1. From the revenue equivalence theorem, we know that the revenues must equal 1/3 in expectation (since the bribe auction is revenue equivalent to a Vickrey auction). Since each bidder pays his or her bid, then the expected revenues are just two times the average amount bid.

\[ 2E[\text{bid}] = 2 \int_0^1 av^2 dv \]

and this amount must be equal to \( \frac{1}{3} \).

Now integrate \( 2E[\text{bid}] \):

\[
2 \int_0^1 av^2 dv = 2a \left( \frac{1}{3} v^3 \right) \bigg|_0^1 = \frac{2}{3} a
\]

Next, set this equal to \( \frac{1}{3} \).

\[
\frac{2}{3} a = \frac{1}{3}
\]

Hence, \( a = \frac{1}{2} \). And this completes the analysis.

2. In a rev share auction with a fixed cash payment \( f \), you should bid up to the point where the inside option = the outside option. Let \( a_i \) be the rev share bid, then

\[
v_i (1 - a_i) - f = 0
\]

Clearly, if \( v_i < f \), you shouldn’t bid while for \( v_i \geq f \), it is a dominant strategy to offer a rev share equal to

\[
a_i = 1 - \frac{f}{v_i}
\]

Since the bids are increasing in values, the highest valued bidder wins the auction.

In a pure cash auction with a starting bid of \( f \), it is a dominant strategy to bid 0 if \( v_i < f \) and to bid up to your value if \( v_i \geq f \).

We are now in a position to compare revenues. Let \( v_1 \) be the value of the highest bidder and \( v_2 \) the value of the lowest bidder. Clearly if \( v_1 < f \) then neither auction produces any bids and both earn 0. When \( v_2 < f \leq v_1 \) then both auctions produce revenues equal to \( f \). The interesting case is where \( f < v_2 \), in that case the rev share
auction produces revenues equal to

\[ R_{royalty} = v_1 a_2 + f \]
\[ = v_1 \left( 1 - \frac{f}{v_2} \right) + f \]
\[ = v_1 - \frac{v_1 - v_2}{v_2} f \]

while the cash auction produces revenues equal to

\[ R_{cash} = v_2 \]

Comparing these two revenues

\[ R_{royalty} - R_{cash} = v_1 - v_2 - \frac{v_1 - v_2}{v_2} f \]
\[ = \left( v_1 - v_2 \right) \left( 1 - \frac{f}{v_2} \right) \]
\[ > 0 \]

since \( v_2 > f \). So it turns out that the rev share always produces greater revenues to the seller.

The key intuition is that the rev share auction ties the amount the seller receives directly to the value generated by the highest bidder.