Supplemental Notes on Wars of Attrition

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Abstract

In this set of supplemental lecture notes, I provide the derivation for a simple version of the war of attrition.

Consider an archetypal war of attrition situation: Two firms compete in a market to become a monopoly. The NPV of future cash flows from becoming a monopoly is v while the cost of fighting each period is one. Suppose that both firms have the same weighted average cost of capital which amounts to a discount factor $\delta < 1$.

The NPV of losses from fighting up to period t is

$$\begin{aligned} L(t) &= -\left(1+\delta+\delta^2+\ldots+\delta^{t-1}\right) \\ &= -\frac{1-\delta^t}{1-\delta} \end{aligned}$$

using a standard formula from finance.

Similarly, the NPV of losses from fighting up to period $t+1$ is

$$L(t+1) = L(t) - \delta^t$$

The NPV of securing the monopoly after t periods of fighting is

$$B(t) = \delta^t v$$

Thus, the net benefit from fighting t periods and becoming a monopoly is

$$F(t) = B(t) + L(t)$$

Stationary Strategies

We are going to look for an equilibrium in "stationary strategies". This is an equilibrium where each firm plays the same strategy in every period. Since firm's are symmetric, we'll look for a symmetric equilibrium in stationary strategies; that is, an equilibrium where bth firms play the same strategy in every period. A simple example of a stationary strategy would be to never concede. More generally, a stationary strategy is characterized by p, the probability of concession in a period.

It is obvious that a symmetric stationary equilibrium cannot consist of both firms always conceding $(p = 1)$ or never conceding $(p = 0)$, so we must look for *mixed strategies*—strategies where a firm is unpredictable.

Suppose the war has lasted t periods and firm 1 is choosing between quitting and staying in one additional period. Suppose also that firm 1's mental model of firm 2 is that firm 2 concedes with probability p. In that case, firm 1 should quit if

$$L(t) > pF(t) + (1-p)L(t+1)$$

and should stay in if

$$L(t) < pF(t) + (1-p)L(t+1)$$

Of course, in a symmetric equilibrium in stationary strategies, firm 1 must randomize between quitting and staying in. That is, both staying in and quitting have to be *best responses* to firm 2's strategy. This requires that firm 1 earn the same expected profits from both strategies. That is, the *equilibrium equation* is

$$L(t) = pF(t) + (1-p)L(t+1)$$

Now, we can fiddle with this equation to get a nice expression for p. First, notice that since $F(t) = B(t) + L(t)$ and $L(t+1) = L(t) - \delta^t$, this equation becomes

$$L(t) = pB(t) + p(L(t)) + (1-p)L(t) - (1-p)\delta^t$$

Now, cancel the sunk cost term, $L(t)$, to obtain

$$pB(t) = (1-p)\delta^t$$

Next, notice that $B(t) = \delta^t v$; hence

$$p\delta^t v = (1-p)\delta^t$$

Dividing through by δ^t, we obtain the expression

$$pv = (1-p)$$

which is the usual marginal benefit = marginal cost condition. In this case, the marginal benefit is the expected value of becoming a monopoly by staying in one more period while the marginal cost is the expected costs of fighting the war for one more period. We can then solve this for p to obtain

$$p^* = \frac{1}{1+v}$$

This expression has a number of properties:

1. Higher stakes lead to longer wars of attrition: As v goes up, the concession probability goes down and the war lasts longer.

2. The length of the war up until time t has no effect on the probability of the war ending.

Property 2 is a curious one stemming from the stationary strategies played by the two firms. Does this have any basis in fact? It turns out that in studies of settlement times in patent litigation in the tech sector, Nathan Myrhvold and collegaues have shown exactly this pattern—the probability of a settlement on date t is independent of how long the litigation has been going on.

Key Psychological Observations

1. Overconfidence bias: There is a tendency to be overconfident of the likelihood of an opponent's conceding in the "next period" of a war of attrition. The tools of game theory can help to calibrate the likelihood of concession.

2. Sunk cost fallacy: There is a tendency to focus on the resources already spent in a war as a determining factor as to whether to proceed. Notice, however, that game theory highlights the fallacy of this reasoning. The sunk costs, $L(t)$, don't figure in the final determination of an appropriate strategy at all.

3. Escalation of commitment: When faced with mounting losses in a war of attrition, there is a tendency to think that, by escalating one's commitment (i.e. pouring resources in at a faster rate), the situation can be remedied. Again, the game theory points out the fallacy of this reasoning. As an exercise, try changing the cost of fighting the war from 1 to 2 per period and recompute the equilibrium. You will find that the expected payoffs are unchanged by this escalation of commitment.