When Do Markets Tip? A Cognitive Hierarchy Approach

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Abstract

The market structure of platform competition is critically important to managers and policy makers. While network effects in these markets predict concentrated industry structures, competitive effects and differentiation suggest the opposite. Standard theory offers little guidance—full rationality models have multiple equilibria with wildly varying market concentration. We relax full rationality in favor of a boundedly rational cognitive hierarchy model. Even small departures from full rationality allow sharp predictions—there is a unique equilibrium in every case. When participants single-home and platforms are vertically differentiated, a single dominant platform emerges. Multi-homing can give rise to a strong-weak market structure: One platform is accessed by all while the other is used as a backup by some agents. Horizontal differentiation, in contrast, leads to fragmentation. Differentiation, rather than competitive effects, mainly determines market structure.

Keywords: Platform competition, tipping, bounded rationality, cognitive hierarchy, vertical and horizontal differentiation

1 Introduction

Theorists have long been fascinated by coordination games. Part of this fascination stems from the fact that standard theory offers little guidance—it predicts that coordination will occur but is silent as to which outcome will be cooperated upon. These limitations are of little practical consequence if one is interested in thought experiments like the famous one proposed by Schelling about strangers trying to meet in New York City. But coordination problems loom large in many high-stakes business settings. Managers and researchers alike stand to benefit from a usable theory that goes beyond the non-predictions of the fully rational framework.

A coordination setting of particular importance concerns competition among online platforms, such as Google and Microsoft in search, what we call the competing matchmakers problem. Unlike standard coordination games where players are typically treated symmetrically, the competing matchmakers problem introduces additional complexity owing to the fact that participants may fundamentally differ from one another. For instance, in online auctions, the value of a platform depends not just on how many buyers it attracts, nor how many sellers, but rather on the combination of the two. Moreover, agents of a given type, such as men in an online dating context, care not just about the number of women on the site, but the number of other men as well, since each represents an additional competitor for a woman's heart. These competitive effects multiply the range of equilibrium possibilities. Indeed, in our baseline model, which nests many of the standard models of platform competition, the main conclusion to be derived from equilibrium under full rationality is that anything can happen: A single platform may dominate the market though the model is silent as to which platform or the market may be fragmented though, again, the model is silent as to who gets what share. For managers or regulators looking to theory as a guide, the full rationality model offers little in the way of help as to the correct business strategy to pursue or policy to implement.

However, full rationality represents an idealization at best for what motivates the choices of market participants. There is a growing body of evidence highlighting situations where seemingly inexplicable behavior (under full rationality) can be readily explained by incorporating limited cognition. One such situation includes behavior in laboratory studies of the famous p-beauty contest game. Unlike most coordination games, full rationality offers a precise prediction for the beauty contest–all subjects will choose the lowest possible action. Actual behavior in various different settings, however, is wildly at odds with this prediction: There is significant dispersion among choices, and few subjects, if any, select the equilibrium. Relaxing full rationality in favor of a model where players differ in their strategic sophistication as suggested by Nagel (1995), what has now come to be known as the cognitive hierarchies model (Camerer, Ho and Chong, 2004), nicely organizes the apparent jumble of data. In these models, non-strategic agents naïvely choose a pre-planned action without analyzing the payoffs. Strategic or sophisticated agents maximize their expected payoff given their beliefs which depend on their cognitive sophistication levels.

We begin with a simple observation: If cognitive hierarchy models are useful in organizing data from coordination games in the lab, perhaps these models might be fruitfully used to offer guidance in more applied coordination settings, such as the competing matchmakers problem. This analysis is the heart of our study.

An important criticism of bounded rationality models is that they open up a Pandora's box of possibilities where "anything goes" and therefore theory loses much of its predictive power. In our setting, the opposite conclusion obtains—while nearly any market share outcome is consistent with equilibrium under full rationality, cognitive hierarchy models produce unique equilibrium predictions. In some instances, these predictions coincide with a particular equilibrium under full rationality, in which case our models may be thought of as a kind of behavioral equilibrium refinement. In other settings, the predictions are completely novel. Thus, in addition to offering more precise predictions, these models are, in principle, empirically distinguishable from their fully rational counterparts.

Before proceeding to describe our main findings, a sketch of the setting is useful. There are N men and N women choosing between two online dating platforms. Platforms may differ in both the fees they charge and the efficacy of their matching processes. Both platforms share the common feature that there are benefits from scale—the larger the participant base at a given platform, the better the expected quality of the resulting matches. This effect pushes the market in the direction of concentration. There is, however, a countervailing competitive force. Men may prefer to be on a smaller platform so as to avoid having to compete as intensely with other men for the attention and affections of the women also located on the larger platform, likewise for women on the smaller platform. Provided that this competitive force is strong enough, platforms of wildly different sizes can coexist in equilibrium under the fully rational model.

When agents must choose a single platform, as would be the case for a seller of a unique object in an online auction, bounded rationality implies that a dominant platform will emerge. All strategic individuals will coordinate on the same platform—regardless of the strength of competitive effects. The particular platform chosen depends on the behavior of the non-strategic agents. In the case where these agents are totally uninformed about the details of the two platforms and choose randomly, the unique equilibrium prediction is that strategic agents will coordinate on the risk dominant platform, an equilibrium refinement first introduced by Harsanyi and Selten (1988) to select among equilibria in stag hunt type games. Roughly speaking, risk dominance implies that the safer platform, the one that better protects its clients from unexpected choices by others, will prevail. This is true even if the safer platform offers a worse experience than its rival when everyone coordinates on a single platform. The market structure of US online auctions, where eBay is the dominant (and the safest) platform, is consistent with this prediction.

Allowing agents to multi-home (i.e., choose to be on both platforms) adds to the set of equilibrium possibilities under full rationality, but still leads to herding under bounded rationality. Again the exact outcome depends on the choices of the naïve agents. Of particular interest is the situation where these agents simply avoid choosing at all and instead multi-home. In that case, strategic agents still coordinate on the single platform, but now select the Pareto dominant rather than the risk dominant choice. In effect, the caution of the naïve players insures the sophisticates against unexpected choices by others. As a consequence, they trade off safety for surplus in coordinating on the more cost-effective platform. Here again bounded rationality acts as a kind of equilibrium refinement, though importantly the refinement selected depends on the particulars of the institutional setting.

When naïve agents randomize their behavior, equilibrium takes a different form: Relatively unsophisticated strategic agents multi-home while sophisticates opt for the Pareto dominant platform exclusively. This equilibrium shares some of the features of credit card markets. While nearly all US credit card holders have a Visa/MasterCard in their wallet, some also carry a Discover card in addition. But the situation is rarely reversed—few people "single home" using Discover. There is no analogous equilibrium under full rationality. Here the boundedly rational model suggests qualitatively different, and more realistic, behavior.

All of these results suggest that competitive forces alone are not sufficient to prevent a dominant platform from emerging. In every case, one of the platforms is accessed by all of the strategic agents (though some may also access a second platform as a kind of backup). While this is a sharp prediction, it is clearly at odds with some market structures arising in real world online markets. For instance, the market for online dating in the US is highly fragmented.

To better understand this phenomenon, we return to the single homing case but now add horizontal differentiation to the mix. Clearly, this provides an additional force allowing both platforms to share the market. Under full rationality, there is an intuitive equilibrium where each agent chooses his or her (horizontally) preferred platform, and the market is split. But there are many other equilibrium possibilities including the emergence of a single dominant platform or a “backwards” equilibrium where every agent chooses her less preferred platform. Relaxing full rationality cuts through the clutter. If naïve agents are weakly more likely to choose their horizontally preferred platform, then the unique equilibrium corresponds to the intuitive case where every agent chooses her (horizontally) preferred platform, and the market is split. The US online dating market is extremely fragmented and horizontally differentiated. Leading sites such as JDate (restricted to Jewish singles), ChristianMingle (restricted to religious Christian singles), and others are all consistent with this emphasis.

Thus the boundedly rational model can account for the variety of market concentrations seen in US platform markets: the dominance of eBay in auctions, the strong-weak division of Visa/MasterCard versus Discover in credit cards, and the severe fragmentation in online dating sites. Moreover, these predictions do not demand that large swaths of the population be naïve. Even arbitrarily small departures from full rationality dramatically sharpen equilibrium predictions in the competing matchmakers problem. The equilibrium multiplicity endemic to coordination games vanishes. More importantly, bounded rationality models highlight the key structural components determining market share. In particular, when platforms are primarily vertically differentiated, it is always the case that one of the platforms is patronized by all agents (though some of these might also visit the rival under multi-homing). This conclusion remains valid regardless of the strength of competitive effects. When platforms are primarily horizontally differentiated, markets are fragmented, even if competitive effects are small or absent altogether.

The model also offers important insights for managers. While the usual business strategy in these markets is to focus mainly on platform quality, our results suggest the critical strategic importance of other considerations. In single-homing contexts, reducing the risk to platform adopters is key: aspects such as 24/7 uptime, backup, and security should be primary considerations. In multi-homing contexts, pricing is critical. The model predicts that a higher quality platform will still falter if it does not pass along enough surplus to its users. Thus, even for successful platforms, monetization at the expense of consumer experience can still lead to grief.

The remainder of the paper proceeds as follows: We conclude this section by placing our results in the context of the extant literature. Section 2 sketches the model. Section 3 characterizes equilibrium in the baseline single-homing model under full and bounded rationality. In section 4, we add multi-homing to the model and explore how this changes choice behavior and market structure. Section 5 adds horizontal differentiation to the model and identifies conditions where platforms coexist. Section 6 studies a dynamic version of the model and shows that our earlier conclusions are not fundamentally altered by this amendment. Finally, section 7 concludes. Some of the proofs are discussed in the main body of the paper before the formal propositions are presented; the rest are contained in Appendix A.

Related Literature

The literature on platform competition has grown in size and importance with the maturation of the Internet. Early studies (see Katz & Shapiro 1994 for a survey) mainly emphasize the concentrating force of network effects. More recently, Ellison and Fudenberg (2003) as well as Ellison, Fudenberg, and Mobius (2004) highlight the power of competitive effects—

competition from agents on the same side of the market—to check network effects and lead to equilibrium coexistence.[1] We relax full rationality and show that the power of competitive effects become greatly attenuated.

A separate strand of the literature studies endogenous pricing decisions by platforms.[2] This literature mostly assumes that competitive effects are absent, platforms are horizontally differentiated and consumers single-home. The main findings characterize how optimal platform pricing varies with demand elasticities on each side of the market. We contribute to this literature by offering a model where scale, differentiation, and competitive effects are all present and where multi-homing is possible. While we mostly abstract away from optimal pricing decisions, Appendix B studies the case where pricing is endogenous.

The empirical literature of platform competition is less well developed. Inspired by David's (1985) influential study, much of this literature examines the QWERTY phenomenon—the possibility that an interior platform might prevail owing to path dependence. Most studies find little evidence of this.[3] Our paper contributes a theoretical rationale for the dearth of QWERTY outcomes.

There is also a small experimental literature on platform competition. In a companion paper, Hossain, Minor, and Morgan (2011) perform laboratory experiments in a single-homing setting using exactly the model outlined below. Unlike the present paper, their main concerns are to use empirical methods to examine the competing predictions of the fully rational model. Moreover, their setting is dynamic—the same group of subjects repeatedly participate in the platform competition game. Their main findings are, however, largely consistent with the predictions under cognitive hierarchies. When platforms are primarily vertically differentiated, the market converges to a single platform, which is the same across groups. Despite the presence of multiple equilibria in the fully rational model, there is remarkable consistency in behavior across subjects. When platforms are primarily horizontally differentiated, the market converse to coexistence where each agent chooses his or her preferred platform.

Ho, Lim, and Camerer (2006) argue that new insights can be gained about firm strategy and market performance by incorporating consumer psychology into choice models.[4] The framework we use, cognitive hierarchies, draws heavily on Camerer, Ho, and Chong (2004), who generalized and expanded upon Nagel's (1995) specification to settings outside the

[1] Ambrus and Argenziano (2009) note that consumers must be non-negligible in size for the competitive effects identified in these to papers to have force.

[2] See, e.g. Caillaud and Jullien (2001, 2003), Rochet and Tirole (2003), Armstrong (2006), Carrillo and Tan (2006) and Damiano and Li (2008).

[3] See, e.g., Liebowitz and Margolis (1990 and 1994), Tellis, Yin, and Niraj (2009), as well as experiments by Hossain and Morgan (2009).

[4] For example, Amaldoss, Bettman, and Payne (2008) show, using laboratory experiments, that behavioral biases by economic agents can, in fact, facilitate coordination.

beauty contest game.[5] This model has proved extremely useful in organizing lab data across a variety of coordination settings.[6] It has also been used successfully in empirical settings including technology adoption by Internet service providers (Goldfarb and Yang, 2009), entry in local telephone markets (Goldfarb and Xiao, 2011), and decision-making by movie distributors (Brown, Camerer, and Lovallo, 2012a) and moviegoers (Brown, Camerer, and Lovallo, 2012b). Ostling *et al.* (2011) apply this model to study the Swedish lottery game LUPI using both field and experimental data. Our paper contributes to this literature by treating the cognitive hierarchy model as an essential tool in applied modeling in more complex settings. The paper also, thus, contributes to the emerging field of applying bounded rationality in industrial organization.[7]

While the cognitive hierarchies framework might be seen as simply a set of principles for organizing data, it also appears to capture fundamental aspects of primate cognition. In fMRI studies, Bhatt and Camerer (2005) find neurological evidence consistent with self-referential thinking models, including cognitive hierarchies. Dorris and Glimcher (2004) find striking similarities between human and monkey behavior in work-shirk games—for both species, shirk rates are consistent with cognitive hierarchies and inconsistent with predictions under full rationality. More broadly, Camerer (2009) offers a survey indicating the mounting evidence for neural underpinnings of behavioral choice models, including our framework.

