

# An Analysis of Stock Recommendations\*

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## Abstract

We study the information content of stock reports when investors are uncertain about a financial analyst's incentives. Incentives may be aligned, in which case the analyst wishes to credibly convey his information, or incentives may be misaligned. We find that: (a) Any investor uncertainty about incentives makes full revelation of information impossible. (b) Categorical ranking systems, such as those commonly used by brokerages, arise endogenously as equilibria. (c) Under certain conditions, analysts with aligned incentives can credibly convey unfavorable information, but can never credibly convey favorable information. (d) Policies that improve transparency of analyst incentives might reduce the information content of stock reports. Finally, we examine testable implications of the model compared to empirical analyses of stock recommendations.

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# An Analysis of Stock Recommendations

## 1 Introduction

In many situations, the economic environment is sufficiently complex that decision makers are uncertain about the impact of their decisions. For instance, a legislature may be uncertain about the economic effect of proposed emission controls. Likewise, investors may be uncertain about the consequences of investing in a particular stock on their retirement savings. In these situations, decision makers often turn to experts for advice and guidance. A key difficulty facing the decision maker is that the *motives* of the expert providing advice may not be transparent. This situation commonly arises in the interaction between investors and financial research analysts.

This paper examines how investor uncertainty about the motives of financial research analysts employed by securities firms affects the information content of their stock reports. Securities firms offer services that include investment banking (such as underwriting the issue of publicly traded companies, raising bank loans, and advising on mergers) and brokerage services (such as investment advice and equity research). Securities firms are required to separate the brokerage and investment banking activities because research analysts in the brokerage division may face undue pressure from the investment banking division to issue stock reports that favor the interests of investment banking clients over those of brokerage clients. To strengthen the “Chinese wall” separating the brokerage and investment banking divisions, Congress amended the United States securities laws in 1988. In the wake of this legislation, the Securities and Exchange Commission (SEC) issued guidelines and the National Association of Securities Dealers and the New York Stock Exchange issued a joint memorandum in 1991 endorsing this separation.

Nevertheless, from time to time, research analysts face pressure to “breach” the Chinese wall and issue upwardly biased stock reports that favor the interests of the firm’s investment banking clients.<sup>1</sup> Indeed, recently the SEC expressed renewed concern about the incentives that analysts face to bias their disclosure to investors; the SEC’s office of compliance, inspections and examinations is currently investigating the policies and procedures securities firms have in place to ensure analysts are appropriately shielded from the other divisions in the firm (Burns [2000]). Congress too is concerned about the potential conflict of interest and has recently been holding hearings to establish whether a conflict exists between analysts’ investment banking and personal stockholding interests and their fiduciary responsibility to investors. Congress is also considering implementing more stringent disclosure requirements for analysts (Schroeder [2001] and Schack [2001]). Further, in response to these concerns, the Securities Industry Association released “Best Practices” guidelines to enhance

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<sup>1</sup>See, for instance, Dugar and Nathan [1995, 1996], Laderman [1998], Lin and McNichols [1998], McGee [1997], McNichols and O’Brien [1997], Michaely and Womack [1999], and Schipper [1991].

analyst credibility (Opdyke [2001a]). The high profile given to these issues highlights the fact that investors remain uncertain about the analyst’s incentives when reporting on a specific company.

We model a setting where an analyst, through his expertise, obtains a private and non-verifiable signal about the firm’s value. The analyst is also privately informed about the nature of the incentives he faces at that moment; for instance, whether there is the possibility of winning future investment banking business or whether he has an equity stake in the firm. The analyst, who is not obliged to truthfully report his private information, releases a stock report. Investors value the firm upon observing this report. The analyst’s payoffs depend on the firm’s stock price, its underlying value, and the presence of investment banking opportunities or personal stockholdings in the firm. The analyst’s incentives are said to be aligned with those of investors when payoffs are maximized by a stock price that exactly reflects the analyst’s information about the firm’s value. Conversely, incentives are misaligned with those of investors when an analyst prefers to induce a higher stock price than is warranted by his information.

Our main findings are as follows:

- In Proposition 1, we show that any investor uncertainty about incentives makes full revelation of information impossible – even when an analyst has incentives perfectly aligned with those of investors.
- Proposition 3 shows that categorical ranking systems, such as those commonly used by brokerages to rank stocks (e.g., buy/hold/sell), arise endogenously as equilibria. Further, Proposition 9 shows that these equilibria have the property that *all* analysts tend to issue more favorable reports with greater frequency than less favorable reports – even those with incentives perfectly aligned with those of investors. Nevertheless, analysts whose incentives are misaligned tend to issue favorable reports even more frequently.
- Under conditions identified in Proposition 2, another class of equilibria arises where analysts with aligned incentives can credibly convey unfavorable information about a firm’s value, but can never credibly convey favorable information. Proposition 6 shows that, compared to categorical ranking systems, this class of equilibria provides much more information to investors.
- We show in Proposition 5 and Examples 1 and 2 that policies currently under consideration that require transparency of analyst incentives can actually reduce the information content of stock reports. Sufficient conditions for transparency to improve information content are offered in Proposition 8.
- The validity of our results may be tested empirically. We highlight testable implications of our model and offer a number of statistical tests using existing data.

The nearest antecedent to our paper is Benabou and Laroque [1992], who also consider the problem of the incentives of analysts to misreport their information in a cheap talk framework. Our paper differs from theirs in two ways. First, whereas in their paper, some types of analysts are constrained to make truthful reports, in our model, reporting strategies of all types of analysts are determined endogenously. As we show, this distinction matters – there is no equilibrium in our model where an analyst whose incentives are perfectly aligned with investors simply discloses his information at face value. A second key difference between the models centers on reputational concerns. Benabou and Laroque are mainly concerned with the dynamics of the disclosure strategies when analysts have misaligned incentives. As a result, their static model is simpler than ours: their model consists of a binary state space, a binary message space, and a binary action space that determines stock prices. Since our concerns are with the impact of transitory investment banking opportunities on analyst incentives, our focus is on the static game, but in a richer context. In our model, states, messages, actions, and the degree of misalignment of preferences are all continuous. This modeling framework allows us to explore certain comparative static properties that are institutionally relevant, but that cannot be addressed in Benabou and Laroque’s framework.

Other work in this area has focused on situations where analysts are not directly concerned with the stock price induced by their reports, but rather are concerned with convincing investors of their expertise in forecasting. Trueman [1994] considers a reporting environment where analysts with different forecasting abilities are motivated to build reputations for forecasting accuracy. He finds that analysts with strong forecasting abilities truthfully reveal their information whereas those with weak predictive abilities try to mimic the strong type. Ottaviani and Sorensen [1999] study information transmission in a model that has some application to analyst reporting. As in Trueman, analysts are solely concerned with investors’ perception of their forecasting ability. Consistent with Benabou and Laroque, and in contrast to these papers, our model is applicable to situations where an analyst is mainly concerned with the impact of his report on the price of the firm’s stock.

Admati and Pfleiderer [1986, 1988, 1990] also study the impact of information transmission in financial markets. Specifically, Admati and Pfleiderer [1986] consider a setting where a monopolist sells information to buyers who subsequently use this information to make investment decisions. The monopolist is endowed with some private information about an asset’s value and may add noise before selling it. The statistical properties of the information are common knowledge and the seller reports information truthfully. Within a perfectly competitive noisy rational expectations framework, they show that a seller may prefer to add noise to his private information to counter the dilution in the information’s value due to its leakage through informative prices. Admati and Pfleiderer [1990] extend this earlier work and allow the information monopolist either to directly sell the information (or a noisy version of it) to buyers who then trade in a speculative market or to indirectly sell the infor-

mation by creating a portfolio and then selling shares in the portfolio. They show that the optimal selling arrangement is dependent upon the amount of information revealed in the asset prices. Admati and Pfleiderer [1988] address similar issues to those they considered above but within a setting where traders submit market orders and take into account their effect on price. Their analyses purposely ignore the incentive problems between the seller and buyers: in particular, they assume that the seller truthfully provides information if sold directly and makes the promised investment if the information is sold indirectly. Incentive issues are focal in our study.

From a purely theoretical perspective, our paper may be viewed as extending the model of Crawford and Sobel [1982] to the case where there is uncertainty about the degree of divergence in preferences between the sender and the receiver. Crawford and Sobel are interested in information transmission between a single sender and a single receiver when there is no uncertainty about the sender's incentives. They find that all equilibria are partitional. Thus analysts are unable to fully reveal their private information. In our model where receivers (or investors) are uncertain of the sender's (or analyst's) incentives, we find a class of equilibria that is partitional and a class of equilibria where analysts with aligned incentives can credibly convey unfavorable information about a firm's value, but can never credibly convey favorable information. Further, because our model explicitly recognizes investor uncertainty about an analyst's incentives, it allows us to explore the efficiency implications of a policy that regulators and legislators are currently formulating to reduce investor uncertainty about analyst incentives. These policy proposals create more stringent requirements for disclosing incentives when stock recommendations are offered (Knox [2000] and Opdyke [2001a]). We suggest that these policy proposal might not necessarily have the desired consequence of enhancing the quality of investor information.

Finally, our paper is also somewhat related to Sobel [1985] and Morris [2001]. These papers also consider information transmission between a single sender and receiver when the receiver is uncertain about the sender's incentives but focus on the dynamics of reputation formation. As a consequence of this focus on dynamics, their modeling environments differ substantially from ours.

On the empirical front, there is considerable work on the effect of analyst incentives on their reporting behavior: representative studies include Dugar and Nathan [1995], Francis and Soffer [1997], Lin and McNichols [1998], Michaely and Womack [1999], and Womack [1996]. These papers document reporting outcomes consistent with analysts having incentives to upwardly bias their reports. There is little extant literature that relates these empirical findings to a game-theoretic model with fully optimizing agents. An important contribution of this paper therefore is to develop a model explaining these findings.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 examines issues related to full revelation of analyst information. Section 4 studies a class of equilibria that correspond closely to the equity ranking categories that brokerages use to rank stocks. Section 5 studies equilibria where unfavorable informa-

tion about firm values can be credibly conveyed in stock reports. Section 6 contains empirical implications of this analysis. Section 7 concludes. Unless otherwise noted, proofs which are not in the text are in the appendix.

## 2 Model

We study a financial market setting consisting of many investors and an equity analyst. The analyst is employed in the brokerage division of a securities firm that offers brokerage and investment banking services. The participants in this setting have identical diffuse prior beliefs about a firm's value,  $\theta$ , lying in some bounded interval, which we normalize to be  $[0, 1]$ . The analyst privately observes  $\theta$ , but this information is neither contractible nor verifiable.<sup>2</sup> The analyst can communicate some or all of his information about  $\theta$  by costlessly issuing a stock report  $m \in \mathfrak{R}$ . The analyst's information about  $\theta$  is "soft" in the sense that he is unconstrained in making his report; in particular, his reports may be vague or even misleading.<sup>3</sup> Investors observe the stock report and then value the firm.<sup>4</sup> The market for the firm's stock is efficient and investors are risk-neutral; hence the stock price of the firm,  $y$ , is equal to its expected value given all publicly available information including any information contained in the analyst's report  $m$ . Lastly, the analyst's compensation is determined.

There are generally two primary components of an analyst's compensation (Stickel [1992] and Michaely and Womack [1999]). The first component is an analyst's performance. The *Institutional Investor* All-American Research Team poll, based on a survey of money managers and institutions, is widely viewed as a measure of an analyst's standing in the industry. This poll ranks analysts on their stock picking and earnings forecasting ability, industry knowledge, client service and the like. Directors of equity research at securities firms often consider an analyst's ranking in this poll when setting his compensation (Michaely and Womack [1999]). Most of the factors the poll considers when evaluating an analyst's performance serve to align his interests with those of investors.

The second component of the analyst's compensation is the analyst's ability to generate investment banking business. Analysts who help win investment banking business may receive a portion of the fees generated, or more commonly, a bonus that is two to four times that of analysts who do not win business (Michaely and Womack [1999]). Analysts often win this investment banking business by issuing positive

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<sup>2</sup>Of course, the analyst might have to exert effort to obtain this information. We are not concerned with incentive schemes inducing optimal effort on the part of the analyst. Such schemes are explored in Osband [1989], Hayes [1998], and Dewatripont and Tirole [1999].

<sup>3</sup>Since false recommendations are allowed, ours is not a "game of persuasion" (see Shin, 1994).

<sup>4</sup>Since we wish to focus on the information that analysts communicate via their stock reports, we ignore the possibility that analyst may trade on their own account. Admati and Pfleiderer [1986 and 1990] make a similar assumption.

recommendations that boost a firm’s stock price.<sup>5</sup> This component of compensation encourages an analyst to be optimistic about a firm’s prospects and often creates a conflict of interest between the analyst and investors.

To reflect the “juggling act” caused by the conflict between the fiduciary responsibility to investors and the responsibility to investment banking clients (or potential clients), we model the analyst’s objective function as consisting of two components – a benefit associated with inflating the stock price above its true value and a cost associated with poor performance. Specifically, we suppose that the analyst’s objective function is:

$$U(y, \theta, \beta) = 2\beta y - (\theta - y)^2.$$

The parameter  $y$  reflects the firm’s stock price following the analyst’s report,  $\theta$  denotes the firm’s true value, and  $\beta$  parametrizes the effect that investment banking concerns have on the analyst’s payoff. Specifically,  $\beta$  is the amount above the true value that an analyst wishes to inflate the stock price. Thus, when  $\beta = 0$ , the analyst has purely performance based concerns. That is, the analyst’s payoffs are maximized by inducing a stock price that is equal to the firm’s value,  $y = \theta$ . Since the analyst desires to have no difference between the induced stock price and the firm’s value, we say that the analyst’s incentives are *aligned* with those of investors. In contrast, when  $\beta = b$ , the analyst has mixed motives. Here, his payoffs are maximized by inducing a stock price that is somewhat above the firm’s value, that is,  $y = \theta + b$ . Since the analyst desires a stock price that exceeds the firm’s value, we say that the analyst’s incentives are *misaligned* with those of investors.

To ensure that the trade-off between the analyst’s responsibility to investors and to investment banking clients (or potential clients) is non-trivial, we assume that  $b < 1$ . If  $b \geq 1$ , then the incentives to “bump up” the stock price completely dominate the analyst’s fiduciary responsibility to investors. To see this, notice that the highest stock price that will ever prevail is  $y = 1$  (since  $\theta \leq 1$ ), and therefore, the analyst’s payoff is maximized when  $\beta = b \geq 1$  by inducing the highest feasible stock price *regardless of the realization of  $\theta$* . Thus, fiduciary or quality concerns are effectively absent.

