# Financing Public Goods by Means of Lotteries

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**Abstract**

When viewed as taxes, lotteries are routinely criticized as being both inequitable and inefficient. But is this an entirely fair comparison? Frequently lotteries are used in lieu of voluntary contributions by private charities and governments when taxes are not feasible. In this paper, a model of equilibrium wagering behavior in lotteries, whose proceeds will be used to fund a public good, is considered. Relative to voluntary contributions, wagers in the unique lottery equilibrium (a) increase the provision of the public good, (b) are welfare improving, and (c) provide levels of the public good close to first-best as the lottery prize increases.

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# 1 Introduction

For more than 500 years, governments, private charities, and civic groups have turned to lotteries as a means of financing public goods. This, despite the fact that, over the same span, alternatives to the use of lotteries became cheaper, more sophisticated, and more efficient. Governments devised significantly more effective methods of tax collection and deficit financing, while private charities benefited from advances in marketing and communications, which opened many new channels for fund-raising. Today global lottery revenues amount to $75 billion annually. By any standard, lotteries are a huge industry and show no signs of being completely usurped in favor of other methods of financing public goods. In current economic research, however, there remains extensive debate about both the equity (see Cook and Clotfelter [7] and Karcher [23]) and efficiency (see Borg and Mason [5] and Gulley and Scott [18]) of lotteries as fund-raising instruments. Much of the analysis of these questions examines lotteries relative to other *tax* instruments. By this criteria, researchers have largely concluded that, viewed as tax instruments, lotteries do not appear to be a particularly equitable or effective means of revenue generation.

But is this an entirely fair comparison? Lotteries are often held by *private* charities lacking tax power. Currently in Britain, private charities raise about 8% (or £500 million) of their income through lotteries.[1] In the US in 1992, among 26 reporting states, about $6 billion was raised by private charities through lotteries.[2] Among state governments, legal restrictions such as Proposition 13 in California and the Headlee Amendment in Michigan as well as popular resistance to tax increases of any sort place both *de jure* as well as *de facto* limitations on the taxation schemes available to states wishing to increase revenues. Thus, lotteries may not be a substitute for confiscatory tax schemes when these are politically feasible; rather, lotteries are often used in lieu of other *voluntary* contribution schemes.

Historically, this has also been the case. The quote below describes authorization of colonial lotteries by state governments.

> As a rule [the authorization of lotteries] followed a definite pattern. Generally some group of citizens would feel the need for an improvement that could not be financed readily by voluntary contributions. They then petitioned the General Assembly for permission to set up a lottery; the legislature, unwilling to levy new taxes, authorized it, and established the rules for the drawing. (Ezell [14], p. 30.)

Viewed in this light, a much different set of questions about the performance of lotteries is suggested: Are lotteries more effective at financing public goods than other voluntary schemes? If so, how much more effective are they? Can lotteries be used to

[1] Douglas [13], Table IV, p. 87.

[2] Douglas [13], p. 357. Since only 26 states report revenues from charitable gambling, the $6 billion represents a substantial underestimate of the size of the phenomenon.

finance undesirable public goods? Is it “irrational” to participate in a lottery? These questions form the central focus of the paper.

A model of equilibrium wagering behavior in lotteries whose proceeds (net of prize amounts) are being used to finance a public good is considered. The agents choosing to participate (or not) in the lottery are assumed to be fully rational and risk-neutral in wealth. In such an environment, lotteries are shown to be an effective means of financing public goods relative to other commonly used voluntary mechanisms in a variety of ways. Precisely how lotteries are effective will be made clear below.

**Overcoming the Free Rider Problem**

In this paper, lotteries are viewed as a practical means of trying to overcome the free rider problem in the decentralized allocation of public goods. It is well known that simply asking each agent in an economy to contribute to the public good generally results in the underprovision of the public good relative to first-best levels (see Bergstom, Blume, and Varian [2] and Andreoni [1], for example). A wide variety of solutions have been offered for this problem (see Moore [26]). For instance, the mechanisms suggested by Groves [16], Groves and Ledyard [17], Walker [31], and D'Aspremont and Gerard-Varet [10] all implement first-best public goods allocations under a variety of information and equilibrium assumptions. Thus, purely as a theoretical exercise, the decentralized public goods provision problem has largely been solved. Nonetheless, most of these schemes require some tax or transfer power on the part of the organization providing the public good, which might prove problematic for organizations with limited ability to impose taxes such as charities, civic groups, or in public broadcasting. Such schemes also rely on the cognitive ability of consumers to play the equilibrium strategy. However, in laboratory settings, consumers routinely fail to play dominant equilibrium strategies even for simple mechanisms such as second-price sealed bid auctions (see Davis and Holt [11], p. 284). Finally, these mechanisms involve having individuals disclose their value (willingness to pay) for the public good. Under incentive compatible schemes, such contingent valuations may not be invariant to framing effects (see Kahneman, Knetsch, and Thaler [22]) and hence not entirely satisfactory in determining public goods allocations. Perhaps for these reasons, such mechanisms appear to be seldom employed in practice.

In contrast, lotteries continue to be a popular and widespread decentralized mechanism for financing public goods. Thus, a model highlighting the strengths and weaknesses of the lottery “mechanism” would seem to be of inherent interest despite the fact that the free rider problem has, arguably, been solved. In the analysis below, the equilibrium properties of lotteries are contrasted with other commonly used voluntary schemes. It is shown that relative to the standard voluntary contributions mechanism, lotteries (a) increase the provision of the public good; (b) are welfare improving ; and (c) provide levels of the public good close to first-best as the size of the lottery prize increases.

There are both positive and normative aspects to the results of the paper. On a positive level, the paper provides a formal model which explains the fund-raising capabilities of lotteries in a society consisting of risk-neutral, non-altruistic expected utility maximizers–an environment where one might not expect lotteries to flourish. On a normative level, the finding that certain types of lotteries act to alleviate, though not eliminate, the free rider problem in funding public goods voluntarily suggests their usefulness in fund-raising.

Some care is required in these interpretations. The assumptions of the formal model serve to highlight the role of the link between lotteries and public goods. Our intention is not to rule out other explanations of lottery play (such as risk-preference based arguments) or voluntary giving (such as other-regarding preferences), but rather to focus on aspects of the lottery mechanism unique to public goods provision roles of state run and charitable lotteries. Specifically, we show that lotteries introduce a favorable *compensating externality* serving to ameliorate the free rider problem: an effect which, to our knowledge, has not previously been considered in this literature.

**Linking Lotteries and Public Goods**

Since the link between the lottery and the provision of some public good is crucial to the results highlighted in the paper, it is perhaps useful to briefly consider the history of lotteries, highlighting the connection between lotteries and public goods.

In the 15th century, faced with the repeated predations of the Hundred Years War, French towns in Burgundy and Flanders introduced the first European lotteries as a means of paying for much needed fortifications. By 1539, the French monarchy, on the brink of bankruptcy, authorized a "loterie" to raise funds for a bridge connecting the Louvre and the Fauborg St. Germain. This proved so effective that additional lotteries were subsequently instituted to pay for defense costs in the religious wars plaguing France at that time. Likewise in England, lotteries were used to finance colonization efforts by the Virginia Company and to finance capital improvements, such as bridges and aqueducts, for the city of London.[3]

Lotteries for financing public goods also proved extremely popular in colonial America. In the period 1744-1774, lotteries were authorized to build roads and bridges, fortify cities, construct harbors and wharves, and establish schools. In Sandy Hook and New London, lotteries were also used to build lighthouses, the quintessential public good. During the American Revolution, faced with the virtual impossibility of effective tax collection, the Continental Congress turned to lotteries as a means of raising more than one million dollars for the troops in the field. Throughout the nineteenth century, lotteries and raffles were used extensively by southern states to finance infrastructure improvements and, following the Civil War, reconstruction efforts.[4]

Currently, state-run lotteries are operated in 36 US states and the District of Columbia. In fiscal year 1995, revenues from these amounted to just under $32

[3] Ezell [14], pp. 1-11.
[4] Ezell [14], pp. 29-59.

billion, about $17 billion of which represented proceeds of the lottery in excess of prizes and administrative costs.[5] Of this amount, more than half was used to fund education, while much of the remainder was used to fund other public goods such as urban renewal, infrastructure development, and environmental preservation.[6] In short, state-sponsored lotteries are still (mostly) being used to fund public goods.

