

Notes and Comments on Search Costs and
Equilibrium Price Dispersion

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Based on “Information, Search and Price Dispersion”

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Various models of information acquisition, or search, have sought to identify how price dispersion equilibrium can arise in perfectly competitive markets with homogenous goods when economic theory predicts a so-called “law of one price.” The first such group of these models are search theoretic models of price dispersion. Search costs in these models consist of a consumer’s opportunity cost of time in searching for a lower price. These costs are referred to as “shoe-leather” costs.

With the rise of the Internet, and the creation of online clearinghouses, which completely reduce search costs down to the click of a mouse, there was a renewed interest in models of equilibrium price dispersion. These revisited models show that positive search costs are indeed not a necessary condition to support equilibrium price dispersion.

I will begin by analyzing Stigler’s 1961 model of equilibrium price dispersion in a homogenous market. Sigler’s model and most models of equilibrium price dispersion begin with a general environment that consists of a continuum of price setting firms that compete in a market selling an identical homogenous product. These firms have unlimited capacity to supply this product at a constant marginal cost, m . There is a continuum of consumers interested in purchasing the product, with μ consumers per firm. Each consumer has quasi-linear utility function $u(q)+y$, so demand for the product is $q(p)=-v'(p)$. To acquire the product, the consumer must first get a price quote. There is a cost of search, c per quote.

Sigler’s 1961 model adds a couple of additional assumptions to this general environment. Sigler assumes that each consumer purchases $K \geq 1$ units of the product. The distribution of prices is given by an exogenous non-degenerate cdf

$F(p)$ on $[\underline{p}, \bar{p}]$. The consumer uses a fixed sample search where prior to searching the consumer decides to search n firms where n is chosen to minimize the total expected costs.

$$\begin{aligned}
 E[C] &= KE \left[p_{\min}^{(n)} \right] + cn \\
 &= K \int_{\underline{p}}^{\bar{p}} p dF_{\min}^{(n)}(p) + cn \\
 &= K \left[\underline{p} + \int_{\underline{p}}^{\bar{p}} [1 - F(p)]^n dp \right] + cn
 \end{aligned}$$

It is important to notice that the term in the brackets is decreasing in n . The last term is increasing in n . So an optimizing consumer will search a finite number of times, and will generally stop before he obtains the lowest price in the market.

The distribution of prices obtained by the consumers in the market is the distribution of the lowest n^* (where n^* is the optimal number of searches) draws from $F(p)$. This distribution is:

$$F_{\min}^{n^*}(p) = 1 - (1 - F(p))^{n^*}$$

So Stigler concludes that price dispersion arises as a consequence of costly search.

Stigler further investigates how prices and search intensity relate to the quantity of the item being purchased. The expected benefit to a consumer who increases her sample size from $n-1$ to n is:

$$E[B^{(n)}] = \left(E[p_{\min}^{n-1}] - E[p_{\min}^n] \right) K,$$

which is decreasing in n . Also, expected benefits are increasing in K , so the expected benefits from search are increasing in the quantity of goods bought. Since the costs

of search are independent of K , but the benefits are increasing in K , we know that search intensity, n , must be increasing in the number of goods desired, K . Also, since a firm charging p is visited by μn^* consumers, and since this firm offers the lowest price with probability $(1-F(p))^{n^*-1}$, a firm's demand is :

$$Q(p) = \mu n^* K (1 - F(p))^{n^*-1},$$

which is decreasing in p .

The Stigler model also provides some interesting comparative statics concerning how dispersed prices affect the expected transaction price. Firstly, if G is a mean preserving spread of a price distribution F , the expected transaction price of a consumer obtaining $n > 1$ price quotes is strictly lower under price distribution G than F . Intuitively, this result stems from the longer tails of the more dispersed price distribution. Consumers only care about the lowest of the price quotes they receive. The presence of more prices located in the left tail, lower expected prices, while the presence of additional high prices located in the right tail have no effect.

Stigler's simple model provides us with some very interesting comparative statics, and shows how costly search leads to equilibrium price dispersion.