Whether or not the analyst’s interests are aligned with those of investors depends upon the particular circumstances prevailing when the analyst issues a stock report. These circumstances change over time as changing market conditions affect the fir-

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<sup>5</sup>The effect of an analyst’s recommendation on the likelihood of winning underwriting business is nicely illustrated by the Atmel Corporation case. Atmel is a global semiconductor company. Following its initial public offering, it began receiving visits from a number of investment banks. Atmel wanted greater Wall Street coverage and told the banks that if they were not prepared to cover it, they would not win its banking business. An equity analyst, Mark Edelstone, knowing that his employer, Prudential Securities, wanted Atmel’s business, investigated the firm and initiated coverage with a “buy” recommendation. Atmel’s management liked his stock report. Subsequently, when Atmel made a seasoned equity offering, Prudential Securities was one of its underwriters. Those involved with the offering openly acknowledge that Mark Edelstone’s coverage was key to his firm winning the new business (see Nocera [1997, 108] for further details).

m's and analyst's prospects. Since the presence of investment banking opportunities is highly variable, investors generally are unaware of the degree to which these opportunities are present (Michaely and Womack [1999]). Likewise, investors may be unaware of the analyst's personal stock holding in the firm. Thus, investors are uncertain about an analyst's precise incentives even when an analyst is concerned about how investors perceive his performance. To capture this key feature of the analyst reporting environment, we assume that  $\beta$  is distributed as follows:

$$\beta = \begin{cases} 0 & \text{with probability } p \\ b & \text{with probability } 1 - p \end{cases} ,$$

where  $b > 0$  and  $p \in (0, 1)$ . The analyst is privately informed about the realization of  $\beta$  prior to issuing a stock report.

In analyzing the information content of stock reports, we restrict attention to pure reporting strategies. A strategy for an analyst with parameter  $\beta$  is a function  $\mu_\beta(\theta)$  mapping the analyst's private information about firm value into a report. A report  $m$  induces beliefs about the firm's value on the part of investors given by the cumulative distribution function  $P(\theta|m)$ . Since the stock price equals the expected value of the firm given all publicly available information, the stock price following report  $m$  is

$$y(m) = \int_0^1 \theta dP(\theta|m) .$$

We study *Perfect Bayesian Equilibria* of this model. These require that:

1. Investors' beliefs,  $P(\cdot|m)$ , are formed using Bayes' rule whenever possible;
2. Given beliefs,  $\mu_\beta(\theta)$  maximizes the analyst's payoff.

We can succinctly define the stock price occurring in equilibrium when the analyst receives signal  $\theta$  and has incentives  $\beta$  as  $Y_\beta(\theta) \equiv y(\mu_\beta(\theta))$ . Likewise, let  $X_\beta(\theta)$  denote the difference between the equilibrium price and the firm's value, i.e.  $X_\beta(\theta) \equiv Y_\beta(\theta) - \theta$ .

We measure of the amount of information transmission in an equilibrium as the variance of  $X_\beta$  weighted by the probability of each realization of  $\beta$ . That is, the *informational efficiency* of a stock price,  $\Phi$ , is

$$\Phi = p \text{Var}(X_0) + (1 - p) \text{Var}(X_b) .$$

Higher values of  $\Phi$  reflect lower efficiency. When  $\Phi = 0$ , prices are informationally efficient. This measure of information efficiency is a natural one because the investors' posterior distribution of  $\theta$  is fully characterized by its mean and variance. Further, it corresponds to measures of expected price efficiency often used in the financial rational expectations literature (see, for instance, Kim and Verrecchia [1994]).



At this point, we discuss some features of our model. The reporting space is  $m \in \mathfrak{R}$  while the type space consists of two components:  $\theta \in [0, 1]$  and  $\beta \in \{0, b\}$ . This reporting space might seem restrictive; nonetheless, it is sufficiently rich to allow the analyst to disclose all of his private information. For example, were the analyst to report  $m = \theta$  when  $\beta = 0$  and report  $m = \theta + 2$  when  $\beta = b$ , investors can perfectly infer the analyst’s private information.

A second consideration in modeling stock reports is that the legal requirements of the Investment Advisers Act of 1940 and the Standards of Professional Conduct for the Association of Investment Management and Research require analysts to disclose the presence of investment banking and other interests in their stock reports. Interestingly, to comply with this requirement, securities firms mainly use a boilerplate clause to cover litigation contingencies and typically disclose that they *may* have investment banking relations with a company for which a report is issued.<sup>6</sup> This disclosure probably stems from imperfect observability of  $\beta$  and the prospect of adverse litigation outcomes in the event that no disclosure is made. Since all stock reports make the same disclosure along this dimension, we do not model this aspect of reporting explicitly.<sup>7</sup>

Third, we have assumed that the quality of the signals that the analyst receives about a firm’s value are commonly known.<sup>8</sup> This is not likely to be the case for new and unproven analysts. Thus, our model is more appropriate for studying the information content of stock reports issued by well-established analysts.

Fourth, our model examines the influence of a report by a single analyst. In the case of issuing stock recommendations, this representation seems to roughly approximate the institutional environment. For instance, Womack [1996] finds that temporal clustering of recommendations from competing analysts is rare. In the case of the issuance of earnings forecasts, clustering is more prevalent. Thus, to the extent that interaction among analysts is important to the perceived information content of their stock reports, our model is more appropriate for stock recommendations than for earnings forecasts.

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<sup>6</sup>For instance, Bear Sterns included the following caveat on a stock report they issued on McKesson Corporation dated September 4, 1996: “... Bear Sterns may make markets and effect transactions, including transactions contrary to any recommendation herein, or have positions in the securities mentioned herein (or options with respect thereto) and may also have performed investment banking services for the issuers of such securities. ...” Similarly, Morgan Stanley included the following statement in a stock report on Cardinal Health dated October 29, 1996: “... Morgan Stanley & Co. Inc. and others associated with it may have positions in and effect transactions in securities of companies mentioned and may also perform or seek to perform investment banking services for those companies.”

<sup>7</sup>Recognizing that the value of this disclosure is questionable, SEC regulators are currently formulating proposals to strengthen these written disclosure rules (see Knox [2000] and Schack [2001]).

<sup>8</sup>In light of Michaely and Womack’s [1999] finding that analyst optimism is not attributable to differences in an analyst’s predictive ability but rather to incentives arising from the need to satisfy investment banking concerns, we view this perspective as conforming to the long-run outcome in the market for analyst advice.

Finally, we have assumed that the analyst receives a perfect signal about the firm's value; however, our analysis would be unaffected if we were to assume instead that the analyst receives a noisy signal. This would preclude the possibility of investors using differences in the report and the realization of  $\theta$  as a disciplining device for analysts (outside the model). It, however, would increase notational complexity.

### 3 Price Responsiveness

A key concern of regulators in proposing rules and procedures designed to preserve the independence of analysts is that even a *potential* conflict of interest may undermine the information content of their stock reports. In this section, we consider whether it is possible for an analyst's stock report to credibly convey information about firm value.

First, we study the responsiveness of stock prices to reports. Intuitively, a responsive stock price impounds, at least partially, all new information about a firm's value. More formally,

**Definition 1** *A stock price is fully responsive if, for some realization of  $\beta$ , the stock price,  $Y_\beta(\theta)$ , is continuous and strictly increasing over the entire unit interval.*

*A stock price is semi-responsive if, for some realization of  $\beta$  and some non-degenerate interval  $(\underline{\theta}, \bar{\theta})$ , where  $\theta \in (\underline{\theta}, \bar{\theta})$ , the stock price,  $Y_\beta(\theta)$ , is continuous and strictly increasing.*

In the remainder of this section, we establish that, while it is possible for stock prices to be semi-responsive to recommendations, a fully responsive stock price can never occur in equilibrium if there is any positive probability that an analyst's incentives are misaligned. In making this argument, the following lemma, which establishes that more positive information about a firm's value never leads to a strictly lower stock price following an analyst's report, is helpful.

**Lemma 1** *In any equilibrium, the stock price,  $Y_\beta(\theta)$ , is non-decreasing in  $\theta$ .*

When it is common knowledge that incentives are aligned ( $\beta = 0$ ), there exists an equilibrium where the stock price is fully responsive.<sup>9</sup> In particular, suppose that an analyst truthfully discloses the firm's value; i.e.,  $\mu_0(\theta) = \theta$  for all  $\theta$ . In this case, using Bayes' rule to form posterior beliefs on the part of investors,  $y(m) = m$  for all  $m \in [0, 1]$ . This then implies that  $Y_0(\theta) = \theta$  for all  $\theta$ ; hence the stock price is fully responsive. Moreover, an analyst does strictly worse than the equilibrium strategy by choosing any message  $m \neq \theta$  when the firm's value is  $\theta$ .

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<sup>9</sup>Of course, there are many other equilibria arising in this case. At the other extreme, babbling is also an equilibrium.

Next, consider the case where it is common knowledge that incentives are misaligned ( $\beta = b$ ). This is covered by the model of Crawford and Sobel [1982].<sup>10</sup> It follows immediately from their Lemma 1 that, in this case, even a semi-responsive stock price is impossible.

When incentives are uncertain, which is the focus of this paper, the situation is different. To see that a fully responsive stock price is impossible, first suppose that the stock price is fully responsive when  $\beta = b$ . This case is ruled out by arguments analogous to those given for the case where it is common knowledge that  $\beta = b$ . Next, consider the case where the stock price is fully responsive when  $\beta = 0$ . Fix a firm value  $\theta' > b$ . Since the stock price is fully responsive, it follows that  $Y_0(\theta') = \theta'$  (otherwise there would be a profitable deviation). Let  $m'$  be the stock report inducing this price. When  $\beta = 0$ , the only firm value where the stock price  $Y_0(\theta')$  is induced is  $\theta'$ . When  $\beta = b$ , an analyst who learns that the firm's value is  $\theta'' = \theta' - b$  will prefer to induce price  $y = \theta'$  over all other prices. Moreover, analogous to the case where  $\beta = 0$ , it must be the case that the only time an analyst with  $\beta = b$  induces this price is when  $\theta = \theta''$ . Thus investor beliefs upon hearing the message  $m'$  are  $E(\theta|m') = p\theta' + (1-p)(\theta' - b)$ . But this is a contradiction since  $E(\theta|m') < \theta'$ .

More generally, we have shown that:

**Proposition 1** *There is no equilibrium where there is an interval  $(\underline{\theta}, \bar{\theta})$ ,  $\bar{\theta} > b$ , where stock prices are semi-responsive.*

It is worth noting that the arguments in Proposition 1 hold quite generally. Indeed, an almost identical argument can be used to show that no semi-responsive equilibrium exists for arbitrary distributions of firm values and where analyst's have preferences  $U(y, \theta, \beta)$  such that  $U_{11} < 0$ ,  $U_{12} > 0$  and  $U_{13} > 0$  and where for each  $\theta$ , there is a unique  $y$  maximizing  $U(y, \theta, \beta)$ .

A key implication of Proposition 1 is that uncertainty makes it impossible for an analyst to fully reveal his information in a credible fashion. One might have conjectured that when there is high probability that incentives are aligned, an analyst will disclose  $\theta$  truthfully and investors would act on the report as though it were truthful.<sup>11</sup> Proposition 1 rules this out. Indeed, an implication of this proposition is that when an analyst receives a relatively favorable signal about firm value (i.e.,  $\theta > b$ ), it is impossible for his stock report to *credibly* convey nuanced information. Instead, in any equilibrium, when an analyst receives a favorable signal about the firm's value, slight differences in the signal almost never will be reflected in stock prices.

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<sup>10</sup>Note, however, that the leading example of Crawford and Sobel [1982] is a special case of our model when  $p = 0$ .

<sup>11</sup>This is an important distinction that our model shares with Morris [2001] – all types of agents behave optimally in their cheap talk reports. The models of Sobel [1985] and Benabou and Laroque [1992] do not have this feature. Some agents are constrained to report truthfully in their models.

The above proposition shows that it is impossible for stock prices to be responsive to reports when the analyst's signal about firm value is relatively favorable. This does not mean, however, that stock prices are never responsive. In particular, we have thus far said nothing about the case where  $\bar{\theta} < b$ . In the next proposition, we offer sufficient conditions for the stock price to be responsive to unfavorable realizations of firm value.<sup>12</sup>

**Proposition 2** *Define  $\theta^* = \frac{1}{p} (1 - \sqrt{1-p})$  and let  $b \geq \theta^*$ , then the following is a semi-responsive equilibrium:*

*If  $\beta = 0$ , then the reporting strategy of an analyst is*

$$\mu_0(\theta) = \begin{cases} \theta & \text{if } \theta < \theta^* \\ \theta^* & \text{if } \theta \geq \theta^*. \end{cases}$$

*If  $\beta = b$ , then the reporting strategy of an analyst is*

$$\mu_b(\theta) = \theta^*.$$

*The pricing strategy is*

$$y(m) = \begin{cases} m & \text{if } m < \theta^* \\ \theta^* & \text{otherwise.} \end{cases}$$

While the precise characterization of  $\theta^*$  in the above proposition depends on the details of the distribution of  $\theta$  and the structure of analyst preferences, the argument may be straightforwardly generalized for an arbitrary continuous distributions of  $\theta$  on the unit interval and analyst payoffs given by  $U(y, \theta, \beta)$  where  $U_{11} < 0$ ,  $U_{12} > 0$  and  $U_{13} > 0$  and where for each  $\theta$ , there is a unique  $y$  maximizing  $U(y, \theta, \beta)$ . In particular, suppose that an analyst with aligned incentives maximizes payoffs when the stock price equals the true value of the firm. When incentives are misaligned, let  $y^*(\theta, b) > \theta$  be the stock price that maximizes payoffs in state  $\theta$ . If for some  $\theta^* \leq y^*(0, b)$ , where

$$\frac{p(1 - F(\theta^*))}{p(1 - F(\theta^*)) + (1 - p)} E(\theta | \theta \geq \theta^*) + \frac{(1 - p)}{p(1 - F(\theta^*)) + (1 - p)} E(\theta) = \theta^*,$$

then a semi-revealing equilibrium of the form characterized in Proposition 2 exists. Semi-responsive equilibria are discussed further in Section 5.