But does the link between lotteries and public goods affect ticket sales? While no definitive study exists, evidence from several sources seems to suggest that it might. First, the perceived importance of linking the proceeds of a lottery with funding some public good is demonstrated by the fact that 21 of 36 lottery states explicitly earmark the proceeds of lotteries to support certain types of public goods.[7] For instance, eleven states specifically require that 100% of lottery proceeds be used to fund education. While there is considerable debate over the effectiveness of earmarking as a budgeting device due to the fungibility of state revenues (see Buchanan [6], Borg and Mason [4], and Goetz [15]), the psychological importance of the linkage may perhaps be observed by comparing wagering behavior in earmarking versus non-earmarking states.

(FIGURE 1 HERE)

As Figure 1 shows, states which earmark have higher average per capita lottery expenditures than states which do not earmark; thus suggesting that consumers pay some attention to the public goods benefits in deciding how much to bet.[8]

Advertising expenditures by states also suggest the linkage is important. Approximately 4% of television lottery advertising content consists of information concerning the public benefit from lotteries.[9] In some lottery states, such as Pennsylvania, where the lottery proceeds are particularly narrowly targeted, this percentage is much higher. While 4% is by no means a large percentage of advertising time to devote to the link to public goods, it is far more than one would expect if bettors paid no attention whatsoever to the purpose for which the lottery proceeds were being used. Additional anecdotal evidence illustrating the importance of this linkage may be seen in the comments of a Pennsylvania revenue official, who notes that, "One of the secrets of the Pennsylvania lottery is having targeted the proceeds. And having the public know where the money goes really seems to help ticket sales."[10] To sum up, lotteries are frequently linked to the provision of public goods, and this linkage does seem to affect betting behavior.

The paper proceeds as follows. Section 2 outlines the basic model and analyzes two benchmark cases: the socially optimal provision of the public good and the provision via voluntary contributions. In section 3, equilibrium wagering behavior with the introduction of a fixed-prize raffle is characterized. Relative to voluntary

[5] LaFleur [24], Lottery FAQ.
[6] Statistical Abstract of the United States, 1995 [30].
[7] LaFleur [24], Lottery FAQ.
[8] Sources: Statistical Abstract of the United States [30] and La Fleur [24], Lottery FAQ.
[9] Douglas [13], p. 355.
[10] Douglas [13], p. 365.

contributions, the introduction of a raffle increases the provision of the public good. Section 4 examines equilibrium behavior in a pari-mutuel raffle where prize amounts are determined endogenously. In section 5, betting behavior in lotto, the most popular lottery game, is considered, and an equivalence between lottos and raffles is shown. Finally, section 6 highlights the empirical and experimental implications of the results of the paper. Proofs to the main theorems and propositions are contained in an appendix.

# 2 Preliminaries

Suppose that an economy consists of a set $N = \{1, 2, ..., n\}$ of consumers (also referred to as bettors when appropriate) with quasi-linear utility functions of the form

$$U_i = w_i + h_i(G)$$

$w_i$ is a numeraire good which denotes the wealth of consumer $i$, and $G \in \Re_+$ denotes the level of the public good provided. Consumers experience diminishing marginal utility from the provision of the public good; hence, $h_i'(\cdot) > 0$ and $h_i''(\cdot) < 0$ for all $i$.

The public good is generated by transforming the numeraire good into $G$, on a one-for-one basis. This is, for the most part, without loss of generality since $h_i(\cdot)$ may be viewed as a composition of most transformation technologies and the utility derived from the public good. Finally, consumers are assumed to maximize their expected utility.

**Social Optimum**

Consider the social optimization problem which seeks to maximize aggregate surplus in the economy. The social optimization problem is to choose the total wealth[11] in the economy to be transformed into the public good. Formally, a social planner chooses $G \leq \sum_{i=1}^{n} w_i$ to maximize

$$W = \sum_{i=1}^{n} (w_i + h_i(G)) - G$$

At an interior solution,[12] the optimal amount of the public good, $G^* > 0$ solves

$$\sum_{i=1}^{n} h_i'(G^*) = 1 \qquad (1)$$

[11]Notice that with the given utility specification, the particular consumers required to contribute wealth to the public good are irrelevant.

[12]Throughout the paper, wealth constraints are assumed to be non-binding for all consumers. While binding wealth constraints are known to have significant strategic effects in auction settings, consideration of such effects would merely confound the analysis of the economic forces generated by the introduction of a raffle. As a practical matter, wealth constraints do not appear to play a significant role in determining betting behavior in lotteries.

which is the well-known Samuelson Criterion for welfare maximization. When $G^* > 0$, the public good is said to be *socially desirable.*

However, if

$$\sum_{i=1}^{n} h_i'(0) < 1$$

then it is not optimal to provide positive amounts of the public good, i.e. $G^* = 0$, and the public good is said to be *socially undesirable.*

Social desirability proves to be a useful criterion in assessing the public goods provision abilities of raffles and lottos as compared to voluntary contributions.

**Voluntary Contributions without a Provision Point**

Now suppose that the government or charitable organization chooses to rely on voluntary contributions for the provision of the public good. There are numerous instances of such voluntary contribution schemes being employed in practice such as Public Broadcasting System (PBS) fund drives, university telemarketing campaigns, direct mail requests for funds from environmental and civic groups, and street corner solicitations by little leaguers; moreover, many of these groups also rely on lotteries for fund-raising. Thus, it seems useful to compare voluntary contributions to lotteries.

Let $x_i$ denote the amount of wealth contributed by $i$, and let $x(S)$ denote the sum of contributions by a set $S \subseteq N$ of consumers. Thus, $x(N)$ denotes the total contributions of all consumers.

Given the contributions of all other consumers, $i$ chooses $x_i \in [0, w_i]$ to maximize

$$U_i = w_i - x_i + h_i(x(N))$$

A Nash equilibrium to the voluntary contribution game consists of an $n - tuple$ $\left(x_1^V, x_2^V, ..., x_n^V\right)$ of contribution amounts. The equilibrium public goods provision is then given by $G^V = x^V(N)$. In the quasi-linear framework of the model, the following well-known result characterizes equilibrium contributions (see Bergstrom, Blume, and Varian [2] for details).

**Proposition 1** *Voluntary contributions without a provision point underprovide the public good relative to first-best levels.*

Contributors do not internalize the benefit conferred upon all other consumers when deciding how much to contribute to the public good; thus, each consumer tends to undercontribute relative to what would be socially optimal. In the aggregate, this leads to a systematic underprovision of the public good. Indeed, when the public good is socially desirable, extreme free-riding is possible . For instance, suppose that for all $i$, $h_i'(0) = \frac{1}{n-1}$, then even though the public good is socially desirable, no consumer will contribute to it.

Voluntary contributions also suffer a multiplicity of equilibria; specifically, if more than two or more consumers ($i$ and $j$, say) contribute positive amounts of the public good, then there are a continuum of equilibria such that $x_i^V + x_j^V = G^V$.

**Voluntary Contributions with a Provision Point**

A simple adaptation of the above scheme leads to equilibria where the socially optimal amount of the public good may be provided. Consider the following scheme

$$G = \begin{cases} 0 & \text{if } x(N) < G^* \\ x(N) & \text{if } x(N) \geq G^* \end{cases}$$

That is, in the event that contributions are less than first-best levels, none of the public good is produced; otherwise, an amount of the public good equal to contributions is produced. In the event that none of the public good is produced, suppose that all of the contributions of consumers are refunded.

