Entrepreneurship and Loss-Aversion in a Winner-Take-All Society

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Abstract

We study entrepreneurship in a setting where identical, loss-averse individuals choose between a risky entrepreneurial path and a safe outside option. The combination of effort and luck determine the single winner of the entrepreneurship market. We obtain a closed form solution to equilibrium entry and effort decisions. The novel implications of the model are: (1) Entrepreneurial effort increases with the return to the outside option, all else equal. (2) In high stakes settings with sufficient loss aversion, a rise in employment wages intensifies competition despite a fall in the number of competitors.

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1 Introduction

Charlie Vergos was an unlikely star. The son of Greek immigrants, he quit high school to serve his parents' adoptive country as a soldier in World War II. When he was discharged, rather than taking advantage of the benefits of the GI Bill to complete his education and join a profession, Charlie set up shop running a bar and restaurant in an alley in Memphis, Tennessee. Initially, the menu was simple: beer and ham and cheese sandwiches. But soon thereafter, Charlie discovered a coal chute on the property and outfitted it so as to be able to grill meats. Finding beef too expensive, a friend suggested that he try grilling pork ribs, as this was a much cheaper cut. Charlie hit upon the idea of adding a Greek mix of spices and paprika, and the Memphis dry rub barbecue was born.

Success did not come instantly. At the start, Charlie sold a box of ribs a week at his restaurant. Today, Rendezvous Ribs serves a ton of ribs in a single day. Lauded by fellow barbecue restaurateur Tony Neely as "the Michael Jordan of ribs," Charlie's ribs were so loved by fellow Memphisite Elvis Presley that he would have them flown first class to wherever he happened to be singing.

A combination of effort, drive, and sheer luck catapulted Charlie Vergos from anonymity to the prestige of being hailed one of America's 50 most influential restaurateurs.[1] Yet he never forgot his humble start. When asked about the path to entrepreneurial success, Charlie offered simple, homespun advice: "Go to work every morning and work hard."[2]

Many studies of entrepreneurship focus on what makes entrepreneurs different from the rest of us–the creativity of Steve Jobs, the vision of Mark Zuckerberg, or the business acumen of Bill Gates. Yet the vast majority of entrepreneurs—individuals opening their own restaurants, beauty salons, online stores, and so on—are completely ordinary, like Charlie Vergos. What motivates these individuals to sacrifice the certainty of employment or the benefits of the GI Bill to undertake the risk of entrepreneurship? What determines the effort these individuals put forth in hopes of attaining success?

In this paper, we investigate how an individual's outside option—the prospect of returns from employment—impact both the decision to become an entrepreneur and the degree of effort un-

[1] The information on the life of Charlie Vergos given here can be found in the Congressional Record, 111th Congress (2009-2010) article 72 of 74 (http://thomas.loc.gov/cgi-bin/query/D?r111:72:./temp/~r111ys0lK4::), on http://paulsvalleydailydemocrat.com/featuresbbqmyway/x2108374578/Elvis-favorite-Rendezvous-Ribs and on http://www.commercialappeal.com/news/2010/mar/27/rendezvous-founder-charlie-vergos-dies/?partner=RSS, all accessed 11 April 2012.

[2] http://www.societyofentrepreneurs.com/bio.asp?ID=64 accessed 11 April 2012.

dertaken to make the venture successful. We model the entrepreneurial market as one in which a combination of hard work and luck interact to produce a single winner from the set of competitors. Individuals in the model are identical—compared to others, our entrepreneurs are neither risk-seeking nor overly optimistic. They are, however, susceptible to one critical foible: they are loss averse. We analyze how loss aversion affects entrepreneurship and effort decisions. As we describe below, adding loss aversion leads to a number of novel findings, many of which are helpful in explaining aspects of empirical and laboratory studies of entrepreneurship and winner-take-all settings.

For instance, it might seem as if there is no relation between the returns to the employment option and the degree of entrepreneurial effort. After all, these returns are sunk at the time effort decisions are undertaken. When individuals are loss averse, however, the returns from employment continue to have a lasting influence through the reference point, the benchmark by which gains and losses are measured. This provides a novel link between the (sunk) wages from employment and entrepreneurial effort.

Our main findings are as follows: First, we show that the loss averse model is quite tractable—we identify a unique symmetric equilibrium in closed form even when the reference point is endogenous. Our model shares the sensible properties of classical models—individuals work harder when the prize is larger and fewer individuals choose entrepreneurship when the outside option is better.

Our model also offers novel implications concerning entrepreneurial decisions. (1) The higher are the returns to employment, the greater the effort undertaken by entrepreneurs, all else equal. The idea is that "the road not taken" by entrepreneurs influences their reference point, which in turn motivates effort. Indeed, sufficiently lofty aspirations can lead loss averse entrepreneurs to overinvest relative to risk neutrality. (2) In high stakes settings where individuals are sufficiently loss averse, higher employment wages intensify entrepreneurial competition despite reducing the number of competitors. This never occurs under risk neutrality or risk aversion—it is unique to loss averse settings.

Why Study Entrepreneurship?

After many years of debate, economists mainly agree that entrepreneurship is a major source of long-run growth in developed countries (e.g. Carree and Thurik, 2010). Yet, the forces that spur entrepreneurship are less well understood. One strand of the entrepreneurship literature emphasizes the role of (over)optimism. Several studies find that optimistic individuals are more likely to enter into entrepreneurship and also provide more effort after having entered (Arabsheibani et. al., 2000; Cooper et al, 1988; Wright et al, 1997; and Manove, 2000).[3] Another strand emphasizes the risk propensity of entrepreneurs though evidence in favor of risk-seeking attitudes is mixed (see, e.g., Caliendo, Fossen and Kritikos, 2009 and Wu and Knott, 2006; for two opposing viewpoints). Our loss averse model combines risk preferences and expectations in a single parsimonious framework: Future expectations affect entry and effort through the reference point while the weight entrepreneurs place on losses determines their sensitivity to risk.

The stakes of the entrepreneurial market are also an important determinant for the expected earnings of entrepreneurs. In small stakes settings entrepreneurship is typically not very lucrative (e.g. Hamilton, 2000). In large stakes settings, entrepreneurs enjoy excess returns even after controlling for risk (e.g. Cochrane, 2005). Our model can rationalize both findings—we show that the difference in pecuniary returns from entrepreneurship compared to the outside option is increasing in the stakes of the entrepreneurship gamble.

As with other models of loss aversion, our model shares the feature that behavior depends crucially on the reference point. To provide more precise predictions, we endogenize the reference point using rational expectations. In our setting, the returns to the employment path offer a natural benchmark for entrepreneurs to build expectations and hence reference points. This creates a linkage between the inside and outside option that is unique to our framework.