To summarize, when there is no uncertainty about the analyst's incentives, the possibility of stock price responsiveness depends on whether incentives are aligned. When incentives are aligned, fully responsive stock prices can occur in equilibrium;

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<sup>12</sup>In this proposition and elsewhere, we shall write the equilibrium pricing strategy without reference to the investors' posterior beliefs. Along the equilibrium path, the beliefs are implied by the strategy chosen by the analyst. Off the equilibrium path, however, beliefs are chosen freely. This implies that any stock price  $y \in [0, 1]$  may be sustained for stock reports not made in equilibrium.

whereas, when incentives are misaligned, stock prices are never responsive. In contrast, when there is uncertainty about incentives, the situation is a mixture of the above two extremes: Fully responsive stock prices are impossible, and, indeed, stock prices cannot be responsive to favorable information about the firm’s value (i.e.  $\theta > \theta^*$ ). However, stock prices may be responsive to relatively unfavorable information (i.e.  $\theta < \theta^*$ ). This is consistent with the conventional wisdom that the market reacts much more strongly to very unfavorable stock recommendations or earnings forecasts than it does to more favorable recommendations and forecasts (see, for instance, Elton, Gruber, and Grossman [1986] and Womack [1996], among others). We summarize these differences in the following table:

Table 1. Relationship between Incentives and Price Responsiveness

<u>Incentives and Information</u>	<u>Fully Responsive</u>	<u>Semi-Responsive</u>
Aligned - Common Knowledge	Yes	Yes
Misaligned - Common Knowledge	No	No
Possibly Aligned - Private Information	No	Possibly

## 4 Categorical Ranking Systems

In studying the information contained in stock reports, empirical work has typically focused on stock recommendations as a summary of the analysts’ opinions and other qualitative information contained in the stock report.<sup>13</sup> In this section, we show that an intuitive condition – that investors remain uncertain about an analyst’s incentives even after having read his report – guarantees that this distillation of the complexities of a stock report to an equity ranking characterizes description of equilibrium behavior. We first show that there are always only finitely many stock price responses. We then establish that these equilibria always can be mapped into a categorical ranking system, such as those commonly observed in practice (e.g., buy/hold/sell). Lastly, we examine the effect of uncertainty about the analyst’s incentives on informational efficiency.

### 4.1 Equilibrium Characterization

We now proceed to characterize equilibria in our model when investors remain uncertain about the analyst’s incentives, even after having read his report. Formally, we restrict attention to equilibria satisfying the following condition: for any stock report,  $m$ , investors (correctly) infer that  $\Pr(\beta = 0|m) \in (0, 1)$ .

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<sup>13</sup>The stock recommendation is the final output of the analyst’s activity. The other tasks that the analyst performs, such as the issuance of earnings forecast, are subordinate to the task of issuing a stock recommendation (see Schipper [1991], Womack [1996], amongst others).

Nevertheless, a number of studies examine both the recommendation and the earnings forecast of an analyst in assessing the information content of stock reports. (See, for instance, Francis and Sofer [1997]).

It is easy to see that equilibria satisfying this condition always exist. To see this, suppose that regardless of  $\beta$  and  $\theta$ , an analyst issues the report  $m = \frac{1}{2}$ . Investors will not update their beliefs on the basis of this report. Further, suppose investors do not update their beliefs following *any* report issued by the analyst. In this case, an analyst can do no better than issue the report  $m = \frac{1}{2}$ . Since stock prices reflect the expected value of the firm given the information contained in the stock report and since the posterior beliefs of investors are formed using Bayes' rule where possible, this constitutes an equilibrium for all  $p$  and  $b$ . In this equilibrium, there is only a single price response ( $y = \frac{1}{2}$ ) to the analyst's report. It is obvious that an equity ranking system consisting of a single category (such as "buy") is outcome equivalent.

Now, consider an equilibrium where the analyst's report induces more than one price. Since investors cannot infer an analyst's incentives from his report (or are uncertain of the analyst's incentives), it follows that stock prices are never semi-responsive. Hence, there are only a countable, but not necessarily finite, number of equilibrium stock prices. Without loss of generality, assume that the stock report  $m_i$  induces stock price  $y_i$ ; that is,  $y(m_i) = y_i$ . The following observation (and notation) is used throughout the remainder of the analysis. Consider two equilibrium stock prices  $y_i$  and  $y_{i+1}$  where  $y_i < y_{i+1}$ . It follows from the concavity of the analyst's objective function that there exists a firm value  $\theta = a_i^\beta$  such that an analyst with incentives  $\beta$  is indifferent between  $y_i$  and  $y_{i+1}$  for firm value  $a_i^\beta$ . Thus  $a_i^\beta$  is such that

$$2\beta y_i - \left(a_i^\beta - y_i\right)^2 = 2\beta y_{i+1} - \left(a_i^\beta - y_{i+1}\right)^2. \quad (1)$$

We shall refer to equation (1) as a "no arbitrage" condition and firm value  $a_i^\beta$  as the  $i$ th "cut point" for an analyst with incentives  $\beta$ . It is useful to note that if, for a given pair of stock prices,  $y_i$  and  $y_{i+1}$ , an analyst with incentives  $\beta = 0$  is indifferent for some firm value  $\theta = a_i^0$ , then an analyst with incentives  $\beta = b$  will be indifferent for a firm value that is lower by exactly  $b$ , that is, his incentive to bump up the stock price. Hence,  $a_i^b = a_i^0 - b$ . Using these observations, we show:

**Lemma 2** *If investors cannot infer the analyst's incentives from his report, there are only a finite number of equilibrium stock prices.*<sup>14</sup>

Intuitively, if equilibrium stock prices are close to one another, then there will exist a report  $m_i$  that induces a price  $y_i$  such that an analyst with aligned incentives

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<sup>14</sup>Since the statement of this lemma appears closely related to that of Lemma 1 of Crawford and Sobel [1982, 1436], it is worth noting that the presence of uncertainty about incentives creates complications such that the method of proof in their paper does not apply here. Instead, we must rely on deriving a contradiction near a limit point. Further, as we showed above, in our model, some equilibria entail an infinite number of actions; whereas this is never the case absent uncertainty about incentives. Although this class of equilibrium we study has in common some properties of models where there is no uncertainty, as we show below, there are important differences in the policy and empirical implications of the competing models.

will choose to induce price  $y_i$  only when firm values are arbitrarily close to  $y_i$ . Further, since investors are unable to infer an analyst's incentives from the stock report, then for an analyst with misaligned incentives, it must be the case that  $y_i$  is only induced when firm values are arbitrarily close to  $y_i - b$ . Thus, investors, upon hearing report  $m_i$ , infer that the firm's expected value is a convex combination of  $y_i$  and  $y_i - b$ . This suggests that analysts with aligned incentives would not have induced  $y_i$  in the first place. The only way to avoid this contradiction is if equilibrium stock prices are sufficiently far apart. Indeed, this argument may be readily extended to show that Lemma 2 holds more generally.

Combining Lemmas 1 and 2, we may order the finite number,  $N$ , of equilibrium stock prices  $\{y_i\}_{i=1}^N$  such that they are increasing in  $i$ . In such an equilibrium, an analyst will strictly prefer to induce the stock price  $y_i$  over all other prices for any  $\theta$  which lies in some interval  $(a_{i-1}^\beta(N), a_i^\beta(N))$ . Moreover, these intervals will partition the space of firm values and be increasing in  $i$ . Using these observations, Proposition 3 shows that when investors are unable to infer the analyst's incentives from the stock report, then categorical ranking systems arise endogenously as equilibria.

**Proposition 3** *Suppose that investors cannot infer an analyst's incentives from the report. Then there exists a positive integer  $N(p, b)$  such that every  $N = 1, 2, \dots, N(p, b)$  is associated with exactly one categorical ranking system equilibrium consisting of  $N$  ranking categories constructed as follows:*

1. For all  $\beta$  and for all  $i = 1, 2, \dots, N - 1$ ,  $a_i^\beta(N)$  satisfies

$$2\beta y_i - \left(a_i^\beta(N) - y_i\right)^2 = 2\beta y_{i+1} - \left(a_i^\beta(N) - y_{i+1}\right)^2,$$

$$\text{and } 0 = a_0^\beta(N) < b < a_1^\beta(N) < \dots < a_N^\beta(N) = 1.$$

2. For each  $\beta$  and  $\theta \in [a_{i-1}^\beta(N), a_i^\beta(N))$ , the stock report  $m_i = y_i$  is issued. For  $\theta = 1$ ,  $m_N = y_N$  is issued.
3. For all  $i = 1, 2, \dots, N$ ,

$$y_i = \pi(m_i) E[\theta | \theta \in [a_{i-1}^0(N), a_i^0(N)]] + (1 - \pi(m_i)) E[\theta | \theta \in [a_{i-1}^b(N), a_i^b(N)]] , \quad (2)$$

where

$$\pi(m_i) \equiv \frac{p(a_i^0(N) - a_{i-1}^0(N))}{p(a_i^0(N) - a_{i-1}^0(N)) + (1 - p)(a_i^b(N) - a_{i-1}^b(N))}.$$

4. For all  $m \notin \{m_i\}_{i=1}^n$ ,  $y(m) = y_1$ .

5.  $N(p, b)$  is the largest integer  $N$  satisfying

$$\begin{aligned} & \frac{1}{2}b(4N^2 - 10N + 7 - 4p(N-1)(N-2)) \\ & + \frac{1}{2}b\sqrt{(8p(N-1)(2Np - 4N - 4p + 7) + (4N - 5)^2)} \\ & \leq 1. \end{aligned}$$

6. Further, all other equilibria are outcome equivalent to a categorical ranking system equilibrium.

Several aspects of this equilibrium characterization are worth noting. First, even though the message space available to an analyst is rich enough to allow him to convey all of his detailed information about the firm's value, we find in equilibrium that an analyst, even one guided solely by performance considerations, *endogenously* eschews revealing his detailed information in favor of conveying a summary statistic that provides only a rough guide as to the firm's value. Indeed, an implication of this proposition is that if the message space were *exogenously* restricted to only  $N(p, b)$  elements, this would not adversely affect information transmission. Thus, an important implication of this proposition is that the exogenous restriction to conventional ranking categories (e.g., buy/hold/sell) often imposed upon analysts by brokerages does not necessarily lead to a significant (or indeed any) loss of information.

Second, the proposition highlights the fact that uncertainty about incentives affects the number of equilibrium prices – even in the limit. To see this, consider the case where the probability that an analyst has aligned incentives goes to zero compared to when the analyst is known to have misaligned incentives with certainty. From Crawford and Sobel [1982], the maximum number of equilibrium prices,  $N$ , when incentives are  $b$  and there is no uncertainty satisfies

$$2N(N-1)b < 1.$$

In contrast, from part 5 of our Proposition 3, when  $p$  approaches 0 the largest number of equilibrium prices,  $N'$ , satisfies

$$(2N' - 1)(N' - 1)b < 1.$$

It is obvious that  $N' \geq N$  for all  $b$ .

Despite the fact that there can be strictly higher numbers of equilibrium prices induced in the limit, the equilibrium payoffs to the analyst under the two different scenarios converge. Notice, however, that the increase in the number of equilibrium actions does highlight the possibility, which we discuss later, that the presence of uncertainty about incentives may be informationally superior to the case where there is no such uncertainty.



## 4.2 Policy Implications: Removing Uncertainty about Incentives

The issue of uncertainty about analyst incentives is becoming more salient as the Chinese wall separating the brokerage division from the corporate finance division of securities firms is perceived to be increasingly porous (see, for instance, Laderman [1998] and Knox [2000]).<sup>15</sup> The SEC's office of compliance, inspections and examinations is currently investigating the policies and procedures securities firms use to separate research and investment banking so as to mitigate the pressure research analysts face from the investment banking division to bias their stock reports. Despite the best regulatory efforts, however, it seems clear that divorcing entirely the research and investment banking activities of a securities firm is not possible.

An additional concern is that analysts often have a direct financial interest in the success of the companies on which they issue reports. To address this issue, the Securities Industry Association, the SEC and the National Association of Securities Dealers have recently proposed rules that require analysts and brokerage houses to state definitively whether or not they own shares in a company on which they are opining. In addition, the US House Finance subcommittee on capital markets is currently holding hearings on these issues (see Opdyke [2001b]).

Taken together, most policy solutions to deal with conflicts of interest faced by analysts emphasize the need for fuller disclosure of potential (and actual) conflicts of interest. That is, these policies view making the analyst's incentives more transparent as an important way of improving information contained in stock recommendations and thereby ultimately benefiting investors.

In this subsection, we investigate conditions where transparency indeed improves the informativeness of stock reports. In particular, we compare equilibria arising in our model with two polar cases that we view as natural benchmarks. First, we compare the case where there is uncertainty about analyst incentives with the case where the analyst is required to completely and truthfully disclose his incentives (i.e.,  $\beta$  is known to investors) and where with probability  $p$  the analyst has aligned incentives. Next, we compare our model to the case where all analysts have an average amount of incentive misalignment (i.e.,  $\beta = (1 - p)b$  with probability 1). This case might occur under a compromise policy where full disclosure of incentives is required, but rules governing participation in investment banking are liberalized.

In studying this question, notice that investors face two kinds of uncertainty in assessing stock reports. First, investors are unsure about the information the analyst

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<sup>15</sup>In fact, over the past decade, research analysts have participated more fully in the corporate finance activities of securities firms. In the past when providing underwriting services, securities firms used employees from the corporate finance division for marketing the issue and the due diligence investigation of the issuer. More recently, however, securities firms have co-opted employees with specialized industry knowledge from the brokerage division to participate in underwriting process. This strategy has reduce the need to duplicate skills in the brokerage and corporate finance divisions of the securities firm (Michaely and Womack [1999]).

has regarding firm value. Second, investors are uncertain of the issuing analyst's incentives. This second kind of uncertainty creates an adverse selection problem in the sense that if favorable reports positively influence stock prices, then they are more likely to be issued by analysts with misaligned incentives, and further, the firm values for which such recommendations are issued are lower on average. This selection problem seems qualitatively similar to that first pointed out by Akerlof [1970]. He showed that the addition of this type of uncertainty to a complete information setting can lead to severe efficiency consequences. Thus, one might expect that a policy initiative that improves transparency would be efficiency enhancing.