Let $\mathbf{x}^{PP} = \left\{x_1^{PP}, x_2^{PP}, ..., x_n^{PP}\right\}$ denote an equilibrium $n$-tuple of contribution amounts in this game, and let $x^{PP}(N)$ denote the sum of these equilibrium contributions, then

**Proposition 2** *When the public good is socially desirable, there exists a continuum of equilibrium allocations* $\mathbf{x}^{PP}$ *such that the efficient level of the public good is provided. In addition, there also exists a continuum of equilibrium allocations* $\mathbf{x}^{PP}$ *such that* ***none*** *of the public good is provided.*[13]

While the introduction of a provision point introduces a continuum of efficient equilibria, it also creates the possibility of equilibria that are *worse* than when there was no provision point.[14] In addition to the obvious problem of the existence of a continuum of inefficient equilibria, there are a number of other difficulties with this mechanism. First, without having any information about preferences, it is difficult to see how the government or charitable organization determines the correct public goods level to pick as the provision point. Also, the ability of the organization providing the public good to commit to this particular course of action proves crucial. An organization concerned solely about maximizing social welfare will renege on its commitment to refund the contributions if given the opportunity. When a public good is socially desirable, clearly producing some of the good will be preferred by the organization to producing none of the good. Consumers will naturally anticipate

[13] To see this, notice that since the sum of the utility functions is greater under $G = G^*$ than when $G = 0$. Since $h_i'(\cdot) > 0$ for all consumers (none of them view the object as a public bad), there exists a vector of contribution amounts $\left(x_1^{PP}, x_2^{PP}, ...x_n^{PP}\right)$ summing to $G^*$ such that for all $i$ ,

$$w_i - x_i^{PP} + h_i(G^*) \geq w_i + h_i(0)$$

thus, $\left(x_1^{PP}, x_2^{PP}, ...x_n^{PP}\right)$ constitutes an equilibrium. Moreover, there exists an open set of such contribution vectors; hence a continuum of efficient equilibria.

Other parts of Propositions 3 and 4 are straightforward exensions of well-known results for a discrete public good; hence no proof is given.

[14] One might argue that since the inefficient equilibria are Pareto dominated, they ought not to be played. However, in a laboratory setting, Isaac, Schmidtz, and Walker [21] found that, for some versions of the provision point mechanism, inefficient equilbria were frequently played.

the incentives to renege, and the mechanism will devolve into voluntary contributions without a provision point. Finally, even when commitment is credible, Pauzner [29] shows that if symmetric consumers make non-infinitesimal mistakes in their contribution amounts, then voluntary contributions with a provision point (which rely on discontinuities in payoff functions to sustain equilibrium) will again yield contributions identical to voluntary contributions without a provision point, even as the number of consumers becomes infinite. That is, the provision point mechanism is not "tremble-proof."

It is also important to recognize the distinction between voluntary contributions with provision points and fund-raising strategies with suggested contribution amounts or posted funding targets. Both suggested contributions and funding targets are, in effect, "cheap talk" on the part of the fund-raiser in the sense that they have no direct effect on the payoffs each consumer faces when deciding on how much to contribute.[15] In the event that (say) United Way fails to meet its funding target, consumers do not expect that their contributions will be returned. In contrast, the provision point mechanism does change the payoffs of the consumers.

# 3 Fixed-Prize Raffles

The raffle is one of the oldest and simplest types of lottery consisting of little more than some numbered tickets and a pre-announced prize. The simplicity and fairly low expense in setting up and administering a raffle no doubt account for the continued popularity of this game. In addition, the well-known games Bingo and Keno, also often used by charities and state lotteries for fund-raising, represent straightforward variations on the raffle idea.

A fixed-prize raffle is modeled as follows: The government or charitable organization chooses a prize, denominated in wealth, of some fixed amount $R$. The value of the prize is the same for all bettors and is commonly known. The $i$th bettor chooses a wager of $x_i \in [0, w_i]$ which, given the wagers of the other contestants, $x_{-i}$ , yields a probability of $i$ winning the contest of

$$\pi(x_i, x_{-i}) \equiv \frac{x_i}{x(N)}$$

where, as before, $x(N)$ denotes the sum of all wagers and $x(N \setminus i)$ denotes the sum of the wagers of all bettors excluding $i$.[16]

Since the charity must pay for the prize, the public good provision consists of the

[15] This is not to say that cheap talk does not matter in this setting. Experimental work by Isaac and Walker [20] suggests that communications among consumers can increase contribution rates. However, the effect of cheap talk on the part of the public goods provider remains an open question.

[16] Weesie [33] studies a similar competitive structure in a model explaining volunteerism. In his model, individuals compete in voluntary effort to obtain recognition and social status.

excess of wagers over $R$; that is

$$G = x(N) - R$$

Suppose that charity has access to an arbitrarily small amount of deficit financing of an amount $\delta$. If the wagers are within $\delta$ of $R$, then the charity proceeds with the raffle as usual.[17] If the wagers are insufficient to cover the cost of the prize, then the charity calls off the raffle and returns each bettor's wager.

Thus, given $x_{-i}$, the expected utility of $i$ from a wager $x_i$ is

$$EU_i = w_i - x_i + \frac{x_i}{x(N)} R + h_i(x(N) - R)$$

provided that the raffle is held.[18]

Differentiating with respect to $x_i$ yields the first-order condition

$$\frac{x(N \setminus i)}{(x(N))^2} R - 1 + h_i'(x(N) - R) \leq 0 \tag{2}$$

It is useful to observe that when $R = 0$; this is identical to the first-order condition for voluntary contributions without a provision point.

The next set of results characterizes the Nash equilibrium of the fixed-prize raffle for the case in which the public good is socially desirable. All proofs are relegated to the Appendix.

**Proposition 3** *The fixed-prize raffle has a unique equilibrium.*

Unlike voluntary contributions, which are plagued with a multiplicity of equilibria, the fixed-prize raffle has a unique equilibrium as a result of the introduction of competition to win the private prize among bettors.[19]

### Raffles in Lieu of Voluntary Contributions

Suppose that a raffle is used to replace an ordinary voluntary contributions campaign, such as a PBS fund drive. The potential contributors to PBS tend to be relatively well educated and are probably sophisticated enough to see through the

[17] This assumption eliminates the possibility of an unstable equilibrium, where, knowing the raffle will be called off anyway, all bettors contribute zero. Notice that in the case of voluntary contributions with provision points, such a device would not eliminate the inefficient equilibria.

[18] Provided that $x(N \setminus i) - R \geq 0$, then $EU_i$ is globally concave in $x_i$; hence the simultaneous satisfaction of first-order conditions is sufficient for an equilibrium. If $x(N \setminus i) - R < 0$, then, noting that equilibrium payoffs are positive (whereas we can normalize payoffs to zero if the raffle is called off), simultaneous satisfaction of first-order conditions is still sufficient under these circumstances.

