Game Theoretical Frameworks to Determine Rent Allocation Among Roommates

Game Theory Among Friends

MBA 211 Game Theory Final Project, Professor John Morgan

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SITUATION OVERVIEW

Many Americans have had the experience of living with roommates before marriage. At Haas, living with roommates is the norm: 85% of students live with one or more roommates and 28% live with two or more roommates[1]. During the housing group formation, there is always a point in which the roommates must decide on allocation of the rooms and the pricing split between those rooms. Sometimes, this is very simple – a house with equal sized rooms often is split equally among all roommates – but sometimes this can become significantly more complicated when a house has heterogeneous rooms and different demand for each room. There are often awkward, light-footed discussions around the selection and pricing, as each roommate hopes to secure a good deal for himself without appearing to have a self-interested motive for the sake of social unity. We believe this can lead to a suboptimal outcome, and that game theory can be used to help the roommates come to a happy equilibrium solution. In this paper, we examine in depth one scenario of five potential roommates who hope to share one house and apply our proposed game theoretic solution.

The House

A group of four girls found one house they all loved: a large, beautifully-furnished, well-located house that was owned by Asian antique art dealers who were currently traveling in Asia. However, the 4-bedroom, 2 bathroom, 2 half-bathroom house was priced at $5,000 a month, and that was more than the girls could afford. The girls, however, were desperate not to give up on the house, with its Steinway grand piano and convection ovens, and recruited a fifth girl to join in the fun.

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[1] Days at Haas Student and Housing Panel, May 1, 2010

While the house had four official bedrooms, the girls agreed that there was potential to convert the master bedroom and its sitting room into two separate rooms. They agreed that at an average price of $1,000 per person per month, each girl was comfortable with moving ahead with the lease-signing, and that they would decide the final room allocations over the next few weeks. The "five" rooms that were created had significantly different qualities, and the girls each thought through which room they wanted.

Master bedroom

The Master: Although the original master bedroom was going to be split into two rooms, the inner room was still considered the Master Bedroom. It was the second largest room in the house with east facing windows and an entire wall of closet space. The room came furnished with a king-sized bed and an antique dressing mirror. There was no private balcony and no views, but easy access to stairs that led to the backyard.

Walk-Through

The Walk-Through: The original master bedroom had a sitting bedroom, which the girls converted into the Walk-Through Bedroom. The room was only slightly smaller than the Master Bedroom and had its own clothing closet. In addition, the Walk-Through had a built-in entertainment center with a 40-inch flat screen TV. However, the Walk-Through lacked privacy since one had to cut through the corner of
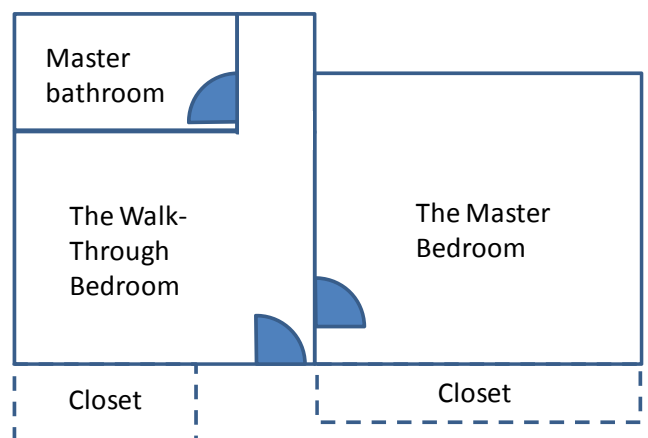
the Walk-Through to enter the Master, and since it shared a common wall with the bathroom, shared by the Master and the Walk-Through.

The Twins: The twins were two small rooms that mirrored each other in size and quality. Both had wall-length closets, and both had private balconies. They were nearly identical except that Twin #1 had a better view and Twin #2 shared a wall with the hallway bathroom.