We first compare the informationally most efficient categorical ranking system equilibrium in our model with the most informationally efficient equilibrium when the realization of  $\beta$  is public information. The following Proposition establishes sufficient conditions for transparency to improve informational efficiency.

**Proposition 4** *Suppose  $b \geq \frac{2}{3+\sqrt{9-8p}}$ , then transparency about incentives strictly improves informational efficiency.*

Roughly speaking, if the degree of misalignment is severe when an analyst has misaligned incentives, then the conditions of Proposition 4 hold. Moreover, policies encouraging disclosure are informationally beneficial even if they do not reduce activities constituting a conflict of interest. That being said, it is not the case that transparency is always informationally beneficial. We next offer two examples where the conditions of Proposition 4 do not hold and where transparency reduces informational efficiency: in Example 1, the degree of misalignment is such that an analyst with  $\beta = b$  cannot credibly convey information when incentives are transparent; Example 2 is one where such an analyst can credibly convey information. Along similar lines, we offer two further examples, Examples 3 and 4, showing that the conditions of Proposition 4 are sufficient but not necessary.

**Example 1.** Suppose  $b = \frac{1}{4}$  and  $p = \frac{1}{5}$ . In this case, the most informationally efficient categorical ranking system equilibrium consists of two reports and leads to informational efficiency of  $5.95 \times 10^{-2}$ . When incentives are transparent, an analyst with incentives  $\beta = b$  cannot credibly communicate whereas an analyst with incentives  $\beta = 0$  can fully reveal his information. In this case, informational efficiency is  $6.67 \times 10^{-2}$ . Thus, transparency, by undermining the incentives of an analyst with misaligned incentives, harms informational efficiency.

**Example 2.** Suppose  $b = \frac{1}{5}$  and  $p = \frac{1}{5}$ . Once again, the most informationally efficient categorical ranking system equilibrium consists of two categories and leads to informational efficiency of  $4.76 \times 10^{-2}$ . Under transparency, an analyst whose incentives are  $\beta = b$  can also credibly issue reports falling into two categories. Nonetheless, the resulting informational efficiency under transparency is  $4.87 \times 10^{-2}$ . Thus, even

though transparency does not reduce the number of categories of reports being made by an analyst with misaligned incentives, his reports contain less information than when there is uncertainty. The upshot is that informational efficiency is decreased under transparency.

**Example 3.** Suppose  $b = \frac{1}{4}$  and  $p = \frac{3}{5}$ . Once again the most informationally efficient categorical ranking system equilibrium consists of two categories and leads to informational efficiency of  $3.68 \times 10^{-2}$ . When incentives are transparent, an analyst with incentives  $\beta = b$  can only babble; nonetheless, informational efficiency under transparency is  $3.33 \times 10^{-2}$ , which is an improvement upon the case where there is uncertainty about incentives.

**Example 4.** Suppose  $b = \frac{1}{5}$  and  $p = \frac{3}{5}$ . Here, the most informationally efficient categorical ranking system equilibrium consists of two categories and leads to informational efficiency of  $3.14 \times 10^{-2}$ . When incentives are transparent, an analyst with incentives  $\beta = b$  can credibly report in two categories, and informational efficiency under transparency is  $2.43 \times 10^{-2}$ . Thus, transparency enhances informational efficiency in this situation.

To see why it is the case that transparency can sometimes reduce informational efficiency, recall that uncertainty enables an analyst with misaligned incentives to credibly convey more information than would otherwise be the case. At the same time, uncertainty is harmful to the credible communication of information by an analyst with aligned incentives. Our examples suggest that the proportion of analysts with misaligned incentives is critical to whether or not transparency helps. When it is relatively unlikely that an analyst's incentives are misaligned, as in Examples 3 and 4, transparency is helpful. On the other hand, when most analysts have conflicts of interest, as in Examples 1 and 2, the loss of information from these analysts under transparency leads to a net reduction in informational efficiency.

Next, we turn to the case where all analyst's have an average amount of incentive misalignment. That is,  $\beta = (1 - p)b$  with probability 1. The following proposition highlights circumstances where transparency (in this form) improves informational efficiency and where it reduces efficiency.

**Proposition 5** *In comparing the informational efficiency of uncertainty versus certainty about analyst incentives:*

1. *Suppose that  $p \in (0, \frac{1}{16}(9 - \sqrt{17}))$  and  $b \in [\frac{1}{4(1-p)}, \frac{2}{3+\sqrt{9-8p}})$ , then uncertainty about incentives leads to greater informational efficiency than when all analysts exhibit the average amount of incentive misalignment.*
2. *Suppose that  $p \in (\frac{1}{16}(9 - \sqrt{17}), 1)$  and  $b \in [\frac{2}{3+\sqrt{9-8p}}, \min\{\frac{1}{4(1-p)}, 1\})$ , then uncertainty about incentives leads to lower informational efficiency than when all analysts exhibit the average amount of incentive misalignment.*

When both the probability that incentives are aligned and the degree of misalignment is relatively low, then uncertainty improves informational efficiency. In contrast, when both the probability that incentives are aligned and the degree of misalignment is high, then uncertainty reduces informational efficiency. These regions are summarized in Figure 1 (on page 42). In region I of Figure 1, uncertainty about incentives improves informational efficiency. Roughly speaking, this is because the dominant effect in this region is the positive spill-over from analysts with aligned incentives, which reduces the distortion in the reports of those with misaligned incentives. In region II, incentives are sufficiently misaligned that stock reports are completely uninformative, regardless of uncertainty about incentives. In region III, uncertainty about incentives reduces informational efficiency. Roughly speaking, this is because the dominant effect is the negative spill-over of analysts with misaligned incentives, which increases the distortion in reports offered by analysts with aligned incentives and reduces information transmission relative to the no uncertainty case. Region IV is ambiguous with respect to which policy leads to greater efficiency. We demonstrate this by returning to Examples 1 and 2 above.

Under Example 1, when all analysts exhibit the average degree of incentive misalignment the most informative equilibrium consists of two reports and leads to informational efficiency of  $6.08 \times 10^{-2}$ . This compares unfavorably to the case where there is uncertainty about incentives. On the other hand, in Example 2 analysts still issue only two reports when they exhibit the average degree of incentive misalignment, but informational efficiency is  $4.64 \times 10^{-2}$ , which is an improvement over the case where incentives are uncertain.

To summarize, while it is likely that changes in disclosure requirements would affect the proportion or average degree of incentive misalignment (contrary to the assumptions of our benchmark cases), our results suggest that caution is needed in determining whether such policies will be beneficial to investors. Indeed, if conflicts of interest are relatively widespread, as testimony in the US House Financial Services Subcommittee hearings seems to suggest (Opdyke [2001b]), then the impact of these policy proposals needs careful study in light of the possibilities raised in Examples 1 and 2.

## 5 Semi-Responsive Equilibria

As we highlighted in Section 3, it is possible for analysts with aligned incentives to credibly reveal their (unfavorable) information about a firm's value. In this section, we compare the informational efficiency of these equilibria with categorical ranking systems and offer sufficient conditions for semi-responsive equilibria to dominate. This, however, does not imply that a categorical ranking system equilibrium is always dominated by some other equilibrium in our model. Indeed, under certain parameters of our model, semi-responsive equilibria do not exist. We identify sufficient conditions on the parameters for non-existence, and therefore, conditions where cate-

gorical ranking systems are informationally dominant. Finally, we revisit the impact of uncertainty about the analyst's incentives on informational efficiency by comparing semi-responsive equilibria with equilibria arising when there is no uncertainty about the analyst's incentives.

Recall that in Proposition 2, we established conditions for a semi-responsive equilibrium to exist. Under these conditions, analysts with aligned incentives truthfully convey unfavorable signals about a firm's value and offer a vague report for more favorable firm values. Analysts with misaligned incentives imitate an analyst with aligned incentives by always offering the same vague report. As a consequence, investors view an analyst's report that a firm's value is low credibly and price the stock accordingly, whereas they view a more favorable report skeptically. Interestingly, when  $b \geq \frac{1}{p}(1 - \sqrt{1-p})$ , it is not possible for a categorical ranking system equilibrium to convey *any* information. This is because, when investors view reports skeptically, there is simply too much optimism needed in the report of an analyst with aligned incentives to undo the effect of investor skepticism. It follows immediately that the semi-responsive equilibrium identified in Proposition 2 informationally dominates all categorical ranking system equilibria. We summarize this observation in the following proposition:

**Proposition 6** *For a given  $p$ , suppose  $b \geq \frac{1}{p}(1 - \sqrt{1-p})$ , then a semi-responsive equilibrium is more informationally efficient than all categorical ranking system equilibria.*

Suppose that  $b$  and  $p$  lie outside this parameter range. Is it still the case that a categorical ranking system equilibrium is dominated by a semi-responsive equilibrium? The following proposition implies that it is not. In fact, for some parameter values, a semi-responsive equilibrium does not exist.

**Proposition 7** *For every  $p$ , if  $b \in \left(\frac{2}{3+\sqrt{9-8p}}, \frac{1}{p}(1 - \sqrt{1-p})\right)$ , then a semi-responsive equilibrium does not exist.*

The reason there are no semi-responsive equilibria for this parameter range is that analysts with aligned incentives would still like to reveal low firm values and have their reports believed. However, because the degree of incentive divergence is not too great for analysts with misaligned incentives, they will imitate the unfavorable reports of aligned analysts when firm values are extremely unfavorable. Of course, investors anticipate this possibility and discount any report accordingly. This unravels any semi-responsive equilibrium but leaves categorical ranking systems as a means of conveying information.

The conditions offered in Proposition 7 are sufficient but not necessary for the non-existence of semi-responsive equilibria. For instance, when  $b = \frac{1}{5}$  and  $p = \frac{1}{5}$ , no

semi-responsive equilibrium exists.<sup>16</sup>

We now return to the policy question of whether transparency improves informational efficiency taking account of semi-responsive equilibrium. We begin by comparing semi-responsive equilibria with the case where the realization of the  $\beta$  parameter is common knowledge. Recall that in Proposition 4, when  $b \geq \frac{2}{3+\sqrt{9-8p}}$ , transparency always improves efficiency compared to categorical ranking system equilibria. The following proposition shows that whenever the conditions for existence of semi-responsive equilibria identified in Proposition 6 hold, transparency still improves informational efficiency.

**Proposition 8** *Suppose  $b \geq \frac{2}{3+\sqrt{9-8p}}$  and incentives are uncertain, then transparency improves informational efficiency over either a categorical ranking system equilibrium or a semi-responsive equilibrium.*

Next, consider our other benchmark case for assessing the impact of transparency; here we compare the case where there is uncertainty about the analyst's incentives (i.e.,  $\Pr(\beta = 0) = p$  and  $\Pr(\beta = b) = 1 - p$ ) to the case where there is no uncertainty about incentives (i.e.,  $\beta = (1 - p)b$  for all analysts). Here, we find that the inclusion of semi-responsive equilibria substantially increases the parameter values where transparency actually reduces informational efficiency.

This relationship between information transmission and the parameter values of the model is summarized in Figure 2 (on page 43). Figure 2 contains six regions: regions I and IV are the same as in Figure 1. Regions II and III from Figure 1 each have been divided into two smaller regions according to whether or not a semi-responsive equilibrium exists. Regions with an "a" designation indicate parameter values where a semi-responsive equilibrium always exists. In regions with a "b" designation, a semi-responsive equilibrium never exists.

In region IIa, no information transmission occurs in a categorical ranking system equilibrium when all analysts exhibit the average degree of incentive misalignment. In contrast, information transmission occurs in the semi-responsive equilibrium. Thus, uncertainty is efficiency enhancing in this region. In region IIIa, there are up to a countably infinite number of partition elements in an equilibrium where all analysts exhibit the average degree of incentive misalignment. Thus, it is not immediately clear whether or not a semi-responsive equilibrium is more efficient than equilibria where all analysts have the average degree of incentive misalignment. Numerical analysis shows that the semi-responsive equilibrium is indeed the most efficient equilibrium in this region.<sup>17</sup> Therefore, it is only in region IIIb, that transparency unambiguously improves informational efficiency.

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<sup>16</sup>This is shown by checking that for all  $N < N(p, b)$ , there does not exist a semi-responsive equilibrium followed by a categorical ranking system equilibrium of size  $N$ . Calculations are available from the authors upon request.

<sup>17</sup>The numerical analysis was performed using a grid size of  $1 \times 10^{-13}$ .

## 6 Testable Implications

In addition to the policy implications described previously, our model also has some key testable implications, which are explored using existing data. The first implication we explore concerns the predicted frequency of various types of recommendations. The second examines stock price movements in response to recommendations. Since the data used in the empirical analysis of stock recommendations is in the form of categorical ranking systems, our implications will be restricted to properties of this class of equilibria.

First, notice that our characterization in Proposition 3 implies certain testable properties of the frequencies with which various recommendations occur. In particular, more favorable recommendations are issued more frequently than less favorable recommendations – regardless of analyst incentives. That is, even when analyst incentives are aligned, recommendations tend to be “optimistic” in the sense that more favorable reports are issued more frequently than less favorable reports. Moreover, the frequency with which non-extreme recommendations are issued is invariant to the incentives of analysts. Formally,

**Proposition 9** *In any categorical ranking system equilibrium where  $N \geq 2$ :*

- 1. More favorable recommendations are issued more frequently than less favorable recommendations even when incentives are aligned;*
- 2. The cumulative distribution of recommendations by analysts with aligned incentives first-order stochastically dominates that of analysts with unaligned incentives when recommendations are ranked according to increasing favorability; and,*
- 3. Each non-extreme recommendation is issued with the same frequency regardless of incentives; that is, for all  $m_i \in \{m_2, \dots, m_{N-1}\}$ ,  $\Pr(m_i)$  is independent of  $\beta$ .*

Notice that these implications differ from models where there is no uncertainty about incentives about the analyst’s incentives and his securities firm restricts him to use a categorical ranking system. When there is no uncertainty, an analyst with aligned incentives issues recommendations in all categories with equal frequency.<sup>18</sup> Further, when there is no uncertainty, the frequency of non-extreme category recommendations, such as hold recommendations, varies with the degree of incentive misalignment.