[19] As Cornes and Sandler [9] show, relative to voluntary contributions without a provision point, the set of equilibria can change dramatically with the introduction of a joint private-public good, such as a lottery.

incentives in a simple raffle. This being the case, should PBS replace its fund drive with a fixed-prize raffle? On the one hand, the introduction of a fixed-prize raffle will introduce the chance to win a private prize which will naturally increase contributions amounts by viewers. Unfortunately for PBS, having to pay for the prize represents an additional cost which will reduce the amount of contributions available to pay for programming. Thus, it seems unclear that the raffle will be an improvement. However,

**Theorem 1.** *The fixed-prize raffle provides more of the public good than the voluntary contribution mechanism.*

Intuitively, the problem with the voluntary contributions campaign is that the public good exhibits positive externalities which are not accounted for in the contribution decisions of each individual. The introduction of a fixed-prize raffle creates an investment fund with a negative externality component. That is, when a consumer purchases more raffle tickets, he reduces the chances of winning of all other bettors. This negative externality compensates for the positive externality; thus reducing the gap between the private and social marginal benefit of contributions. As a result, equilibrium contributions increase.[20]

But how much do contributions increase? Perhaps, like the introduction of a provision point, the fixed-prize raffle creates the possibility of *over*providing the public good. In fact, this is never the case with the fixed-prize raffle. Instead, the fixed-prize raffle provides less of the public good than the social optimum and hence is *always* welfare improving relative to voluntary contributions.

To see this, suppose that the fixed-prize raffle provided exactly the socially optimal amount of the public good. If, in this equilibrium, only the first $n'$ bettors wagered positive amounts then

$$\sum_{i=1}^{n'} h_i'(G^*) - n' + (n'-1)\frac{R}{R+G^*} = 0 \tag{3}$$

However, we also know that $\sum_{i=1}^{n} h_i'(G^*) = 1$ when the public good is socially optimal. Thus,

$$\begin{aligned} & \sum_{i=1}^{n'} h_i'(G^*) - n' + (n'-1)\frac{R}{R+G^*} \\ \leq \; & \sum_{i=1}^{n} h_i'(G^*) - n' + (n'-1)\frac{R}{R+G^*} \end{aligned}$$

[20] There are several other examples of the compensating externality motif in economic models. For instance, Lazear and Rosen [25] introduce a competitive tournament among workers to overcome moral hazard problems, and Hollander [19] considers models with multiple externalities in motivating volunteerism by individuals.

$$
\begin{aligned}
&= (n' - 1)\left(\frac{R}{R + G^*} - 1\right) \\
&< 0
\end{aligned}
$$

hence the raffle must provide *less* of the public good than the first-best levels. It then readily follows that the introduction of a fixed-prize raffle is unambiguously welfare improving over voluntary contributions since it always provides levels of the public good which are closer to first-best.

Although the introduction of the fixed-prize raffle reduces the gap between the private and social marginal benefits of bettors' contribution decisions, the structure of the game is such that the negative externalities are never sufficient to completely cancel the positive externalities from giving to the public good. These compensating externality effects may be easily quantified for the case of $n$ identical bettors who value the public good according to $h(\cdot)$. The social marginal benefit of providing $G$ of the public good is simply $nh'(G)$; whereas the private marginal benefit is $h'(G)$. Thus, the positive externality associated with contributing the $G$th dollar to the public good is $(n-1)h'(G)$. The introduction of a fixed-prize raffle creates a compensating negative externality. Now, by choosing to contribute $x$ when the total wagers by other bettors are $y$, a bettor reduces the expected payoffs of the other bettors by $\frac{y}{(x+y)^2}R$. In equilibrium, this term simplifies to $\frac{n-1}{n}\frac{R}{R+G}$, and the net externality, $E(R)$, as a function of the prize offered becomes

$$E(R) = \frac{(n-1)}{n}\left(nh'(G) - \frac{R}{R+G}\right) \tag{4}$$

Since $R = 0$ under voluntary contributions, the introduction of a fixed-prize raffle reduces the gap between the private and social incentives.

**First-Best Raffles**

How does the level of public goods provision in a raffle compare with the social optimum? While a fixed-prize raffle cannot generate the socially optimal provision of the public good, it can come arbitrarily close to the first-best outcome. The intuition behind this is that the need to award a prize to some bettor, creates a "wedge" between the optimal provision of the public good and the amount provided by the raffle. This wedge may be reduced by making the prize amounts sufficiently large to induce bettors to bid closer to first-best levels, but it cannot be completely eliminated.

**Theorem 2.** *Given any $\epsilon > 0$, there exists an economy of size $\sum_{i=1}^{n} w_i^*$ and a raffle with prize $R^*$ such that the public goods provision induced by the raffle lies within $\epsilon$ of the first-best outcome.*

If we again return to the identical $n$ bettor case, notice that the gap between the private and social marginal benefit, equation (4), is decreasing as a function of the prize amount of the raffle, $R$. As $R$ becomes arbitrarily large, $\frac{R}{R+G} \to 1$; moreover, as

$G \to G^*$, $nh'(G) \to 1$. Thus, increasing the prize reduces the gap between private and social incentives and generates public goods provisions arbitrarily close to first-best levels.

In interpreting the above results, it is useful to keep in mind that, due to the assumption of risk-neutrality in wealth, a deterministic "rebate game" with rebate amount $R$ will lead to an identical conclusion. If we more realistically assume that some agents in the economy have a preference for lottery games (either through risk preferences or utility from gambling), then the lottery will provide strictly more of the public good than such a rebate game. Moreover, lotteries with finite prize levels may be chosen to exactly implement the first-best outcome. In this case, the compensating externality motives present in the risk-neutral case are operating in tandem with agents' innate preferences for lottery games to increase public goods provision over voluntary contributions.

**Incomplete Crowding Out**

In the private provision of public goods, the possibility of "crowding out" of private donations by government contributions is well-known. Bergstrom, Blume, and Varian [2] and Warr [32] show that government contributions crowd out private donations on a one-for-one basis in a general setting. Andreoni [1] and Bernheim [3] demonstrate the neutrality of a wide range of policies when a public good is funded through voluntary contributions without a provision point. In contrast, when a public good is funded by means of a fixed-prize raffle, outside donations are shown not to be neutral.

In the case of a fixed-prize raffle, given a small donation $D$, the sum of first-order conditions for positive bettors becomes

$$\sum_{i=1}^{N'} h_i'(\bar{x}(N) + D - R) - N' + (N' - 1)\,\frac{R}{\bar{x}(N)} = 0$$

Differentiating

$$\frac{\partial \bar{x}(N)}{\partial D} = \frac{-\sum_{i=1}^{N'} h_i''(\bar{x}(N) + D - R)}{\sum_{i=1}^{N'} h_i''(\bar{x}(N) + D - R) - (N' - 1)\,\frac{R}{(\bar{x}(N))^2}} \geq -1$$

and since, in absolute value, the denominator of this expression is larger than the numerator, crowding out is less than one-for-one in this circumstance. Thus

**Proposition 4** *In a fixed-prize raffle, small donations will increase the total provision of the public good.*

Incomplete crowding out suggests that an organization might wish to make an outside donation to increase the level of the public good. One common way in which such donations are made is by contributing directly to the prize pool rather than to the public good directly. Is this the most effective manner of contributing? Combining the results of Proposition 4 and Theorem 2 yields

**Corollary 1** *In a fixed-prize raffle, small donations to the prize pool provide more of the public good than direct contributions.*

To see this, observe that a direct contribution, $D$, to the public good results in the following optimization for $i$.

$$\max_{x_i} U_i = \frac{x_i}{x(N)} R - x_i + h_i (x(N) - R + D)$$

In contrast, a contribution to the prize pool results in the optimization

$$\max_{x_i} U_i = \frac{x_i}{x(N)} (R + D) - x_i + h_i (x(N) - R)$$

which may be rewritten as

$$\max_{x_i} U_i = \frac{x_i}{x(N)} (R + D) - x_i + h_i (x(N) - R - D + D)$$

Letting $R + D = R'$,then

$$\max_{x_i} U_i = \frac{x_i}{x(N)} R' - x_i + h_i (x(N) - R' + D)$$

But this is equivalent to donating $D$ directly and increasing the prize offered from $R$ to $R'$. Since crowding out is less than one for one and public goods provision is increasing in the size of the prize offered, the result then follows.

