to\infty} n\{q_{c,n}\} = L < \infty$$

Recall that the incentive compatibility condition for the right-most centrist may be written as

$$F^{-1}(1-q_r) = \frac{\begin{array}{c}\sum_{k=0}^{n-1} \Pr[\langle n-1,k\rangle |s=0] (E[\theta| \langle n,k+1\rangle] - E[\theta| \langle n,k\rangle]) \\ \times (E[\theta| \langle n,k+1\rangle] + E[\theta| \langle n,k\rangle] - 2E[\theta| \langle n-1,k\rangle, s=0])\end{array}}{2\sum_{k=0}^{n-1} \Pr[\langle n-1,k\rangle |s=0] (E[\theta| \langle n,k+1\rangle] - E[\theta| \langle n,k\rangle])}$$

Since $\lim_{n\to\infty} \{q_{c,n}\} = 0$ then $\lim_{n\to\infty} \{q_{r,n}\} = \frac{1}{2}$ (from the symmetry of F) and hence the left-hand side of the above expression equals zero.

Since there are only a finite number of centrists in the limit, a given constituent's signal affects his posterior beliefs about the state for a given outcome of the poll. This implies

$$\lim_{n\to\infty}\left(E\left[\theta|\left\langle n,k\right\rangle\right]-E\left[\theta|\left\langle n-1,k\right\rangle,s=0\right]\right)>0$$

and hence

$$\lim_{n\to\infty}\left(E\left[\theta|\left\langle n,k+1\right\rangle\right]+E\left[\theta|\left\langle n,k\right\rangle\right]-2E\left[\theta|\left\langle n-1,k\right\rangle,s=0\right]\right)>0$$

for all k. Let

$$\delta=\min_{k}\left(\lim_{n\to\infty}\left(E\left[\theta|\left\langle n,k+1\right\rangle\right]+E\left[\theta|\left\langle n,k\right\rangle\right]-2E\left[\theta|\left\langle n-1,k\right\rangle,s=0\right]\right)\right)$$

then, it follows that

$$\begin{aligned}\lim_{n\to\infty} RHS &\geq \frac{\delta}{2}\\ &> 0\end{aligned}$$

which is a contradiction. ■

Proof of Proposition 14:

Suppose not. Then there exists a centrist-extremist equilibrium where $c>\bar{n}$ constituents are centrists and the remainder are extremists. Suppose that constituent i with ideology $b_i=b$ and signal $s_i=0$ is a centrist. Then, under the putative equilibrium, such a constituent must weakly prefer to reveal truthfully than to report $m_i=1$. This choice requires that

$$\begin{aligned}&\sum_{k=0}^{c-1}\Pr\left(\sum_{j\neq i}s_j=k|s_i=0\right)\int_0^1\left(U\left(y\left(k\right),\theta,b\right)\right)g\left(\theta|\left\langle n,k\right\rangle\right)d\theta\\ \geq\;&\sum_{k=0}^{c-1}\Pr\left(\sum_{j\neq i}s_j=k|s_i=0\right)\int_0^1\left(U\left(y\left(k+1\right),\theta,b\right)\right)g\left(\theta|\left\langle n,k\right\rangle\right)d\theta.\end{aligned}$$

In the quadratic loss specification, this inequality reduces to the condition that

$$\sum_{k=0}^{c-1}\Pr\left(\sum_{j\neq i}s_j=k|s_i=0\right)\left(\frac{1}{c+\alpha+\beta}-2b\right)\left(\frac{1}{c+\alpha+\beta}\right)\geq 0,$$

which further reduces to

$$\frac{1}{c+\alpha+\beta}\geq 2b.$$

However, since $c>\bar{n}$, it follows that

$$\frac{1}{c+\alpha+\beta}\leq\frac{1}{\left(\bar{n}+1\right)+\alpha+\beta},$$

and, by the definition of $\bar{n}$,

$$\frac{1}{(\bar{n}+1)+\alpha+\beta} < 2b.$$

Therefore,

$$\frac{1}{c+\alpha+\beta} < 2b,$$

which is a contradiction.

An analogous contradiction is obtained for the case where constituent i has signal $s_i = 1$ and ideology $b_i = -b$. ■

Proof of Proposition 9:

We first establish that ϕ is strictly increasing when $k > \frac{n}{2}$ and strictly decreasing when $k < \frac{n}{2}$; the case where $k = \frac{n}{2}$ is impossible since n is odd. Fix two values, k and k', and evaluate $\phi(\cdot)$ at these points. Notice that

$$\phi(k) - \phi(k') = 2\left((n-r)^2 + 3r^2\right)(k'-k)(n-k-k').$$

When $k < k' < \frac{n}{2}$, then $\phi(k) - \phi(k') < 0$. When $\frac{n}{2} < k < k'$, then $\phi(k) - \phi(k') > 0$.