We now show that Proposition 9 is consistent with some empirical properties of stock recommendations. Lin and McNichols [1998] examined stock recommendations issued in the three-year period preceding and following companies’ seasoned equity

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<sup>18</sup>This reporting strategy maximizes the informational efficiency of the stock price.

offerings. They compare, for a given company, the favorableness of stock recommendations of analysts employed by investment firms underwriting the offering (hereafter, affiliated analysts) to that of analysts who have no such affiliation (hereafter, unaffiliated analysts). As a reference, Lin and McNichols [1998] code a recommendation from most favorable (1) to least favorable (5) in a five-category ranking system.

It is useful to think of the ex post realization of the identity of affiliated analysts as a proxy for their incentives,  $\beta$ , prior to the offering. That is, for reports issued prior to the offering, one might imagine that affiliated analysts have more misaligned incentives than unaffiliated analysts, but that this fact is not apparent to investors. We should stress that, as a test of the predictions of our model, the following arguments are offered cautiously. We require a number of assumptions for our model to be applicable to this data. First, and most importantly, we are assuming that investors are uncertain about the analyst's incentives prior to the equity offering. Second, we are assuming that the misalignment of incentives is largely caused by the desire to win investment banking business. Finally, our model literally assumes that if the analyst's employer is not awarded the company's investment banking business, then the analyst's incentives were never misaligned in the first place. The last assumption, however, may be relaxed (with added complication to the equilibrium characterization but with little change in testable implications) by assuming that the incentives of analysts whose firms lose the investment banking business are *less* misaligned on average than those whose firms win. With these caveats in mind, we now turn to the data.

Figure 3 (on page 44) compares the empirical cumulative distribution functions of recommendations by analysts from lead underwriting firms with recommendations issued by unaffiliated analysts. In each case, attention is confined to the latest report issued before the equity offering. As the figure shows, for analysts affiliated with underwriting firms, the predicted increasing frequency of recommendations holds. For unaffiliated analysts, more favorable forecasts are issued more frequently than less favorable forecasts with one exception: hold recommendations are issued more frequently than buy recommendations. Thus, the predictions of part 1 of Proposition 9 are largely supported. Further, the figure shows that the hypothesized stochastic dominance relationship is present in the data. This observation may be formalized by employing a chi-square test of the null hypothesis of equality of the two distributions. In performing such a test, Lin and McNichols find a test statistic of 43.5, which enables us to confidently reject the null hypothesis of equality in favor of the one-sided alternative hypothesis of stochastic dominance. Thus, part 2 of Proposition 9 is also largely consistent with the data.

Part 3 of Proposition 9 predicts that the distribution of observations in intermediate categories is the same for affiliated and unaffiliated analysts. To test this prediction, we perform a Wilcoxon sum of ranks test on observations in the buy, hold, and sell categories for the two types of analysts. Our null hypothesis is that these observations are drawn from the same underlying distribution. We obtain a  $z$



statistic of 5.161, which rejects the null hypothesis at conventional levels. Thus, this implication of our model is inconsistent with the data.

We can also use the data to distinguish between our model and one where there is no uncertainty about analyst incentives. The empirical cumulative distribution of lead underwriting analysts' recommendations is consistent with both models. Figure 4 (on page 45) compares the empirical cumulative distribution function of unaffiliated analysts with the theoretical distribution predicted in the absence of investor uncertainty. Notice that, absent investor uncertainty, the theoretical prediction is for recommendations to simply be uniformly divided by quintile into the five categories.<sup>19</sup> It is apparent that this theoretical prediction is not supported by the data. We can formalize this observation by performing a chi-square test of the null hypothesis that the theoretical prediction generated the observed data. Such a test yields a chi-square statistic of 233.65, which confidently rejects the null hypothesis. Note that while we have compared the empirical distribution to the theoretical prediction that incentives are perfectly aligned, given the size of the chi-square statistic, it seems likely that we would reject the null hypothesis for any relatively small degree of incentive misalignment.

Another implication of our model, which is inconsistent with a model in which there is no uncertainty about analyst incentives, is the following:

*The short-run stock price response to a given recommendation is independent of the incentives of the issuing analyst.*

Lin and McNichols [1998] offer some evidence on this implication. They study three-day returns around the issuance of a recommendation for recommendations issued one to two years following the seasoned equity offering. That is, these stock recommendations are issued after the identity of the lead underwriting firm is known. Thus, if the identity of the underwriting firm revealed the incentives of analysts, one would expect differential returns to recommendations. However, for strong buy and buy recommendations, there is little difference in short-run returns whereas in the case of a the union of hold, sell, and strong sell recommendations, returns are more negative for analysts affiliated with the underwriting firm. Thus, one possible explanation for this result is that there remains uncertainty even after the identity of the underwriting firm is revealed. Of course, a sharper test of our hypothesis would be to look at differential returns for recommendations *preceding* the equity offering. Here we would expect little difference in short-run returns across analyst types. This test remains for future research.

A considerable amount of empirical work on price responses to stock reports codes recommendations into three categories; thus, it is useful to examine the implications of our model in a three-category equilibrium. Given prior beliefs of investors, the stock price before the issuance of a recommendation is  $E(\theta) = \frac{1}{2}$ . We characterize

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<sup>19</sup>This prediction is independent of the uniform distribution over  $\theta$  employed in this paper but assumes the analyst with aligned incentives wishes to maximize the informational efficiency of the firm's stock price.

downward or upward price movement in response to recommendations. Our model predicts:

**Proposition 10** *In any three-category equilibrium:*

1. *Sell recommendations lead to greater downward price movement than hold recommendations. Buy recommendations always lead to upward price movement.*
2. *The magnitude of a price movement in response to a sell recommendation is greater than that in response to either a hold or buy recommendation.*

These predicted price responses are consistent with the findings of Womack [1996], Francis and Soffer [1997], and Barber *et al.* [2001]. The magnitudes of the predicted price responses are also consistent with the asymmetric market reaction to added-to-buy and added-to-sell recommendations documented empirically by Elton, Gruber, and Grossman [1986] and Womack [1996], among others.

## 7 Discussion

The broad array of conflicting incentives affecting financial analysts employed by securities firms leads to an environment where investors are skeptical of the motives of any analyst issuing a stock report. As a consequence of this skepticism, strategic “filtering” of the information contained in the report to correct for bias often occurs. We establish that the presence of uncertainty about an analyst’s incentives and the strategic responses to it on the part of both investors and analysts lead to a situation where an analyst’s information about firm value is not fully impounded in stock price even if most analysts have aligned incentives.

Two classes of equilibria emerge in this situation. The first class, which we call categorical ranking system equilibria, always exist and correspond to the equity ranking categories (e.g., buy/hold/sell) used by brokerages to rank stocks. These equilibria have the property that *all* analysts tend to issue more favorable reports with greater frequency than less favorable reports – even those with incentives perfectly aligned with investors. Nevertheless, analysts whose incentives are misaligned tend to issue favorable reports even more frequently. These and several other implications of our model accord well with empirical findings in this area.

The second class, which we call semi-responsive equilibria, have the property that analysts with aligned incentives are able to effectively communicate unfavorable information about a firm’s value, but not favorable information. This is because reports of favorable information are imitated by analysts with misaligned incentives whereas unfavorable reports are not. Thus, it is only for unfavorable reports that all relevant information is impounded in stock price. This information can be considerable: we identify conditions where semi-responsive equilibria are more informationally efficient

than all categorical ranking system equilibria. In this case, brokerages imposing restrictions on the recommendations that an analyst may issue leads to informational loss.

Uncertainty about incentives is crucial to all of these results. A natural policy question is whether the elimination of this uncertainty is desirable. To study this question, we compare informational efficiency under uncertainty with two benchmark cases: the first case is one where the proportion of analysts with aligned or misaligned incentives is unchanged, but information about an analyst's incentives is common knowledge. Here we find that if the degree of misalignment is severe, then a policy of transparency improves the informational efficiency of stock reports. When incentive misalignment is less severe, then transparency may be harmful. In particular, when analysts mostly have misaligned incentives, but the degree of misalignment is relatively mild, then it can be the case that a policy of transparency actually reduces informational efficiency. The second benchmark case is one where there is no uncertainty about incentives and all analysts exhibit the average degree of incentive misalignment. In contrast to the results of the first benchmark case, here we find that if the degree of incentive misalignment is large, then transparency unambiguously reduces informational efficiency. Thus, the details of how one implements a policy of transparency lead to important differences in whether or not such policies improve information disclosure in stock reports.

As a first step to understand the effects of investor uncertainty about the analyst's incentives, we analyzed a simple setting. However, several extensions of the model seem warranted. Specifically, investors sometimes report cross-checking analyst reports with other data in order to determine the degree of bias more precisely. Such cross-checking is absent from our model; however, one reasonable way in which it might be added would be to allow for the existence of two or more analysts simultaneously issuing reports. Likewise, there is interaction between the forecasts by the company about whom recommendations are being made and analyst disclosure. Management incentives and information obviously differ from those of analysts (and investors) and hence extending the model to allow investors to integrate information from these two sources seems fruitful. Exploring these avenues remains for future research.

In conclusion, we see the analyst reporting environment as a natural setting in which to explore strategic information transmission when there is uncertainty about a sender's incentives to report information. Nevertheless, the model we examine is sufficiently general that our findings about the nature and amount of information transmission in this context are applicable to a number of other institutional settings.

# A Appendix

Proof of Lemma 1:

Suppose not. Then there exists a realization of  $\beta$  and a pair of signals  $\theta'$  and  $\theta''$  such that  $\theta' < \theta''$  and  $Y_\beta(\theta') = y' > Y_\beta(\theta'') = y''$ . Denote the reports that induce each of these prices by  $m'$  and  $m''$ , respectively. For these to be sent in equilibrium, it must be the case that given signal  $\theta'$ , the analyst prefers to induce  $y'$  to  $y''$ . That is,

$$2\beta y' - (y' - \theta')^2 \geq 2\beta y'' - (y'' - \theta')^2.$$

Equivalently,

$$2\beta y' - (y' - \theta')^2 - \left(2\beta y'' - (y'' - \theta')^2\right) = (y' - y'')(2(\theta' + \beta) - (y'' + y')) \geq 0.$$

Since  $y' > y''$ , the above condition requires that

$$2(\theta' + \beta) \geq (y'' + y'). \quad (\text{A1})$$

Likewise given signal  $\theta''$ , the analyst prefers to induce  $y''$  to  $y'$  if and only if

$$2\beta y'' - (y'' - \theta'')^2 - \left(2\beta y' - (y' - \theta'')^2\right) = (y'' - y')(2(\theta'' + \beta) - (y'' + y')) \geq 0.$$

For this condition to hold requires that

$$(y'' + y') \geq 2(\theta'' + \beta). \quad (\text{A2})$$

Combining (A1) and (A2) yields

$$2(\theta' + \beta) \geq (y'' + y') \geq 2(\theta'' + \beta).$$

But this set of inequalities can only hold if  $\theta' > \theta''$ . This is a contradiction. ■

Proof of Proposition 1:

Follows directly from the argument in the text. ■

Proof of Proposition 2:

To prove that the above strategies indeed comprise an equilibrium, notice that when  $\beta = 0$ , an analyst can do no better than to induce a price  $y = \theta$  when  $\theta < \theta^*$  and prefers  $\theta^*$  to any lower price when  $\theta \geq \theta^*$ . It follows therefore that investors are able to perfectly infer the firm's value from any stock report  $m \in [0, \theta^*)$  and the stock price is responsive to this information. In contrast, when  $\beta = b$ , the analyst prefers to induce price  $\theta^*$  to any price below  $\theta^*$  for all realizations of  $\theta$ . ■

Proof of Lemma 2:

Before beginning with the proof, the following facts are useful. Using equation (1), one may readily verify that:

Fact 1.  $a_i^0 - a_i^b = b$ .

Next notice that when  $\theta$  and  $\beta$  are such that  $y_i = \theta + \beta$ , the analyst does strictly worse by inducing a stock price other than  $y_i$ . That is, for firm value  $\theta$ , the most preferred stock price for an analyst with incentives  $\beta$  is  $y_i$ . Hence,

Fact 2. In any equilibrium,  $\mu_\beta(y_i - \beta) = m_i$ .

We now proceed with the proof. Suppose to the contrary that there exists an equilibrium with an infinite number of equilibrium stock prices. Define the set of all equilibrium prices to be  $\mathcal{Y}$ . Since all prices lie in the interval  $[0, 1]$ , it follows from the Bolzano-Weierstrass Principle (Shilov, 1996, 75) that the set  $\mathcal{Y}$  has a limit point  $y^*$ . Hence, there exist an infinite number of distinct equilibrium prices in a  $\gamma$ -neighborhood of  $y^*$  for arbitrarily small  $\gamma > 0$ .

Let  $y_1 < y_2 < y_3$  be three such prices. Let  $a_1^\beta$  and  $a_2^\beta$  be the ‘‘cut points’’ associated with these prices for an analyst with incentives  $\beta$ . Since  $\frac{\partial^2(2\beta y - (\theta - y)^2)}{\partial y^2} < 0$ , it follows that for all  $\theta < a_1^\beta$ , an analyst with incentives  $\beta$  strictly prefers  $y_1$  to  $y_2$  or  $y_3$ . Likewise for all  $\theta \in (a_1^\beta, a_2^\beta)$ , an analyst with incentives  $\beta$  prefers to induce  $y_2$  to  $y_1$  or  $y_3$ , and so on. Now, as  $\gamma \rightarrow 0$ ,  $y_1 \rightarrow y_3$ . Hence  $a_1^0 \rightarrow a_2^0$  and  $a_1^b \rightarrow a_2^b$ . Hence  $y_2 \rightarrow pa_1^0 + (1 - p)a_1^b$ . From Fact 1,  $a_1^b = a_1^0 - b$ , hence  $y_2 \rightarrow a_1^0 - (1 - p)b$ . From Fact 2, an analyst with aligned incentives will induce  $y_2$  when  $\theta = a_1^0 - (1 - p)b$ . Notice, however, that  $a_1^0 - (1 - p)b < a_1^0$  for  $p, b > 0$ . This contradicts the fact that  $y_1$  is preferred by an analyst with aligned incentives to  $y_2$  for all  $\theta < a_1^0$ . ■

### Proof of Proposition 3:

First, observe that for any set of  $N$  equilibrium prices, the set of states for which an analyst with aligned (misaligned) incentives most prefers an equilibrium stock price  $y$  constitutes an interval. The set of all such intervals for an analyst with aligned (misaligned) incentives constitutes a partition of  $[0, 1]$ . Moreover,  $a^0(N)$  must satisfy the ‘‘no arbitrage’’ condition:

$$2\beta y(m_j) - (a_j^0(N) - y(m_j))^2 = 2\beta y(m_{j+1}) - (a_j^0(N) - y(m_{j+1}))^2 \quad (\text{A3})$$

for all  $j = 1, 2, \dots, N - 1$ . Likewise  $a^b(N)$  must satisfy the indifference condition

$$2\beta y(m_j) - (a_j^b(N) - y(m_j))^2 = 2\beta y(m_{j+1}) - (a_j^b(N) - y(m_{j+1}))^2 \quad (\text{A4})$$

for  $j = 1, 2, \dots, N - 1$ .