Rothschild (1973) criticizes Stigler's model for two reasons. Firstly, he remarks that the search procedure in Stigler's model may not be optimal, and does not take into account new information obtained during the search. This critique while valid does not concern me as much as the fact that the distribution of prices $F()$ is exogenously specified. Since $F()$ is exogenously specified and firms are not optimizing, firms are forced to price according to $F()$ and price dispersion occurs even without search costs. Stigler's model while interesting does not demonstrate that equilibrium

price dispersion arises from information costs. In fact, Diamond (1971) proposed a model with costly search where the unique equilibrium involves all firms charging the monopoly price, and all consumers optimally searching one firm.

In Diamond's model, consumers have identical downward sloping demand, and engage in optimal sequential search. Additionally, a firm acting as a monopoly would optimally charge monopoly price of p^* . Consumers charged p^* still earn sufficient surplus to obtain a single price quote. Given that consumers engage in optimal sequential search, the optimal strategy for all firms is to charge $p=p^*$. Given this pricing strategy, the optimal strategy by consumers is to search once, and purchase the good at $p=p^*$. To see that this is the optimal pricing strategy, note that if a firm were to charge $p < p^*$ then it would be beneficial for the firm to increase price to $\min\{p+c, p^*\}$. Given the consumers strategy, any consumer that visits said firm will buy since the marginal benefit of an additional search is less than the marginal cost. Finally to conclude that this is an equilibrium, note that there is no profitable deviation by either the consumer or the firm given the others strategy.

Remarkably, Diamond has shown that it is possible to achieve monopoly pricing in a perfectly competitive market by including search frictions. This makes intuitive sense. It is possible to think of the search costs as segmenting the market to the point where for any given consumer, there is really only one provider of the good. The search costs make it too costly for the consumer to locate another provider of the good, and drive costs down to marginal cost pricing through competition.

Reinganum (1979) presents a nearly identical model to Diamond. However, Reinganum allows for firm heterogeneity in marginal cost. This one change drives his results since even if all firms price enact monopoly pricing, the monopoly price for each firm will differ according to that firms marginal costs. On its own, the Reinganum model's contribution to the literature is small. However, in conjunction with the MacMinn (1980) model, which assumes fixed sample search, and privately observed heterogeneous marginal costs, the contributions are much greater.

In the Reinganum model, a reduction in search costs decreases the variance of equilibrium prices. However, in the MacMinn model, a reduction in search costs increases the variance of equilibrium prices. In the Reinganum model, a reduction in search costs reduces the reservation prices of consumers. This induces firms with monopoly costs above the new reservation price to lower their price to the reservation price. Firms with monopoly prices below the new reservation price make no change. This leads to a reduction in the variance of equilibrium prices. In the MacMinn model a reduction in search costs leads to consumers sampling more firms. This increases competition and leads to greater price dispersion. In my view, the important contributions to the literature of these two models is in showing how a change in search mechanism can lead to search costs having completely different effects on equilibrium price dispersion.

Burdett and Judd (1983) present us with a model of equilibrium price dispersion where firms and consumers are ex ante homogeneous. In addition to the basic framework, Burdett and Judd assume that consumers have unit demand up to price v . Additionally, consumers engage in optimal fixed sample search. Each firm

in our model has marginal costs, m , and would ideally charge all consumers monopoly price $p^*=v$. Finally, Burdett and Judd assume that a consumer that is charged monopoly price earns sufficient surplus to cover the cost of obtaining a single price quote.

Interestingly, Burdett and Judd's setup has an internal contradiction. They specify that consumers have unit demand up to price v . Notice that monopoly price is v , so all consumer surplus is taken away when price is monopoly price. At price v , a consumer would be better off not searching. However, we specify that a consumer that is charged monopoly price earns sufficient surplus to cover the cost of obtaining a single price quote.