2 The Model

Consider a market where there are two competing platforms labeled A and B, serving two types of agents. In terms of exposition, we shall think of these platforms as competing matchmakers and shall refer to the agents as *women* and *men*. There are exactly N of each type of agent. The role of the platform is to match agents of one type with agents of the other, i.e., to match men with women. To perform this service, each platform i charges an up-front access fee $p_i > 0$ where $i \in \{A, B\}$.

All agents simultaneously decide which platform to access. For the moment, we assume that only one of the two platforms may be chosen (i.e., no multi-homing) though we relax this assumption later. We also assume that the benefits and fees of the platforms are commonly known and that all agents prefer to participate rather than opting out entirely.

Payoffs for each agent consist of gross payoffs from the match technology of the platform less the cost of the access fee. Let $u_i\left(n_{i1}, n_{i2}\right)$ denote the gross payoff from accessing platform

[5]Technically, our model slightly generalizes Camerer, Ho, and Chong by allowing the distribution of cognitive types to be arbitrary rather than Poisson distributed.

[6]See, e.g. Stahl and Wilson (1995), Camerer, Ho, and Chong (2004), as well as Costa-Gomes and Crawford (2006).

[7]See Ellison (2006) for an excellent survey.

i when n_{i1} agents of the same type and n_{i2} agents of the opposite type access the platform. For instance, when n_{i1} women and n_{i2} men access platform i, then each woman enjoys a gross payoff of $u_i(n_{i1}, n_{i2})$ and a net payoff of $u_i(n_{i1}, n_{i2}) - p_i$. Similarly, the payoffs to a man when n_{i1} men and n_{i2} women accessed site i would be identical.

We focus on the agents' platform choices rather than the strategy of the platforms themselves; thus, we restrict attention to non–discriminatory pricing schemes where the access fee for men and women is the same. Platforms can also charge non-discriminatory fees for a successful match, which are accounted for in the gross payoff functions. We assume that platforms exhibit standard competition and network effects. Formally,

Assumption 1 (market size effect): Gross payoffs are increasing in the number of agents of the opposite type. For all n_1 and n_2, $u_i(n_1, n_2 + 1) > u_i(n_1, n_2)$.

Assumption 2 (market impact effect): Gross payoffs are decreasing in the number of agents of own type. For all n_1 and $n_2 > 0$, $u_i(n_1, n_2) > u_i(n_1 + 1, n_2)$.

Assumption 3 (positive network externality): Gross payoffs increase when the number of agents of both types on the platform increase equally. For all n_1 and n_2, $u_i(n_1 + 1, n_2 + 1) > u_i(n_1, n_2)$.[8]

Assumption 4: For all $n \in \{1, 2, \ldots, N\}$ and $i \in \{A, B\}$, $u_i(n, 0) = 0$.

We maintain these assumptions throughout the paper. Assumptions 1 and 2 guarantee that women benefit from a greater choice of men on the platform and suffer from more competing women (and vice-versa for men.) Assumption 3 guarantees that, all else equal, a larger platform is preferred to a smaller platform. Assumption 4 says that women are unaffected by competition when there are no men on the platform. We normalize this payoff to zero for simplicity. These assumptions do not provide a complete ranking of the gross payoffs for all possible platform choices by the agents. Indeed, the model is flexible enough to accommodate most models of competing platforms in the extant literature.

Finally, to rule out knife-edge or pathological cases, we restrict attention to generic net payoffs, so that it is not the case that for all $i,j \in \{A, B\}$ and $n_1, n_2 \in \{1, 2, \ldots, N\}$, $u_i(n_1, n_2) - p_i = u_j(n_1, n_2) - p_j$ and assume that access fees are such that agents make positive net payoffs if all of them coordinate on a single platform, i.e., $u_i(N, N) - p_i > 0$.

3 Equilibrium

We now examine equilibria arising in the model under full rationality, restricting attention to pure strategy Nash equilibria. We then relax this assumption, allowing for differences in the strategic sophistication of agents, using the cognitive hierarchy framework proposed

[8] Our results are unchanged if we recast Assumption 3 as multiplicative. Specifically, it may be replaced by the assumption that, for all $(n_1, n_2) >> 0$, $u_i(kn_1, kn_2) > u_i(n_1, n_2)$ for $k > 1$.

by Camerer, Ho, and Chong (2004). As mentioned above, this model has proved useful in organizing data in a wide array of coordination games and is consistent with neurological evidence regarding choice behavior. Later, we provide a detailed description of how the cognitive hierarchy model works and what aspects of bounded rationality it is meant to capture. Our main result is to show that, while a wide array of market share distributions can arise as equilibria under full rationality, adding even a vanishingly small fraction of strategically unsophisticated agents yields a unique prediction—a single dominant platform is selected by all strategic types.

Full Rationality

We first characterize equilibria in the model under the usual assumption of full rationality. Recall that the gender ratio of the market as a whole is 1 to 1.[9] The following lemma shows that in any Nash equilibrium, the gender ratio of agents at each platform is the same as that of the market as a whole. Formally,

Lemma 1 *In any Nash equilibrium, the same number of agents of each type select a given platform.*

While the result is intuitive for the case where both platforms are identical, Lemma 1 shows that, despite asymmetries across platforms, all equilibria remain symmetric in the sense that the gender ratio is the same across platforms. To see this, suppose more women than men join platform A in equilibrium. This implies that the fee difference $p_B - p_A$ is large enough to offset any gain in payoff a woman located at platform A would enjoy from switching to the platform B, which has relatively more men. This, however, implies that a man on platform B would benefit from switching for the same reasons.

The scale effect contained in Assumption 3 implies that these markets are, in a sense, natural monopolies. All else equal, agents benefit from coordination on a single platform. Formally, we say that the market has *tipped* when only one platform is active, i.e., all agents opt for a single platform. When both platforms are active, we say that they *coexist.*

The next proposition shows that tipping is always an equilibrium although it is silent as to which platform will be the "winning" one. To see this, suppose that, in equilibrium, all agents locate on platform i and earn payoffs $u_i(N, N) - p_i > 0$, where the inequality follows by assumption. Now, if an agent deviates to platform j, she earns $u_j(1, 0) - p_j < 0$ since $u_j(1, 0) = 0$ and $p_j > 0$; therefore such a deviation is not profitable. It then follows immediately that

[9] Assuming equal numbers of agents facilitates Lemma 1 below, which considerably simplifies the equilibrium characterization. The qualitative features of equilibrium under full rationality—multiplicity and the possibility of both tipping and coexistence—hold more generally. Detailed analysis of the case where the gender ratio is not equal to one is available upon request from the authors.

Proposition 1 *Tipping to either platform is an equilibrium. Formally, it is a Nash equilibrium for all agents to select a single platform $i \in \{A, B\}$.*

One might think that something like Assumption 3 is *necessary* for tipping to comprise a Nash equilibrium. This is not the case. Even if platforms exhibited diseconomies of scale, Proposition 1 would still hold owing to Assumption 4 and the fact that coordinating on a single platform yields non-negative surplus. The reason is that, unlike most standard coordination games, deviations by *both* types of agents are needed to unlock surplus from the inactive platform.

While Assumption 3 is not necessary for tipping, Assumption 2 is required for coexistence (in a pure strategy equilibrium). To see this, define the magnitude of the market impact effect in market i with n agents of each type to be

$$\delta_{i,n} = u_i(n, n) - u_i(n+1, n).$$

Consider an equilibrium where n agents of each type go to platform A with the remainder going to platform B. The difference in equilibrium utility for agents going to platform A versus those going to platform B is

$$\Delta U_n = u_A(n, n) - u_B(N-n, N-n) - (p_A - p_B).$$

Suppose that $\Delta U_n \geq 0$. Clearly, agents located on A cannot profitably deviate to B since their payoffs are less than $u_B(N-n, N-n) - p_B$ (owing to positive market impact effects). Thus, we only need to show that agents located on B have no wish to deviate. Incentive compatibility requires that

$$u_B(N-n, N-n) - p_B \geq u_A(n+1, n) - p_A.$$

Subtracting $u_A(n, n) - p_A$ from both sides of the inequality, we obtain

$$-\Delta U_n \geq -\delta_{A,n}$$

or, equivalently, that market impact effects for platform A must be sufficiently large, i.e., $\delta_{A,n} \geq \Delta U_n$.

The case where $\Delta U_n < 0$ yields the analogous condition that the market impact effects for platform B must be sufficiently large, i.e., $\delta_{B,N-n} \geq -\Delta U_n$. To summarize, we have shown

Proposition 2 *Any market share split is consistent with equilibrium provided market impact effects are sufficiently large.*

Formally, n agents of each type locating on platform A with the remainder choosing platform B comprise a coexisting equilibrium provided that: (1) $\delta_{A,n} \geq \Delta U_n$ when $\Delta U_n \geq 0$ and (2) $\delta_{B,N-n} \geq -\Delta U_n$ when $\Delta U_n < 0$.

A different way to see that market impact effects are necessary for coexisting equilibria to arise is to consider the case where the two platforms are identical. Suppose that platform A enjoys a smaller market share than platform B. In that case, the net payoff to men and women located on A is smaller than that enjoyed by their counterparts on B. What prevents a man on A from profitably deviating is that, were he to switch, the additional competition among men on B would lower the payoffs of men on that platform through the market impact effect. If this effect is large enough to overwhelm the gains from scale offered by B, then such a deviation is not profitable. Essentially, this is the force leading to equilibrium coexistence in the model of Ellison and Fudenberg (2003).

One might worry that coexisting equilibria arising in this model are "knife-edge" in the sense that any small perturbation in agent strategies leads to tipping. This is not the case. Generically, the coexisting equilibria we identify above are *strict Nash equilibria* and hence are robust to small perturbations. The following example illustrates how the model works.

Example 1 *Suppose the matching technology is such that when a man joins a platform that has at least as many female participants as male participants (including himself), the market impact effect is relatively small. However, when there are fewer females than males on the platform, competition between men becomes more acute leading to a larger market impact effect. A simple gross payoff function based on this matching technology can be described by:*

$$u_A(n_1, n_2) = u_B(n_1, n_2) = \begin{cases} 100 \times \max\left\{\frac{n_2}{N} - \gamma_1 \frac{n_1 - 1}{N-1}, 0\right\} & \text{if } n_1 \leq n_2 \\ 100 \times \max\left\{\frac{n_2}{N} - \gamma_2 \frac{n_1 - 1}{N-1}, 0\right\} & \text{if } n_1 > n_2 \end{cases}$$

where $0 < \gamma_1 < \gamma_2 < \frac{N-1}{N}$ *and* $n_1 \geq 1$. *Here,* γ_1 *and* γ_2 *represent the magnitudes of the market impact effects. This market satisfies all of the assumptions above. Women gain with an increase in the fraction of men located on a given platform. They lose in proportion to the fraction of women on the same platform, and the effect is more pronounced when women on the platform outnumber men. When* $N = 10$, $\gamma_1 = 0.05$, $\gamma_2 = 0.6$, $p_A = 2$ *and* $p_B = 0.01$, *there are five coexisting equilibria of this market consisting of equal market shares, a 60-40 split in favor of either platform, and a 70-30 split in favor of either platform. The remaining equilibria consist of tipping to either platform.*

Another worry is that coexistence is an artifact of the assumption of exogenous access fees. One might reason that a platform with higher match quality could simply compete Bertrand style in access fees and thereby capture the entire market. The flaw in this intuition is that a platform is only valuable to the extent that it can induce *multilateral* deviations. Regardless of price, it does not pay to switch to a higher quality platform where few other agents are present. In the Appendix, we formalize this intuition and show that coexistence is consistent with equilibrium even when fees are endogenous.

Cognitive Hierarchy

The previous analysis relied on the full rationality of market participants. In particular, the choices made by each agent depend on expectations about the choices made by all other agents, which in turn depend on expectations of expectations, and so on. Clearly, this level of sophistication is an idealization at best—some participants are likely to be more naïve and make choices without fully reflecting on the selections of other agents. To capture this idea, we use a model of cognitive hierarchies. Cognitive hierarchy models are meant to capture heterogeneities in the strategic sophistication of participants in the market. Specifically, some fraction of agents are non-strategic. Their choices are determined by rules or heuristics and made irrespective of beliefs about the choices of others. Other agents have limited strategic reasoning. Their expectations are formed based on (flawed) models of the choice behavior of all other agents.

Formally, each agent has a cognitive sophistication level of $l \in \left\{0, \ldots, \overline{L}\right\}$. For simplicity, we assume that the true distribution of the levels of cognitive sophistication is the same for women and men. An agent is of cognitive sophistication level l with probability $f(l) > 0$ where $\sum_{l=0}^{\overline{L}} f(l) = 1$. Note that we impose no additional structure on f. As such, our results hold for a broad class of distributions including the normalized Poisson (with any finite value of its parameter τ), which has frequently been used to analyze experimental data.

Level-0 agents are non-strategic. They make no inference about the behavior of others around them to determine the correct choice and instead rely on rules or heuristics to guide their choices. Rather than imposing a specific heuristic for these types, we remain agnostic about their strategy and assume that these agents choose platform A with probability $\lambda_A \in [0,1]$ and platform B with probability $\lambda_B = 1 - \lambda_A$. Agents of level $k \geq 1$ believe that all others have sophistication levels strictly below k and best respond accordingly. Formally, a level-k woman assumes that all N men and the remaining $N-1$ women are of level $k-1$ or below. Moreover, she perceives that the population fraction of level l is $f(l) / \sum_{t=0}^{k-1} f(t)$ for $l \leq k-1$ and is 0 for $l \geq k$. A level-k man has analogous beliefs about others. Even though these agents are strategic, their beliefs about the strategic sophistication of the population are incorrect, instead reflecting a form of overconfidence. Each agent perceives that he or she is more strategically sophisticated than others making choices.