While, to our knowledge, we are the first to study loss aversion in entrepreneurial settings, technically, our paper is closely related to Cornes and Hartley (2003), though their concerns are very different. They examine a model of rent-seeking where individuals are loss averse and compete in a lottery contest. Our entrepreneurship game is mathematically isomorphic to a lottery contest; however, unlike Cornes and Hartley, we endogenize both entry and the reference point.

Our paper adds to the vast and growing literature on prospect theory. While the general theory since Kahneman and Tversky (1979) is well-developed, the use of prospect theory in applied settings has lagged somewhat, presumably for reasons of tractability and precision of predictions. An important contribution of our paper is to show how such models do not lead to excessive technical complexity nor to imprecise predictions. Indeed, by endogenizing the reference point, our model is as parsimonious as a standard risk averse model.

The remainder of the paper proceeds as follows: In section 2 we sketch the model. Section 3 examines equilibrium and comparative static implications. Our main findings are contained in Propositions 2, 6, and 8. Finally, section 4 concludes. All proofs are contained in the Appendix.

[3] See Parker (2006) for a survey of entrepreneur optimism.

2 The Model

Consider a situation where $N \geq 2$ identical individuals are making a career choice. Each individual begins with initial wealth W. By choosing employment, an individual enjoys an additional fixed wage w. We assume that the set of individuals is small relative to the size of the (unmodeled) labor market, so each person takes the wage as given, and the market is large enough to accommodate all N individuals should they choose this path.

Alternatively, an individual may choose entrepreneurship. Obviously, there are many differences between entrepreneurship and employment. Entrepreneurs do not have to answer to a boss, nor work on a set schedule of hours. An important difference concerns the connection between effort and outcomes. Whereas an employee who works or shirks will have some small effect on the success or failure of the firm for which she is working, for entrepreneurs, the connection between effort and outcomes is more direct. Of course, effort alone is no guarantee of success. Entrepreneurs also face strategic risks from their competitors. For instance, a restaurateur opening shop in a busy downtown area will likely not be alone. Success or failure also depends on the efforts of rival restaurants. Finally, there is a considerable element of luck determining success in entrepreneurship. While one restaurateur may be fortunate enough to run the "hot" restaurant in town, another, who gave equal effort, may find her tables empty on a Saturday night purely as a result of the fashions of the times.

To capture the effects of effort, strategic risk, and luck in determining outcomes, we model the entrepreneurship path as a winner take all market that awards a "prize" equal to $R > w$ to the winning individual. The prize, R, should be understood as the long-run value of becoming a dominant player in the particular market. That is, it represents the net present value of future cash flows that result from success early on. Individuals choosing the entrepreneurship path simultaneously choose costly effort e. Effort, together with a noise term ε, which is an iid draw from an extreme value (Weibull) distribution F, determine an individual's performance; that is, the performance of individual i is $p_i = e_i \times \varepsilon_i$ where e_i is that individual's effort and ε_i represents their realization of the noise term. The highest performing individual receives the prize while all the other entrants receive nothing. This implies that, when n individuals choose to become entrepreneurs, the probability of success for individual i undertaking effort e_i is simply $e_i / \sum_{j=1}^{n} e_j$ where, without loss of generality, we have assumed that the n individuals with the lowest indices choose to become entrepreneurs.[4] This lottery success function appears in many applied settings, including much of the literature on rent-seeking (see, e.g., Nitzan, 1994 for a survey). We use it here since it tractably captures the combination of effort, strategic risk, and luck on outcomes. Costs are assumed to be linear in effort, and there is no upper bound on the amount of effort an individual might choose to undertake.

We assume that there is an exogenously specified order in which individuals make career choices and that earlier choices are observable. Without loss of generality, assume that individuals get to choose their career paths in order of their indices; thus, individual $i = 1$ chooses first, followed by $i = 2$, and so on.[5] We will show below, that this results in the first n individuals becoming entrepreneurs with the remainder choosing employment. Entry will occur up to the point where the gain/loss utility from entrepreneurship just equals that of the outside option.[6]

The model divides the entry decision, which takes place earlier, from the effort decision, which takes place after all entry has been determined. The rationale for this modeling assumption is twofold. First, we think of the effort decision as unfolding over a period of time long after entry decisions have been made. For instance, once an individual decides to open a restaurant, it will be months before this decision translates into a grand opening. It will be even longer yet before the restaurant will have gained enough public attention to possibly become "hot." Meanwhile, entrepreneurial effort is occurring over all this time. Second, linking the effort and entry decisions leads to issues of commitment and preemptive effort in an attempt to erect barriers to future entrepreneurial entry. While these are interesting issues, they are not our main focus. Instead, we are interested in how the preferences of individuals, specifically loss aversion, influence the decision to become an entrepreneur and the subsequent effort undertaken.

Obviously, entrepreneurship has many aspects besides the ones modeled here. Moreover, depending on the field of endeavor, a winner-take-all model may be more or less appropriate. For instance, it seems a reasonable approximation for a model of restaurants, where a single hot restaurant often emerges. The "winning" restaurant may not have the best chef, the nicest ambiance or

[4] See Fu and Lu (2011) for a formal proof that a tournament where performance is determined by the product of investment and a random draw from an extreme value (Weibull) distribution is equivalent to a lottery contest.

[5] Our model prevents individuals from postponing career choices to a later time; however, this restriction is of no consequence. If instead, we permitted individuals to select their careers later, it is easy to show that it is a weakly dominant strategy to choose at the earliest possible moment.

[6] The modeling assumption is inconsequential in arriving at this conclusion. For instance, if we instead assumed that entry is simultaneous, then having the n entrants with the lowest indices enter up to the point where inside and outside payoffs are equalized remains an (asymmetric) equilibrium.

other products of entrepreneurial effort; it may simply be fortunate enough to capture the zeitgeist of the moment. Likewise, the model seems reasonable for startups in network markets, such as the internet. In these markets, there is a tendency for a single firm to emerge as the dominant "platform" from a sea of competitors. As with restaurants, the winning platform is not always the best platform. The model is less appropriate for other markets, such as realty or corner stores, that are clearly not winner-take-all.

So far, this is merely a standard model of entry into a winner-take-all market. The key novelty in our model is to make our individuals loss averse. While there are many specifications that loss aversion might take, we choose a simple linear specification with a kink at some reference point r. Given this reference point and an ending wealth state ω, the payoffs to each individual are

$$U(\omega) = \begin{cases} \beta(\omega - r) & \text{if } \quad \omega > r \\ \alpha(\omega - r) & \text{if } \quad \omega \leq r \end{cases}$$

where $0 < \beta \leq 1 \leq \alpha$. The parameter α represents the weighting on losses while β represents the weighting on gains relative to the reference point. As in Kahneman and Tversky (1979), we assume that losses are more painful than gains are pleasurable; hence the ordering on α and β. Notice that, when $\alpha = \beta$, we obtain a standard entry model with risk neutral individuals.

