Spectrum Auctions Debrief

The goal of this exercise was to design an auction where it was a *dominant strategy* to bid truthfully. Two key features any such auction must have: (1) Payments cannot depend directly on own bids; and (2) Allocation rule should be efficient (i.e. the highest valued bidders get the object assuming truthful bidding).

When there is only a single unit being auctioned, the answer is the “it’s a wonderful life rule”—give the object to the highest bidder and have that bidder pay the amount of the “damage” done to the other bidder. For instance, if bidder 1 values the object at 5 and bidder 2 values it at 3, then bidder 1 should get the object and pay 3 since this is the damage done to the other bidders by 1 having shown up for the auction. The auction implementation for this is simple: Have everyone submit bids, high bidder wins and pays second highest bid. This is called a *second price sealed bid auction*. It is a dominant strategy to bid truthfully since my bid only determines whether I get the object or not, and I only want to get the object when my payment is below my true value.

Suppose there are two identical units being auctioned. Bidders desire more than one unit, but get declining value from each additional unit. The logical extension of the second price sealed bid auction is the *uniform price auction*. Bids are ordered from highest to lowest and all winning bidders pay the market clearing price—the highest losing bid. This auction is used to sell some types of trestry securities. It is also the mechanism used by WR Hambrecht in its IPO auctions. Notice, however, that it has a problem. Since a bidder’s payment may be determined by its own bid (if it is the highest losing bid) it is no longer a dominant strategy to bid truthfully. Instead, an optimal strategy is to bid truthfully on the first unit and shade bids on
How should we auction in this setting for truth-telling to be a dominant strategy. Here’s how: First, order the bids from highest to lowest and determine how much “value” each bidder gets. (A bidder’s value is equal to the submitted winning bid). Next, remove all of bidder #1’s bids and recompute the value to all other bidders. The difference in the value other bidders get when 1 is present versus when 1 is absent is the amount of “damage” bidder 1 does to the other bidders. It is also 1’s payment. Repeat this exercise for other bidders.

Example: Suppose that bidder 1 values the 1st unit at 10 and the 2nd unit at 9. Bidder 2 values the first unit at 8 and the 2nd at 1. Suppose 2 submits bids of $y_1 > y_2$ (which may or may not be truthful). Absent bidder 1, bidder 2 is assigned surplus of $y_1 + y_2$. Suppose bidder 1 bids $x_1 > x_2$ for his two bids. If bidder 1 gets one of the objects, it must be that $x_1 > y_2$. When would bidder 1 be happy about this? Only when $y_2 < 10$; therefore, bidder 1 can do no better than to choose $x_1 = 10$, his true value. If bidder 1 gets both objects, it must be that $x_2 > y_1$. For the same reason, bidder 1 can do no better than to bid $x_2 = 9$, his true value.

In the auctions with synergies game the idea is the same. Let $v_A^1$ denote the value to bidder 1 of obtaining object $A$, etc. Suppose we award both items to bidder 1. In that case, the damage to bidder 2 is $v_{AB}^2$, therefore, under the it’s a wonderful life principle, bidder 1 should pay the package bid of 2 if 1 wins both. Suppose that 1 only wins object $A$. Then it must be that

$$v_A^1 + v_B^2 > v_{AB}^2$$

In other words, it was better to split the items than to award both to bidder 2. In that case, the damage bidder 1 does is $v_{AB}^2 - v_B^2$, in other words, the difference in surplus $B$ gets when awarded both versus being awarded only object b. Hence, a payment rule
when bidder 1 pays $y_{AB}^2 - y_{B}^2$ (where $y$ denotes the bid of bidder 2), bidder 1 can do no better than to bid truthfully.

This class of auctions are known as *Vickrey-Clarke-Groves (or VCG) auctions.* (They were invented independently in different contexts by three different game theorists.)

So why aren’t VCG auctions used much in practice? Three reasons

1. **Complexity to implement:** With a large number of items with synergies, the problem of allocating items and determining payments in a VCG auction is *NP hard* meaning that the most efficient possible algorithm cannot solve this problem in polynomial time.

2. **Complexity to explain:** Bidders need to be educated about the VCG auction. Even then, the price mechanism is sufficiently opaque that bidders may not trust it.

3. **Fairness:** Since bidders pay "damages" which are not determined by their own bids, high bidders may easily pay lower prices than low bidders who still win. This strikes many as unfair.

4. **Money left on the table:** The VCG auction makes very explicit the amount of money left on the table (the difference between the winning bidder’s value and payment). This can lead to embarrassment as the New Zealand government learned in 1989. That year, it auctioned a single spectrum license (NZ is not all that large) using a VCG auction. The winning bid was $NZ 7 million. The second highest bid was $NZ 5,000. Explaining this to voters proved difficult for the government.