To analyze the game, the following notation proves helpful: Let $U_i(\lambda)$ denote the expected gross payoff to an agent from choosing platform i when all other agents independently select this platform with probability λ. That is,

$$U_i(\lambda) = \sum_{s=1}^{N} \sum_{t=0}^{N} \binom{N-1}{s-1} \binom{N}{t} \lambda^{s-1+t} (1-\lambda)^{2N-s-t} u_i(s,t). \tag{1}$$

Clearly, $U_i(\lambda)$ is continuously differentiable, $U_i(\lambda) > U_i(0)$ for all $\lambda \in (0,1]$, and $U_i'(0) > 0$. To make cross-platform comparisons with respect to λ requires some additional structure on payoffs. To ensure that payoffs satisfy the familiar single-crossing condition with respect to λ, it suffices to ensure that the payoffs for each platform are single-peaked in λ. Formally,

Assumption 5: If $U_i'\left(\widehat{\lambda}\right) = 0$ then $U_i'(\lambda) < 0$ for all $\lambda > \widehat{\lambda}$.

Assumption 5 guarantees that there is a unique λ^* solving

$$U_i(\lambda^*) - p_i = U_j(1-\lambda^*) - p_j.$$

Moreover, for all $\lambda' > \lambda^*$,

$$U_i(\lambda') - p_i > U_j(1-\lambda') - p_j$$

for $i \in \{A, B\}$, which are the usual single-crossing conditions.

With this notation in hand, let us consider the best responses for each agent. From the perspective of a level-1 agent, all other agents are selecting platforms at random, thus, her expected payoff from choosing platform i is simply $U_i(\lambda_i) - p_i$.[10] Naturally, such an agent chooses platform i over j if and only if

$$U_i(\lambda_i) - p_i > U_j(1-\lambda_i) - p_j \tag{2}$$

Level-1 agents choose platform i provided there is a sufficiently high chance of encountering level-0 agents there. A level-2 agent believes that all other agents go to platform i with probability $\frac{\lambda_i f(0)+f(1)}{f(0)+f(1)} > \lambda_i$ as she believes all other agents are of level 0 or 1. That is, she believes a larger fraction of agents are choosing platform i than does a level-1 agent. The single-crossing property implies that she too prefers platform i to j. (Notice that absent Assumption 5, one might encounter the rather implausible situation where an agent who is convinced that platform i enjoys a higher market share is less likely to choose it compared to an agent who believes that i enjoys a smaller market share.) The same logic obtains for agents with ever higher levels of sophistication. As a consequence, the market will tip to the platform satisfying equation (2). Formally,

Proposition 3 *Under cognitive hierarchy, all agents with sophistication level $l > 1$ choose the same platform as level-1 agents. Level-1 agents choose the platform i satisfying equation* (2).

Like many models with behavioral types, the choices of level-0 types profoundly influence the decisions of more sophisticated agents, even when level-0 agents are relatively scarce in the population as a whole. Of particular interest is the situation where level-0 agents choose either platform with equal probability, i.e. $\lambda_i = \frac{1}{2}$. In that case, there is a useful link between cognitive hierarchy and the risk dominance notion of equilibrium selection first

[10] We ignore the non-generic case where λ_i happens to leave level-1 types indifferent between the two platforms.

introduced by Harsanyi and Selten (1988). Harsanyi and Selten were motivated by the game stag hunt. It is well-known that there are two pure strategy equilibria in stag hunt, one corresponding to the "safe" strategy of hunting hare and the other corresponding to the "risky" strategy of hunting stag. Of course, in equilibrium, neither strategy is truly risky in that the behavior of the others is perfectly anticipated. Yet, in a real sense, hunting stag is riskier—an agent's payoff could be lower if the other player chose an unexpected action. Harsanyi and Selten sought to capture this notion through the risk dominance equilibrium refinement. Specifically, given two pure equilibria, E and E' of a bi-matrix game, equilibrium E is said to be *risk dominant* if the expected payoff to each agent is higher under E than under E' given random (equiprobable) play on the part of others. So long as the downside of hunting stag is sufficiently large, hunting hare is the risk dominant equilibrium in the game. The same holds true in our setting and hence:

Remark 1 *Suppose that $\lambda_i = \frac{1}{2}$ and $f(0) \to 0$, then the unique equilibrium under cognitive hierarchy converges to the risk dominant equilibrium.*

While the cognitive hierarchy outcome corresponds to risk dominance under the specific assumption of equiprobable choice behavior by level-0 agents, the model predicts herding—all more sophisticated agents will mimic the choices of level-1 agents—regardless of the particular specification of level-0 behavior. Indeed, this herding phenomenon is quite robust. While we derived the herding effect using the Camerer-Ho-Chong specification of beliefs in the cognitive hierarchy model, this property is shared by all other specifications used in this literature. For instance, in the Nagel-Stahl-Wilson specification, a level-k agent believes that all other agents have cognitive sophistication level of $k-1$. Obviously, the behavior of level 1 agents is unchanged under this specification. Naturally, all other cognitive types will choose the same platform as level-1 agents. Indeed, Assumption 5 is no longer needed for this specification of beliefs.

Notice also that the results are *independent* of the distribution of strategic types. Even if level-0 types are rare, strategic agents (who in past laboratory studies accounted for most of the population) choose the risk dominant platform. The herding result is also robust to relaxing the assumption that the gross payoffs treat men and women symmetrically. So long as the expected payoff maximizing platform is the same for both types of agents, the cognitive hierarchy model will again predict a unique equilibrium where all agents will herd on the choice of the level-1 agents. Likewise, the result straightforwardly extends to the case where there are more than two competing platforms.

From a managerial perspective, this suggests that an emphasis on safety is called for as agents are likely to choose the safer platform over a high-return but high-variance platform. This is illustrated in activities of several major platforms. For instance, eBay implemented a number of policies to protect sellers and buyers against non-performance by the counterparty. Microsoft emphasizes the security of its operating systems (albeit with mixed results). Facebook likewise emphasizes data security, privacy, and 24/7 uptime.

4 Multi-Homing

The previous section follows much of the literature on platform competition by restricting agents to choose a single platform. In practice, however, there are many circumstances where such an assumption is patently unrealistic. For instance, if one were interested in applying the model to study credit card markets, assuming that merchants only accept a single type of card or that consumers only have one card in their wallets is clearly at odds with reality. One reason for restricting attention to the single-homing case is tractability. As we saw, equilibrium multiplicity was a serious problem in the fully rational model even under single-homing. The analysis only grows more complex with the addition of multi-homing. A second reason for such a restriction is that the single-homing assumption might be innocuous—the analysis may be fundamentally unchanged despite the added complexity.

In this section, we amend the model to allow for multi-homing. Formally, each agent's choice set now consists of $\{A, B, AB\}$ where AB denotes subscribing to both platforms. We show that, in the fully rational case, this additional option is not innocuous—the set of co-existing equilibria change when multi-homing is permitted. However, this added complexity does not change the simplicity of the cognitive hierarchy approach. There remains a unique equilibrium, but the character of the equilibrium does change. In particular, even when level-0 agents choose each available option with equal probability, it is no longer the case that the risk dominant platform prevails in the market. Indeed, the addition of multi-homing tends to favor the "better" platform in the sense of Pareto dominance. Thus, the assumption of single-homing is a meaningful restriction, regardless of the assumed level of rationality.

Assumptions 1-4 imply that one of the platforms will be Pareto dominant—payoffs for all participants are maximized when everyone chooses this platform exclusively. Let platform i denote the Pareto dominant platform, and note that this implies that $u_i(N, N) - p_i > u_j(N, N) - p_j$.

Amending the model to allow for multi-homing requires more than merely adding this option to the choice sets of each agent. It also requires some specification of how the matching process works (and hence payoffs are generated) when agents choose to multi-home. We assume that agents follow a lexicographic rule: First, they go to the better (Pareto dominant) platform and enjoy payoffs from whoever else is at that platform. That is, an agent enjoys payoffs $u_i(n_{i1}, n_{i2}) - p_i$. Next, they go to the worse platform and enjoy payoffs from any *new* individuals of the opposite type they encounter. Of course, they still suffer costs from competition associated with *all* individuals of the same type visiting the worse platform. That is, an agent who multi-homes enjoys incremental payoffs of $u_j\left(n_{j1}, n_{j2}^E\right) - p_j$ where $n_{j2}^E = N - n_{i2}$.[11] Thus, the net payoff for a multi-homing agent is

$$u_i\left(n_{i1}, n_{i2}\right) + u_j\left(n_{j1}, n_{j2}^E\right) - p_A - p_B$$

This type of rule is intuitive in a dating market context. It makes sense that a woman will first search for matches on the better dating platform, collecting contact information for the attractive men located there. Having obtained this information, she then visits the less attractive dating platform. Obviously, the only additional value such a visit provides is the contact information for *new* attractive men not already encountered on the better platform. Of course, she faces competition from all of the women located at each platform regardless of duplication.[12]

Full Rationality

Tipping to either platform remains an equilibrium even when we add the option of multi-homing. To see, this, suppose women all choose platform $i \in \{A, B\}$ exclusively, then men have no incentive to join platform j or to multi-home since there is no benefit to visiting a platform which is devoid of women. The same is true of women when men join platform i exclusively.

Likewise, under some parameter values, it remains an equilibrium for n agents of each type join platform A and the remaining $N - n$ agents of each type join platform B, the analog to coexisting equilibria under single homing. Proposition 4 formalizes this.

Proposition 4 *When agents can multi-home, tipping to either platform is an equilibrium. Furthermore, any market share split is consistent with equilibrium provided market impact effects are sufficiently large.*

Formally, there exists an equilibrium where all agents choose platform $i \in \{A, B\}$. There exists an equilibrium where n agents of each type choose platform $i \in \{A, B\}$ with the remainder choosing platform j provided that:

$$\delta_{A,n} \geq u_A\left(n, n\right) - p_A \geq 0 \textit{ and } \delta_{B,N-n} \geq u_B\left(N-n, N-n\right) - p_B \geq 0 \qquad (3)$$

Since the multi-homing option is not exercised for the coexisting equilibria characterized in Proposition 4, we can examine how multi-homing affects the chance that platforms coexist. Define $\bar{\delta}_{A,n}^{MH} \equiv u_A\left(n, n\right) - p_A$ to be the critical threshold for market impact effects on platform

[11] The superscript E is a mnemonic for the *extra* agents of the other type encountered at platform j.

[12] While this rule seems intuitive, it is not required for our main result (Proposition 6)—that under cognitive hierarchy the better platform prevails. The result would still hold if we instead assumed that all agents visited the worse platform first.

A to sustain coexistence in an equilibrium where n agents choose platform A under multi-homing. That is, the market impact effect, $\delta_{A,n}$ must be $\bar{\delta}^{MH}_{A,n}$ or more for this configuration to be an equilibrium. Under single-homing, the relevant critical threshold is $\bar{\delta}^{SH}_{A,n} \equiv \Delta U_n$. The critical thresholds for the market impact effects on platform B are analogous. Now, since $u_B (N - n, N - n) - p_B \geq 0$, it follows immediately that $\bar{\delta}^{MH}_{A,n} \geq \bar{\delta}^{SH}_{A,n}$ and similarly $\bar{\delta}^{MH}_{B,n} \geq \bar{\delta}^{SH}_{B,n}$—market impact effects must be larger to sustain coexistence under multi-homing than under single-homing. The option to multi-home undermines the prospects of equilibrium coexistence (for the class of equilibria where the multi-homing option is not exercised). This is intuitive in that multi-homing offers an additional possibility for deviation from equilibrium, namely collocating on both platforms. The required conditions to rule such deviations out are, accordingly, more stringent.

Of course, Proposition 4 only considers the set of equilibria in which the option to multi-home is not exercised. Equilibrium coexistence might also arise when it is an equilibrium for one or both types of agents to multi-home. One can easily rule out the possibility that all agents multi-home. To see this, notice that, since all women are on both platforms, there is no incremental benefit to men from visiting the worse platform. Moreover, such visits are costly. Hence, men can profitably deviate by single-homing at the better platform and likewise for women. Similarly, it can never be an equilibrium for all men to choose platform i and some men to multi-home. Under this circumstance, all women would choose to visit platform i exclusively and hence the multi-homing men derive no benefit from also accessing platform j. (An identical argument rules out the case where all women visit platform i and some multi-home.)

There are, however, coexisting equilibria where some agents of each type exclusively use each of the platforms while others multi-home. For instance, some men exclusively use platform A, others exclusively use platform B, while the remainder multi-home and symmetrically for the women. Since some men find it optimal to multi-home, one may wonder why it is not profitable for a man currently using a single platform to deviate by multi-homing. What prevents this is the market impact effect—by adding a second platform, competition among men on this platform is increased—which deters such deviations. Proposition 5 formalizes the exact conditions where the market impact effects can sustain this type of coexistence.