Owing to the variety of entrepreneurship situations we have in mind, we do not model the specifics of any particular situation, rather our specification tries to capture, in a reduced form way, how luck, effort and competition are the key factors leading to success or failure. Likewise, the reward structure is intended to capture, in a simple way, the idea that the distribution of payoffs is highly skewed in favor of a few successful entrants.

3 Equilibrium

In this section, we characterize pure strategy equilibria in the game. Since all individuals are identical in the model, it follows that, post entry, there is nothing to distinguish competing entrepreneurs. Therefore, we restrict attention to the unique symmetric equilibrium arising in each subgame as a function of n, the number of entrants. We then work backwards to solve for the equilibrium number of entrants, n^*. As usual, equilibrium entry will occur up to the point where an additional entrant expects a lower utility from the entrepreneurship path than from employment. Since entry provides weakly higher gain/loss utility than non-entry, it follows that the first n^* individuals will opt for entrepreneurship with the remaining individuals pursuing employment.

A critical difference between characterizing equilibrium under loss aversion compared with standard models is the role of the reference point in individual decisions and hence the equilibrium of the game. First generation loss aversion models treat the reference point as exogenous. More recently, a number of approaches have been suggested for endogenizing the reference point.[7]

In our setting, we use a rational expectations approach to endogenize the reference point. Rational expectations imply that an individual's gain/loss utility will be equal to zero under employment, as she will correctly anticipate the outcome from this path. Free entry then implies that gain/loss utility from entrepreneurship must likewise equal zero. We discuss this approach in more detail in Section 3.3.

An important comparison, which we focus on throughout the paper, is the difference between the average monetary payoffs of employees versus entrepreneurs. This statistic has the advantage that is it measurable (and has been measured) empirically, thus permitting us to develop implications that can be tested to support or falsify the loss averse model of career choice.

3.1 Post-Entry Decisions

Assume that the first $n > 1$ individuals choose the entrepreneurship path, and, for the moment, treat the reference point as exogenous. Note, however, that in the full game both n and r will be determined endogenously. The location of the reference point plays a critical role in the analysis. The interesting case occurs when the following assumption is satisfied:

Assumption 1: $W < r < W + \frac{\beta n+\alpha(n-1)^2}{\beta n+\alpha n(n-1)}R$.

Assumption 1 says that an individual pursuing entrepreneurship aspires to a return exceeding her initial wealth level but less than a scaled value of the prize. This merely ensures that a winning individual enjoys pecuniary payoffs that exceed the reference point while a losing individual lands below the reference point. As we discussed above, when we endogenize the reference point, the link between the aspiration level and the employment option ensures that something like Assumption 1 must hold.

If Assumption 1 fails to hold, i.e. if the winning and losing outcomes are both above or both below the reference point, there is no kink in the utility function, and the analysis is identical to the risk neutral case.

[7] The seminal paper by Koszegi and Rabin (2006) is one such approach.

The gain/loss utility to entrepreneur i who exerts effort e_i is then

$$EU_i = \frac{e_i}{\sum_{j=1}^{n} e_j} \beta \left(W + R - r - e_i\right) + \left(1 - \frac{e_i}{\sum_{j=1}^{n} e_j}\right) \alpha \left(W - r - e_i\right) \tag{1}$$

Differentiating with respect to e_i and symmetrizing (i.e., setting $e_i = e_j = e^*$ for all i, j) yields the equilibrium condition on effort:

$$(n-1)\left(\beta\left(W + R - e^* - r\right)\right) - \beta n e^* - (n-1)\alpha n e^* + (n-1)\alpha\left(e^* + r - W\right) = 0 \tag{2}$$

which may be rewritten in closed form as

$$e^* = \frac{n-1}{(2n-1) + \frac{\alpha}{\beta}(n-1)^2} \times \left(R + (\frac{\alpha}{\beta} - 1)(r - W)\right). \tag{3}$$

Of course, this derivation is merely heuristic. Our next proposition formally establishes that a unique symmetric equilibrium exists. This follows from the fact that equation (2) is linear in e^*. Checking endpoint conditions and applying the intermediate value theorem yields existence, while linearity implies uniqueness, regardless of the reference point r or the number of entrants n. Formally,

Proposition 1 *Suppose that Assumption 1 holds. Then, for a given r and n, equilibrium effort in the unique symmetric equilibrium is given by equation* (3).

Having established that there is a unique symmetric equilibrium, we can perform comparative analyses to see how effort responds to the parameters of the model. From equation (3), it may be readily observed that effort fall when there are more competitors and rises as the stakes (R) increase. Of course, both of these statics are also true of expected utility models.

A key difference in a loss averse setting is the effect of changes in the reference point. The more optimistic are entrepreneurs about the returns from entrepreneurship (i.e. the higher is the reference point), the greater their effort. Intuitively, there is more to lose for an entrepreneur with higher aspirations. Since losing is costly relative to winning, the entrepreneur will strive hard to avoid this outcome by exerting more effort.

Comparison to Risk Neutrality and Risk Aversion

Risk neutrality offers a natural benchmark. This case is nested in the model by $\alpha = \beta$, and equilibrium effort reduces to

$$e^{RN} = \frac{n-1}{n^2} R$$

where the superscript RN denotes the fact that the analysis occurs under risk neutrality. How does this compare to effort under loss aversion? Under both sets of preferences, effort is a linear function of the value of the prize; however, in the loss averse setting, this effort is reduced owing to the fact that gains are less pleasurable than losses are painful. To see this, notice that the denominator under risk neutrality may be rewritten as $(2n-1)+(n-1)^2$ and

$$(2n-1)+(n-1)^2 < (2n-1)+\frac{\alpha}{\beta}(n-1)^2$$

when $\alpha > \beta$, as is the case under loss aversion. Of course, the right-hand side of the above equation is simply the denominator of equation (3), the equilibrium effort under loss aversion. Notice that the more pronounced the "kink" in the gain/loss function (i.e. the larger is the α/β ratio), the greater the loss of motivation from winning. Were this the only effect, we could conclude that loss aversion unambiguously lowers effort compared to the risk neutral benchmark. This, however, ignores the aspirational motivation of the reference arising under loss aversion. In principle, this effect can dominate and lead to greater effort compared to risk neutrality. Computing $e^* - e^{RN}$ and simplifying reveals:

Proposition 2 *Fix the number of competitors n, the reference point r, and suppose that Assumption 1 holds. Compared to risk neutrality, equilibrium effort is higher under loss aversion if and only if $r > W + R\left(\frac{n-1}{n}\right)^2$.*

The condition offered in the Proposition is intuitive. Effort is higher under loss aversion when the aspiration level is higher, competitors are poorer or fewer in number, or the prize is smaller.