Twin #2
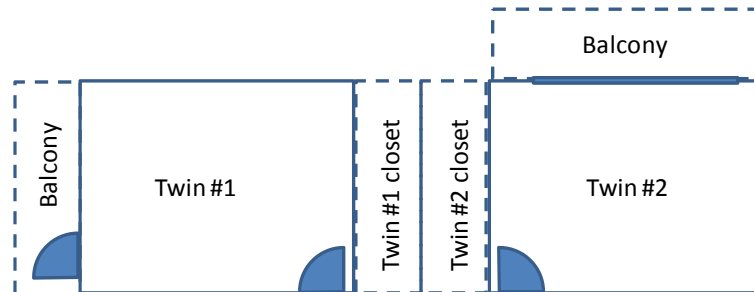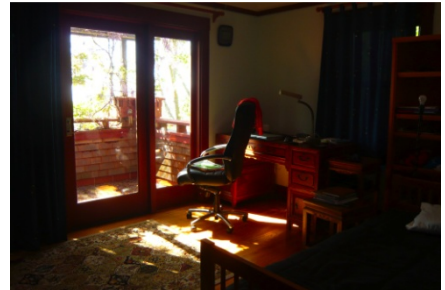
The Treehouse: In the attic was the Treehouse. It was the largest room of the house and had tons of light, brand-new full carpet, and a unique charm. However, the ceiling sloped on both sides of the room, making it uncomfortable for anyone who was tall or claustrophobic. In addition, the attic would get the hottest during the summer and whoever lived there had the added responsibility of chasing out squirrels that would sometimes climb through the windows when open. While there was no real balcony, there was easy access to the rooftop which one could relax on. And finally, there was no closet in the Treehouse. The Twins and the Treehouse all shared the same hallway bathroom.

The Treehouse

The Girls

The girls living in the house were similarly heterogeneous and diverse in their preferences and requirements. Three were under 5' 3" and could easily fit into the Treehouse, two were in some stage of a relationship and needed more privacy than the Walk-Through provided, and two were sensitive to cost. Most held similar waking hours, with one extreme early bird and no extreme night-owls. In addition to these individual requirements, the girls were also at varying levels of friendship and comfort with each other, and this discrimination was revealed through bidding disparities and altered willingness to pay when certain pairs were matched for the Walk-Through/Master combo.

A summary of the girls is described below.

	Jia Jia	Tina	Elise	Abi	Siddhi
Relationship	Single	Semi-relationship (local)	Single	Single	Long-distance relationship
Financial	Flexible	Flexible	Price-sensitive	Flexible	Price-sensitive
Current rent (pre-Arch St.)	$750	$1,000	$850	$1,200	$1,000
Height	5' 3"	5' 3"	5' 6"	5' 8"	5' 1"
Other	Can be stubborn	Focused on household harmony	5th roommate	Most polite	Concerned with delta in room costs

While most girls preferred either the Master or the Twin rooms, Tina and Elise appeared to be most open of the five girls to the possibility of living in either the Treehouse or Walk-Through.

THE ORIGINAL AUCTION

The five girls first used a serial first price ascending auction, similar to the one we conducted in class for the OPEC simulation. Although there was not a determinable "payout" structure like the OPEC auction, there was consensus in the overall ranking of the rooms in terms of desirability. The rationale to follow this auction structure was that the person who was willing to pay the most for the most desirable room should get it, assuming they know some internal value they hold for the less desirable rooms. The order of the auction was pre-determined as follows:

1. Master bedroom
2. Twin #1
3. Twin #2
4. Tree house
5. Walk-through

The auction started and the first room was bid up quickly, as were the second and third rooms. However, when only the last two rooms were left, neither bidder had a strong preference for either room. In the OPEC auction, the second to last country still had a small degree of competitive bidding, which determined its price. If that held true for this scenario, there could have been a potential surplus in the sum of the total bids. Instead, there was no incentive to bid anything for the last two rooms, and both girls instead felt pressured (instead of volunteering) to "make up the difference" for the $5,000 total sum required, regardless of their feelings of value for the rooms. The end result was that one of these last two bidders felt they were paying too much.