**Extreme Free Riding**

As was mentioned earlier, voluntary contributions also suffer from the drawback that socially desirable public goods may go unfunded entirely. While the fixed-prize raffle provides more of the public good than voluntary contributions, it might still be the case that some socially desirable public goods also go entirely unfunded by raffles. However, it turns out that the negative externality effect of introducing the raffle is always sufficient to generate bets in excess of the prize for all socially desirable public goods and never sufficient to generate bets in excess of the prize for socially *undesirable* public goods. More formally

**Theorem 3.** *The fixed-prize raffle provides positive amounts of the public good if and only if the good is socially desirable.*

# 4 Pari-Mutuel Raffles

In this section, the prize amount of the raffle is made endogenous. That is, instead of designating a fixed-prize amount, the government or charitable organization designates a percentage of total wagers to be placed in a prize pool. Specifically, in a pari-mutuel raffle, some percentage, $p$, of the "handle" (the total bets) is rebated in the form of prizes. As with voluntary contributions, assume that there is no cost to administer the raffle; hence $(1-p)$ of the handle is used to fund a public good.

In a pari-mutuel raffle, the utility maximization problem becomes to choose $x_i$ (given the wagers of all other bettors, $x_{-i}$) to maximize

$$EU_i = w_i + \left(\frac{x_i}{x(N)}\right) pB - x_i + h_i((1-p)B)$$

where $B \equiv x(N)$, the total wagers of all bettors.

Differentiating with respect to $x_i$ yields

$$p - 1 + h_i'((1-p)B)(1-p) \leq 0$$

Thus, $i$ chooses a wager satisfying

$$h_i'((1-p)B) - 1 \leq 0$$

but this is identical to the optimization conditions under voluntary contributions without a provision point. The following result is immediate; hence no proof is given.

**Proposition 5** *The equilibrium public goods provision in a pari-mutuel raffle is exactly the same as that obtained through voluntary contributions without a provision point.*

Intuitively, the pari-mutuel prize structure "dilutes" the negative externality effects of a fixed-prize raffle. Additional bets continue to reduce the chance of winning for all other bettors, so the negative externality component continues to be present. However, such bets simultaneously *increase* the prize pool available to all other bettors thus introducing an additional positive externality. In the quasi-linear case, these two externalities exactly offset each other, and only the positive externality effects of contributing to the public good remain. Naturally, this leads to contributions yielding an outcome identical to voluntary contributions without a provision point. For more general utility specifications, the increase in the prize pool will not exactly cancel the reduction in winning chances; however, the dilution of the negative externality will still be present. As a result, a pari-mutuel raffle will continue to be less effective at providing public goods than its fixed-prize counterpart.

# 5 Lotto

Lotto is the most popular and fastest growing of the lottery games offered in the US. Lotto ticket sales account for more than half of all lottery ticket revenues in the US and almost three quarters of lottery revenues globally. Typically, the prize structure in lotto is hybrid, having both fixed prize and a pari-mutuel components. The fixed-prize components of lotto games come from two sources, guaranteed minimum payouts regardless of ticket sales and prize money rolled over from previous drawings in which there was no winner. Once ticket sales for a lotto drawing exceed some threshold, the prize pool is augmented in pari-mutuel fashion with a fixed percent of the handle being added to the prize pool.

To most easily see the structure of the lotto game, it is helpful to begin by considering a pure fixed-prize version of the game. In a fixed-prize lotto, a government or charitable organization offers $K$ possible winning numbers (or combinations of numbers) and a fixed prize $R > 0$.

Bettors who bet on a number $k$ which wins the lotto, then share the prize in proportion to their bets as a percentage of the total amount bet on $k$. With $n$ bettors, the utility maximization problem of bettor $i$, given the wagers of the other bettors is to choose a $K$ vector of bets $x_i = (x_{i1,}x_{i2}, ..., x_{iK})$ corresponding to each of the $K$ numbers which maximize

$$EU_i = w_i + \sum_{k=1}^{K} \left( \frac{1}{K} \left( \frac{x_{ik}}{x_{ik} + y_k} \right) R - x_{ik} \right) + h_i \left( \sum_{k=1}^{K} (x_{ik} + y_k) - R \right)$$

where $y_k = \sum_{j \neq i} x_{jk}$ denotes the sum of the wagers of the other bettors on the $k$th number in the lotto.

Notice that, conditional on the winning number $k$, the payoff structure for the lotto is identical to that of the raffle. Thus, if all of the bettors knew what the winning number would be, their bets would be identical to those of the raffle. In reality, bettors are symmetrically uninformed about which number will be the winning one. Since all numbers have an equal probability of being winners, bettors simply divide their bets (knowing the winning number) evenly among all the numbers. Thus, their betting behavior in the lotto game is identical to the raffle, and the two games are *outcome equivalent* in the sense that total equilibrium wagers by each bettor are identical to a similarly structured raffle.[21] It is a simple matter to show that outcome equivalence between lottos and raffles extends to equilibrium betting on pari-mutuel forms of each game. As a result, all of the results derived for raffles also hold for lottos.

Gulley and Scott [18] observe that an empirical regularity of lotto play is that the occurrence of rollovers generates much increased betting on lotto, often in excess of the amount of the increase in the prize amount. DeBoer [12] and [18] used this variation in the prize pool to recommend changes in the odds structure to induce additional rollovers and generate greater lotto revenue. Notice, however, that the changes in betting behavior associated with rollovers correspond to changing the

[21] Some care is needed here. Outcome equivalence follows from risk neutrality. In general, raffles and lottos will not be outcome equivalent; however, the compensating externality effects which yield increased provision of the public good with the introduction of a fixed-prize raffle will still be present with the introduction of a fixed-prize lotto.

level of the fixed prize component of the lotto. As was shown earlier, increasing the fixed prize leads to increased provision of the public good net of prize amounts; that is, bets increase in excess of the amount of the prize. Thus, the observed behavior is consistent with the predictions of the model. In light of this, the data might be interpreted in support of a much different policy prescription. That is, reducing the pari-mutuel component of the prize structure while increasing the fixed prize component (with no change in the odds) should also yield higher lotto revenues.

# 6 Empirical and Experimental Implications

The results of the model offer a number of unique predictions about how lottery wagering behavior will change with changes in both the structure of the lottery games as well as the desirability of the public good being financed. In this section, a number of these implications are highlighted and tests using data from the field and in the lab are suggested.

- In a laboratory setting where the marginal utility associated with provision of the public good is known, the model makes specific predictions about the level of equilibrium wagering behavior which will occur in the lottery under approximately risk-neutral preferences. Lottery wagering explanations based solely on risk-preference arguments (absent public good considerations) predict systematically lower levels of lottery giving.
  - Morgan and Sefton [27] conducted fixed-prize raffles in a laboratory setting by extending standard voluntary contribution mechanism experimental designs. They found that average lottery bets were indistinguishable from the Nash equilibrium prediction.
- Holding fixed the public good being provided, the introduction of a fixed-prize lottery increases the provision of public good relative to voluntary contributions.
  - In [27], public goods provisions from a fixed-prize raffle were compared to the voluntary contribution mechanism. Depite “excessive giving” in the voluntary contribution mechanism, the raffle still increased the provision of the public good.
- For identical equilibrium prize awards, fixed-prize lotteries outperform pari-mutuel lotteries.

- Rollovers in lotto games provide a natural experiment of this prediction. As was mentioned above, Gulley and Scott [18] and others observe increases in lottery wagering in excess of the amount of the rollover, consistent with the theory. A straightforward experimental test of this prediction would be to structure a pari-mutuel and fixed-prize lottery such that equilibrium prize amounts are identical and observe giving behavior in each game.

- Large lotteries provide public good levels closer to first-best than small lotteries.

  - Certainly, the increased proceeds from lotto with increased prize amounts are qualitatively consistent with this prediction. In a laboratory setting, [27] confirm this qualitative prediction by coparing public goods provision in lotteries of varying prize amounts.