While the reference point effect is unique to the loss averse model, the form of preferences we specify can be thought of as a linearization of risk averse preferences with a single kink point. Thus, one might suspect that similar tradeoffs will arise in a risk averse model as well. Konrad and Schlesinger (1997) highlight that the same ambiguity in the comparison of effort relative to the risk neutral benchmark also arises under risk aversion. Again, this is the product of two competing effects. An increase in effort reduces the upside from winning the market; however since the marginal utility of wealth is decreasing, the trade-off between additional wealth and additional probability of winning favors the latter under risk aversion. They refer to this as the contraction effect. This same effect is responsible for increased bidding in a first-price auction under risk aversion. Unlike a first price auction, where only the winning bidder pays, increased effort in an entrepreneurship setting also has the effect of making the downside from losing more pronounced. Since the marginal utility of wealth is higher in this region, this countervailing force favors reduction in effort. Konrad and Schlesinger refer to this countervailing force as the spread effect. They offer the following condition for when the contraction effect dominates the spread effect:

$$EU' < \frac{u_{win} - u_{lose}}{R}$$

Here, the left-hand side represents a weighted average of the marginal utility of wealth at the winning and losing payoffs evaluated at the risk neutral efforts. The right-hand side is the difference between the utility levels from winning and losing, again evaluated at risk neutral efforts and normalized by the size of the prize. Simply inspecting the condition, it is difficult to tell what types of risk averse preferences and what prize levels satisfy it. In our setting, however, the expression becomes

$$\frac{1}{n}\beta + \frac{n-1}{n}\alpha < \frac{\beta\left(R + W - r - e^{*}\right) - \alpha\left(W - r - e^{*}\right)}{R}$$

which, after simplification, reduces to the condition given in Proposition 2. That is,

Remark 1 *Fix n and r and suppose that Assumption 1 holds. Furthermore, suppose that individuals are risk averse with utility function:*

$$u = \begin{cases} \alpha W & \text{if } W < r \\ \alpha r + \beta\left(W - r\right) & \text{if } W \geq r \end{cases}$$

then equilibrium effort under risk aversion exceeds that under risk neutrality if and only if $r > W + R\left(\frac{n-1}{n}\right)^2$.

Remark 1 highlights the equivalence between conditions for excess effort (compared to risk neutrality) under loss aversion and risk aversion. This implies that, simply by looking at effort data, the two models are observationally equivalent. Perhaps more importantly, one need not appeal to considerations like overoptimism or risk-loving behavior to justify the extremes of effort that entrepreneurs sometimes undertake. Even when entrepreneurs are loss or risk averse, equilibrium can still produce effort in excess of the risk neutral benchmark. It is interesting to note that, while standard forms of loss averse preferences, such as the one we use here, can produce excess effort, standard forms of risk averse preferences, such as CRRA, cannot. One contribution of Remark 1 is to provide a simple form of risk averse preferences where excess effort can arise.

Intensity of Loss Aversion

From Proposition 2, we saw that the conditions needed for loss aversion to produce greater effort than risk neutrality were independent of the intensity of loss aversion. This is not to say that the intensity of loss aversion has no effect on equilibrium effort. Indeed, casual inspection of equation (3) reveals that it does. Here, we study how changes in the intensity of loss aversion affect equilibrium effort. Our main finding is that the condition needed for greater intensity of loss aversion to lead to greater effort is *identical* to that given in Proposition 2. Formally,

Proposition 3 *Given $n > 1$ entrepreneurs and Assumption 1, an increase in the intensity of loss aversion increases effort if and only if $r > W + R\left(\frac{n-1}{n}\right)^2$.*

It is interesting to compare this with the findings of Kong (2008) who analyzes the behavior of loss averse contestants in an experiment. Kong finds that contest expenditures decrease in the degree of loss aversion. This would correspond to our finding where subjects' reference points are not very high. Our theory predicts that inducing higher expectations should lead to a reversal of this finding—more loss averse subjects will compete more fiercely in the contest.

3.2 Entry

Next, we turn to entry. We assume that the entrepreneurship market is sufficiently large that we can ignore integer constraints. In that case, entry occurs up to the point where the gain/loss utility from entrepreneurship equals the gain/loss utility from employment.[8] From Proposition 1, it is a simple matter to show that the gain/loss utility from entrepreneurship, for a given n and r, is

$$EU(n, r) = \frac{\beta^2 R + (W - r)\left((n-1)\alpha + \beta\right)^2}{\beta(2n-1) + \alpha(n-1)^2} \tag{4}$$

As one would expect, competition reduces equilibrium utility from entrepreneurship: equation (4) may easily be seen to be decreasing in n. Of greater interest is how changes in the reference point affect equilibrium utility from entrepreneurship. As we saw above, a more ambitious reference point leads to more effort by entrepreneurs. Collectively, this additional effort *reduces* equilibrium

[8] Accounting for integer constraints requires that we choose the integer floor of the value of n solving for this equality and, as a result, makes the analysis more cumbersome without adding economic intuition. In circumstances where there are a large number of entrepreneurs, as in the examples we offered to motivate the model, there is little difference between the integer floor and the value of n solving for the exact equality; hence, we exclude this from consideration.

utility. Thus, the more optimistic are entrepreneurs, i.e., the higher is r, the lower their gain/loss utility in equilibrium.

Of course the number of entrepreneurs is not exogenous, it is determined by an entry condition comparing the expected gain/loss utility from entrepreneurship to that from employment. Formally, for a given r, and treating n as continuous, at an interior solution, entry will occur up to the point where gain/loss utility is equalized across the two career paths. That is, n^* solves

$$EU\left(n^*, r\right) = \frac{\beta^2 R + (W - r)\left(\left(n^* - 1\right)\alpha + \beta\right)^2}{\beta\left(2n^* - 1\right) + \alpha\left(n^* - 1\right)^2} = \theta\left(W + w - r\right) = EU_0 \tag{5}$$

where EU_0 denotes the expected gain/loss utility from the employment and $\theta \in \{\alpha, \beta\}$ represents the relevant gain or loss parameter depending on whether the payoffs from employment lie above or below the reference point. The interesting case is where there is *some* competition in the entrepreneurship market, i.e., $n^* \geq 2$. This requires a high enough payoff from winning the market and a high enough reference point that the payoffs from employment are not coded as gains. A sufficient condition is

Assumption 1a: $W + w \leq r < W + \frac{1}{2}\left(\frac{\alpha+2\beta}{\alpha+\beta}\right) R$ and $R \geq w\left(\frac{\alpha}{\beta} + 1\right)^2$.

We are now in a position to state the main result for this section:

Proposition 4 *Suppose that Assumption 1a holds, then there is a unique $n^* \geq 2$ consistent with a symmetric effort equilibrium, where*

$$n^* = \min\left(N, 1 - \frac{\beta}{\alpha} + \frac{\sqrt{\beta}}{\alpha}\sqrt{\frac{\left(R\beta + (\alpha - \beta)(r - W - w)\right)}{w}}\right) \tag{6}$$

Furthermore, the set of parameter values in which Assumption 1a holds is non-empty.