At the same time, certain "non-rational" behavioral influences affected the other bidders. Although each bidder had the opportunity to decide on her own values before the

bidding began, once they saw the low prices of the last two rooms, there was a feeling of the "winner's curse". Similar to the OPEC game where the first place Saudi Arabia teams held an overly optimistic mental model about potential collusion and positive outcomes, some of the top three bidders felt that they had been overly optimistic about the resulting bidding for the last two rooms. When seeing the price differential, two of the top three bidders wanted to change their bids.

Although this buyer's remorse was fine for our OPEC game, this disparity did not sit well with the girls. Whether the original high bids were caused from mistaken calculations or irrational calculations before the bidding, the girls needed to solve this. Other factors such as household harmony and interpretations of "fairness" were needed to make this outcome stick. An equilibrium at this point was not reached. The girls determined that this auction had failed and they needed to find another way to determine prices.

The girls then engaged in negotiations to try to come to a fair and more equitable range of prices. Again, behavioral reactions influenced the pricing. For instance, several girls' first reaction was to peg the price of each room as $1,000, and then consider a price deviation as a premium or discount to that average price. After discussing numerous different options, they came up with a much tighter band of prices as shown below. The range now was only $250 between the highest and lowest rent payments.[2] However, the Stalemates do not believe that this is the optimal solution for the girls.

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[2] An interesting note is that the current 4 residents of the house pay $1400 for the Master, $1250 each for the Twins, and $1100 for the Tree-House. This is a similarly small range of $300 in rent payments.

Jia Jia	Abi	Siddhi	Tina	Elise
Master	Twin	Twin	Tree house	Walk through
$1,100	$1,025	$1,025	$1,000	$850

RECOMMENDED APPROACH

Modifying the Auction Design Leads to Optimal Outcome

In determining the optimal approach to allocating rooms and rent, our criteria three fold: create a solution the roommates found "fair"; maximize household harmony; and generate the required $5,000 revenue to make the total rent payment for the house. Our recommendation is to use an iterated game of conducting sealed bid first-price auctions. The auction design rules are as follows:

- Sealed first price bid
- Each bidder must bid on all 5 rooms totaling $5,000
- Winning bids are determined based on highest spread
- Total rent scaled down proportionally to $5,000
- Any bidder can require the auction to be repeated
- Continue until all parties are satisfied with results

Under these auction rules, the optimal bidding strategy for the initial bid is to bid true indifference point for all five rooms. In order to understand this further, we can look for potential deviations to determine if any other equilibrium bidding strategy exists. We assume that each roommate wants to maximize household harmony and minimize total rent payment. By definition, a roommate who bids her indifference point would not want to bid any higher than her true willingness to pay. Such a bid could result in a successful winning bid for a room

above the indifference point meaning that she would have preferred to switch rooms with another bid.

Now, let's examine a scenario in which a roommate believes that she can underbid her most desired room in order to reduce her total rent payment. While the possibility of a profitable deviation exists, the bidder will be bidding above her indifference point for a less desired room. In this scenario, the bidder has two potential outcomes: 1) successful bid for most desired room below true WTP or 2) successful bid for less desirable room above true WTP. In the latter case, the roommate has the option to force an iteration of the game. However, this action carries reputational costs with other roommates especially if others perceive that the bidder has attempted to take advantage of the auction mechanism in order to minimize her rent payment. So, because the reputational cost of low-ball bidding offsets the benefit of reduced rent, we believe bidding true value is the optimal strategy.