Proposition 5 *Provided market impact effects are sufficiently large, coexistence where some agents multi-home is an equilibrium.*

Formally, suppose that A is the Pareto dominant platform and that

$$\begin{aligned} \delta_{A,n_A} + \delta_{B,n_B} &\geq p_A - p_B + u_B(n_B, n_B) - u_A(n_A + 1, n_A) \geq 0 \\ \overline{\delta}_{B,n_B} &\geq p_B - u_B(n_B + 1, N - n_A) \geq 0 \\ \delta_{A,n_A} &\geq p_A - u_A(n_A + 1, n_A) + u_B(n_B, n_B) - u_B(n_B, N - n_A) \geq 0 \end{aligned}$$

where $\overline{\delta}_{B,n_B} \equiv u_B(n_B, N - n_A) - u_B(n_B + 1, N - n_A)$. *Then it is a Nash equilibrium for* $N - n_B$ *agents of each type locate only on platform* A, $N - n_A$ *agents of each type locate only on platform* B, *and* $n_A + n_B - N$ *agents of each type multi-home.*

Proposition 5 reveals that the multi-homing behavior seen in practice in credit card markets is consistent with a coexisting equilibrium under full rationality. Moreover, it is essential that not all individuals on the same side of the market make the same choice. Some consumers will use Visa/MasterCard exclusively while others will also carry Discover card. Likewise, not all merchants will accept both cards. One counterfactual aspect of the equilibrium is that it requires that some merchants and some consumers use/accept Discover card exclusively. While the exclusive acceptance of Discover was, at one time, the policy of both Sears and Sam's Club, this is no longer the case. Thus, a coexisting equilibrium is capable of rationalizing some but not all behavior with respect to multi-homing. Perhaps more importantly, such equilibria are ruled out (by assumption) by limiting attention to the single-homing case.

Taken together, Propositions 4 and 5 point out that equilibrium still offers little guidance as to what market structures emerge with platform competition under full rationality and multi-homing. Indeed, if anything, the picture is even more muddled than under single homing. For instance, one can easily choose parameter values such that the addition of multi-homing merely expands the (already considerable) set of equilibria that previously arose under single homing.

Cognitive Hierarchy

We saw that relaxing the assumption of full rationality in favor of the arguably more realistic cognitive hierarchy formulation substantially clarified predictions about market structure under single-homing regardless of the assumptions made about the behavior of level-0 agents. Multi-homing introduces additional possibilities for modeling the choices made by these individuals. Now the probabilistic mix is multi-dimensional rather than single dimensional. Assuming single peakedness (Assumption 5) guaranteed that the problem of best responses for level 1 and higher agents was well-behaved thus facilitating full characterization under single-homing. The situation is more nuanced in the multi-homing case. Thus, rather than characterizing equilibria under arbitrary choices of level-0 agents, we temporarily restrict attention to circumstances where these choices are in pure strategies. Later, we relax this assumption to allow for symmetric randomization behavior by these agents; that is, level-0 agents (stochastically) choose either of the platforms with equal probability and otherwise multi-home.

Pure Strategy Choices by Level-0 Agents

Consider the case where level-0 agents avoid choosing between competing platforms; they simply multi-home. We claim that all strategically sophisticated agents choose the better platform. When level-0 agents multi-home, level-1 agents, who view all agents as being level-0, believe that everyone will be present on both platforms. There is, effectively, no risk associated with choosing either platform and, as a consequence, level-1 agents select the better (i.e. Pareto dominant) platform. A level-2 agent believes that all agents are level-1 or level-0 and hence believes that all agents will be present on the Pareto dominant platform. As a consequence, such agents are best served by mimicking the choices of the level-1 agents. The same holds of all agents with higher levels of strategic sophistication. Formally, we may conclude:

Proposition 6 *Suppose that all level-0 agents multi-home. Under cognitive hierarchy, all agents with sophistication level $l \geq 1$ choose the Pareto dominant platform.*

Proposition 6 reinforces the notion that, by allowing for some degree of bounded rationality, market impact effects are not enough to sustain equilibrium coexistence—one of the platforms will enjoy 100% market share of sophisticated agents while the rival platform gets 0% market share. Moreover, it sharpens the prediction as to the identity of the winning platform. In particular, it suggests that the QWERTY phenomenon—the possibility of agents getting locked in to the inferior platform—does not arise. Put differently, lock-in at the inferior platform does not arise *despite* the hyper-sophistication assumed in the fully rational model, but rather relies upon this sophistication in an essential way. It is perhaps for this reason that examples of this type of lock-in are rare.

Next, consider the case where all level-0 agents choose platform i exclusively. Clearly level-1 agents will follow suit. There is no gain to accessing platform j either exclusively or through multi-homing since no agents are believed to be present on the platform. The same logic applies to all agents with higher levels of sophistication. Thus, we have shown that

Proposition 7 *Suppose that all level-0 agents choose platform i. Then, under cognitive hierarchy, the market tips to platform i—all agents utilize this platform exclusively.*

Propositions 6 and 7 highlight several key properties of bounded rationality and platform competition. First, the "herding" effect where all agents of higher levels of sophistication mimic the choices of level-1 agents is a robust feature of the model. Second, despite the option to multi-home, all agents of higher levels of sophistication opt for a single platform. Third, and most importantly, even in the presence of multi-homing, a single dominant platform emerges as the equilibrium market structure.

Stochastic Level-0 Agent Choices

One may worry, however, that the tendency toward tipping is purely an artifact of our restriction to pure strategy behavior on the part of level-0 agents. We now partially relax this assumption to allow for non-deterministic behavior on their part. Specifically, we assume that level-0 agents choose to access platform i exclusively with the same probability as platform j. With remaining probability, level-0 agents multi-home.

Before proceeding with the analysis, we need to introduce some additional notation to account for stochastic choices on the part of other agents. As usual, let i be the Pareto dominant platform. Suppose that an agent of a given level of rationality believes that all other agents select platform i (exclusively) with probability λ_i, select platform j with probability λ_j, and multi-home with the remaining probability $1-\lambda_i-\lambda_j$. In that case, her payoff from multi-homing when exactly $s-1$ agents of the same type choose i, r multi-home, and t of the opposite type choose platform i (either exclusively or through multi-homing) is simply $\left(u_i\left(s+r,t\right)+u_j\left(N-s+1,N-t\right)\right)$. The probability of this event happening is

$$\Pr[s,r,t]=\binom{N-1}{s-1}\binom{N-s}{r}\binom{N}{t}\lambda_i^{s-1}\left(1-\lambda_i-\lambda_j\right)^r\lambda_j^{N-s-r}\left(1-\lambda_j\right)^t\lambda_j^{N-t}.$$

Summing over all possible events yields the expected utility from multi-homing,

$$U_{mh}\left(\lambda_i,\lambda_j\right)=\sum_{s=1}^{N}\sum_{r=0}^{N-s}\sum_{t=0}^{N}\Pr[s,r,t]\left(u_i\left(s+r,t\right)+u_j\left(N-s+1,N-t\right)\right).$$

When an agent chooses platform i exclusively, on the other hand, she gets payoff from all other agents who join platform i, exclusively or not. That is, she believes that an agent will locate on platform i with probability $1-\lambda_j$. Therefore, her expected payoff from joining platform i exclusively is

$$\begin{aligned}U_{sh,i}\left(\lambda_i,\lambda_j\right) &= \sum_{s=1}^{N}\sum_{t=0}^{N}\binom{N-1}{s-1}\binom{N}{t}\left(1-\lambda_j\right)^{s-1}\lambda_j^{N-s}\left(1-\lambda_j\right)^t\lambda_j^{N-t}u_i\left(s,t\right)\\ &= \sum_{s=1}^{N}\sum_{t=0}^{N}\binom{N-1}{s-1}\binom{N}{t}\left(1-\lambda_j\right)^{s+t-1}\lambda_j^{2N-s-t}u_i\left(s,t\right)\end{aligned}$$

While the delineation of λ_i and λ_j is needed in determining the payoffs under multi-homing, it is not strictly necessary under single homing. Indeed, $U_{sh,i}\left(\lambda_i,\lambda_j\right)=U_i\left(1-\lambda_j\right)$ as defined in equation (1). Similarly, when an agent chooses platform j exclusively, she earns

$$U_{sh,j}\left(\lambda_i,\lambda_j\right)=\sum_{s=1}^{N}\sum_{t=0}^{N}\binom{N-1}{s-1}\binom{N}{t}\left(1-\lambda_i\right)^{s+t-1}\lambda_i^{2N-s-t}u_j\left(s,t\right)=U_j\left(1-\lambda_i\right).$$

Each of these functions is well-defined and continuously differentiable in λ_i and λ_j. The case where $\lambda_i = \lambda_j = 0$ corresponds to the situation where all other agents are perceived to multi-home. As we saw in the proof of Proposition 6, an agent's best response was to select the better platform exclusively given these beliefs. That is,

$$U_{sh,i}(0,0) - p_i > U_{mh}(0,0) - p_A - p_B \tag{4}$$

For multi-homing to be a viable best response to symmetric choices by level-0 agents, we assume that

$$U_{mh}\left(\frac{1}{2},\frac{1}{2}\right) - p_A - p_B > \max\left[U_{sh,i}\left(\frac{1}{2},\frac{1}{2}\right) - p_i, U_{sh,j}\left(\frac{1}{2},\frac{1}{2}\right) - p_j\right] \tag{5}$$

This assumption merely guarantees that, if all other agents single home with equal probability for each platform, then the benefits of encountering all of the agents of the opposite type exceed the costs of multi-homing.

Finally, the analysis is greatly simplified if we extend the notion of Pareto dominance to situations where platforms enjoy less than 100% market share. Specifically, we say that platform i is *super dominant* if, for a given market share, payoffs are higher on platform i than on platform j. For instance, were j to enjoy 60% market share, then payoffs to those on platform j would be lower than to agents on platform i when i enjoys this same market share. Formally, we assume that, for all λ, λ'

$$U_{sh,i}(\lambda, \lambda') - p_i > U_{sh,j}(\lambda', \lambda) - p_j.$$

Obviously, super dominance implies Pareto dominance.

As for the single-homing case, we require some additional structure to ensure that the expected payoff functions are well-behaved. Analogous to Assumption 5, we assume that $U_{mh}(\lambda, \lambda)$ is single-peaked in λ. Moreover, we assume that relative attractiveness of multi-homing over single-homing at platform i is decreasing in the probability of an agent choosing platform i and is increasing in the probability of an agent choosing platform j. Note that Assumption 5 already implies that $U_{sh,i}(\lambda_i, \lambda_j)$ is single-peaked in λ_j. Formally,

Assumption 6: If $U'_{mh}\left(\widehat{\lambda}, \widehat{\lambda}\right) = 0$ then $U'_{mh}(\lambda, \lambda) < 0$ for all $\lambda > \widehat{\lambda}$. Moreover, $U_{mh}(\lambda_i, \lambda_j) - U_{sh,i}(\lambda_i, \lambda_j)$ is decreasing in λ_i and increasing in λ_j.

With these assumptions, we can now analyze the behavior of level-1 agents. Let $\lambda_i = \lambda_j = \tilde{\lambda}$ denote the choice probabilities of level-0 agents. Clearly, if $\tilde{\lambda}$ is small, then the best response for a level-1 agent is to single-home, exclusively choosing the super dominant platform. This follows from continuity and the inequality in equation (4). Similarly, if $\tilde{\lambda}$ is close to 50%, then the best response for a level-1 agents is to multi-home, which follows from continuity and the inequality in equation (5). Thus, there exists for intermediate probability,

$\tilde{\lambda} = \lambda^*$, where level-1 agents are exactly indifferent between single and multi-homing. Clearly, level-1 agents multi-home if and only if $\tilde{\lambda} \geq \lambda^*$.[13]

When $\tilde{\lambda} < \lambda^*$, level-1 agents choose the super dominant platform. Naturally, this makes this platform more attractive for higher level agents, and we obtain the familiar herding result—more sophisticated agents mimic the behavior of level-1 agents and choose the super dominant platform exclusively.

Of greater interest is the case where $\tilde{\lambda} \geq \lambda^*$. Here, level-1 agents choose to multi-home and thus, from the perspective of a level-2 agent, the fraction of other agents choosing to be exclusively on platform i or j falls to $\lambda' < \lambda^*$. As a consequence, multi-homing is now less attractive. Eventually, there exists a level-$\underline{k}$ agent for whom λ' has fallen sufficiently that it is now below the critical threshold, λ^*. This agent then chooses to visit the super-dominant platform exclusively and, as usual, all more sophisticated agents follow suit.

While the above sketches the essence of the proof, it omits a number of technical details needed to ensure that the intuitive behavior described above is, indeed, optimal. Proposition 8 presents a formal statement of the result. The detailed proof is contained in Appendix A.

Proposition 8 *Suppose Assumptions 1-6 hold. Then, under cognitive hierarchy:*

If level-0 agents single-home on each platform with probability $\tilde{\lambda} < \lambda^*$, all strategically sophisticated agents choose the super-dominant platform.

If level-0 agents single-home on each platform with probability $\tilde{\lambda} \geq \lambda^*$, then there exists $\infty > \underline{k} > 1$ such that all agents of sophistication levels $\{1, 2, ..., \underline{k} - 1\}$ multi-home while more sophisticated agents choose the super-dominant platform.

Proposition 8 highlights that the addition of multi-homing offers the possibility of a much richer set of choice behavior in equilibrium under cognitive hierarchy. While it remains the case that bounded rationality leads to unique predictions that entail herding behavior where more sophisticated agents mimic the choices of less sophisticated agents, it is no longer the case that there is a single, dominant platform selected by sophisticated agents. When the fraction of level-0 agents who single-home is high enough, relatively less sophisticated strategic agents respond by multi-homing while sophisticates choose the better platform exclusively. This behavior is qualitatively consistent with what one sees in the credit card market—some people carry Visa/MasterCard and Discover in their wallet while others use Visa/MasterCard exclusively. Likewise for merchants—Discover cards are not universally accepted while Visa/MasterCards are. It is also unlike *any* equilibrium under full rationality. Thus, in principle, the distinction between the two models is empirically testable.

[13] We assume that, when there is a tie between single and multi-homing, level-1 agents choose to multi-home. The particular tie-breaking rule is inessential to the result.

More broadly, in multi-homing contexts, value is the key. This has led to a benefits war in credit card markets where competitors vie to provide consumers with rewards such as cash back, airline miles, and so on to induce them to use a particular card. Likewise, interest rate cuts and fee waivers are used to entice customers to switch away from rivals. Similarly, in search engines Google is ubiquitous; however for some queries, particularly those related to shopping for a particular product, some individuals will multi-home, using both Google and Amazon. Even though search engines are free to consumers, there is a constant battle over quality. For instance, Microsoft's search engine Bing distinguished itself with faster incorporation of social data, such as Twitter feeds, into its search results. It also, for a time, paid consumers to use its engine for queries.

