Some Notes on Bidding for MBA Classes

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1 Overview

The auction used in bidding on MBA classes is a two round first price auction with budget constraints. The budget available to bidders in the second round depends on the number of successful bids made in the first round. In this document, we try to apply some of the insights of game theory to develop several strategies for successful bidding.

Let us start with the simplest possible interesting bidding situation. Suppose that there are 2 desirable classes labeled a and b, each of which has a capacity of 1. Suppose that there are 3 bidders each with an identical points budget Z. Suppose that all bidders value the desirable class at 1 but are otherwise indifferent between the 2 classes. An undesirable class (the outside option) is worth 0. In the event of a tie amongst high bids, the seat is allocated randomly among the winning bidders.

What does optimal bidding look like? First, we begin with a mental model. Suppose that bidder 1 (say) believes that the high bid for class a will be x while the high bid for class b will be y. A best response is simple, if $x < y$ then bid everything on class a; otherwise, bid everything on class b. Notice that there is no point in dividing bid between the two classes since this simply reduces the probability of getting 1 class. Of course, then we end up in a situation where, in equilibrium, bidders choose a class at random and bid their entire budget on it. Suppose that bidder 1 bid her entire budget on class a, then her expected payoff is

$$\begin{aligned} E\pi &= \binom{2}{2}\left(\frac{1}{2}\right)^2\frac{1}{3}+\binom{2}{1}\left(\frac{1}{2}\right)^2\frac{1}{2}+\binom{2}{0}\left(\frac{1}{2}\right)^2 1 \\ &= \frac{7}{12} \end{aligned}$$

Suppose instead, such a bidder deviated and bid 1 on class b with the remainder of her budget being devoted to class a. In that case, she only wins either class if no one else has bid on it. Here expected payoff here is

$$\begin{aligned} E\pi' &= \binom{2}{0}\left(\frac{1}{2}\right)^2 1+\binom{2}{2}\left(\frac{1}{2}\right)^2 1 \\ &= \frac{1}{2} \end{aligned}$$

which is worse.

Remark 1 *In situations where 2 classes have the same value (which exceeds all other classes), bidding max points on 1 of the classes is best.*

A practical example of this situation is were 2 sections of the same class are offered and this class is preferred to all others. In that case, dividing points is worse than loading all points onto a single section.

Now, let us amend the model a little. Suppose that class a is more desirable than class b. In particular, suppose that a is valued at 2 by all bidders while b is valued at 1. Undesirable courses are still valued at 0. First, suppose that everyone bid everything on the desirable class. If bidder did likewise, her payoff would be 2/3 (the value times the chance of getting a seat). Clearly, such a bidder would find it profitable to deviate and bid on class b; thereby ensuring a payoff equal to 1.

Let us look for a symmetric equilibrium in split bets where all bidders bid z for class a and $Z - z$ for class b. Under such an equilibrium, each bidder would obtain an expected payoff equal to 1. (2 x 1/3 + 1 x 1/3). If instead, bidder 1 spent her entire budget on class a, she would win this seat and earn 2. Thus, there is no equilibrium of this kind.

Let us look for an equilibrium in mixed strategies. Suppose that each bidder spends her entire budget on class a with probability p. Then the expected payoff frm a bet on a would be

$$\begin{aligned} E\pi_A &= 2\left[\binom{2}{2}(p)^2\frac{1}{3} + \binom{2}{1}p(1-p)\frac{1}{2} + \binom{2}{0}(1-p)^2\right] \\ &= 2\left(\frac{1}{3}p^2 - p + 1\right) \end{aligned}$$

whereas

$$\begin{aligned} E\pi_B &= \binom{2}{2}(p)^2 + \binom{2}{1}p(1-p)\frac{1}{2} + \binom{2}{0}(1-p)^2\frac{1}{3} \\ &= \frac{1}{3}p^2 + \frac{1}{3}p + \frac{1}{3} \end{aligned}$$

Equalizing these two expressions, we obtain and equilibrium p of

$$\begin{aligned} p &= \frac{7}{2} - \frac{1}{2}\sqrt{29} \\ &\approx .81 \end{aligned}$$

We need only check that a split bet cannot do better

$$\begin{aligned} E\pi_{split} &= \binom{2}{2}(p)^2 + 2\binom{2}{0}(1-p)^2 \\ &= 0.726\,10 \end{aligned}$$

and it does not. Thus, we've found an equilibrium.

Remark 2 *Even if everyone prefers one course to another, choose one class to make a big bet on. If most bidders are placing big bets on the more popular course, a bet on the second most popular can lead to higher rewards.*

In practice, there may be two courses that stand above all of the others. A bad strategy is to go for both. A better strategy is to place one big bet. If you sense enough herding into the top course, the second best course might be a cheaper and surer bet.

We could probably tease out some more insights from this model, but enough for now. Instead, let's just observe where the cutoff prices were in last year's fall auction. Notice that, of the less popular courses, the losing bid was most often 1. This suggests large amounts of bunching with throwaway bids of 1 clustering at the bottom. Thus, if you're going to make a "safety" bid on a course less likely to fill, it's better to bid 2 rather than 1 thereby avoiding this cluster at the bottom.

Summary

1. Bid a mental model of winning bid distributions for the courses you want.

2. Best respond given your model.

3. Make a big bet on your top (or 2nd favorite) course. Don't be greedy by placing two biggish bets on the two most popular courses and expect to win.

4. If your favorite course has two sections, make a big bet on one–don't spread bets.

5. For bets on less popular courses, a bet of 2 provides better value than a bet of 1.

6. Competitor intelligence helps. In developing a mental model, it's a good idea to ask around and see if there is some consensus strategy. Incorporate this into your model and then best respond accordingly.