In absence of the reputational factors and desire for household harmony, we do not believe this would be an optimal bidding strategy. If the goal was to get the best personal deal, one would want to make their initial bid in such a way as to hide their true willingness to pay, so as not to reveal information on the first round. Optimal opening bidding strategies in this instance could include, bidding $1,000 for each, bidding randomly or bidding in ways to disguise your room preference and willingness to pay. This is a serious problem, since people acting rationally would never be able to reach an agreement, as it is never optimal to reveal your preferences knowing that the others can force a re-bid. Hence, the reputational factors are critical in this scenario, as no girl wants to be known as deceptive to her future roommates, and

repeated dishonest bidding may result in her being asked to leave the house. It is important to remember this consideration when contemplating other uses for an auction such as this.

We conducted the auction based on the rules described above with the following results:

- Room allocation remains the same
- Rent price spread increased to $500
- Iterations were required to achieve satisfactory outcome

The bids and results of the first round of the auction game are summarized in the table below:

Bidder Name	Master	Walkthrough	Treehouse	Twin	Twin
Jia Jia	1,300	550	1,050	1,050	1,050
Abi	1,150	700	950	1,100	1,100
Tina	1,100	800	1,000	1,050	1,050
Siddhi	1,200	800	750	1,125	1,125
Elise	1,100	900	1,000	1,000	1,000
Winning Bid	1,300	900	1,000	1,100	1,125
Recommended Rent	1,198	829	922	1,014	1,037

The recommended room matches is highlighted in blue, with the recommended price allocation at the bottom of the table. However, we should note that in a normal auction Jia Jia would actually have also won the Treehouse and both Twins would have gone to Siddhi. However, because no girl can live in two rooms, we eliminated the option of any girl winning more than one room.

We believe this auction game design offers several advantages compared to the original auction and negotiation combination used. First, our recommended auction is fast and takes

only a few minutes to conduct. Second, the game has clear rules that are easily understood, unlike the negotiation, which lacked clear guidelines.

ITERATION & EQUILIBRIUM

Ideally, we would have sat all five girls down, and had them participate in as many iterations as were needed to reach a equilibrium rent split that they all felt comfortable with. Unfortunately, due to factors such as limited time and limited incentive for the girls who are currently getting a good deal to participate we were unable to complete this step of the auction. However, if we make the assumption that the girls' initial bids were their true willingness' to pay then we can simulate how the iterations would play out and determine an equilibrium. We acknowledge that this assumption may not necessarily be correct, as the girls' perceptions of value have been skewed by the previous negotiated agreement and they may have felt comfortable being completely honest with their valuations; however we believe it is a reasonable place to begin. The actual numbers in the end are not as important to the conclusion, since a true iterative process would be based on the girls' own knowledge of their valuations and hence would incorporate the true values.

As shown above, the initial round of bidding gave us the following values:

Bidder Name	Master	Walkthrough	Treehouse	Twin	Twin
Jia Jia	1,300	550	1,050	1,050	1,050
Abi	1,150	700	950	1,100	1,100
Tina	1,100	800	1,000	1,050	1,050
Siddhi	1,200	800	750	1,125	1,125
Elise	1,100	900	1,000	1,000	1,000
Winning Bid	1,300	900	1,000	1,100	1,125
Recommended Rent	1,198	829	922	1,014	1,037

This led to the following surplus for each girl, with surplus being defined as Willingness to Pay (initial bid) less Recommended Rent:

Bidder Name	Master	Walkthrough	Treehouse	Twin	Twin
Jia Jia	102	-279	128	36	13
Abi	-48	-129	28	86	63
Tina	-98	-29	78	36	13
Siddhi	2	-29	-172	111	88
Elise	-98	71	78	-14	-37

At this point, we believe rebidding would be forced, and a few things would occur. First, Abi and Siddhi would reduce their bids to $1,075 ($25 bid increments used for simplicity) since it is the lowest bid they can make where they feel certain they would win their desired rooms. Jia Jia would act similarly, dropping her bid to $1,225, because at the current level she would rather have the Treehouse, but she knows she can get the Master for $1,225. Elise would also drop her bid for the walkthrough to $775 for the same reason. The girls would increase their bids on rooms they believe they are unlikely to win in order to offset their decreased bids on their desired rooms. The new bids would be as follows:

Bidder Name	Master	Walkthrough	Treehouse	Twin	Twin
Jia Jia	1,225	700	975	1,050	1,050
Abi	1,150	725	975	1,075	1,075
Tina	1,150	750	1,000	1,050	1,050
Siddhi	1,175	750	925	1,075	1,075
Elise	1,150	775	975	1,050	1,050
Winning Bid	1,225	775	1,000	1,075	1,075
Recommended Rent	1,189	752	971	1,044	1,044

This would lead to the following surplus amounts:

Bidder Name	Master	Walkthrough	Treehouse	Twin	Twin
Jia Jia	111	-202	79	6	6
Abi	-39	-52	-21	56	56
Tina	-89	48	29	6	6
Siddhi	11	48	-221	81	81
Elise	-89	148	29	-44	-44

At this point, every girl except Tina is in the room which maximizes their surplus within the framework of this bid. One last iteration would lead us to Tina reducing her value on the attic, knowing that Jia Jia and Elise would match her reduction as they would have greater utility from winning the Master and Walk-Through respectively. Jia Jia would also lower her rent to $1,200 since that is the lowest winning bid for the Master.   This leads to the following bids:

Bidder Name	Master	Walkthrough	Treehouse	Twin	Twin
Jia Jia	1,200	750	950	1,050	1,050
Abi	1,150	750	950	1,075	1,075
Tina	1,175	750	975	1,050	1,050
Siddhi	1,175	750	925	1,075	1,075
Elise	1,175	775	950	1,050	1,050
Winning Bid	1,200	775	975	1,075	1,075
Recommended Rent	1,176	760	956	1,054	1,054

This gives each girl the following surplus amounts:

Bidder Name	Master	Walkthrough	Treehouse	Twin	Twin
Jia Jia	124	-210	94	-4	-4
Abi	-26	-60	-6	46	46
Tina	-76	40	44	-4	-4
Siddhi	24	40	-206	71	71
Elise	-76	140	44	-54	-54

This represents one possible equilibrium, since every girl is in the room which maximizes her surplus given the existing rent split. No girl has an incentive to try to win any other room in this situation. It is interesting to note that surplus is driven by the girls' willingness to pay, so Siddhi can have a higher surplus than Abi while paying the same rent for a nearly identical room. In addition, those with the greatest spreads between their bid and the next highest bid have the strongest leverage to capture surplus. In this third iteration, Abi, Tina and Siddhi have less surplus than in the first iteration, but they have no way to get more since that would require they increase their bid price on another room, and winning another room at a negative surplus.

In this game, there are many possible equilibria. For example, Jia Jia could rebid for a lower rent, knowing that everyone else has a significantly higher surplus in their current room/rent combinations than they would if they won the Master, and hence that they would lower their bid on the Master to accommodate. However, calculating all of the equilibria is beyond our modest capabilities and we believe in real life that a game would most likely end after only a couple iterations. If, for example, Jia Jia made an aggressive move to capture more surplus like that, she would probably be viewed in an extremely negative light by her roommates, who may choose not to accommodate on the basis of hurt feelings.

BEHAVIORAL FACTORS

Illusion of Fairness

Over the next year, the ladies of the Arch house will have to live together, set rules, negotiate and compromise on multiple issues. Given the high level of repeated interaction, the

auction must leave a feeling of fairness so all parties are willing to coordinate in the future. Behavioral factors influence the perception of fairness and must be taken into account when designing the auction. The iterations discussed above address the behavioral factors that influence the illusion of fairness.