5 Horizontal Differentiation

While models with full rationality offered little in the way of predictions about market structure, bounded rationality models offered more precise predictions. Specifically, regardless of the size of market impact effects, vertical differentiation, or single versus multi-homing, a ubiquitous platform always arose in equilibrium. Under single-homing, this implied that there was a single, dominant platform selected by all strategic agents. Under multi-homing, both platforms might coexist, but one of the platforms would be "universal" in the sense that all sophisticated agents chose it either exclusively or through multi-homing. While this matches many platform competition situations where there is a single big agent, in other situations the market is more fragmented. In this section, we enrich the model to account for differences in individual preferences across platforms, i.e. to permit horizontal as well as vertical differentiation across platforms.

Up until now, we have assumed that the payoffs for all individuals of a given type choosing a given platform were the same. Thus, while players might view the platforms Match.com and eHarmony as different, all men and women feel the same way about each platform. Clearly, this is an unrealistic assumption. One key dimension along which Match and eHarmony differ is whether the user browses to find the right match versus whether the site provides the user with a short list of suitable matches. A user visiting Match.com can browse the profiles of all others signed up to the site and decide who to contact. Browsing, however, is not permitted on eHarmony. Instead, the user receives a list of a small set of potential matches based on compatibility algorithms at the website. Some users prefer the do it yourself approach of Match while others prefer the top-down approach of eHarmony.

To model this, we suppose that each agent has a horizontally preferred platform. By choosing the preferred platform, the agent receives a discount of $\theta > 0$ off of the access fee. Suppose platform i is a given man's preferred platform where $n_1 - 1$ other men and

n_2 women has joined and the remaining men and women have joined his non-preferred platform j. Then his payoff from joining platforms i and j will be $u_i(n_1, n_2) - p_i + \theta$ and $u_j(N - n_1 + 1, N - n_2) - p_j$, respectively. The model is uninteresting when the discount is so large as to induce the agent to go to his preferred platform even when he or she is alone on the platform. Thus, we assume that if platform i is the preferred platform then

$$\theta < u_j(N, N) - p_j + p_i \tag{6}$$

Suppose $\widetilde{n_A}$ men and $\widetilde{n_A}$ women have a preference for platform A and $\widetilde{n_B} = N - \widetilde{n_A}$ agents of each type have a preference for platform B. To examine the pure effect of horizontal differentiation, we revert to the case where only single-homing is allowed.

Full Rationality

We do not characterize all equilibria for this model. However, we show that both tipping and coexistence occur in equilibrium. Importantly, adding horizontal differentiation admits a new possibility—for generic parameter values, it may be that neither platform is Pareto dominant. Pareto dominance requires that horizontal differentiation be relatively unimportant. Formally, a Pareto dominant platform exists if and only if

$$\theta \leq u_i(N, N) - u_j(N, N) - p_i + p_j \tag{7}$$

for some i. It may be readily verified that the inequality given in equation (7) is more stringent than that given in equation (6). Thus, the model covers parameter values where horizontal differentiation is small, so a Pareto dominant platform exists, or large, so it does not. Regardless of whether the inequality in equation (7) holds, tipping to either platform remains an equilibrium. If all agents are located on platform i, even an agent whose preferred platform is j cannot benefit from unilaterally switching to platform j given the upper bound on θ as specified in equation (6). Thus, we have shown

Proposition 9 *Under horizontal differentiation, tipping to either platform is a Nash equilibrium.*

Under horizontal differentiation, coexisting equilibria continue to exist. The most intuitive of these is one where each agent goes to her (horizontally) preferred platform; however, there are many other classes of equilibria where platforms coexist. For instance, for some parameter values (shown formally below) it is an equilibrium for everyone to choose their non-preferred platform. A mixture, where some agents choose their preferred platform and others their non-preferred, is also possible. As usual, the key to equilibrium coexistence is the size of market impact effects. For the intuitive equilibrium, the magnitude of the required effect is reduced by the discount θ. It is raised by this same amount for the "backwards" equilibrium. The following proposition derives formal conditions on market impact effects for equilibrium coexistence to arise. The broader point is that adding horizontal differentiation merely exacerbates the equilibrium multiplicity already present under the baseline model where horizontal differentiation is absent. Formally,

Proposition 10 *Platform coexistence is consistent with equilibrium under horizontal differentiation provided that market impact effects are large enough. Specifically,*

I. All agents joining their preferred platforms is a coexisting equilibrium if (1) $\delta_{A,\widetilde{n_A}} \geq \Delta U_{\widetilde{n_A}} - \theta$ *when* $\Delta U_{\widetilde{n_A}} \geq 0$ *and (2)* $\delta_{B,\widetilde{n_B}} \geq -\Delta U_{\widetilde{n_A}} - \theta$ *when* $\Delta U_{\widetilde{n_A}} < 0$.

II. All agents joining their non-preferred platform is a coexisting equilibrium if $\delta_{B,\widetilde{n_A}} \geq -\Delta U_{\widetilde{n_B}} + \theta$ *and* $\delta_{A,\widetilde{n_B}} \geq \Delta U_{\widetilde{n_B}} + \theta$.

III. Moreover, $\widetilde{n_A} - m$ *pairs of men and women choosing their preferred platform A,* m *pairs of men and women choosing their non-preferred platform B and* $\widetilde{n_B}$ *pairs of men and women agents choosing their preferred platform B for some* $m \in \{1, 2, \ldots, \widetilde{n_A} - 1\}$ *is an equilibrium if* $\delta_{A,\widetilde{n_A}-m} \geq \Delta U_{\widetilde{n_A}-m} + \theta$ *and* $\delta_{B,\widetilde{n_B}+m} \geq -\Delta U_{\widetilde{n_A}-m} - \theta$.

We can illustrate multiple coexisting equilibria under horizontal differentiation using Example 1 with the additional assumptions that $\widetilde{n_A} = \widetilde{n_B} = 5$ and $\theta = 10$. Equal market shares for both platforms as well as 60-40 and 70-30 splits in favor of either platform constitute coexisting equilibria. Within these market share splits, any combination of agents choosing their preferred or non-preferred platforms constitute an equilibrium. Moreover, an 80-20 split in favor of either platform where two pairs of men and women choose their preferred platform and all other agents choose the other platform (which is the preferred platform for five men and five women located there) is an equilibrium. In this example, the possible set of coexisting equilibria under horizontal differentiation is strictly larger than that of the baseline model.[14]

To summarize, adding horizontal differentiation to the single-homing model under full rationality does little to clarify predictions about market structures or offer insights about business strategies. Depending on the type of equilibrium, market impact effects and horizontal differentiation can interact in peculiar ways. In a coexisting equilibrium where agents choose their preferred platform, horizontal differentiation aids in sustaining coexistence whereas in an equilibrium where agents choose non-preferred platform, market impact effects must be especially strong to overcome horizontal differentiation. Regardless, equilibrium coexistence is by no means assured—tipping remains an equilibrium.

Cognitive Hierarchy

Once again we relax the full rationality assumption. Our main result in this section is to show that the cognitive hierarchy model predicts a unique outcome—provided horizontal

[14]The set of equilibria depends on the size of θ. If the discount is large, i.e., $\theta = 100$, all agents choosing their preferred platforms is the only coexisting equilibrium.

differentiation is sufficiently important, each strategic agent chooses her preferred platform and hence both platforms coexist in equilibrium.

While we were agnostic about the behavior of level-0 agents when horizontal differentiation was absent, here we place some (mild) additional structure on their choices: We assume that level-0 agents are weakly more likely to choose their preferred platform than their non–preferred platform. This rules out bizarre cases where being horizontally preferred reduces the chance that a platform is selected by a non-strategic agent.

The interesting case arises when the degree of horizontal differentiation (θ) is relatively large. Our baseline model is, in effect a special case of the horizontal differentiation model where $\theta = 0$. As we showed, in that case a single, dominant platform is chosen by all strategic agents. By continuity, if θ is small, this continues to be the case. The interesting situation arises when:

Assumption 7: $\theta > U_j\left(1 - \min\left\{\frac{1}{2}, \frac{\widetilde{n}_i}{N}\right\}\right) - U_i\left(\min\left\{\frac{1}{2}, \frac{\widetilde{n}_i}{N}\right\}\right) - (p_i - p_j)$ for $i \in \{A, B\}$.
Assumption 7 is fairly weak. Among other things, it merely ensures that when the choices of all other agents are random, it is better for an agent to choose her preferred platform over the non-preferred platform. With this assumption, we are now in a position to state our main result of this section:

Proposition 11 *When horizontal differentiation is sufficiently large, platforms coexist under cognitive hierarchy.*

Formally, suppose level-0 agents weakly choose their preferred platform and Assumptions 1-5, and 7 hold. Then strategically sophisticated agents choose their preferred platform in the unique equilibrium.

We sketch the proof below, but leave the formal analysis to Appendix A. When level-1 agents are determining which platform to select, they anticipate that level-0 agents are weakly more likely to choose their preferred platform. Notice that, even when level-0 agents are selecting randomly, Assumption 7 implies that level-1 agents optimally select their preferred platform. Likewise, when level-0 agents are always selecting their preferred platform, level-1 agents find it optimal to do so as well (since this is a Nash equilibrium under full rationality). Assumption 5 guarantees that, for any convex combination of these two extremes, it remains optimal for level-1 agents to choose their preferred platform. Level-2 agents likewise face a convex combination of random choice and selection based on preferred platforms and respond identically to level-1 agents. And so on for more sophisticated agents.

Comparing Propositions 3 and 11 reveals striking differences in market structure under bounded rationality. When horizontal differentiation is only a secondary consideration, there is a strong tendency toward industry concentration—all strategic agents choose the same platform regardless of market impact effects. Once horizontal differentiation becomes an important consideration, the industry tends to remain fragmented regardless of the magnitude of positive network externalities. Thus, the cognitive hierarchy model is capable of rationalizing the vast difference in the market structure of online auctions (extremely concentrated) and online dating markets (extremely fragmented). While the technology used by platforms in both of these markets is similar, idiosyncratic match characteristics (horizontal differentiation) are much more important in selecting a date or a life partner than they are in selecting a Beanie Baby or a new golf club. Differences in the market structure for video game consoles (fragmented) versus office software and high definition optical disc format (concentrated) can also be explained along the same lines. From a managerial perspective, this suggests that emphasizing the unique identity of culture of users of a given platform can be a more successful marketing strategy than one that emphasizes the quality of the matchmaking process or the value of the site.

The results of laboratory studies offer formal evidence supporting the predictions of the cognitive hierarchy model. Hossain, Minor, and Morgan (2011) examine the dynamics of platform competition under single homing, varying the degree of horizontal and vertical differentiation, as well as the strength of competitive effects. When horizontal differentiation is small or absent altogether, they find strong evidence in favor of market tipping toward the risk dominant platform (regardless of competitive effects).[15] When horizontal differentiation is strong, platform coexistence emerges with agents choosing their preferred platform.

6 Market Dynamics

Our model follows much of the extant literature in treating platform competition as a simultaneous game. Yet, for many online markets, perhaps the most significant feature of the business landscape has been the phenomenal growth in the number of users. In this section, we extend the baseline model to allow for rudimentary market dynamics. Specifically, we divide the platform competition game into two stages—an initial stage marked by a small number of users, followed by a maturation stage with a larger influx of new users. Payoffs for all users are realized following the maturation stage.

A standard intuition is that markets with network effects, such as those that we study, exhibit strong path dependence—platform choices at the initial stage dictate the winning and losing platform as the market matures. In a sense, the herding by sophisticated types under cognitive hierarchies has some of the flavor of this agglomeration dynamic. As we will show, however, such forces carry no particular weight under full rationality. Indeed, our

[15] Some care is needed here. In most treatments, the risk dominant platform was also Pareto dominant. In treatments where it was not, markets were more likely to tip to the risk dominant platform when subjects were inexperienced, but the Pareto dominant platform thereafter.

main result in this section is that the "anything goes" feature of the baseline model under full rationality carries through almost entirely in a dynamic setting, even when all agents coordinate on a single platform in the initial stage.

The formal model is as follows: During the initial stage $d \in \left\{1, 2, ..., \frac{N}{2}\right\}$ agents of each type simultaneously select a platform.[16] At the maturation stage, their choices are revealed to all the remaining agents. These $N - d$ pairs of agents then simultaneously make platform choices. The timing of moves is exogenously specified; thus, an agent cannot choose to wait or go early. At the conclusion of the game, payoffs for all agents are determined based on the total number of agents of each type attracted to each platform exactly as specified previously.

Throughout both periods, prices, match efficiency, and access fees for each platform remain fixed; thus, the model rules out penetration pricing or other time varying strategies by platforms. This is done to allow a direct comparison to the simultaneous game, but is, admittedly, not a realistic feature. The situation we have in mind is where d is small relative to N although the analysis requires no such restriction in that regard.

Full Rationality

As usual, we restrict attention to pure strategy equilibrium though subgame perfection obliges us to admit mixed strategies off the equilibrium path. We begin by establishing the analog of Lemma 1 for the sequential version of the model.

Lemma 2 *In any subgame perfect equilibrium of the sequential game, the same number of agents of each type select a given platform.*

Lemma 2 considerably simplifies the equilibrium characterization. We are now in a position to report the main result of this section.

Proposition 12 *Fix a gross payoff function and suppose that n (resp. $N - n$) agents of each type subscribe to platform A (resp. B):*

(1)Then there exists a pair of access fees (p_A, p_B) such that these market shares comprise an equilibrium of both the sequential and simultaneous games.