- If the benefit of the public good is not positively correlated with wealth, then *both* lotteries and voluntary contributions are regressive.

  - The regressivity of lotteries has been well documented; however, if lotteries are being used as a replacement for voluntary contributions, then the model predicts that voluntary contributions should also be regressive. In Figure 2, data on lottery and voluntary contribution expenditures as a percentage of household income are compared with results that appear consistent with the theoretical prediction.

    (FIGURE 2 HERE)

- Socially undesirable lotteries lose money. Notice that risk-preference based predictions of lottery gambling have no such prediction.

  - [27] compare experimentally two lotteries with identical prize levels while varying the social desirability of the public good being funded. They find that lottery wagering when the lottery is funding a socially undesirable public good.

It is important to notice that these implications and (in some cases, laboratory results) differ sharply from models in which lottery play is motivated by risk-seeking or love of gambling. In these models, one would not expect changes in betting behavior to vary with the social desirability of the public good nor to be correlated with voluntary giving. It would also be difficult to say how changes in the prize structure of the lottery should affect betting behavior. Thus, the theoretical predictions of this paper in combined with observations from both the field and the laboratory suggest that the link between individual wagering and public goods provision is important in understanding the success of lotteries.

# A Appendix

## A.1 Proof of Proposition 3

Before proceeding with the proof, the following preliminary results are helpful.

Define

$$\psi_i(G;R) \equiv h_i'(G) - 1 + \frac{R}{R+G}$$

Notice that $\psi_i$ is everywhere decreasing in $G$ and increasing in $R$.

Given a prize $R$, let $G_i$ solve

$$\psi_i(G_i;R) = 0$$

and suppose without loss of generality that $G_1 \geq G_2 \geq ... \geq G_n$.

**Lemma 1** *An equilibrium exists for the fixed-prize raffle.*

**Proof.** Since the set of $i's$ actions is compact and convex, and his payoffs are continuous and quasi-concave, the standard existence conditions are satisfied. See Theorem 20.3 Osborne and Rubinstein [28]. ■

For circumstances in which an equilibrium provides positive amounts of the public good, the following lemmas hold.

**Lemma 2** *Any equilibrium generating $G$ of the public good consists of bets $x_i > 0$ for all $i$ such that $G_i > G$ and zero bets from all other bettors.*

**Proof.** Suppose there exists a bettor such that $G_i > G$ and $x_i = 0$, then $\psi_i(G;R) > 0$ and it is profitable for $i$ to make a small positive bet. Similarly, suppose there exists a $j$ such that $G_j \leq G$. then $\psi_j(G;R) \leq 0$ and it is not profitable to bet any positive amount. ■

Combining the fact that $G_1 \geq G_2 \geq ...G_n$ with Lemma 2 implies that for any level of $G$ only the first $n'$ bettors, such that $G_{n'} \geq G > G_{n'+1}$, will wager positive amounts in any equilibrium.

**Lemma 3** *Given an equilibrium where the first $n'$ bettors wager positive amounts when all other bettors wager zero, then the wagers of all players and the public goods provision are uniquely determined.*

**Proof.** Summing equations (2) for $i = 1, ..., n'$, $G_{n'}$ solves

$$\sum_{i=1}^{n'} h_i'(G) - n' + (n'-1)\frac{R}{R+G} = 0$$

And since the left-hand side of this expression is decreasing, the solution is unique.

The tuple of wagers must also simultaneously satisfy (2) with equality for all $i = 1, ..., n'$. Fixing $G = G_{n'}$ then, for $i = 1, ..., n'$, (2) becomes

$$h_i'(G_{n'}) - 1 + \frac{x(N \setminus i)}{R + G_{n'}} = 0$$

But this is a linear system in $(x_1, x_2, ..., x_{n'})$, and, since the matrix of coefficients is nonsingular, the solutions are uniquely determined. ■

It is useful to show that there are no equilibria in which the raffle fails to be held.

**Lemma 4** *There is no equilibrium set of wagers* $(x_1, x_2, ...x_n)$ *such that* $x(N) - R < -\delta$.

**Proof.** Suppose the contrary is true. Then there is an equilibrium in which the raffle is called off. In this equilibrium, each bettors' utility is simply $w_i$.

Now, consider a deviation by bettor $i$. Suppose that $i$ alters his bet such that $x(N) - R = -\delta$. Then the raffle is held and $i$ earns

$$\begin{aligned} U_i &= w_i - x_i + \frac{x_i}{x(N)}R \\ &= w_i + x_i\left(\frac{R}{R-\delta} - 1\right) \\ &> w_i \end{aligned}$$

since $\left(\frac{R}{R-\delta} - 1\right) > 0$. Thus, this deviation is profitable for $i$ which contradicts the original supposition. ■

We now proceed to prove Proposition 3

**Proof.** From Lemma 1, we know that an equilibrium exists, and from Lemma 4, we know that, in equilibrium, the raffle is held.

Suppose a set $N' \subseteq N$ players bidding positive amounts constitutes an equilibrium. Then, for all $i \in N'$

$$h_i'(G_{N'}) - 1 + \frac{Rx(N \setminus i)}{(R + G_{N'})^2} = 0$$

for all $j \in N \setminus N'$

$$h_j'(G_{N'}) - 1 + \frac{R}{R + G_{N'}} \leq 0$$

By Lemma 3, there is a unique equilibrium involving $N'$ positive bettors.

Combining the fact that $G_1 \geq G_2 \geq ... \geq G_n$ with Lemma 2, attention may be restricted to subsets and supersets of $N'$. This follows from the fact that any equilibrium provision of the public good must either result in $N'$ and some additional bettors wagering positive amounts, or marginal players in $N'$ dropping out according to the size of $G_i$ relative to the equilibrium $G$.

*Case 1.* Suppose that there exists a set $N'' \subseteq N'$ that also constitutes an equilibrium. Then $N''$ satisfies

$i \in N'' \subseteq N'$

$$h_i'(G_{N''}) - 1 + \frac{Rx(N \setminus i)}{(R + G_{N''})^2} = 0$$

For $k \in N' \setminus N''$

$$h_k'(G_{N''}) - 1 + \frac{R}{R + G_{N''}} \leq 0 \tag{5}$$

and for $j \in N \setminus N'$

$$h_j'(G_{N''}) - 1 + \frac{R}{R + G_{N''}} \leq 0$$

Notice that for (5) to hold, then $G_{N''} > G_{N'}$.

Recall that $G_{N''}$ solves

$$\sum_{i \in N''} h_i'(G) - N'' + (N'' - 1) \frac{R}{R + G} = 0$$

evaluating this expression at $G_{N'}$ yields

$$\sum_{i \in N''} h_i'(G_{N'}) - N'' + (N'' - 1) \frac{R}{R + G_{N'}} < 0 \tag{6}$$

since, summing (2) over the set $N''$ of players yields

$$\sum_{i \in N''} h_i'(G_{N'}) - N'' + [(N'' - 1) + x(N' \setminus N'')] \frac{R}{R + G_{N'}} = 0$$

but the inequality in (6) implies $G_{N''} < G_{N'}$ which is a contradiction.