Inspecting equation (6) reveals that, at an interior solution, entry is increasing in the reward for being the winning entrepreneur and decreasing in the payoffs from employment. These implications are intuitive and identical to expected utility models.

The key difference between these models and our loss averse model arises through the reference point. Equation (6) shows that higher reference points are associated with *more* entry.

This implication is surprising in light of our earlier finding that, for a fixed n, higher reference points led to lower gain/loss utility from entrepreneurship. To rationalize these findings, one needs to account for the impact of the reference point on employment as well. An increase in the reference point decreases the gain/loss utility from employment by more than that from entrepreneurship.

A marginal increase in the reference point reduces employment utility by α whereas it reduces entrepreneurship utility by

$$-\frac{\partial EU\left(n,r\right)}{\partial r}=\frac{\left(\left(n^{*}-1\right)\alpha+\beta\right)^{2}}{\beta\left(2n^{*}-1\right)+\alpha\left(n^{*}-1\right)^{2}}$$

One may verify that this expression is smaller than α. Thus, an increase in the reference point makes entrepreneurship relatively more attractive even though the level of gain/loss utility is now lower. The reduction in utility stems both from an increase in the number of entrepreneurs as well as from an increase in the effort undertaken by each entrepreneur.

3.3 Endogenizing the Reference Point

Up until now, we have treated the reference point as exogenous. We close the model by endogenizing the reference point. To do so, we assume that individuals form beliefs about the reference point using rational expectations. The idea is the following: an individual choosing a given career path will have a relatively clear idea about the possible outcomes she faces. Thus, she should not systematically experience mostly gains nor mostly losses. The reference point os chosen so that, on average, the expected utility from gains equals that from losses. Thus, individual expectations about gains and losses are, on average, correct.

In our model, the returns from employment are fixed; hence an individual with rational expectations adjusts her reference point such that her gain/loss utility is equal to zero when considering this option. Since she expects to earn $W+w$, the reference point should equal this amount. In an interior equilibrium under free entry, the gain/loss utility from entrepreneurship must equal that of employment. We assume that there is a sufficiently large pool of potential entrepreneurs, N, that we obtain interior solutions for n^{*}.[9] This implies that the endogenous reference point is

$$r^{*}=W+w \tag{7}$$

Having determined r^{*}, it is straightforward to determine equilibrium effort, e^{**}, and number of entrants n^{**}. Substituting r^{*} into equation (6) yields a unique solution to the equilibrium number of entrepreneurs (ignoring integer constraints), n^{**}. The two star notation indicates an equilibrium value with an endogenous reference point. Substituting the value of n^{**} into equation (3) then yields the equilibrium effort per entrepreneur. Formally, we have shown that:

[9] Allowing for corner solutions implies that the reference point is pinned down entirely by the zero gain-loss condition under entrepreneurship with N entrants. In this case the analysis with fixed N in Section 3.3 applies. Otherwise, the analysis is identical. Calculations are available upon request from the authors.

Proposition 5 *Suppose that Assumption 1a holds and that the reference point, r^*, is determined endogenously via rational expectations. Then there is a unique symmetric equilibrium of the model where*

$$n^{**} = 1 + \frac{\beta}{\alpha}\left(\sqrt{\frac{R}{w}} - 1\right)$$

individuals become entrepreneurs and choose effort

$$e^{**} = \sqrt{wR} - w$$

Proposition 5 illustrates that, even when we endogenize the reference point, the loss averse model is surprisingly simple to work with. The key choice variables in equilibrium, e^{**} and n^{**}, can be characterized purely as a function of the primitives of the model. The addition of the reference point does not lead to a model that "explains everything." Indeed, the only additional parameters of the loss averse model are α and β, which characterize the degree of loss aversion; these are the analog of the degree of risk aversion in an expected utility model. Moreover, certain ambiguities with respect to comparisons with the risk neutral case are resolved.

Compared to the risk neutral case, we have: (1) Fewer individuals choose entrepreneurship under loss aversion; and (2) For a fixed number of entrepreneurs, equilibrium effort is less under loss aversion. To see why (2) holds, notice that the equilibrium effort is identical for loss averse and risk neutral individuals. However, since there are fewer entrants under loss aversion and since effort is decreasing in the number of entrants, then, if we reduced the number of risk neutral entrepreneurs to be the same as in the loss averse level, risk neutral equilibrium effort would increase.

This resolves the ambiguity in Proposition 3, which showed that the comparison of equilibrium effort depended crucially on the magnitude of the reference point. Endogenizing the reference point reveals that the reference point never becomes sufficiently high to lie in the region where loss averse individuals exert more effort than risk neutral individuals—expectations (and hence the reference point) are disciplined by the free entry condition.

Next, we turn to comparative analyses to examine how the number of entrepreneurs and effort change with the parameters of the model. Increasing the returns to entrepreneurship, R, leads to more entry while increasing the wage from employment, w, leads to less. The more loss averse are potential entrants (i.e., the higher is α/β), the smaller the number of entrepreneurs attracted to the market.

Increasing the returns to entrepreneurship produces more effort. The change in effort when the employment wage increases, however, depends on the returns to entrepreneurship. Assumption 1a ensures that R is sufficiently high so that higher wages produce more effort. The mechanism by which this operates is more complex than under risk neutrality. There, an increase in w reduces the number of entrepreneurs and this slackening of competition increases the returns to effort. The same holds true under loss aversion; however, there is an additional mechanism as well. An increase in w also leads to an increase in the reference point, and this provides an additional motive for increasing effort.

If instead of examining individual effort, we studied overall effort, $E = n^{**}e^{**}$, then, when bidders are risk neutral or risk averse, an increase in the returns to employment, w, unambiguously decreases overall effort.[10] The argument is straightforward: An increase in w reduces the number of entrants and overall effort is an increasing function of the number of entrants. With loss aversion, the reference point effect can, under some conditions, dominate the "numbers effect" and produce the opposite result—the higher are the wages from employment, the higher is overall effort in the entrepreneurship market. Proposition 6 below offers the exact condition required for this to be the case.

Proposition 6 *Suppose that Assumption 1a holds and that the reference point, r^*, is determined endogenously via rational expectations. Then an increase in w raises overall effort if and only if*

$$\left(\frac{\alpha}{\beta} - 1\right)\left(2 - \sqrt{\frac{R}{w}}\right) + \sqrt{\frac{R}{w}} < 0 \tag{8}$$

The condition given in equation (8) is more likely to be satisfied the higher are the returns from entrepreneurship and, surprisingly, the higher the intensity of loss aversion. This last effect is the result of complementarity between the reference point and the degree of loss aversion. The more loss averse are individuals, the higher the marginal utility of effort with respect to a change in the reference point. The intuition is that, for individuals with greater intensity of loss aversion, the pain of losses is greater and therefore the willingness to work hard to avoid loss outcomes is higher; hence, these individuals respond more to an increase in the reference point than do individuals whose loss aversion is less intense.