(2) Furthermore, for any pair of access fees, if these market shares comprise a subgame perfect equilibrium of the sequential game, they also form a Nash equilibrium of the simultaneous game.

Proposition 12 highlights that equilibrium multiplicity under full rationality is as problematic in the sequential game as in the simultaneous game. The proof of part 1 of the proposition is by construction. The idea is as follows: choose access fees such that the market impact effect is large enough to sustain a market where n pairs of agents choose

[16] Here we assume N to be even. For odd N, we can assume that $d \leq \frac{N-1}{2}$.

platform A with the remainder choosing B. This ensures that these market shares arise in a Nash equilibrium of the simultaneous game. To ensure subgame perfection requires the additional condition that the market share of the platform producing higher equilibrium payoffs contains at least d pairs of players. To complete the construction, suppose that all players choose the higher payoff platform during the initial stage while the remainder fill out each platform up to its equilibrium share in the maturation stage. Clearly, maturation stage players cannot profitably deviate for reasons identical to the simultaneous game. If anyone deviates in the initial stage, his or her "slot" will simply be filled by a maturation stage agent of the same type, so this too is unprofitable.

For the special case where $d = 1$, we can use this same construction to make a sharper statement:

Remark 2 *When $d = 1$, an outcome is a subgame perfect equilibrium of the sequential game if and only if it is a Nash equilibrium of the simultaneous game.*

This case is primarily of interest as a robustness check. It shows that if we slightly perturb the simultaneous game by allowing one pair of players to move ahead of the others, the set of equilibria is completely unaffected. In a way, this is surprising. One might think that the first move confers some commitment power as in the other strategic settings. The key here is the twosidedness of platform markets. While deviations by *pairs* of agents can alter the strategic situation, unilateral deviations cannot since any such deviation in the initial stage can be undone in the maturation stage.

When $d > 1$, the sets of equilibria in the two games do not perfectly coincide. The following example demonstrates a situation where an equilibrium of the simultaneous game does not survive in the sequential model.

Example 2 *Suppose that $N = 4$, $d = 2$, and we use the matching technology from Example 1. Then 3 pairs of men and women joining platform A with the remaining pair joining platform B occurs in an equilibrium of both the simultaneous and sequential games if $(p_A, p_B) = (45, 5)$ and only the simultaneous game if $(p_A, p_B) = (70, 5)$.*

The key to the example is variation in the access fees on each platform. Depending on these fees, the set of subgame perfect equilibria in the sequential game can be a strict subset of the set of Nash equilibria in the simultaneous game. To sum up, even when all agents coordinate on a single platform during the initial period of the life of the market, this is no guarantee that the "anointed" platform will dominate in the maturation phase. As with the baseline model, coexistence or tipping to either platform are all consistent with an equilibrium under full rationality.

Cognitive Hierarchy

Next, we study outcomes under the cognitive hierarchy model. Recall that expectations about the play of level-0 agents were key in determining behavior of more sophisticated types. These expectations still play a role in the sequential game, but the analysis is now complicated by the fact that agents in the maturation stage get to observe earlier choices, including those of the level-0 agents choosing during the initial stage. Thus the *realizations* of random play by level-0 agents also affect play.

To reduce this complication and isolate the pure effect of timing on choice behavior, consider a situation where the probability of a level-0 type, $f(0)$, goes to zero in the limit. Specifically, let $f_z(k)$ be a sequence of probabilities over the levels of strategic sophistication where:

$$\lim_{z\to\infty} \frac{f_z(0)}{f_z(1)} = 0 \tag{8}$$

The idea here is that level-0 agents comprise a small fraction of the population. A special case of this assumption occurs when one considers a small perturbation from full rationality where higher cognitive types are strictly and exceedingly more likely than lower cognitive types. To maintain expositional simplicity of our analysis, we restrict attention to the case where all level-0 agents choose each platform with equal probability; that is, $\lambda_i = \frac{1}{2}$. We shall refer to the combination of equiprobable platform choice and probabilities satisfying equation (8) as the "limit cognitive hierarchy" model. We are now in a position to characterize equilibrium in this setting.

Proposition 13 *In the limit cognitive hierarchy model, all strategic types choose the risk dominant platform in the unique equilibrium at the limit.*

Proposition 13 shows that the behavior of the cognitive hierarchy model is unchanged with the addition of dynamic entry. Restricting attention to the limit case, where realizations from random behavior by level-0 agents at the initial stage do not affect subsequently outcomes enables a precise statement, but qualitative behavior is easily characterized outside this case. Specifically, if the risk dominant platform enjoys sufficient market share in the first period, then all strategic types in the second period will again coordinate on this platform. When the risk inferior platform enjoys high market share during the initial stage, strategic types will switch and coordinate on this platform instead. This latter situation can arise if a large fraction of agents are level-0, and the realizations of their random choices favor the risk inferior platform. Behavior of strategic types during the initial stage is simpler—strategic types will opt for the risk dominant platform for the usual reasons. The difficulty lies in determining the exact market share realizations that tip the balance between the two platforms at the maturation stage. These thresholds depend on the distribution of cognitive levels, the size of the initial and maturation phases, as well as the particular payoffs under each platform. Since this adds little to understanding of the qualitative features of the cognitive hierarchy model, we eschew a detailed analysis.

7 Conclusion

While models of bounded rationality have been strongly embraced in interpreting data from laboratory experiments, their acceptance in applied settings has been much more limited. A compelling objection against their use is that the very flexibility that makes these models attractive for organizing lab data undermines their ability to make sharp predictions. For instance, quantal response equilibrium is a commonly used solution concept for analyzing experimental data, but, as shown by Haile, Hortacsu, and Kosenok (2008), its use is clearly problematic in applied settings as it can, under mild conditions, rationalize any set of observed choices.

Under platform competition, we showed that the situation is exactly reversed. The standard, fully rational model can justify a wide range of market structures owing to the combination of network and competitive effects. In contrast, the boundedly rational cognitive hierarchy model yields unique predictions. Moreover, by varying key features of the platform competition setting, such as the ability to multi-home or the degree to which the platforms are horizontally differentiated, we can identify which structural features lead to industry concentration versus those that lead to fragmentation. In particular, competition among agents of the same type, such as sellers on an online auction platform, does little to prevent the emergence of a dominant platform. Horizontal differentiation, however, leads to fragmentation even if the degree of differentiation is relatively modest.

From a managerial perspective, the model offers key insights about successful platform strategy. Competing in single-homing markets where differentiation is difficult, managers should focus on reducing the risks to platform users. Quality of service, security and privacy of data, as well as refunds in the event that performance falls short all play a critical role in determining the risk ranking of a platform relative to its rivals. The model points out that this risk ranking is key to market share. This is broadly consistent with the business strategies pursued by eBay. EBay implemented a scheme through its PayPal subsidiary ensuring both buyers and sellers against non-performance by the counter-party thus reducing the risk associated with eBay auctions. EBay also changed their reporting on bid histories to better protect the privacy of users. Finally, eBay emphasizes 24/7 uptime for its site. Under multi-homing, quality and user value should be emphasized. For instance, in the credit card market, there has been a proliferation of cash-back benefits and low interest rates to capture market share. In online markets, short-run monetization strategies that come at the expense of the consumer experience offer a Faustian bargain: While profits may initially increase, such strategies open the door to a higher value platform to gain dominance in the long-run. Where horizontal differentiation has the potential to outpace vertical differentiation, the former should be emphasized. This strategy may be seen by the recent advertising campaigns of eHarmony and ChristianMingle, two online dating sites. The former differentiates itself from other sites by its concern with long-run compatibility rather than short-run opportunities for sexual access. ChristianMingle emphasizes the shared values of its user base—committed heterosexual Christians looking for a match literally made in heaven.

It is, however, worth noting that our cognitive hierarchy model shares a defect common to many models of bounded rationality—the choice behavior of non-strategic players is a free variable and, even when these types are a vanishingly small fraction of the population, their choices play a critical role in the resulting decisions of strategic players. The situation is analogous to that of behavioral types in the reputation literature (see, e.g. Kreps *et al.*, 1982). Despite this, several key qualitative features of industry structure, notably the emergence of a single platform accessed by *all* strategic types absent horizontal differentiation, occur regardless of the assumed behavior of naïve types.

Saying more requires judgment about the motives of non-strategic types. One interpretation is that these types are completely uninformed about the particulars of each platform and hence choose at random. In the single homing model, we showed that this connected the cognitive hierarchy model to a much older equilibrium refinements literature—choice behavior of strategic types corresponds to a risk dominant equilibrium. Thus, one (modest) contribution of the paper is to provide a behavioral micro-foundation for this refinement. But the predictions under bounded rationality do not always coincide with risk dominance. Allowing for multi-homing does not change the identity of the risk dominant platform but substantially changes the behavior of strategic types. They now respond with a combination of multi-homing and exclusively choosing the Pareto dominant platform.

Compared to theory offerings, the empirical literature on platform competition is relatively sparse. Certainly, the complexity of these models combined with the resulting equilibrium multiplicity is not helpful in this regard. Perhaps our most important contribution is to show how allowing for bounded rationality gives rise to clear, testable predictions about how the structural features of platform competition translate into resulting market share performance. While our results are consistent with data from laboratory experiments and with key features of real-world platform markets, an important next step is to carefully examine these predictions empirically. This remains for future research.

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A Proofs

Proof of Lemma 1

Proof. Suppose to the contrary that, in an equilibrium, s women and t men enter platform i. Without loss of generality, we assume that $s > t$. Since women in platform i have no incentive to move to platform j,

$$\begin{aligned} u_i(s,t) - p_i &\geq u_j(N-s+1, N-t) - p_j \\ \Rightarrow p_j - p_i &\geq u_j(N-s+1, N-t) - u_i(s,t). \end{aligned}$$

The assumption of $s > t$ implies

$$u_i(s,t) \leq u_i(t+1,t) < u_i(t+1,s)$$

and

$$u_j(N-(s-1), N-t) \geq u_j(N-t, N-t) > u_j(N-t, N-s).$$

Therefore,

$$\begin{aligned} u_j(N-s+1, N-t) - u_i(s,t) &> u_j(N-t, N-s) - u_i(t+1,s) \\ \Longrightarrow p_j - p_i &> u_j(N-t, N-s) - u_i(t+1,s) \\ \Longrightarrow u_i(t+1,s) - p_i &> u_j(N-t, N-s) - p_j. \end{aligned}$$

However, this implies that men in platform j will have incentives to move to platform i. Therefore, if s women and t men entering platform i is an equilibrium, then $s = t$. ■

Proof of Proposition 3

Proof. Suppose we draw $U_A(\lambda) - p_A$ and $U_B(1-\lambda) - p_B$ on the same graph for $\lambda \in [0,1]$. Given the market size and positive network externalities effects,

$$U_i(1) - p_i = u_i(N,N) - p_i > U_j(0) - p_j = u_j(1,0) - p_j$$

for $i, j \in \{A, B\}$. If both U_A and U_B are increasing functions of the probability of an agent choosing that platform, then that immediately implies single-crossing of the two curves. Otherwise, $U_B(\lambda) - p_B$ and $U_A(1-\lambda) - p_A$ will intersect at most twice given Assumption 5. However, if they intersect twice then $U_i(1) - p_i$ must be smaller than $U_j(0) - p_j$ with $i \neq j$ for at least one i. Given the upper bound on p_i and Assumption 4, this is impossible. This implies that $U_A(\lambda) - p_A$ and $U_B(1-\lambda) - p_B$ intersect exactly once and there is a unique λ^* such that $U_A(\lambda^*) - p_B = U_B(1-\lambda^*) - p_B$. Moreover, $U_A(\lambda) - p_A < U_B(1-\lambda) - p_B$ for all $\lambda < \lambda^*$ and $U_A(\lambda) - p_A > U_B(1-\lambda) - p_B$ for all $\lambda > \lambda^*$.