*Case 2.* Suppose that there exists a set $N'' \supseteq N'$ of players constituting an equilibrium. Then

$i \in N'$

$$h_i'(G_{N''}) - 1 + \frac{Rx(N \setminus i)}{(R + G_{N''})^2} = 0$$

For $k \in N'' \setminus N'$

$$h_k'(G_{N''}) - 1 + \frac{Rx(N \setminus i)}{(R + G_{N''})^2} = 0$$

and for $j \in N \setminus N''$

$$h_j'(G_{N''}) - 1 + \frac{R}{R + G_{N''}} \leq 0$$

To induce additional bidders to join the bidding requires $G_{N''} < G_{N'}$

Since

$$\sum_{i \in N'} h_i'(G_{N'}) - N' + (N'-1)\frac{R}{R+G_{N'}} = 0$$

then if $G_{N''} < G_{N'}$

$$\sum_{i \in N'} h_i'(G_{N''}) - N' + (N'-1)\frac{R}{R+G_{N''}} > 0 \tag{7}$$

Summing the FOCs over the $N'$ players

$$\sum_{i \in N'} h_i'(G_{N''}) - N' + \frac{R}{(R+G_{N''})^2}\left[\sum_{N'} x(N \setminus i)\right] = 0 \tag{8}$$

We can rewrite the left-hand side of (8) as:

$$\sum_{i \in N'} h_i'(G_{N''}) - N' + \left[(N'-1) + x(N'' \setminus N')\right]\frac{R}{R+G_{N''}} > $$
$$\sum_{i \in N'} h_i'(G_{N''}) - N' + (N'-1)\frac{R}{R+G_{N''}} > 0$$

by equation (7). But this is a contradiction.

Since cases 1 and 2 are exhaustive, the proof is complete. ■

## A.2 Proof of Theorem 1

Before proceeding, the following Lemma proves useful

**Lemma 5** *In a fixed-prize raffle, if the number of positive bettors remains fixed, then the provision of the public good is increasing in the amounts of the prize.*

**Proof.** By Proposition 3, there is a unique set of $N'$ bettors with equilibrium wagers $(\bar{x}_1, \bar{x}_2, ..., \bar{x}_n)$ such that summing (2) over the first $n'$ bettors yields

$$\sum_{i=1}^{n'} h_i'(\bar{x}(N) - R) - n' + (n'-1)\frac{R}{\bar{x}(N)}$$

Differentiating

$$\frac{\partial \bar{x}(N)}{\partial R} = \frac{-\sum_{i=1}^{n'} h_i''(\bar{x}(N) - R) + (n'-1)\frac{1}{\bar{x}(N)}}{-\sum_{i=1}^{n'} h_i''(\bar{x}(N) - R) + (n'-1)\frac{1}{\bar{x}(N)}\frac{R}{\bar{x}(N)}} \geq 1$$

where the inequality follows from the fact that $\frac{R}{\bar{x}(N)} < 1$; hence the denominator is smaller than the numerator.

Recall that $\frac{\partial \bar{G}}{\partial R} = \frac{\partial(\bar{x}(N)-R)}{\partial R} = \frac{\partial \bar{x}(N)}{\partial R} - 1$. Hence

$$\frac{\partial \bar{G}}{\partial R} \geq 0$$

with strict inequality provided $n' > 1$. Thus, the claim is proven. ■

We now proceed to prove Theorem 1.

**Proof.** Recall that under voluntary contributions $x^V(N)$ solves

$$\sum_{i=1}^{n^V} h_i'(x^V(N) - R) - n^V + \left(n^V - 1\right) \frac{R}{R + G^V} = 0$$

when $R = 0$ and $i = 1, ..., n^V$ bettors contribute positive amounts.

*Case 1.* If the set of positive bettors is unchanged by the introduction of a raffle, then, by Lemma 5, the provision of the public good is higher.

*Case 2.* Suppose that the set of positive bettors changes. Suppose bettor $i$ drops out with the introduction of the raffle, then

$$\psi_i(G^V; 0) > 0$$

and

$$\psi_i(\bar{G}; R) < 0$$

and since $\psi_i$ is decreasing in $G$ and increasing in $R$, then $\bar{G} > G^V$ is necessary for this to happen.

Suppose that the introduction of the raffle induces bettors $n^V+1$,...,$n^V+k$ to bet positive amounts, then $G$ must also increase.

To see this note that $G^V$ solves

$$\psi_i(G^V; 0) = 0$$

and with the fixed-prize raffle $\bar{G}$ solves

$$\sum_{i=1}^{n^V+k} \psi_i(\bar{G}; R) - \frac{R}{R + \bar{G}} = 0$$

Differencing yields

$$\sum_{i=1}^{n^V} \left(h_i'(\bar{G}) - h_i'(G^V)\right) + \sum_{i=n^V+1}^{n^V+k} \psi_i(\bar{G}; R) + \left(n^V - 1\right) \frac{R}{R + \bar{G}} = 0$$

but if $\bar{G} \leq G^V$ then all the terms in this expression are positive, which is a contradiction. Thus, $\bar{G} > G^V$.

Since cases 1 and 2 are exhaustive, the proof is complete. ■

## A.3 Proof of Theorem 2

The following Lemma is helpful in the proof.

**Lemma 6** *In a fixed-prize raffle, if a prize $R \geq \bar{R}$ is offered, then all bettors wager positive amounts.*

**Proof.** Recall that it is profitable for $i$ to wager a positive amount provided

$$\psi_i(G; R) > 0$$

Let $\bar{R}$ solve

$$\psi_n(G^*; \bar{R}) = 0$$

By Proposition **??**, all equilibrium provisions of the public good are less than $G^*$. Hence, for any equilibrium $\bar{G}$,

$$\psi_n(\bar{G}; \bar{R}) > 0$$

and since $G_1 \geq G_2 \geq ... \geq G_N$, then for all $i$

$$\psi_i(\bar{G}; \bar{R}) \geq \psi_n(\bar{G}; \bar{R})$$

and all bettors will wager positive amounts. Since, $\psi_i$ is increasing in $R$, then for all $R \geq \bar{R}$, all bettors will likewise wager positive amounts. ■

We proceed to prove Theorem 2.

**Proof.** Without loss of generality, let $R^* > \bar{R}$, then, by Lemma 6, attention can be restricted to strictly interior equilibria.

The $G$ generated by $R^*$ solves

$$\sum_{i=1}^{n} h_i'(G) - n + (n-1)\frac{R}{R+G} = 0$$

As $R \to \infty$, $G \to G^*$; moreover, by Lemma 5, $G$ is increasing in $R$. Hence, for some $\bar{G} \in [G^* - \epsilon, G^*)$ , there exists $R^*$ such that

$$\sum_{i=1}^{n} h_i'(\bar{G} - \epsilon) - n + (n-1)\frac{R^*}{R^* + \bar{G}} = 0$$

and provided $\sum_{i=1}^{n} w_i^*$ is sufficiently large, $R^*$ is a feasible prize. ■

## A.4 Proof of Theorem 3

First, suppose the raffle funds the public good.

If $(\bar{x}_1, \bar{x}_2, ..., \bar{x}_n)$ are the equilibrium wagers then the equilibrium equation of bettor $i$ for the raffle is

$$h_i'(x(N) - R) = 1 - \left( \frac{x(N \setminus i)}{(x(N))^2} \right) R$$

provided that $\bar{x}_i > 0$. Suppose $n' < n$ players wager positive amounts, then summing over all $n'$ players yields

$$\sum_{i=1}^{n'} (h_i'(x(N) - R)) = n' - (n' - 1) \frac{R}{x(N)} \tag{9}$$

If a raffle funds the public good, i.e., $R \leq \bar{x}(N)$ then (9) implies

$$\sum_{i=1}^{n'} (h_i'(0)) \geq 1$$

and hence

$$\sum_{i=1}^{n} (h_i'(0)) \geq 1$$

Thus, from (1), $G^* > 0$, and the public good is socially desirable.