[10] The result is well-known for the risk neutral case. For the risk averse case, Cornes and Hartley (forthcoming, Propositions 5.1 and 5.2) show that, under certain regularity conditions on risk averse preferences, overall effort also decreases.

3.4 Returns to Entrepreneurship

Having closed the model by endogenizing the reference point, we know that the marginal individual is indifferent between employment and entrepreneurship. However, this determination is based on gain/loss utility relative to the reference point. A central, and more empirically relevant, question turns on the monetary returns to entrepreneurship. In the risk neutral model, the monetary returns are the same under both career paths. As we show below, this is not true under loss aversion.

Proposition 7 *When individuals are loss averse, the monetary returns from entrepreneurship always exceed those of employment. Moreover, the higher are the stakes of winning the entrepreneurial market, the larger is the payoff gap.*

The result is intuitive: Entrepreneurs are exposed to the possibility of losses, which are more painful relative to the equivalent gains. Thus, if the monetary payoffs from entrepreneurship were the same or lower than the outside option, the entrepreneur's gain/loss utility would also be lower and exit would occur. This process will continue up to the point where the gains are sufficiently high to compensate for losses, and this, in turn, implies that monetary returns must be higher.

The result that the payoff gap is increasing in the stakes of the entrepreneurship market is broadly consistent with empirical findings. The returns relative to the outside option are small in low stakes markets like the restaurant industry (Business Week Online, 2003). In higher stakes settings, the returns to entrepreneurship vastly exceed those of employment (see, e.g. Cochrane, 2005). Moreover, these findings are similar to what one observes in laboratory settings. Morgan, Sefton, and Orzen (forthcoming) study contests with endogenous entry where they varied the prize of the contest. When the contest prize, R, is high, the returns from entering the contest (i.e. entrepreneurship) vastly exceed those of the outside option. This is not the case when the stakes are low.

3.5 Sunk Cost Fallacy

Proposition 7 illustrates that a risk premium is required to motivate loss averse individuals to become entrepreneurs. The same, of course, is true under risk aversion. Empirically, one could use variation in the risk premium to recover risk aversion parameters; however, this same data can be used to recover the degree of loss aversion $(\alpha - \beta)$. Thus, if one only examined this metric, the two models are observationally equivalent.

Where the two models differ is with respect to effort choice following a change in the employment wage. Expected utility models predict this variation is irrelevant to post-entry behavior since it represents a sunk cost. In our loss averse model, however, the reference point is determined by the employment wage. Clearly, the higher is w, the greater the sacrifice to become an entrepreneur and hence the higher their expectations about the reward from entrepreneurship.

We can formalize this as follows: Fix the number of entrepreneurs, n, and consider the effect of a marginal change in the employment wage. In expected utility models, this has zero effect, and one would be tempted to ascribe any non-zero change to a failure in rationality through sunk cost fallacy. In contrast, the change in effort under loss aversion is

$$\frac{\partial e^*}{\partial w} = \frac{\partial}{\partial w} (n-1) \frac{(\beta R + (\alpha - \beta)(r^* - W))}{\beta (2n-1) + \alpha (n-1)^2}$$

where r^* is the endogenous reference point. The derivative takes the sign of $(\alpha - \beta)$, the intensity of loss aversion. Since losses are more painful than gains are pleasurable, it then follows that $\frac{\partial e^*}{\partial w} > 0$. Thus, we have shown

Proposition 8 *In the loss averse model with an endogenous reference point and a fixed number of entrepreneurs, a change in the employment wage increases the effort of entrepreneurs.*

Proposition 8 illustrates that behavior that appears to stem from sunk cost fallacy is, in fact, consistent with optimizing equilibrium choice under loss aversion.

4 Conclusions

The decision to pursue entrepreneurship—founding an internet startup, pursuing an invention, or opening a restaurant—is a momentous choice. Fundamentally, this choice is about risk: the payoffs from entrepreneurship are potentially large, but also much more uncertain, compared to the safer employment path. In part, these risks are inherent to the entrepreneurship environment. But the dice of fate are, in effect, loaded depending on the level of competition. Success is more likely the fewer the number of competitors pursuing the same idea.

A large literature examines the risk preferences of entrepreneurs.[11] However, within the broader economics literature, an important alternative, advanced initially by Kahneman and Tversky, is to use prospect theory in place of expected utility theory as a means of analyzing risky choice.

[11] See, e.g. Kihlstrom and Laffont (1979) for an early theory of risk preference selection into entrepreneurship. Nguyen-Chyung (2011) studies the interaction of talent and risk attitudes. Parker (2009) offers a brief survey of empirical studies of risk preferences and entrepreneurship.

In this paper, we offer a model of entrepreneurship in winner-take-all markets when individuals are loss averse. Risks stem from both competition and luck. A common criticism of loss averse models is that the conclusions hinge critically on the choice of the reference point. In our model, we endogenize the reference point using a rational expectations approach.

This produces a simple model yielding novel implications concerning the connection between the wage from employment and entrepreneurial effort. While the employment wage represents an irrelevant sunk cost to entrepreneurs in standard models, in our setting, the outside option drives the reference point and hence continues to affect entrepreneurial choices. Specifically, a rise in the employment wage spurs greater effort on the part of entrepreneurs, all else equal. This effect can be so powerful that loss averse entrepreneurs work harder than their risk neutral counterparts.

Moreover, when the stakes are high and individuals are sufficiently loss averse, an increase in the employment wage fuels fierce competition: Overall effort increases despite the fact that fewer individuals choose to become entrepreneurs. This effect never occurs in expected utility models.

Comparing the financial returns from entrepreneurship and employment reveals that loss averse individuals must be compensated for choosing the risky entrepreneurial path. Furthermore, the higher the stakes, the larger are the excess financial returns to entrepreneurship. This is consistent with empirical findings which show that entrepreneurial ventures with small stakes, such as opening a restaurant, yield far less excess returns than high stakes ventures like internet startups.[12]

Given the primacy of expected utility models in the extant literature, we highlight the similarities and differences between loss aversion and risk aversion. First, "excess" effort undertaken by entrepreneurs is commonly seen as evidence of either over-optimism or risk loving preferences. In fact, neither explanation is necessary. Risk aversion can produce greater effort than risk neutrality under some conditions. The same is true of loss aversion. We derive an equivalence result for the conditions where excess effort occurs in the two settings. As we stressed above, risk aversion and loss aversion deliver differing results when one endogenizes both entry and the reference point. The main difference arises through the impact of the employment wage on the reference point. This channel is absent in a risk averse model.

Of course, our loss averse model is only a first step. Obviously, not all entrepreneurial settings are well-described by winner-take-all markets nor are all competitors likely to be identical. Indeed, an important question, which our model is incapable of answering, concerns selection—who becomes

[12]See Hamilton (2000) for evidence of low (or negative) returns to small business. In contrast, Cochrane (2005) provides evidence of high returns for high stakes technology startups.

an entrepreneur and who does not. Given the simplicity and tractability of the symmetric model, there is reason for optimism in incorporating these additional factors. This, however, remains for future research.