Now we analyze the equilibria under the cognitive hierarchy model. A level-0 agent chooses to join platform i with probability λ_i. As a level-1 agent assumes that all other agents are of level-0, her expected payoff from joining platforms A and B are $U_A(\lambda_A) - p_A$ and $U_B(1-\lambda_A) - p_B$, respectively. First suppose $\lambda_A < \lambda^*$. Then, all level-1 agents will choose to go to platform B. A level-2 agent believes that any of the other agents is of level-0 with probability $\frac{f(0)}{f(0)+f(1)}$ and of level-1 with probability $\frac{f(1)}{f(0)+f(1)}$. Moreover, the agent believes that a level-0 agent chooses platform B with probability $1-\lambda_A$ and a level-1 agent chooses platform B with probability 1. The expected payoff of a level-2 agent from platform A and B are $U_A\left(\frac{\lambda_A f(0)}{f(0)+f(1)}\right) - p_A$ and $U_B\left(\frac{(1-\lambda_A)f(0)+f(1)}{f(0)+f(1)}\right) - p_B$, respectively. As $\frac{\lambda_A f(0)}{f(0)+f(1)} < \lambda_A < \lambda^*$, a level-2 agent will choose platform B. It can easily be shown that, a level-l agent believes that another agent chooses platform B with probability $1 - \frac{\lambda_A f(0)}{\sum_{k=0}^{l-1} f(k)}$ for all $l \geq 1$. As a result, her best response is to join platform B. Similar logic shows that if $\lambda_A > \lambda^*$, then all level-l agents will choose platform A for $l \geq 1$. ■

Proof of Proposition 4

Proof. The proof that tipping is an equilibrium is analogous to the argument in Proposition 1. To establish conditions where coexisting equilibria exist, consider (p_A, p_B) such that

$$u_A(n,n) \geq p_A \geq u_A(n+1,n) \tag{9}$$

$$u_B(N-n, N-n) \geq p_B \geq u_B(N-n+1, N-n) \tag{10}$$

for some $n \in \{1, 2, \ldots, N-1\}$. Note that, for each equation, at least one of the inequalities will be strict because of the market impact effects. Then n players of each type choosing platform A and $N-n$ players of each type choosing platform B is an equilibrium. Under these platform choices, all agents make non-negative payoff. If a female agent on platform B also joins platform A, she will have access to n new male agents while competing with n other female agents and paying an access fee of p_A. However, as $u_A(n+1,n) \leq p_A$, she will have no incentive to multi-home. She will also have no incentive to choose platform A exclusively. Similarly, as $u_B(n+1,n) \leq p_B$, an agent on platform A will have no incentive to switch to platform B or multi-home. Subtracting $u_A(n,n)$ from the inequalities in equation (9) and $u_B(N-n, N-n)$ from the inequalities in equation (10) yields the inequalities in equation (3). ■

Proof of Proposition 5

Proof. Without loss of generality, assume that platform A is Pareto dominant. To ensure that the proposed equilibrium exists, the following conditions need to be satisfied. An agent who is single-homing on platform A will not deviate to single-home on platform B if

$$u_A(n_A, n_A) - p_A \geq u_B(n_B+1, n_B) - p_B \tag{11}$$

and will not multi-home if

$$p_B \geq u_B\left(n_B + 1, N - n_A\right). \tag{12}$$

On the other hand, an agent single-homing on platform B will not single-home on platform A and will not choose to multi-home if

$$u_B\left(n_B, n_B\right) - p_B \geq u_A\left(n_A + 1, n_A\right) - p_A \tag{13}$$

and

$$u_B\left(n_B, n_B\right) - p_B \geq u_A\left(n_A + 1, n_A\right) - p_A + u_B\left(n_B, N - n_A\right) - p_B, \tag{14}$$

respectively. Finally, an agent who chooses to multi-home in this equilibrium will not deviate by choosing just one of the platforms if

$$u_A\left(n_A, n_A\right) + u_B\left(n_B, N - n_A\right) - p_A - p_B \geq u_z\left(n_z, n_z\right) - p_z \tag{15}$$

for $z \in \{A, B\}$.

We next rearrange and simplify these equilibrium conditions. Equations (11) and (13) together imply

$$u_A\left(n_A, n_A\right) - u_B\left(n_B + 1, n_B\right) \geq p_A - p_B \geq u_A\left(n_A + 1, n_A\right) - u_B\left(n_B, n_B\right).$$

Equations (12) and (15), for $z = A$, lead to

$$u_B\left(n_B, N - n_A\right) \geq p_B \geq u_B\left(n_B + 1, N - n_A\right).$$

Equations (13) and (15), for $z = B$, suggest that

$$u_A\left(n_A, n_A\right) + u_B\left(n_B, N - n_A\right) - u_B\left(n_B, n_B\right) \geq p_A \geq u_A\left(n_A + 1, n_A\right) - u_B\left(n_B, n_B\right) + u_B\left(n_B, N - n_A\right).$$

Writing these expressions in terms of the market impact effects yields the set of inequalities in the statement of the proposition. ■

Proof of Proposition 6

Proof. Suppose $U_A\left(N, N\right) - p_A > U_B\left(N, N\right) - p_B$ and all level-0 agents join both platforms A and B. A level-1 agent assumes that all other agents are of level 0. Hence, she believes that all other agents join both platforms. Given that belief, if she joins only platform B, her net payoff is $U_B\left(N, N\right) - p_B$ and her expected payoff if she joins only platform A is $U_A\left(N, N\right) - p_A$. If she joins both platforms then she does not gain any benefit from joining platform B as she meets all the agents of the opposite type already at the Pareto dominant platform A. Her net payoff from multi-homing, thus, is $U_A\left(N, N\right) - p_A - p_B$. Hence, all level-1 agents will choose to go to platform A. A level-2 agent believes that any of the other agents is of level 0 with probability $\frac{f(0)}{f(0)+f(1)}$ and of level 1 with probability $\frac{f(1)}{f(0)+f(1)}$. Moreover, she believes that all other agents join platform A and level-0 agents join platform B in addition to joining platform A. Hence, her optimal action is to join only platform A. Similar arguments show that all agents with a higher level of cognitive ability will choose to join only platform A. In the unique equilibrium, a level-0 agent joins both platform and a level-l agent joins only the Pareto dominant platform A for all $l \geq 1$. ■

Proof of Proposition 8

Proof. Suppose A is the super-dominant platform. Then, any agent with sophistication level of 1 or higher will never choose single-homing on platform B over single-homing on platform A. Moreover, given Assumptions 5 and 6, both U_{mh} and $U_{sh,A}(\lambda, \lambda)$ are single-peaked in λ. Note that $U_{mh}(0,0) - p_A - p_B < U_{sh,A}(0,0) - p_A$ and $U_{mh}\left(\frac{1}{2}, \frac{1}{2}\right) - p_A - p_B > U_{sh,A}\left(\frac{1}{2}, \frac{1}{2}\right) - p_A$. Using similar logic to those in the proof of Proposition 3, one can show that there is exactly one λ^* such that $U_{mh}(\lambda^*, \lambda^*) - p_A - p_B = U_{sh,A}(\lambda^*, \lambda^*) - p_A$ and $U_{mh}(\lambda^*, \lambda^*) - p_A - p_B < U_{sh,A}(\lambda^*, \lambda^*) - p_A$ for $\lambda < \lambda^*$ and $U_{mh}(\lambda^*, \lambda^*) - p_A - p_B > U_{sh,A}(\lambda^*, \lambda^*) - p_A$ for $\lambda > \lambda^*$.

Now we analyze the best responses of sophisticated agents given level-0 agents' behavior. First, consider the case that $\widetilde{\lambda} < \lambda^*$; that is, relatively few level-0 agents choose a platform exclusively. Then it is optimal for a level-1 agent to choose only platform A as $U_{mh}\left(\widetilde{\lambda}, \widetilde{\lambda}\right) - p_A - p_B < U_{sh,A}\left(\widetilde{\lambda}, \widetilde{\lambda}\right) - p_A$. A level-2 agent then believes that other agents choose platforms A and B exclusively with probabilities $\frac{\widetilde{\lambda} f(0)+f(1)}{f(0)+f(1)}$ and $\frac{\widetilde{\lambda} f(0)}{f(0)+f(1)}$, respectively and chooses to multi-home with probability $\frac{\left(1-2\widetilde{\lambda}\right)f(0)}{f(0)+f(1)}$. That is, according to her beliefs, more agents join platform A exclusively and fewer agents join platform B exclusively compared to the beliefs of level-1 agents. Given Assumption 6, she gets strictly higher payoff by single-homing on platform A than multi-homing and will choose platform A exclusively in any equilibrium. Similarly, one can show that all level-l agents for $l \geq 1$ will choose platform A when $\widetilde{\lambda} < \lambda^*$.

Next suppose $\widetilde{\lambda} \geq \lambda^*$. Then, it is optimal for level-1 agents to multi-home. A level-2 agent believes that all other agents are of level 0 or 1 and will choose platforms A or B exclusively with probability $\frac{\widetilde{\lambda} f(0)}{f(0)+f(1)}$ each and will multi-home with probability $\frac{\left(1-2\widetilde{\lambda}\right)f(0)+f(1)}{f(0)+f(1)}$. If $\frac{\widetilde{\lambda} f(0)}{f(0)+f(1)} > \lambda^*$ then the level-2 agent will multi-home. Otherwise, she will choose platform A exclusively. In general, suppose $\underline{k} > 1$ is such that $\frac{\widetilde{\lambda} f(0)}{\sum_{k=0}^{\underline{k}-2} f(k)} \geq \lambda^* > \frac{\widetilde{\lambda} f(0)}{\sum_{k=0}^{\underline{k}-1} f(k)}$. Then agents of level l will multi-home for $l < \underline{k}$ and will choose platform A exclusively for $l \geq \underline{k}$ in the unique equilibrium. ■

Proof of Proposition 10

Proof. Suppose all agents choose to join their preferred platform. That is, $\widetilde{n_A}$ pairs of males and females join platform A and $\widetilde{n_B}$ pairs of males and females join platform B. If

$$\Delta U_{\widetilde{n_A}} = u_A\left(\widetilde{n_A}, \widetilde{n_A}\right) - p_A - u_B\left(\widetilde{n_B}, \widetilde{n_B}\right) + p_B \geq 0$$

then, given the benefit from choosing one's own preferred platform (θ) and the market impact effects, an agent located on platform A will have no incentive to join platform B instead. Now, if $\delta_{A,\widetilde{n_A}} + \theta \geq \Delta U_{\widetilde{n_A}}$ then

$$\begin{aligned} u_A\left(\widetilde{n_A}, \widetilde{n_A}\right) - u_A\left(\widetilde{n_A}+1, \widetilde{n_A}\right) + \theta &\geq u_A\left(\widetilde{n_A}, \widetilde{n_A}\right) - p_A - u_B\left(\widetilde{n_B}, \widetilde{n_B}\right) + p_B \\ \Longrightarrow u_B\left(\widetilde{n_B}, \widetilde{n_B}\right) - p_B + \theta &\geq u_A\left(\widetilde{n_A}+1, \widetilde{n_A}\right) - p_A. \end{aligned}$$

In that case, an agent locating on platform B will have no incentive to join platform A instead. Similarly, if $\Delta U_{\widetilde{n_A}} < 0$ then $\delta_{B,\widetilde{n_B}} + \theta \geq -\Delta U_{\widetilde{n_A}}$ ensures that none of the agents will have an incentive to deviate from the strategy of choosing her preferred platform.

Now suppose all agents join their non-preferred platforms. That is, $\widetilde{n_B}$ pairs of males and females join platform A and $\widetilde{n_A}$ pairs of males and females join platform B. An agent on platform A receives a net payoff of $u_A\left(\widetilde{n_B}, \widetilde{n_B}\right) - p_A$. If she decided to join her preferred platform B instead, she can earn a net payoff of $u_B\left(\widetilde{n_A}+1, \widetilde{n_A}\right) - p_B + \theta$. Suppose $\delta_{B,\widetilde{n_A}} - \theta \geq -\Delta U_{\widetilde{n_B}}$. In that case,

$$\begin{aligned} u_B\left(\widetilde{n_A}, \widetilde{n_A}\right) - u_B\left(\widetilde{n_A}+1, \widetilde{n_A}\right) - \theta &\geq -u_A\left(\widetilde{n_B}, \widetilde{n_B}\right) + p_A + u_B\left(\widetilde{n_A}, \widetilde{n_A}\right) - p_B \\ \Rightarrow u_A\left(\widetilde{n_B}, \widetilde{n_B}\right) - p_A &\geq u_B\left(\widetilde{n_A}+1, \widetilde{n_A}\right) - p_B + \theta. \end{aligned}$$

Therefore, an agent located on platform A will have no incentive to locate on her preferred platform B instead. Similarly, agents locating on platform B will have no incentive to locate on platform A if $\delta_{A,\widetilde{n_B}} - \theta \geq \Delta U_{\widetilde{n_B}}$.

Finally, suppose $\widetilde{n_A} - m$ pairs of male and female agents choose their preferred platform A, m pairs of male and female agents choose their non-preferred platform B and $\widetilde{n_B}$ pairs of male and female agents choose their preferred platform B for some $m \in \{1, 2, \ldots, \widetilde{n_A} - 1\}$. Now, if $\delta_{A,\widetilde{n_A}-m} - \theta \geq \Delta U_{\widetilde{n_A}-m}$ then

$$\begin{aligned} -u_A\left(\widetilde{n_A} - m + 1, \widetilde{n_A} - m\right) - \theta &\geq -u_B\left(\widetilde{n_B} + m, \widetilde{n_B} + m\right) - p_A + p_B \\ \Rightarrow u_B\left(\widetilde{n_B} + m, \widetilde{n_B} + m\right) - p_B &\geq u_A\left(\widetilde{n_A} - m + 1, \widetilde{n_A} - m\right) - p_A + \theta. \end{aligned}$$

In that case, an agent who is located on her platform B will have no incentive to join platform A instead no matter whether her preferred platform is A or B. If $\delta_{B,\widetilde{n_B}+m} + \theta \geq -\Delta U_{\widetilde{n_A}-m}$ then

$$\begin{aligned} -u_B\left(\widetilde{n_B} + m + 1, \widetilde{n_B} + m\right) + \theta &\geq -u_A\left(\widetilde{n_A} - m, \widetilde{n_A} - m\right) + p_A - p_B \\ \Longrightarrow u_A\left(\widetilde{n_A} - m, \widetilde{n_A} - m\right) - p_A + \theta &\geq u_B\left(\widetilde{n_B} + m + 1, \widetilde{n_B} + m\right) - p_B. \end{aligned}$$

Therefore, an agent locating on her preferred platform A will have no incentive to switch to platform B. Note that this condition is trivially satisfied when $\Delta U_{\widetilde{n_A}-m} \geq 0$. ■