Equation (A3) (respectively (A4)) ensure that the messages sent by the analysts with aligned (misaligned) incentives are best responses given the investors’ pricing function in expression (2), which is the expected value of the firm given an equilibrium

report where the beliefs of investors about the type of analyst issuing the report are determined using Bayes' rule.

It remains to show that the indifference conditions (A3) and (A4) form a well-defined difference equation that has a solution for any  $N$  such that  $1 \leq N \leq N(p, b)$ . The case where  $N = 1$  is obvious.

First, notice that any sequence  $a^0(N)$  satisfying (A3) has the property that the associated sequence  $a^b(N)$  satisfies (A4). Hence, we restrict attention to sequences satisfying (A3), and, to simplify the notation, we write  $a_i$  to denote  $a_i^0$ .

The following lemma is useful.

**Lemma 3** *A necessary condition for any equilibrium consisting of  $N \geq 2$  prices is that*

$$a_i(\alpha) = i\alpha + (i-1)((1-\pi(m_1))b) + 2(i-1)^2(1-p)b$$

for  $i = 1, \dots, N-1$  and  $\pi(m_1) \equiv \frac{p\alpha}{\alpha - (1-p)b}$ .

**Proof.** Let  $A_j$  denote a non-decreasing partial partition of  $[0, 1]$  consisting of sequence of  $j \geq 2$  "cut points"  $(a_1, a_2, \dots, a_j)$ , where  $a_1 > b$ . Any  $A_j$  satisfying equation (A3) may be equivalently stated as

$$y(m_{j+1}) = 2a_j - y(m_j). \tag{A5}$$

The stock price after observing  $m_1$  is

$$y(m_1) = \frac{a_1}{2} - (1 - \pi(m_1)) \frac{b}{2}.$$

The stock price after observing  $m_j$ , where  $j > 2$ , is

$$y(m_j) = \frac{a_j + a_{j-1}}{2} - (1 - \pi(m_j))b. \tag{A6}$$

A similar expression holds for  $y(m_{j+1})$ . Substituting into (A5) yields

$$\begin{aligned} \frac{a_{j+1} + a_j}{2} - (1 - \pi(m_{j+1}))b &= 2a_j - \frac{a_j + a_{j-1}}{2} + (1 - \pi(m_j))b \\ a_{j+1} - 2(1 - \pi(m_{j+1}))b &= 2a_j - a_{j-1} + 2(1 - \pi(m_j))b. \end{aligned}$$

Now, observe that  $\pi(m_{j+1}) = \pi(m_j) = p$  for all  $N-2 \geq j \geq 2$ . Thus,

$$a_{j+1} = 2a_j - a_{j-1} + 4(1-p)b, \tag{A7}$$

which is a well-defined second-order difference equation having a unique solution which is non-decreasing in  $j$ . Moreover, if the first cut point is  $a_1 = \alpha$ , then we must have that  $a_2$  satisfies

$$y(m_2) = 2\alpha - y(m_1),$$

which we may then simplify to

$$a_2(\alpha) = 2\alpha + (1 - \pi(m_1))b + 2(1 - p)b. \quad (\text{A8})$$

Noting that  $a_1 = \alpha$  and computing recursively using equations (A7) and (A8), the  $i$ th cut point in a size  $N$  partial partition must satisfy

$$a_i(\alpha) = i\alpha + (i - 1)((1 - \pi(m_1))b) + 2(i - 1)^2(1 - p)b \quad (\text{A9})$$

for  $i = 1, \dots, N - 1$ .

Lemma 4  $\frac{\partial a_i(\alpha)}{\partial \alpha} > 0$  for  $i = 2, \dots, N - 1$  where  $a_1 = \alpha$  and  $\pi(m_1) \equiv \frac{p\alpha}{\alpha - (1-p)b}$ .

Proof. First notice that  $\frac{\partial}{\partial \alpha}(\pi(m_1)) = -\frac{p(1-p)b}{(\alpha - (1-p)b)^2} < 0$ . Now differentiate equation (A9) to yield

$$\frac{\partial a_i(\alpha)}{\partial \alpha} = i - (i - 1)b \frac{\partial}{\partial \alpha}(\pi(m_1)) > 0.$$

Let  $a_N(\alpha)$  denote the  $N$ th cut point given by equation (A9). Since,  $a_N^b(N)$  is not shifted  $b$  to the left of  $a_N^0(N)$ ,  $a_N(\alpha)$  is not the actual  $N$ th cut point. We require separate arguments to characterize this point. The following notation is helpful. Denote the terminal cut point when there are exactly  $N$  prices and the first cut point is  $a_1 = \alpha$  by  $\tilde{a}_N(\alpha)$ .

Let

$$\pi(m_N) \equiv \frac{p(\tilde{a}_N(\alpha) - a_{N-1}(\alpha))}{\tilde{a}_N(\alpha) - a_{N-1}(\alpha) + (1 - p)b}. \quad (\text{A10})$$

The price associated with message  $m_N$  is

$$y(m_N) = \frac{\tilde{a}_N(\alpha) + a_{N-1}(\alpha)}{2} - \frac{(1 - \pi(m_N))b}{2}.$$

Using arguments identical to those given above,  $\tilde{a}_N(\alpha)$  satisfies

$$y(m_N) = 2a_{N-1}(\alpha) - y(m_{N-1}).$$

If  $N \geq 3$ , we may use expression (A10) and the definition of  $y(m_N)$  and  $y(m_{N-1})$ , to rewrite the no arbitrage condition as

$$\frac{\tilde{a}_N(\alpha) + a_{N-1}(\alpha)}{2} - \frac{(1 - \pi(m_N))b}{2} = 2a_{N-1}(\alpha) - \left( \frac{a_{N-1}(\alpha) + a_{N-2}(\alpha)}{2} - (1 - p)b \right),$$

or alternatively,

$$\tilde{a}_N(\alpha) = 2a_{N-1}(\alpha) - a_{N-2}(\alpha) + 2(1 - p)b + (1 - \pi(m_N))b. \quad (\text{A11})$$

Lemma 5  $a_{N-1}(\alpha) < \tilde{a}_N(\alpha) < a_N(\alpha)$ .

Proof. We may rewrite expression (A11) as

$$\tilde{a}_N(\alpha) = a_{N-1}(\alpha) + (a_{N-1}(\alpha) - a_{N-2}(\alpha) + 2(1-p)b + (1 - \pi(m_N))b)$$

and it directly follows that  $\tilde{a}_N(\alpha) > a_{N-1}(\alpha)$  since

$$(a_{N-1}(\alpha) - a_{N-2}(\alpha) + 2(1-p)b + (1 - \pi(m_N))b) > 0.$$

Define  $\Delta = \tilde{a}_N(\alpha) - a_{N-1}(\alpha)$ , then

$$\begin{aligned} \Delta &= a_{N-1}(\alpha) - a_{N-2}(\alpha) + 2(1-p)b + (1 - \pi(m_N))b \\ &> b + 2(1-p)b + (1 - \pi(m_N))b \\ &> b. \end{aligned}$$

Rewriting expression (A11) again and using the definition of  $a_N(\alpha)$  we obtain

$$\begin{aligned} \tilde{a}_N(\alpha) &= a_N(\alpha) - 4(1-p)b + 2(1-p)b + (1 - \pi(m_N))b \\ &= a_N(\alpha) - 2(1-p)b + (1 - \pi(m_N))b. \end{aligned} \tag{A12}$$

We claim  $2(1-p) > (1 - \pi(m_N))$ . To see this notice that

$$\begin{aligned} 2(1-p) - (1 - \pi(m_N)) &= \frac{2(1-p)\Delta + 2(1-p)^2b - (1-p)(\Delta + b)}{\Delta + (1-p)b} \\ &= \frac{(1-p)(\Delta + (1-2p)b)}{\Delta + (1-p)b} \\ &\geq \frac{(1-p)(2(1-p)b)}{\Delta + (1-p)b} \\ &> 0, \end{aligned}$$

and the claim follows.

This establishes the lemma.  $\square$

Lemma 6  $\frac{\partial \tilde{a}_N(\alpha)}{\partial \alpha} > 0$ .

Proof. Using equation (A11) and substituting in the definition of  $\pi(m_N)$ , we obtain

$$\tilde{a}_N = 2a_{N-1} - a_{N-2} + 2(1-p)b + \left( \frac{\tilde{a}_N - a_{N-1} + (1-p)b - p(\tilde{a}_N - a_{N-1})}{\tilde{a}_N - a_{N-1} + (1-p)b} \right) b,$$

where we have suppressed the dependence of the cut points on  $\alpha$  to ease the notational burden somewhat. We can rewrite this expression as

$$\begin{aligned} &(\tilde{a}_N - 2a_{N-1} + a_{N-2} - 2(1-p)b)(\tilde{a}_N - a_{N-1} + (1-p)b) \\ &- (\tilde{a}_N - a_{N-1} + (1-p)b - p(\tilde{a}_N - a_{N-1}))b \\ &= 0. \end{aligned}$$



We can differentiate this to obtain an expression for  $\frac{\partial \tilde{a}_N}{\partial \alpha}$ .

$$\frac{\partial \tilde{a}_N}{\partial \alpha} = -\frac{Z}{2\tilde{a}_N - 3a_{N-1}(\alpha) + a_{N-2}(\alpha) - 2(1-p)b}.$$

where

$$\begin{aligned} Z &= (-2a'_{N-1} + a'_{N-2})(\tilde{a}_N - a_{N-1}(\alpha) + (1-p)b) \\ &\quad - a'_{N-1}(\tilde{a}_N - 2a_{N-1} + a_{N-2} - 2(1-p)b) + a'_{N-1}(1-p)b, \end{aligned}$$

and the expressions  $a'_i$  denotes  $\frac{\partial a_i}{\partial \alpha}$ .

We establish that the denominator of  $\frac{\partial \tilde{a}_N}{\partial \alpha}$  is positive. To see this, substitute the expression in equation (A11) for  $\tilde{a}_N$ . Then the denominator becomes

$$a_{N-1} - a_{N-2} + 2(1-p)b + 2(1-\pi(m_N))b > 0$$

since  $a_{N-1} > a_{N-2}$ .

Next we establish that  $Z < 0$ . To see this, rewrite  $Z$  as

$$\begin{aligned} & [(-a'_{N-1})(\tilde{a}_N - a_{N-1} + (1-p)b)] + [(a'_{N-2} - a'_{N-1})(\tilde{a}_N - a_{N-1} + (1-p)b)] \\ & - [a'_{N-1}(\tilde{a}_N - 2a_{N-1} + a_{N-2} - 3(1-p)b)]. \end{aligned}$$

Notice that  $\tilde{a}_N - a_{N-1} + (1-p)b > 0$  by Lemma 5. Moreover, by Lemma 4,  $a'_{N-1} > 0$ ; hence the first bracketed expression is negative. To see that the second bracketed expression is negative, notice that an implication of Lemma 4 is that  $a'_i$  is increasing in  $i$ . Hence  $a'_{N-2} - a'_{N-1} < 0$ , so the second bracketed expression is negative. To see that the third bracketed expression is negative, substitute the expression for  $\tilde{a}_N$  given in equation (A11) into the expression  $\tilde{a}_N - 2a_{N-1} + a_{N-2} - 3(1-p)b$ . This yields  $(1-\pi(m_N))b - (1-p)b > 0$  since  $\pi(m_N) < p$ . Thus we have established that  $Z < 0$ .

Combining these facts, we have shown that  $\frac{\partial \tilde{a}_N}{\partial \alpha} > 0$ .  $\square$

Notice that by Lemmas 4, 5 and 6 for any  $N$  such that  $\tilde{a}_N(b) < 1$ , there exists exactly one  $\alpha$  where  $\tilde{a}_N(\alpha) = 1$  that constitutes an equilibrium consisting of  $N$  prices. Moreover, since  $a_{N-1}(\alpha)$  lies below  $\tilde{a}_N(\alpha)$ , it follows that if there exists an equilibrium consisting of  $N$  prices, there exists an equilibrium of  $i$  prices for  $i = 1, \dots, N-1$ .