Same Room, Same Price. The recommended sealed bid, first price auction fairly allocates the surplus by the same percentage of the bid. In the first round of bidding, each girl got a 6% surplus. However, under this method, the two twin bedroom occupants are paying different prices for identical rooms. Siddhi is paying $1,035 while Abi is paying $1,055 because her willingness to pay was higher. The different rents create an illusion of unfairness, and can lead to ill-will between the girls. To preserve the perception of fairness, Abi and Siddhi should pay the same price. Through the iterations discussed above, we noticed how Siddhi would lower her bid to match that of Abi's bid. At this point, Siddhi is receiving more surplus than Abi. However, due to behavioral factors, the perception of fariness is determined by the equal pricing, not equal allocation of surplus.

Relative Versus Absolute Prices. Individuals often base the fairness of a price not on its absolute value of a good, but rather by its relation to the price of other goods. The girls will therefore evaluate the fairness of the auction based on how much more or less they are paying versus the other girls. Even though the girls bid their initial true values the first round, the overall allocation will change these values. For example, many girls' relative pricing bias led them to believe that the ~$500 spread was too high. Jia Jia may feel she is paying significantly more than Elise to live in the same house. Realizing she can pay less, she will reduce her bid in

the following iterations. This will lead to a smaller spread and increase the harmony in the house through the perception of fairness.

Biases Impacting Auctions

Anchoring and Adjusting. In auctions and negotiations, parties usually tend to anchor heavily onto the initial price and do not adjust sufficiently. Thus setting an accurate anchor is essential for the fairness and accuracy of the auction. In the Serial First Price Ascending Auction the girls anchored to the average price of $1,000. This value did not reflect the different features of the rooms and each girls' willingness to pay. Therefore, the average price was a poor anchoring point. The small range between the highest and lowest prices of $250 reflected the lack of sufficiently adjusting prices to reflect the girls' WTP. In the recommended auction, the girls will anchor to the prices from the first iteration of the auction. We believe that anchoring to these values is better as they more closely reflect the girls' true values for the room. The limited adjustments on the prices will allow the girls to address the issues of perceived fairness, but will not significantly move the girls away from their initial true values.

Reputational Issues. Two factors that may cause the girls to accept terms that do not reflect their true values include their image and their friendship assessment. This is the first major issue that the girls need to agree upon. The auction will also determine each girl's interaction style in future negotiations. No girl will want to seem difficult or stubborn. Thus, in a pure negotiation, the girls may settle prior to reaching the true equilibrium point. Secondly, the girls have invested a lot of time into these friendships. If the girls cannot come to an agreement, the girls also lose their friendship. Accordingly, the girls may take into account the sunk costs of the friendship in evaluating the value of the house and can potentially overpay for

the room and their future friendship. Therefore, once again, it is crucial for the initial auction to anchor to the true values prior to iteration and verbal negotiations, and we believe the recommended auction structure most effectively captures these values.

Unlike typical auction, maximizing individual surplus can lead to social ostracism and decrease in aggregate utility. Given the repeated interaction, the girls' surplus is dependent on the value they get from the room plus the harmony of the household. To achieve this, the girls need to account for these behavioral factors that will affect the perception of fairness.

CONCLUSION

In this paper, we examined how game theory can be applied to a commonly-faced situation of determining rooms and rent-split among roommates. Although this situation is highly sensitive to behavioral responses (with strong emphasis placed on issues like fairness and reputation) we believe a game theoretical framework would have facilitated the team in quickly reaching an optimal solution that made everyone happy. We recommend a pro-rated sealed first-price auction with iterations as a way of reaching one equilibrium rent price. In this type of auction, each bidders optimal strategy is to bid true value due to the reputational costs of low-ball bidding. While this housing scenario had many difficult challenges (such as heterogenous goods with no clear "payout" values, heterogeneous demands, a forced revenue requirement, and strong human tendencies to evaluate prices in relative, not absolute, manner), we believe that an auction structure, altered to fit the situation, would be the best solution, and we will propose this to other rooming groups in the upcoming year.