Proof of Proposition 11

Proof. Suppose each level-0 agent chooses her preferred platform with probability $\widetilde{\lambda} \geq \frac{1}{2}$. Given the bound on θ stipulated by equation (6), $U_i(0) - p_i + \theta < U_j(1) - p_j$ for $i \in \{A, B\}$. Moreover, $U_i(1) - p_i + \theta > U_j(0) - p_j$. Assumption 5 implies single-crossing of $U_i(\lambda) - p_i + \theta$ and $U_j(1-\lambda) - p_j$ for $\lambda \in [0,1]$, $i \in \{A, B\}$ and $j \neq i$. Assumption 7 then implies that for all $\lambda > \min\left\{\frac{1}{2}, \frac{\widetilde{n_i}}{N}\right\}$, $U_i(\lambda) - p_i + \theta > U_j(1-\lambda) - p_j$. Consider a level-1 agent who prefers platform i. She believes that all agents are of level 0 and each of them chooses platform i with probability $\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}$. If $\widetilde{n_i} \geq \widetilde{n_j}$ then $\frac{\widetilde{n_i}}{N} \geq \widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N} \geq \frac{1}{2}$ and if $\widetilde{n_i} < \widetilde{n_j}$ then $\frac{1}{2} \geq \widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N} \geq \frac{\widetilde{n_i}}{N}$. Therefore, a level-1 agent whose preferred platform is i will choose platform i. A level-2 agents believe that level-0 agents choose platform i with probability $\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}$ and level-1 agents choose their preferred platforms. That is, she believes that an agent is likely to choose platform i with probability $\frac{\left(\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}\right)f(0) + \frac{\widetilde{n_i}}{N}f(1)}{f(0)+f(1)}$. Of course, if $\widetilde{n_i} \geq \widetilde{n_j}$ then $\frac{\widetilde{n_i}}{N} \geq \frac{\left(\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}\right)f(0) + \frac{\widetilde{n_i}}{N}f(1)}{f(0)+f(1)} \geq \frac{1}{2}$ and $\frac{1}{2} \geq \frac{\left(\widetilde{\lambda}\frac{\widetilde{n_i}}{N} + \left(1-\widetilde{\lambda}\right)\frac{\widetilde{n_j}}{N}\right)f(0) + \frac{\widetilde{n_i}}{N}f(1)}{f(0)+f(1)} \geq \frac{\widetilde{n_i}}{N}$ otherwise. Therefore, a level-2 agent whose preferred platform is platform i will choose platform i. In general, an agent of sophistication level l for $l > 0$, whose preferred platform is platform i, believes that her expected net payoffs from joining platforms i and j are $U_i(\lambda) - p_i + \theta$ and $U_j(1-\lambda) - p_j$, respectively for some $\lambda \in \left[\frac{1}{2}, \frac{\widetilde{n_i}}{N}\right]$ if $\widetilde{n_i} \geq \widetilde{n_j}$ and for some $\lambda \in \left[\frac{\widetilde{n_i}}{N}, \frac{1}{2}\right]$ otherwise. Thus, all agents with sophistication level $l > 0$ will choose their preferred platform in the unique equilibrium. ■

Proof of Lemma 2

Proof. In period 2, agents will choose platforms that are best responses given the location choices of agents in period 1 and strategies of other players choosing in period 2. Suppose there is a subgame perfect equilibrium where s women and t men join platform i. Without loss of any generality, we assume that $s > t$. That is, platform i has more women than men and platform j has more men than women. First assume that at least one man joins platform j and at least one woman joins platform i in period 2. Using the logic in lemma 1, we can show that this cannot constitute an equilibrium as at least one of these players can profitably deviate. Now suppose all men moving on period 2 join platform i and all women moving on period 2 join platform j. Then, if a man who is supposed to join platform j in period 1 deviate by joining platform i instead, he would compete with at most t other men and have at least s women to choose from on platform i. Since this deviation should not be profitable in equilibrium, we can surmise that

$$u_j\left(N-t, N-s\right) - p_j \geq u_i\left(t+1, s\right) - p_i.$$

Similarly, if a woman who joins platform i in period 1 deviates by joining platform j instead, she would compete with at most $N-s$ other women and have at least $N-t$ men to choose from on platform j. This implies that

$$u_i\left(s, t\right) - p_i \geq u_j\left(N-s+1, N-t\right) - p_j.$$

Given that $s > t$,

$$\begin{aligned} u_j\left(N-t, N-s\right) - p_j &\geq u_i\left(t+1, s\right) - p_i \geq u_i\left(s, s\right) - p_i > u_i\left(s, t\right) - p_i \\ &\geq u_j\left(N-s+1, N-t\right) - p_j > u_j\left(N-t, N-s\right) - p_j. \end{aligned}$$

That is impossible implying that one of these deviations will actually be profitable.

Next, suppose that all men moving on period 2 join platform i but at least one woman moving on period 2 joins platform i. Consider the deviation that one man who is supposed to join platform j in period 1 chooses platform i instead. Note that the number of men in platform i can be at most $t+1$ (including himself). As any reduction in the number of men choosing platform i in period 2 makes this deviation profitable, we assume that men do not change their response. First, suppose that the number of women choosing platform i in period 2 does not decrease as a result of this deviation. However, then we can again use the same logic as above to show that either such a deviation or deviation by one of the women joining platform i in period 2 will be profitable. If the suggested deviation decreases the number of women choosing platform i in period 2, then that implies that one or more of the women will choose platform j instead of platform i in period 2 in response to this deviation. Nevertheless, for such an agent, platform i is more attractive than before as it will have one more man and (weakly) fewer women. Thus, she will not make such a deviation. Similarly, we can show that all women and at least one man joining platform j on period 2 cannot happen in equilibrium either. Hence, under all 4 possible cases, different numbers of men and women cannot choose a platform in an equilibrium of the sequential game. Thus, the equilibrium market shares can be described as $(n, N-n)$ where n pairs of men and women join platform A and the remaining $N-n$ pairs of men and women join platform B. ■

Proof of Proposition 12

Proof. To prove the first statement, suppose there is an equilibrium of the simultaneous game where n pairs of players choose platform A and $N-n$ pairs of players choose platform B. Without loss of any generality, we assume that $n \geq d$ as at least one of the platforms will receive at least as many as d pair of players. Given that this is an equilibrium, we require that

$$u_A(n,n) - p_A \geq u_B(N-n+1, N-n) - p_B$$

and

$$u_B(N-n, N-n) - p_B \geq u_A(n+1, n) - p_A.$$

Note that this equilibrium is supported by a large set of access fees. Let us consider the equilibrium where $p_B = u_B(N-n, N-n)$ and $u_A(n,n) > p_A > u_A(n+1,n)$. This implies that $u_A(n,n) - p_A > u_B(N-n, N-n) - p_B$. Players who choose platform A are better off than players who choose platform B. We now construct this equilibrium in the sequential game. In this equilibrium, all agents moving in period 1 choose platform A. In period 2, $n-d$ pairs of players choose platform A and the rest choose platform B. Consider the strategy profile where the strategy for players in periods 1 is to join platform A. In period 2, players follow the following strategy: if they observe that $n-k_M$ men and $n-k_F$ women have chosen platform A in period 1, then k_M men and k_F women choose platform A and $N-d-k_M$ men and $N-d-k_F$ women choose platform B. Given that $n \geq d$, k_M and k_F can only take positive values. As the equilibrium constraints are satisfied, this constitutes best response from all players moving in period 2. If a player unilaterally deviates in period 1 and chooses platform B instead of platform A, the market share of the two platforms will not change given these strategy profiles. However, she will be strictly worse off because of the deviation as she will be in platform B. Thus, there will be no profitable deviation under this strategy profile.

If $n = N$, let us assume that the access fees are such that tipping to platform A is Pareto dominant. Then, all players moving in period 1 choose platform A. In period 2, all players choose platform A if no player chose platform B in period 1. Otherwise, players choose a (potentially) mixed strategy simultaneous equilibrium given the platform choice of period 1 players. Now, if a period 1 player unilaterally deviates, then the final outcome of the game will be different from market tipping to platform A. However, given that tipping to platform A is Pareto dominant, this will not be a profitable deviation for the player. Thus, the proposed strategy profile will constitute a subgame perfect equilibrium of the sequential game where the market tips to platform A. Therefore, for any $n \in \{0, 1, \ldots, N\}$, there exists a set of access fees such that market shares of $(n, N-n)$ occurs in an equilibrium of both the simultaneous and sequential games.

Next we prove the second statement. Suppose that, given the access fees, tipping to platform A occurs in an equilibrium of the sequential game. Thus, the access fees are such that there is no benefit for a player to unilaterally deviate to platform B in period 2. Then, tipping to platform A will also be an equilibrium of the simultaneous game. Now consider an equilibrium of the sequential game with market shares $(n, N-n)$ with $n \in \{1, \ldots, N-1\}$. None of these players has an incentive to unilaterally deviate. Without loss of any generality, consider a player who joins platform A in period 2. The no deviation condition implies that

$$u_A(n, n) - p_A \geq u_B(N - n + 1, N - n) - p_B. \tag{16}$$

If there is a player who chooses platform B in period 2, then

$$u_B(N - n, N - n) - p_B \geq u_A(n + 1, n) - p_A. \tag{17}$$

If there is no such player, then all players (both men and women) moving in period 2 join platform A and $n \geq N - d$. However, if equation (17) does not hold then a player joining platform B in period 1 will have incentives to unilaterally deviate and join platform A instead. Thus, both equations (16) and (17) must hold. However, that means that there must be an equilibrium of the simultaneous game where n pairs of male and female agents choose platform A and the rest choose platform B. ■

Proof of Proposition 13

Proof. We first construct the putative equilibrium and then we show that it is unique. To see that everyone choosing the risk dominant platform is a limit equilibrium, suppose that the realization in the first period was that everyone chose the risk dominant platform. Then, clearly, all strategic players (cognitive sophistication level of 1 or higher) will choose this platform in the second period. Given this, we now turn to first period behavior: Level-1 agents will choose the risk dominant platform since they view all other players as being level-0 and hence choosing the risk dominant platform is a best response. By equation (8), level-2 agents will anticipate that nearly all other players are level-1; therefore, they too will choose the risk dominant platform. An analogous argument shows that players with higher levels of sophistication will also choose this platform. Finally, equation (8) implies that the probability that all agents moving in period 1 choose the risk dominant platform goes to one in the limit. Therefore, second period players will also choose this platform. This establishes that all strategic agents choosing the risk dominant platform is an equilibrium. Finally, notice that, in any equilibrium, any level-1 player in the first period will choose the risk dominant platform given that she believes that all other players randomly choose a platform with equal probability. All higher level agents will do the same given equation (8).

Subgame perfection requires that period 2 players best respond to the outcome of the first period, therefore all strategic agents will choose the risk dominant platform in that period as well. Hence, the equilibrium identified above is the unique equilibrium of the limit cognitive hierarchy model. ■

B Endogenizing Access Fees

While the model treats access fees as exogenous, in this section we show that coexistence is consistent with equilibrium even when platforms choose fees optimally. Specifically, suppose that platforms simultaneously choose access fees prior to agents deciding on which platform to locate. As is the case in the rest of the model, platforms charge the same access fee to male and female agents. The following proposition shows that the key condition for coexistence is that the magnitude of the market impact effects must be sufficiently large. Formally,

Proposition 14 *Suppose that market impact effects are such that, for some* $n \in \{1, ..., N-1\}$

$$\begin{aligned} \delta_{i,n} &\geq \frac{N-n}{n} u_i(n+1, n) \\ \delta_{j,N-n} &\geq \frac{n}{N-n} u_j(N-n+1, N-n) \end{aligned}$$

Then it is a coexisting equilibrium for n *agents of each type to choose platform* i *with the remainder choosing platform* j *where* i *charges* $p_i = u_i(n, n)$ *and* j *charges* $p_j = u_j(N-n, N-n)$.

Proof. Consider the following proposed equilibrium. First, platforms i and j choose access fees $p_i = u_i(n, n)$ and $p_j = u_j(N-n, N-n)$. Then, agents 1 to n of each type, for some $n \in \{1, 2, \ldots, N-1\}$, follow the following strategy: choose platform i if

$$u_i(n, n) - p_i \geq u_j(N-n+1, N-n) - p_j \text{ and } u_i(n, n) \geq p_i,$$

choose platform j otherwise as long as $u_j(N-n+1, N-n) \geq p_j$ and else choose neither platform. Similarly, agents $n+1$ to N of each type choose platform j if

$$u_j(N-n, N-n) - p_j \geq u_i(n+1, n) - p_i \text{ and } u_j(N-n, N-n) \geq p_j.$$

Then, first n pairs of male and female agents join platform i because they get zero net payoff from platform i and negative net payoff from platform j. The remaining agents join platform j because they get zero net payoff from that platform and negative net payoff from platform i. Now, platforms i and j will have no incentive to change their pricing in the first stage if they cannot raise profit by choosing different access fees. Take platform i: to attract agents who would choose platform j otherwise (agents 1 to n of each type), it needs to charge an access fee of $u_i(n+1,n)$ or lower. In that case, all agents will choose platform i. This is not profitable if

$$\begin{aligned} nu_i(n,n) &\geq Nu_i(n+1,n) \\ \Longrightarrow n\left(u_i(n,n)-u_i(n+1,n)\right) &\geq (N-n)\,u_i(n+1,n) \\ \Longrightarrow n\delta_{i,n} &\geq (N-n)\,u_i(n+1,n) \\ \Longrightarrow \delta_{i,n} &\geq \frac{N-n}{n}u_i(n+1,n)\,. \end{aligned}$$

Similarly, platform j will not try to attract agents otherwise choosing platform i by reducing p_j if

$$\begin{aligned} (N-n)\,u_j(N-n,Nn) &\geq Nu_j(N-n+1,N-n) \\ \Longrightarrow \delta_{j,n} &\geq \frac{n}{N-n}u_j(N-n+1,N-n)\,. \end{aligned}$$

Thus, the proposed strategies constitute a subgame perfect coexisting equilibrium where platforms choose profit maximizing access fees. ■

While Proposition 14 specifies conditions on market impact effects where coexistence can occur in equilibrium, one may worry about whether such conditions can ever be satisfied. To allay this concern, notice that the market in Example 1 supports three coexisting equilibria when platform choose the access fees. Five pairs of men and women joining each platform with $p_A = p_B = 47.78$ is an equilibrium. Moreover, 4 pairs of men and women joining platform i and 6 pairs of men and women joining platform j with $p_i = 38.33$ and $p_j = 57.22$ are equilibria for $i \in \{A, B\}$. Hence, unequal market shares are also consistent with optimal fee choice by platforms.