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5 Appendix

This appendix contains proofs of selected propositions offered in the text.

Proposition 1 *Suppose that Assumption 1 holds. Then, for a given r and n, equilibrium effort in the unique symmetric equilibrium is given by equation* (3).

Proof. Temporarily assume that winning the game is coded as a gain and losing is coded as a loss. (We will verify that this is the case later.) Differentiating equation (1) with respect to e_i yields the first-order condition:

$$\frac{\sum_{k\neq i} e_k}{\left(\sum_{j=1}^{n} e_j\right)^2}\left(\beta\left(W+R-e_i-r\right)+\alpha\left(e_i+r-W\right)\right)+\frac{e_i}{\sum_{j=1}^{n} e_j}\left(\alpha-\beta\right)-\alpha=0$$

It is routine to verify that equation (1) is strictly concave in e_i; hence the first-order condition is both necessary and sufficient.

Solving for a symmetric equilibrium, we have

$$\frac{(n-1)}{n^2 e}\left(\beta\left(W+R-e-r\right)\right)-\frac{\beta}{n}-\left(\frac{n-1}{n}\right)\alpha+\frac{(n-1)}{n^2 e}\alpha\left(e+r-W\right)=0$$

and, taking a common denominator, the condition reduces to

$$(n-1)\left(\beta\left(W+R-e-r\right)\right)-\beta ne-(n-1)\alpha ne+(n-1)\alpha\left(e+r-W\right)=0$$

which yields equation (3).

Finally, we need to verify that, under the equilibrium effort, winning the market constitutes a gain; that is $W+R-e>r$ is satisfied.

$$W+R-(n-1)\frac{\beta R+(r-W)(\alpha-\beta)}{\beta(2n-1)+\alpha(n-1)^2}>r$$

Cross-multiplying

$$\begin{aligned}(W+R-r)\left(\beta(2n-1)+\alpha(n-1)^2\right)-(n-1)\left(\beta R+(r-W)(\alpha-\beta)\right) &> 0\\ R\left(\beta n+\alpha(n-1)^2\right)-(r-W)\left(\beta n+\alpha n(n-1)\right) &> 0\end{aligned}$$

And this condition is satisfied when:

$$r-W<\frac{\beta n+\alpha(n-1)^2}{\beta n+\alpha n(n-1)}R$$

which is consistent with Assumption 1. ■

Proposition 3 *Given $n>1$ entrepreneurs and Assumption 1, an increase in the intensity of loss aversion increases effort if and only if $r>W+R\left(\frac{n-1}{n}\right)^2$.*

Proof. If we denote the intensity of loss aversion by $\rho \equiv \alpha/\beta$, then differentiating equation (3) with respect to ρ yields:

$$\frac{\partial e^*}{\partial \rho} = (n-1)\frac{n^2(r-W) - R(n-1)^2}{\left(2n-1+\rho(n-1)^2\right)^2}$$

The sign of this expression turns on

$$n^2(r-W) - R(n-1)^2$$

or

$$r > W + R\left(\frac{n-1}{n}\right)^2$$

We will show that, depending on the location of the reference point, effort may increase or decrease with ρ. It is straightforward to identify the region where effort decreases. Simply notice that $W + R\left(\frac{n-1}{n}\right)^2 > W$ and hence, for all $r \in [W, W + R\left(\frac{n-1}{n}\right)^2]$, equilibrium effort (weakly) falls with an increase in the intensity of loss aversion. ■

Proposition 4 *Suppose that Assumption 1a holds, then there is a unique* $n^* \geq 2$ *consistent with a symmetric equilibrium, where*

$$n^* = \min\left(N, 1 - \frac{\beta}{\alpha} + \frac{\sqrt{\beta}}{\alpha}\sqrt{\frac{(R\beta + (\alpha-\beta)(r-W-w))}{w}}\right)$$

Furthermore, the set of parameter values in which Assumption 1a holds is non-empty.

Proof. We first show that $n^* \geq 2$. It suffices to establish that:

$$EU(2,r) = \frac{\beta^2 R + (W-r)(\alpha+\beta)^2}{3\beta+\alpha} \geq \alpha(W+w-r) = U_0$$

Now, since the left-hand side is increasing in R, we establish the required inequality when $R = w\left(\frac{\alpha}{\beta}+1\right)^2$. Substituting

$$\begin{aligned} EU(2,r) &\geq \frac{w\beta^2\left(\frac{\alpha}{\beta}+1\right)^2 + (W-r)(\alpha+\beta)^2}{3\beta+\alpha} \\ &= (W+w-r)\frac{(\alpha+\beta)^2}{3\beta+\alpha} \end{aligned}$$

Now since $(W+w-r) \leq 0$, then the required inequality is satisfied iff

$$\frac{(\alpha+\beta)^2}{3\beta+\alpha} \leq \alpha$$

Cross-multiplying and expanding

$$\alpha^2 + 2\alpha\beta + \beta^2 \leq 3\alpha\beta + \alpha^2$$

Simplifying

$$\beta \leq \alpha$$

which always holds. Thus, we have shown that $n^* \geq 2$.

To establish uniqueness of n^*, we will show that $EU(n,r)$ is strictly decreasing in n. To see this, recall that

$$EU(n,r) = \frac{\beta^2 R + (W-r)((n-1)\alpha+\beta)^2}{\beta(2n-1)+\alpha(n-1)^2}$$

and differentiating this expression with respect to n, we obtain

$$\frac{\partial EU(n,r)}{\partial n} = -2\beta \frac{\beta+(n-1)\alpha}{\left(\beta(2n-1)+\alpha(n-1)^2\right)^2}(R\beta+(\alpha-\beta)(r-W)) < 0$$

where the inequality follows from the fact that $n > 2$ and $r > W$. Now, since $EU(n,r)$ is strictly decreasing in n while U_0 is constant in n, this implies that there is a unique number of entrants in equilibrium.

Next, we solve for n^* in closed form. Clearly, if $EU(N,r) \geq U_0$, then $n^* = N$. If $EU(N,r) < U_0$, then n^* solves $EU(n,r) = U_0$, which yields

$$n^* = 1 - \frac{\beta}{\alpha} + \frac{\sqrt{\beta}}{\alpha}\sqrt{\frac{(R\beta+(\alpha-\beta)(r-W-w))}{w}}$$

Thus, we may conclude that there is a unique $n^* \geq 2$ consistent with a symmetric equilibrium where

$$n^* = \min\left(N, 1 - \frac{\beta}{\alpha} + \frac{\sqrt{\beta}}{\alpha}\sqrt{\frac{(R\beta+(\alpha-\beta)(r-W-w))}{w}}\right)$$

Finally we show that the set of reference points satisfying Assumption 1a is non-empty. First, notice that the expression $W + \frac{1}{2}\left(\frac{\alpha+2\beta}{\alpha+\beta}\right)R$ is simply the same inequality as in Assumption 1 setting $n = 2$. Next, notice that the expression $\frac{\beta n+\alpha(n-1)^2}{\beta n+\alpha n(n-1)}$ in Assumption 1 is strictly increasing in n for all $n \geq 2$. Thus, setting $n = 2$ suffices to ensure that the required condition holds for all $n^* > 2$.