Conversely, suppose the good is socially desirable and is not funded by the raffle. Recall that social desirability requires

$$\sum_{i=1}^{n} h_i'(0) \geq 1$$

and, from (9), a necessary condition for a raffle not to fund the good is

$$\sum_{i=1}^{n} h_i'(0) < 1$$

But this is clearly a contradiction. ■

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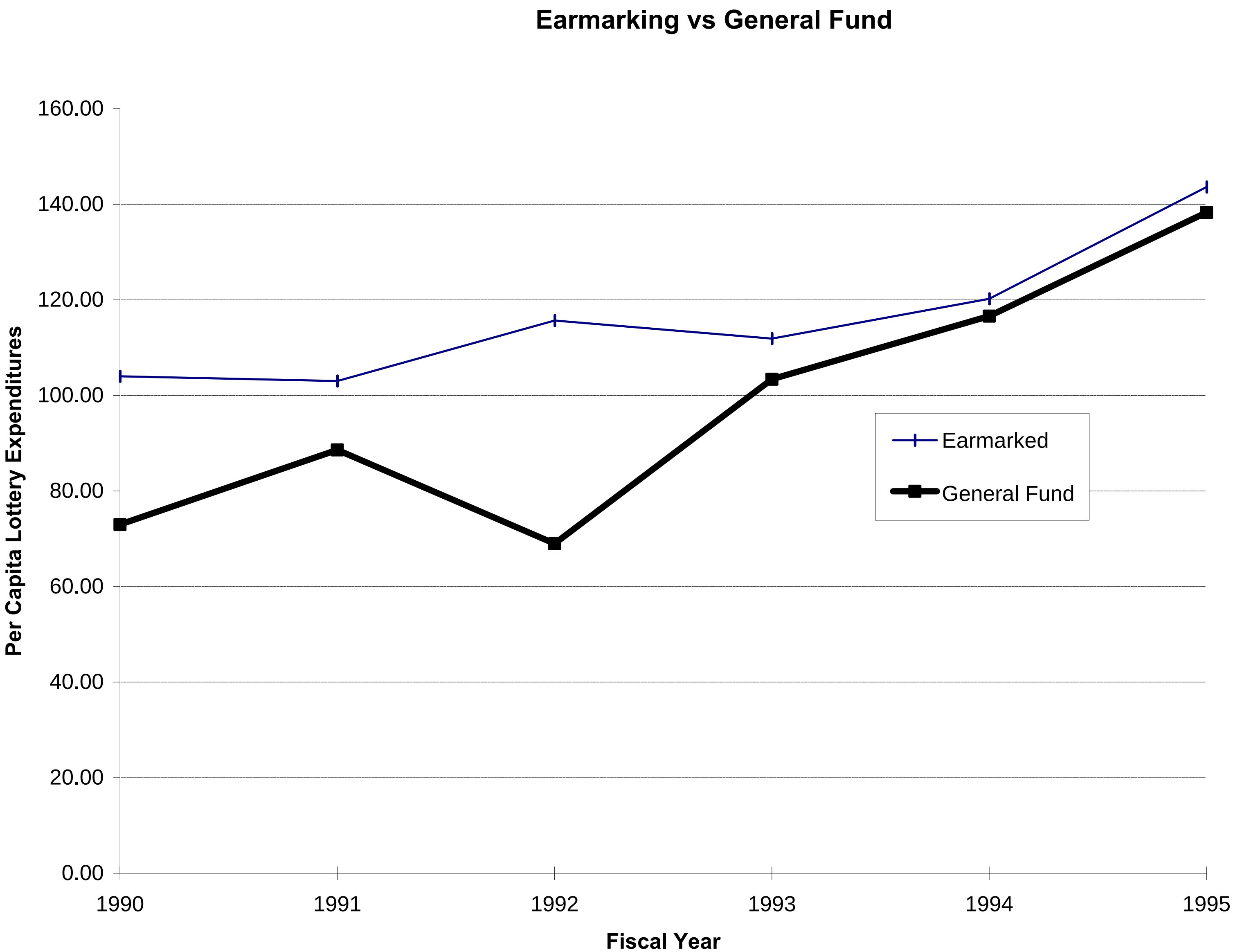
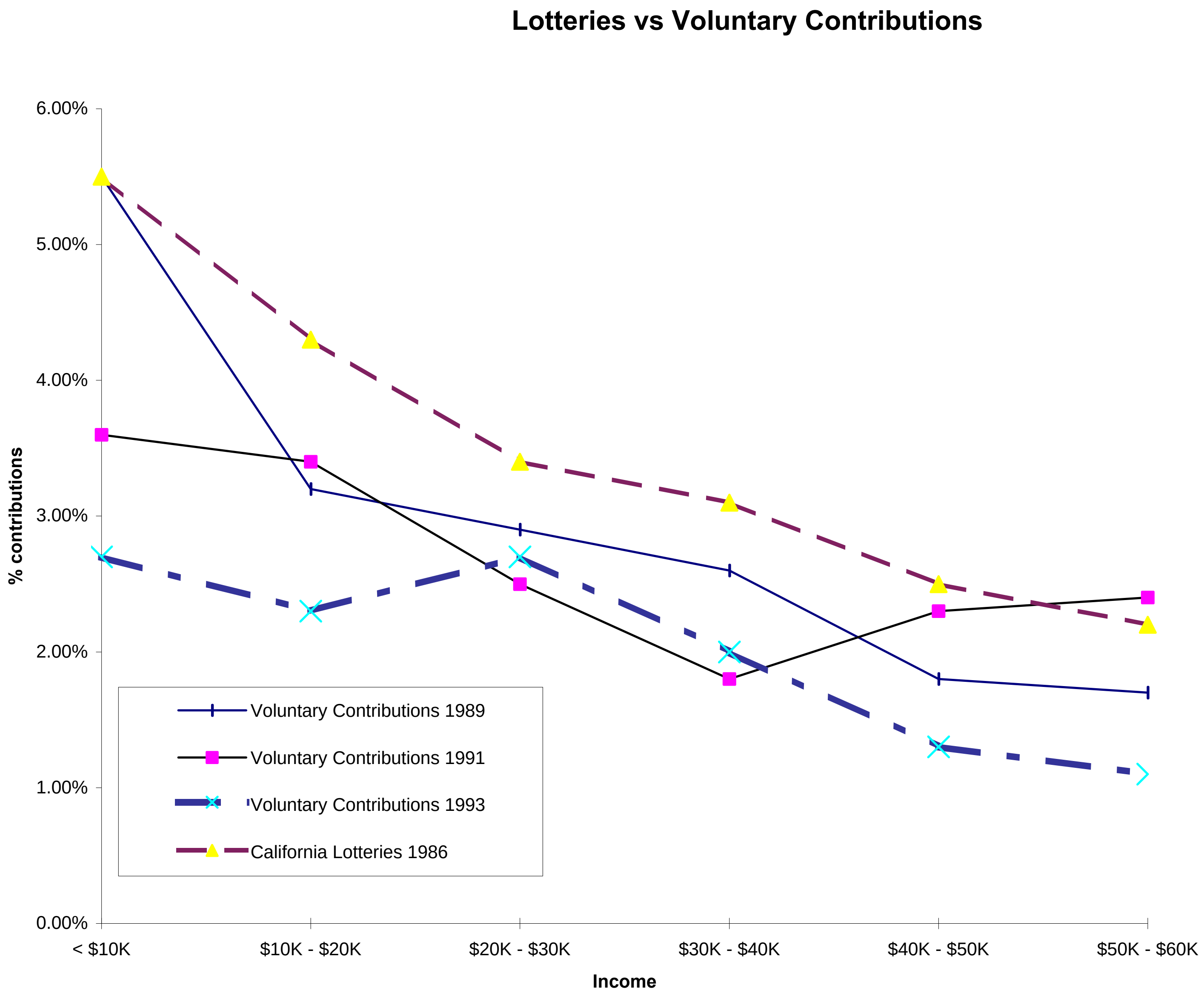
**Figure 1**




Sources:
*Statistical Abstract of the United States, 1991-1995*
*La Fleur's Lottery World*

# Figure 2




Sources:
Clotfelter and Cook, *Selling Hope*
*Statistical Abstract of the United States, 1992-95*