Next we characterize the maximal number of stock prices,  $N(p, b)$ , arising in equilibrium as a function of  $p$  and  $b$ . When  $N \geq 2$ , we may use expression (A12) and expression (A9) to obtain expressions for  $a_{N-1}(\alpha)$ . We may write  $\tilde{a}_N(\alpha)$  as

$$\begin{aligned} \tilde{a}_N(\alpha) &= N\alpha + (N-1)(1-\pi(m_1))b + 2(N-1)^2(1-p)b \\ &\quad + (1-\pi(m_N))b - 2(1-p)b. \end{aligned}$$

Using the definitions of  $\pi(m_1)$  and  $\pi(m_N)$ , we can obtain a closed-form solution for  $\tilde{a}_N(\alpha)$ . When  $\alpha = b$ , this solution reduces to

$$\begin{aligned}\tilde{a}_N(b) &= \frac{1}{2}b(4N^2 - 10N + 7 - 4p(N-1)(N-2)) \\ &\quad + \frac{b}{2}\sqrt{(8p(N-1)(2Np - 4N - 4p + 7) + (4N - 5)^2)}.\end{aligned}$$

Since by Lemma 6,  $\tilde{a}_N(\alpha)$  is increasing in  $\alpha$ , it follows that if  $\tilde{a}_N(b) < 1$ , then there exists an equilibrium consisting of  $N$  prices. This equilibrium is obtained by finding  $\alpha$  that solves  $\tilde{a}_N(\alpha) = 1$ . Therefore, it follows that  $N(p, b)$  is the largest integer  $N$  satisfying

$$\begin{aligned}\frac{1}{2}b(4N^2 - 10N + 7 - 4p(N-1)(N-2)) \\ + \frac{1}{2}b\sqrt{(8p(N-1)(2Np - 4N - 4p + 7) + (4N - 5)^2)} \leq 1.\end{aligned}$$

Next, we establish that there is exactly one categorical ranking system that is equivalent to an equilibrium that is observed in a setting where investors are unable to infer an analyst's incentives from his report. From the properties of any equilibrium in this class, we know that for all  $\theta \in (a_{i-1}^0(N), a_i^0(N))$ ,  $y(\mu_0(\theta)) = y_i$ , and similarly for  $b$  types. Define  $M_i = \{m : y(m) = y_i\}$ . Then, for any  $m \in M_i$ , the stock price is  $y_i$ . Moreover, since the stock price is the same for all  $m \in M_i$ , then

$$\begin{aligned}y_i &= E(\theta|M_i) \\ &= \Pr(\beta = 0|M_i)E(\theta|\beta = 0, M_i) + \Pr(\beta = b|M_i)E(\theta|\beta = b, M_i)\end{aligned}$$

However, since the set of messages  $m \in M_i$  are only played in the interval  $\theta \in (a_{i-1}^\beta(N), a_i^\beta(N))$  and not elsewhere when the analyst has incentives  $\beta$ , it then follows that

$$\begin{aligned}y_i &= \left( \frac{p(a_i^0(N) - a_{i-1}^0(N))}{p(a_i^0(N) - a_{i-1}^0(N)) + (1-p)(a_i^b(N) - a_{i-1}^b(N))} \right) E[\theta|\theta \in [a_{i-1}^0(N), a_i^0(N)]] \\ &\quad + \left( \frac{(1-p)(a_i^b(N) - a_{i-1}^b(N))}{p(a_i^0(N) - a_{i-1}^0(N)) + (1-p)(a_i^b(N) - a_{i-1}^b(N))} \right) E[\theta|\theta \in [a_{i-1}^b(N), a_i^b(N)]] .\end{aligned}$$

But note that this is identical to the price induced in a categorical equity ranking system consisting of  $N$  categories. Hence, all equilibria observed in a setting where investors are unable to infer an analyst's incentives from his report are outcome equivalent to categorical ranking systems. ■

Proof of Proposition 4:

When  $b \geq \frac{2}{3+\sqrt{9-8p}}$ , it may be readily verified that  $N(p, b) = 1$ ; hence the only equilibrium when there is uncertainty about incentives is babbling. In contrast, full information disclosure is always an equilibrium when  $\beta = 0$  and this is common knowledge. Since this event occurs with strictly positive probability, informational efficiency is strictly greater under transparency. ■

Proof of Proposition 5:

When there is no uncertainty about the analyst's incentives  $\beta = (1-p)b$ . If  $b \geq \frac{1}{4(1-p)}$ , then  $\beta \geq \frac{1}{4}$  and Crawford and Sobel [1982] establish that the unique equilibrium is babbling. If  $b < \frac{1}{4(1-p)}$ , then  $\beta < \frac{1}{4}$  and Crawford and Sobel [1982] establish that there are at least two price responses in the most efficient equilibrium. When there is uncertainty about analyst type, if  $b < \frac{2}{3+\sqrt{9-8p}}$ , then there are at least two price responses in the most efficient equilibrium. If  $b \geq \frac{2}{3+\sqrt{9-8p}}$ , then the unique ranking system equilibrium is babbling.

To obtain part 1 of the proposition, observe that if  $p < \frac{1}{16}(9 - \sqrt{17})$ , then  $\frac{1}{4(1-p)} < \frac{2}{3+\sqrt{9-8p}}$  and there are at least two price responses with uncertainty compared to a single price response with no uncertainty. Hence, uncertainty improves informational efficiency. To obtain part 2 of the proposition, observe that if  $p > \frac{1}{16}(9 - \sqrt{17})$ , then  $\frac{1}{4(1-p)} > \frac{2}{3+\sqrt{9-8p}}$  and the reverse result holds. ■

Proof of Proposition 6:

We first show that when  $b \geq \frac{1}{p}(1 - \sqrt{1-p})$ , the unique categorical ranking system equilibrium is characterized by babbling. To see this notice that for  $N(p, b) = 1$ , we require that  $b > \frac{2}{3+\sqrt{9-8p}}$ . Next, notice that for all  $p$ ,  $\frac{1}{p}(1 - \sqrt{1-p}) > \frac{2}{3+\sqrt{9-8p}}$ . To see this, we rewrite this condition as  $(1 - \sqrt{1-p})(3 + \sqrt{9-8p}) - 2p > 0$ . Since the left-hand side of the above expression is increasing in  $p$ , and the left-hand side of the expression is zero when  $p = 0$ .

>From Proposition 2, we know that a semi-responsive equilibrium exists for  $b \geq \frac{1}{p}(1 - \sqrt{1-p})$ . It is trivial to show that the semi-responsive equilibrium is more informative than the categorical ranking system characterized by babbling. ■

Proof of Proposition 7:

First notice that if  $b > \frac{2}{3+\sqrt{9-8p}}$  then for all  $N \geq 2$ ,

$$\begin{aligned} & \frac{1}{2}b(4N^2 - 10N + 7 - 4p(N-1)(N-2)) + \\ & \frac{1}{2}b\sqrt{(8p(N-1)(2Np - 4N - 4p + 7) + (4N - 5)^2)} \\ & > 1. \end{aligned}$$

Therefore,  $N(p, b) = 1$ . Next notice that if  $b < \frac{1}{p}(1 - \sqrt{1-p})$ , then a semi-responsive equilibrium of the form specified in Proposition 2 is impossible. But since  $N(p, b) = 1$ , then the construction given in Proposition 2 is the only possible semi-responsive equilibrium construction. Therefore, a semi-responsive equilibrium does not exist. ■

**Proof of Proposition 8:**

For  $b \in \left(\frac{2}{3+\sqrt{9-8p}}, \frac{1}{p}(1 - \sqrt{1-p})\right)$  the proof follows directly from Propositions 4 and 7. For  $b \geq \frac{1}{p}(1 - \sqrt{1-p})$ , it may be readily verified that the informational efficiency under transparency is

$$\Phi_T = (1-p) \frac{1}{12}.$$

Under a semi-responsive equilibrium, informational efficiency is

$$\Phi_S = p \left( \int_{\theta^*}^1 (\theta - y)^2 d\theta \right) + (1-p) \int_0^1 (\theta - y)^2 d\theta,$$

where  $y = \frac{p(1-\theta^*)}{p(1-\theta^*)+(1-p)} \left(\frac{1+\theta^*}{2}\right) + \frac{1-p}{p(1-\theta^*)+(1-p)} \frac{1}{2}$ .

Hence

$$\Phi_S = \frac{1}{3} \left(\frac{1-p}{p}\right)^2 \left( \frac{\left(1 - \sqrt{1-p}\right)^3}{\sqrt{1-p}} + \frac{\sqrt{1-p}(-5p^2 + 24p - 24) + (2-p)(p^2 - 12p + 12)}{\left(-1 + \sqrt{1-p} + p\right)^2} \right).$$

We aim to show that  $\Phi_S - \Phi_T > 0$  for all  $p \in (0, 1)$ . Substituting in the expression for  $\Phi_S$  and  $\Phi_T$  yields

$$\begin{aligned} & 4(1-p)^2 \left( \left(1 - \sqrt{1-p}\right)^3 \left(-1 + \sqrt{1-p} + p\right)^2 \right. \\ & \left. + \sqrt{1-p} \left( \sqrt{1-p}(-5p^2 + 24p - 24) + (2-p)(p^2 - 12p + 12) \right) \right) \\ & - p^2(1-p) \left(-1 + \sqrt{1-p} + p\right)^2 \sqrt{1-p} \\ & = (1-p)^2 \sqrt{1-p} \left( 2(p^2 + 8p - 16) \sqrt{1-p} + (p-4)(p^2 + 6p - 8) \right) > 0. \end{aligned}$$

Since  $(1-p)^2 \sqrt{1-p} > 0$ , it remains to show that for all  $p \in (0, 1)$ ,  $\Psi > 0$  where

$$\Psi \equiv 2(p^2 + 8p - 16) \sqrt{1-p} + (p-4)(p^2 + 6p - 8).$$

Lemma 7 (see below) establishes that  $\frac{\partial \Psi}{\partial p} > 0$  for all  $p \in (0, 1)$ . Since  $\Psi = 0$  when  $p = 0$ , the proposition follows. ■

Lemma 7 Let  $\Psi \equiv 2(p^2 + 8p - 16)\sqrt{(1-p)} + (p-4)(p^2 + 6p - 8) > 0$ . For all  $p \in (0, 1)$ ,  $\frac{\partial \Psi}{\partial p} > 0$ .

Proof. We seek to establish that for all  $p \in (0, 1)$ ,  $\frac{d\Psi}{dp} > 0$ . Observe that

$$\frac{\partial \Psi}{\partial p} = \frac{-20p - 5p^2 + 32 + \sqrt{(1-p)}(p+4)(3p-8)}{\sqrt{(1-p)}}.$$

Since  $\sqrt{(1-p)} > 0$ , we need only show that the numerator of the above expression is positive. Denote the numerator of the above expression by  $\nu(p)$ . It may be immediately verified that  $\lim_{p \rightarrow 0} \nu(p) = 0$ . Further, for all  $p \in (0, 1)$ ,

$$\frac{\partial \nu(p)}{\partial p} = \frac{\left(-\frac{5}{2}\right)8\sqrt{(1-p)} + 4\sqrt{(1-p)}p + 3p^2 - 8}{\sqrt{(1-p)}} > 0.$$

To see that the above inequality holds, notice that the numerator equals zero at  $p = 0$  and it is routine to verify that the derivative of the numerator is positive for  $p \in (0, 1)$ . Thus, the claim is proved. ■

#### Proof of Proposition 9:

More favorable recommendations are issued more frequently than less favorable recommendations if and only if for all  $i = 1, 2, \dots, N-1$ ,  $a_{i+1}^0 - a_i^0 > a_i^0 - a_{i-1}^0$  and  $a_{i+1}^b - a_i^b > a_i^b - a_{i-1}^b$ . Suppose that the proposition does not hold. Consider the recommendations sent by analysts with aligned incentives, then there exists a  $k$  such that  $a_{k+1}^0 - a_k^0 \leq a_k^0 - a_{k-1}^0$ . Therefore  $a_{k+1}^0 + a_{k-1}^0 \leq 2a_k^0$ .

Since  $y_i$  is the convex combination of the intervals of  $\theta$  reported by analysts with aligned and misaligned incentives, it follows that  $y_{k+1} < \frac{a_{k+1}^0 + a_k^0}{2}$  and  $y_k < \frac{a_k^0 + a_{k-1}^0}{2}$ . Thus,  $y_{k+1} + y_k < \frac{a_{k+1}^0 + 2a_k^0 + a_{k-1}^0}{2} \leq 2a_k^0$ ; the strict inequality follows when the above relation is substituted into the expression. However, the no arbitrage condition  $y_{k+1} - a_k^0 = a_k^0 - y_k$  implies that  $y_{k+1} + y_k = 2a_k^0$ . This yields a contradiction.

The claim that non-extreme recommendations are issued with the same frequency follows directly from the fact that  $a_i^0 = a_i^b - b$  and  $\theta$  is uniformly distributed.

Finally, the claim of stochastic dominance follow from the fact that all intermediate recommendations have the same frequency and that the first and last cut points are shifted  $b$  distance to the left when  $\beta = b$ . ■

#### Proof of Proposition 10:

We prove the proposition in two steps. We begin by proving the first part of the proposition that  $y(m_1) - \frac{1}{2} < y(m_2) - \frac{1}{2} < 0 < y(m_3) - \frac{1}{2}$ . It has already been

established that  $a_1^0 < \frac{1}{3}$  and  $a_2^0 < \frac{2}{3}$ , which arises from the optimism in the analyst's report. Therefore,  $y(m_1) \leq \frac{1}{6}$  and  $y(m_2) \leq \frac{1}{2}$ . Since  $a_2^0 > 0$ , it follows that

$$\begin{aligned} y(m_3) &= \pi(m_3) \left( \frac{a_2^0 + 1}{2} \right) + (1 - \pi(m_3)) \left( \frac{a_2^0 - b + 1}{2} \right) \\ &> \pi(m_3) \left( \frac{a_2^0 + 1}{2} \right) + (1 - \pi(m_3)) \left( \frac{1}{2} \right) \\ &> \frac{1}{2}. \end{aligned}$$

We now prove the second part of the proposition that  $|y(m_1) - \frac{1}{2}| > |y(m_2) - \frac{1}{2}|$  and  $|y(m_1) - \frac{1}{2}| > |y(m_3) - \frac{1}{2}|$ . Consider the first part of the claim:  $y(m_1) < y(m_2)$  implies that  $\frac{1}{2} - y(m_1) > \frac{1}{2} - y(m_2)$ . Consider the second part of the claim. Since  $a_2^0 < \frac{2}{3}$  (from the optimism in the analyst's report), it follows that  $y(m_3) \leq \frac{a_2^0 + 1}{2} < \frac{5}{6}$ . This observation together with  $y(m_1) \leq \frac{1}{6}$  give the result that  $\frac{1}{2} - y(m_1) > y(m_3) - \frac{1}{2}$ , or  $1 > y(m_1) + y(m_3)$ . ■