It remains to show that $r \in [W+w, W+\frac{1}{2}\left(\frac{\alpha+2\beta}{\alpha+\beta}\right)R)$ is non-empty. Since $\frac{1}{2}\left(\frac{\alpha+2\beta}{\alpha+\beta}\right)R$ is increasing in R, it suffices to show non-emptiness when $R = w\left(\frac{\alpha}{\beta}+1\right)^2$. Evaluating $W + \frac{1}{2}\left(\frac{\alpha+2\beta}{\alpha+\beta}\right)R$ at this value of R yields

$$W + w\frac{2\beta+\alpha}{2(\beta+\alpha)}\left(\frac{\alpha}{\beta}+1\right)^2$$

Thus, it suffices to show that

$$\frac{2\beta+\alpha}{2(\beta+\alpha)}\left(\frac{\alpha}{\beta}+1\right)^2 \geq 1$$

Simplifying the left-hand side reveals that the required condition is

$$\frac{1}{2\beta^2}\left(\alpha^2+3\alpha\beta+2\beta^2\right)\geq 1$$

which always holds since $0<\beta<\alpha$. ■

Proposition 6 *Suppose that Assumption 1a holds and that the reference point, r^*, is determined endogenously via rational expectations. Then an increase in w raises overall effort iff*

$$\left((\rho-1)\left(2-\sqrt{\frac{R}{w}}\right)+\sqrt{\frac{R}{w}}\right)<0$$

Proof. Recall that aggregate effort is simply $n^{**}e^{**}$, which is equal to

$$\begin{aligned}
E &= n^{**}e^{**}=\left(1+\frac{\beta}{\alpha}\left(\sqrt{\frac{R}{w}}-1\right)\right)\left(\sqrt{wR}-w\right) \\
&= \sqrt{wR}-w+\frac{\beta}{\alpha}\sqrt{\frac{R}{w}}\left(\sqrt{wR}-w\right)-\frac{\beta}{\alpha}\left(\sqrt{wR}-w\right) \\
&= \left(\sqrt{wR}-w\right)\left(1-\frac{\beta}{\alpha}\right)+\frac{\beta}{\alpha}R-\frac{\beta}{\alpha}\sqrt{wR} \\
&= \frac{R}{\rho}-w\left(1-\frac{1}{\rho}\right)+\sqrt{Rw}\left(1-\frac{2}{\rho}\right)
\end{aligned}$$

Taking the derivative with respect to w and simplifying we get

$$\frac{\partial E}{\partial w}=-\frac{1}{2\rho}\left((\rho-1)\left(2-\sqrt{\frac{R}{w}}\right)+\sqrt{\frac{R}{w}}\right)$$

■

Proposition 7 *When individuals are loss averse, the monetary returns from entrepreneurship always exceed those of employment. Moreover, the higher are the stakes of winning the entrepreneurial market, the larger is the payoff gap.*

Proof. In equilibrium, the expected monetary payoffs from entrepreneurship are:

$$\begin{aligned}
\pi &= W+\frac{1}{n^{**}}R-e^{**} \\
&= W+\frac{1}{n^{**}}R-\left(\sqrt{wR}-w\right)
\end{aligned}$$

The relevant comparison is the difference between this expected payoff and what an individual earns from employment, $W+w$. Thus, the net payoff difference between entrepreneurship and employment is

$$\Delta=\frac{1}{n^{**}}R-\sqrt{wR}$$

Substituting for n^{**} and simplifying, we obtain

$$\Delta = \frac{(\alpha - \beta)\sqrt{wR}}{(\alpha - \beta)\sqrt{w} + \beta\sqrt{R}}\left(\sqrt{R} - \sqrt{w}\right) > 0$$

where the inequality follows from the fact that $\alpha > \beta$ and $R > w$.

To establish that the returns gap increases in R, differentiate Δ with respect to R. This is proportional to

$$\frac{\partial\Delta}{\partial R} \propto \beta\sqrt{w}R + 2\sqrt{R}w(\alpha - \beta) - w^{\frac{3}{2}}(\alpha - \beta)$$

and since $R > w$ (i.e., the returns from winning the entire entrepreneurship market exceed the individual return from the outside option), it follows that $\frac{\partial\Delta}{\partial R} > 0$. ■

Proposition 8 *Fix the number of competitors n, the reference point r, and suppose that Assumption 1 holds. Compared to risk neutrality, equilibrium effort is higher under loss aversion if and only if $r > W + R\left(\frac{n-1}{n}\right)^2$.*

Proof. The difference between effort under loss aversion and risk neutrality is equal to

$$\begin{aligned} e^* - e^{RN} &= \frac{(n-1)\left(R + (\frac{\alpha}{\beta} - 1)(r - W)\right)}{(2n-1) + \frac{\alpha}{\beta}(n-1)^2} - \frac{n-1}{n^2}R \\ &= (n-1)\frac{(\frac{\alpha}{\beta} - 1)\left((r - W)n^2 - R(n-1)^2\right)}{\left((2n-1) + \frac{\alpha}{\beta}(n-1)^2\right)n^2} \end{aligned}$$

which turns on the sign of

$$(r - W)n^2 - R(n-1)^2$$

Thus effort under loss aversion exceeds effort under risk neutrality iff

$$(r - W)n^2 - R(n-1)^2 > 0$$

or

$$r > W + \frac{(n-1)^2}{n^2}R$$

Lastly we need to check whether given Assumption 1 both cases can arise. To see that $e^* < e^{RN}$ is possible notice that $W + R\left(\frac{n-1}{n}\right)^2 < W + R\frac{n-1}{n}$ since $n > 1$. We will now show that the upper bound on r given in Assumption 1 is greater than the RHS of this inequality. That is, we will show that

$$W + R\frac{n-1}{n} < W + \frac{\beta n + \alpha(n-1)^2}{\beta n + \alpha n(n-1)}R.$$

Simplifying, we require only that

$$\frac{\beta n + \alpha (n-1)^2}{\beta n + \alpha n (n-1)} > \frac{n-1}{n}$$

And cross-multiplying yields the condition

$$\beta n^2 > \beta n (n-1)$$

which always holds since $n > 1$. Thus, when the reference point is high enough, or $r \in \left(W + R\left(\frac{n-1}{n}\right)^2, W + R\frac{\beta n + \alpha (n-1)^2}{\beta n + \alpha n (n-1)} \right)$ then $e^* > e^{RN}$. ■