Next, notice that, when $k = r$ or $n-r$, then

$$\phi(k) = r(2r-n)^3 > 0.$$

Hence, $n - r < k_0 < k_1 < r$.

Thus, we have shown that for $k < k_0$ and $k > k_1$, $\phi(k) > 0$ while for $k \in [k_0, k_1]$, $\phi(k) \leq 0$. ■

Proof of Proposition 11:

Following Feddersen and Pesendorfer (1997). Define

$$\begin{aligned} v(\theta, b) &= -(y_1 - (\theta + b))^2 + (y_0 - (\theta + b))^2 \\ &= (y_1 - y_0)(2b + 2\theta - y_0 - y_1) \end{aligned}$$

and notice that this is increasing linearly in θ and b.

Next, observe that, for all θ,

$$v(\theta, -1) < 0$$

while

$$v(\theta, 1) > 0$$

This implies that there is a set of types $[-1, -1+\varepsilon]$ from whom it is a dominant strategy to report $m = 0$ and similarly a set of types $[1-\varepsilon, 1]$ for whom it is a dominant strategy to always report $m = 1$.

Next, notice that the incentive compatibility condition is a type that solves

$$E\left[v\left(\theta,b\right)|\left\langle n-1,k\right\rangle,s=0,\mathbf{b}\right]=0$$

where $\mathbf{b}$ denotes a putative equilibrium strategy, (b_l, b_r), played by all the other constituents. Since v is increasing in b,then, together with the conditions on $v(\theta,-1)$ and $v(\theta,1)$ implies that there exists a single value b_r such that the above expression is equal to zero.

Similarly, there exists a unique value b_l such that

$$E\left[v\left(\theta,b\right)|\left\langle n-1,k\right\rangle,s=1,\mathbf{b}\right]=0$$

Finally, since the posterior distribution $G\left(\theta|\left\langle n-1,k\right\rangle,s,\mathbf{b}\right)$ is stochastically ordered in s,then $b_l < b_r$.

Hence, for any putative equilibrium strategy $\mathbf{b}$, there are a unique pair of cutpoints that result. Next, define the function that yields the unique set of cutpoints associated with the putative equilibrium strategy $\mathbf{b}$.Call this function ψ and notice that $\psi : \mathbf{b} \to \mathbf{b}$. Furthermore, it is apparent that ψ is continuous in the vector $\mathbf{b}$ (from the Theorem of the Maximum). Hence, by Kakutani's fixed point theorem, this mapping has a fixed point which comprises a non-degenerate centrist-extremist equilibrium. ■

Proof of Proposition 12

Let k^n_{piv} denote the k rule used for a poll of size n. It follows from the optimality of the policy maker's choice that, under truth-telling,

$$k^n_{piv}=\left\lfloor\left(\frac{y_1+y_0}{2}\right)(n+\alpha+\beta)-\alpha\right\rfloor$$

We claim that

$$\lim_{n\to\infty}\frac{k^n_{piv}+a}{n+\alpha+\beta}=\frac{y_0+y_1}{2}$$

To see this, notice that, under the definition of k^n_{piv},

$$\frac{k^n_{piv}+a}{n+\alpha+\beta}\leq\frac{y_0+y_1}{2}<\frac{k^n_{piv}+1+a}{n+\alpha+\beta}$$

and notice that

$$\lim_{n\to\infty}\frac{k^n_{piv}+a}{n+\alpha+\beta}=\lim_{n\to\infty}\frac{k^n_{piv}+1+a}{n+\alpha+\beta}$$

hence

$$\lim_{n\to\infty}\frac{k^n_{piv}+a}{n+\alpha+\beta}=\frac{y_0+y_1}{2}$$

In that case, for all $b_i > 0$ observe that

$$\lim_{n\to\infty}\left(y_0+y_1-2\left(\frac{k^n_{piv}+a}{n+\alpha+\beta}\right)-2b_i\right)<0$$

Finally, since, for all n

$$\frac{k_{piv}^n + 1 + a}{n + \alpha + \beta} - \frac{k_{piv}^n + a}{n + \alpha + \beta} = \frac{1}{n + \alpha + \beta}$$

which is decreasing in n. Hence, for n sufficiently large truth-telling is not an equilibrium. ■

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