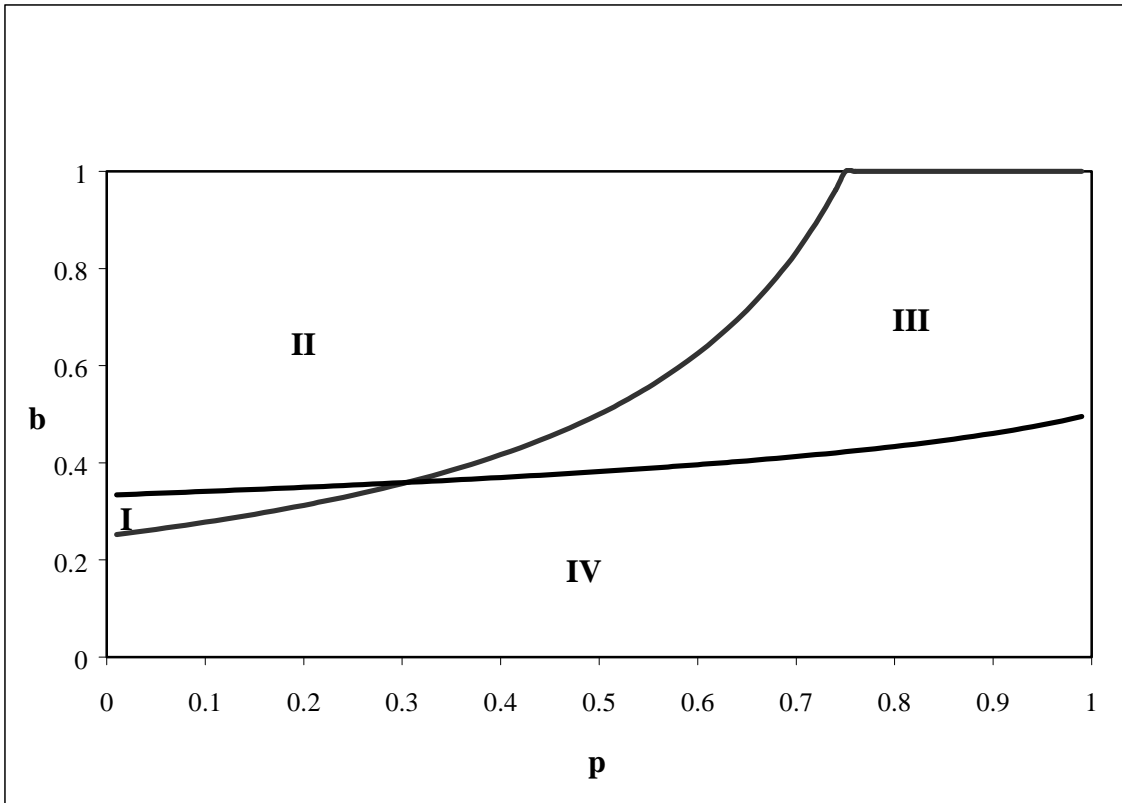
## References

- [1] Admati, A.R. and P. Pfleiderer. “A Monopolistic Market for Information.” *Journal of Economic Theory*, Vol. 39 (1986): 400-438.
- [2] Admati, A.R. and P. Pfleiderer. “Selling and Trading on Information in Financial Markets.” *American Economic Review, Papers and Proceedings*, Vol. 78, No. 2 (May 1988): 96-103.
- [3] Admati, A.R. and P. Pfleiderer. “Direct and Indirect Sale of Information.” *Econometrica*, Vol. 58, No. 4 (July 1990): 901-928.
- [4] Akerlof, G. “The Market for ‘Lemons’: Quality uncertainty and the market mechanism.” *Quarterly Journal of Economics*, Vol. 89 (August 1970): 488-500.
- [5] Barber, B., R. Lehavy, M. McNichols, and B. Trueman. “Can Investors Profit from the Prophets? Security Analyst Recommendations and Stock Returns.” *Journal of Finance*, Vol. LVI, No. 2 (April 2001): 531-63.
- [6] Benabou, R. and G. Laroque. “Using Privileged Information to Manipulate Markets: Insiders, Gurus, and Credibility.” *Quarterly Journal of Economics*, Vol. 107 (August 1992): 921-958.
- [7] Burns, J. “SEC Checks Brokerage Safeguards Against Leaks of Inside Information.” *The Wall Street Journal*, (April 28, 2000): interactive edition.
- [8] Crawford, V.P. and J. Sobel. “Strategic Information Transmission.” *Econometrica*, Vol. 50, No. 6 (November 1982): 1431-1451.
- [9] Dewatripont, M. and J. Tirole. “Advocates.” *Journal of Political Economy*, Vol. 107, No. 1 (1999): 1-39.
- [10] Dugar, A. and S. Nathan. “The Effect of Investment Banking Relationships on Financial Analyst’s Earnings Forecasts and Investment Recommendations.” *Contemporary Accounting Research*, Vol. 12, No. 1 (Fall 1995): 131-160.
- [11] Dugar, A. and S. Nathan. “Analysts’ Research Reports: Caveat Emptor.” *The Journal of Investing*, (Winter 1996): 13-22.
- [12] Elton, E.J. , M.J. Gruber, and S. Grossman. “Discrete Expectational Data and Portfolio Performance” *The Journal of Finance*, Vol. 41, No. 3 (July 1986): 699-713.
- [13] Francis, J. and L. Soffer. “The Relative Informativeness of Analysts’ Stock Recommendations and Earnings Forecast Revisions.” *Journal of Accounting Research*, Vol. 35, No. 2 (Autumn 1997): 193-211.

- [14] Hayes, R.M. “The Impact of Trading Commission Incentives on Analysts’ Stock Coverage Decisions and Earnings Forecasts.” *Journal of Accounting Research*, Vol. 36, No. 2 (Autumn 1998): 299-320.
- [15] Kim, O. and R.E. Verrecchia. “Market liquidity and volume around earnings announcements.” *Journal of Accounting and Economics*, Vol. 17, No. 1/2 (January 1994): 41-67.
- [16] Knox, N. “Wall Street’s secret code spoils investors’ aim.” *USA Today* (December 21, 2000): B1-2.
- [17] Laderman, J.M. “Wall Street’s Spin Game.” *Business Week*, (October 5, 1998): 148-156.
- [18] Lin, H.-W. and M.F. McNichols. “Underwriting Relationships, Analysts’ Earnings Forecasts and Investment Recommendations.” *Journal of Accounting and Economics*, Vol. 25, No. 1 (February 1998): 101-127.
- [19] McGee, S. “After Oracle Misfire, Wall Street’s Research is Blasted.” *The Wall Street Journal*, (December 11, 1997): C1, C22.
- [20] McNichols, M. and P.C. O’Brien. “Self-Selection and Analyst Coverage.” *Journal of Accounting Research*, Vol. 35, Supplement (1997): 167-199.
- [21] Michaely, R. and K. Womack. “Conflict of Interest and the Credibility of Underwriter Analyst Recommendations.” *The Review of Financial Studies*, Vol. 12, No. 4 (1999): 653-686.
- [22] Morris, S. “An Instrumental Theory of Political Correctness.” *Journal of Political Economy*, Vol. 109, No. 2 (2001): 231-265.
- [23] Nocera, J. “Who Really Moves the Market?” *Fortune*, Vol. 136, No. 8 (October 1997): 93-110.
- [24] Opdyke, J.D. “Guidelines Aim to Polish Analysts’ Image.” *The Wall Street Journal*, (June 13, 2001a): C1, C2.
- [25] Opdyke, J.D. “Many Analysts Found to Invest in Companies They Covered” *The Wall Street Journal*, (August 1, 2001b): C1, C10.
- [26] Osband, K. “Optimal Forecasting Incentives.” *Journal of Political Economy*, Vol. 97, No. 5 (1989): 1091-1112.
- [27] Ottaviani M. and P. Sorensen. “Professional Advice.” Working paper, University College London and University of Copenhagen (November 1999).



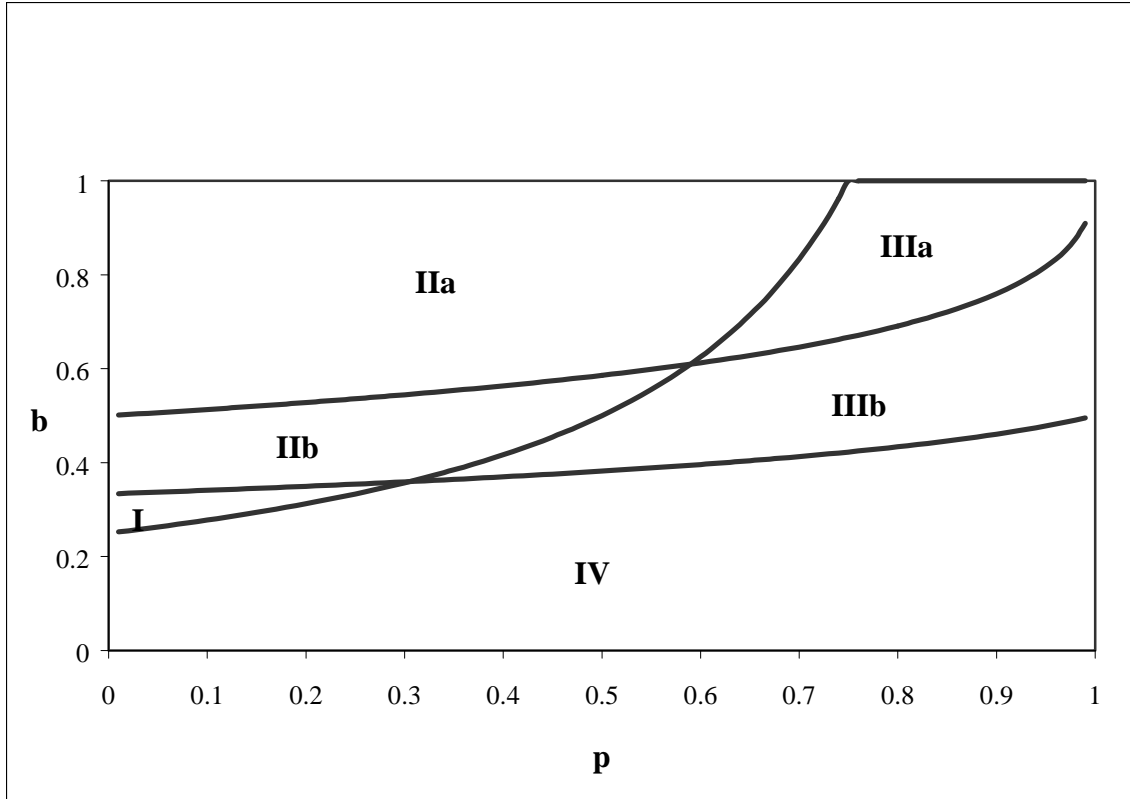
- [28] Schack, J. "Should analysts own stocks they cover?" *Institutional Investor*, (April 2001): 60-71.
- [29] Schipper, K. "Analysts' Forecasts." *Accounting Horizons*, Vol. 5, No. 4 (December 1991): 105-121.
- [30] Schroeder, M. "Congressman Presses for Remedy to Analysts' Conflicts of Interests." *The Wall Street Journal*, (June 14, 2001): A4.
- [31] Shilov, G.E. *Elementary Real and Complex Analysis*. New York: Dover Publications, Inc., 1996.
- [32] Shin, H.S. "News Management and the Value of Firms." *The RAND Journal of Economics*, Vol. 25, No. 1 (Spring 1994): 58-71.
- [33] Shin, H. S. "Disclosures and Asset Returns." mimeo, London School of Economic, February 2001.
- [34] Sobel, J. "A Theory of Credibility." *Review of Economic Studies*, Vol. 52 (1985): 557-573.
- [35] Stickel, S. "Reputation and Performance Among Security Analysts." *The Journal of Finance*, Vol. 47, No. 5 (December 1992): 1811-1836.
- [36] Trueman, B. "Analyst Forecasts and Herding Behavior." *The Review of Financial Studies*, Vol. 7, No. 1, (1994): 97-124.
- [37] Womack, K. "Do Brokerage Analysts' Recommendations Have Investment Value?" *The Journal of Finance*, Vol. 51, No. 1 (March 1996): 137-167.



**Figure 1**

**Effect of Uncertainty on Information Transmission.**

The effect of uncertainty on information transmission is as follows: In Region I, uncertainty about incentives improves informational efficiency. In Region II, stock reports are uninformative regardless of uncertainty. In Region III, uncertainty about incentives reduces informational efficiency. In Region IV, the effect of uncertainty about incentives on information transmission is ambiguous.



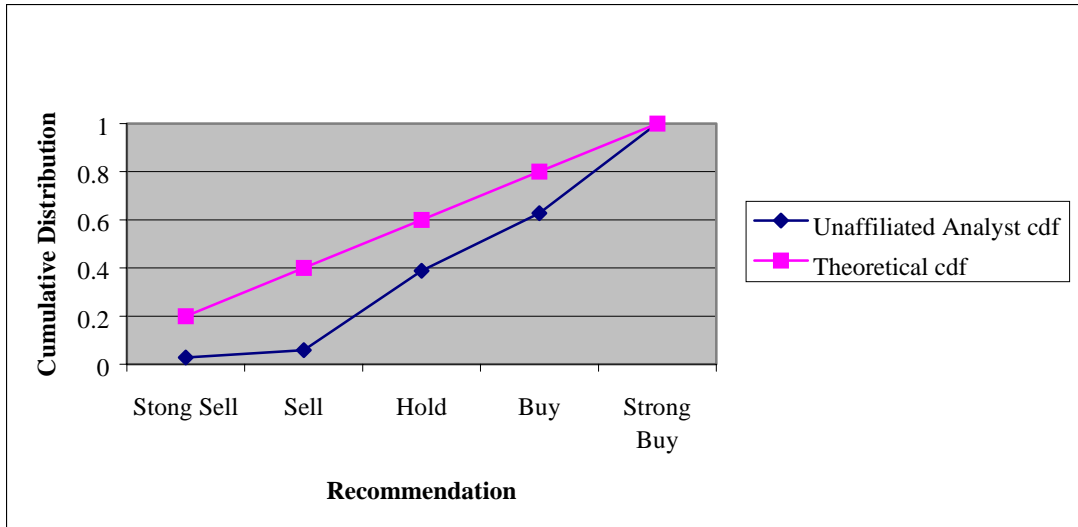
**Figure 2**

**Semi-Responsive Equilibria and Information Transmission.**

The effect of uncertainty on information transmission is as follows: In Region I, uncertainty about incentives improves informational efficiency. In Regions IIa and IIIa, semi-responsive equilibria exist and are more efficient than all other equilibria. In Region IIb, no information transmission occurs in any equilibrium. In Region IIIb, semi-responsive equilibria never exist; hence, uncertainty reduces information transmission. In Region IV, the effect of uncertainty about incentives on information transmission is ambiguous.



**Figure 3**  
**Comparison of Empirical Cumulative Distribution Functions of Lead Underwriters and Unaffiliated Analysts Prior to Equity Offering.**  
 Source: Data from Lin and McNichols [1998].



**Figure 4**  
**Comparison of Empirical Cumulative Distribution Functions of Unaffiliated Analysts with Theoretical Predictions in the Absence of Uncertainty.**  
 Source: Data from Lin and McNichols [1998].