In the Burdett and Judd model, an equilibrium consists of a price distribution $F(p)$ and an optimal search distribution. Let θ_i be the probability that a consumer searches exactly i firms. Equivalently, θ_i can also denote the fraction of consumers in the population that search exactly i firms. Let us denote the expected benefit of increasing the search size from $n-1$ to n firms as:

$$E[B^{(n)}] = E[p_{\min}^{(n-1)}] - E[p_{\min}^{(n)}]$$

If all consumers obtained two or more price quotes, then the optimal strategy for all firms is to price according to marginal cost, as we would have a perfectly competitive market. However, if all firms were setting price equal to marginal cost, then the optimal search strategy of each consumer is to search exactly one firm, so we know that any equilibrium strategy must have $\theta_1 > 0$.

Now assume that each consumer adopts the strategy to only search one firm. The best response pricing strategy for every firm is to charge monopoly price.

While this does constitute an equilibrium, it is not a price dispersion equilibrium.

Thus, we know that if a price dispersion equilibrium were to exist we must have

$$0 < \theta_1 < 1.$$

Since expected consumer benefits from search are decreasing in the search sample size, a consumer must be indifferent between obtaining one price quote and two price quotes. Thus, in a price dispersion equilibrium $\theta_1, \theta_2 > 0$ and $\theta_i = 0$ for all $i > 2$. Since we know that consumers will only search one or two firms let $\theta = \theta_1$ and let $1 - \theta = \theta_2$.

If a firm sets a monopoly price, it will earn positive profits, since there is a positive probability that a consumer will only search one firm and that firm will face no competition. Specifically, the expected profits of a firm setting monopoly price is:

$$E[\pi_i | p_i = v] = (v - m)(\mu\theta)$$

If a firm sets price less than the monopoly price, that firm only makes a sale when its price is lower than the price of the other firm that is sampled. Thus expected profits when setting price less than monopoly price is:

$$E[\pi_i | p_i \leq v] = (p_i - m) \times \mu(\theta + (1 - \theta)(1 - F(p_i)))$$

Since we know that expected profits of setting $p = v$ and $p \leq v$ should be equal we can conclude that for some given distribution of searches, θ , a price dispersion equilibrium is equal to:

$$F(p) = 1 - \frac{(v - p)\theta}{(p - m)(1 - \theta)}$$

Finally, we can determine an equilibrium value for θ . Again since consumers are indifferent between searching one or two firms, $E[B^{(2)}] = c > 0$, and since $E[B^{(2)}]$ is quasi-concave, there are two price dispersion equilibrium when c is sufficiently low. One involves a high fraction of consumers searching once, and a low fraction of consumers searching twice. The other equilibrium involves a low fraction of consumers searching once and a high fraction of consumers searching twice.

We can easily see that $F(p)$ is decreasing in θ , so we are able to conclude that the distribution of prices with a small θ , first order stochastically dominates the distribution of prices with a high θ . So the expected price is lower for consumers in the equilibrium with many consumers searching two firms ($1-\theta \gg \theta$). Thus there is a positive externality between consumers in the sense that each consumer benefits when others possess better market information.

We have progressed from Stigler's model which presented us with a so called "partial-partial equilibrium" solution, to Burdett and Judd's model that demonstrates that there exist price dispersion equilibrium even in the presence of ex-ante homogeneous firms and consumers. There are also many interesting extensions to the Burdett and Judd model. For example, Moraga-Gonzalez et al (2010) show that with a finite number of firms and heterogeneous search costs, the change in average prices and consumer welfare do to the number of firms in the market depend extensively on how dispersed search costs of consumers are.

In contrast to search theoretic models, where the consumer pays a positive cost of obtaining every price quote, information clearinghouse models, aggregate the firms pricing information in one location. The clearinghouse is able to provide

consumers who access it with a list of prices charged by the different firms in the market. One very important difference between the general clearinghouse model setup and the setup we used in analyzing search theoretic models is that clearinghouse models tend to be oligopoly models. Thus, there is no continuum of firms in this setup.

Let us consider an environment with a finite number, $n > 1$, of price setting firms. These firms have unlimited capacity to supply a homogenous product at a constant marginal cost, m . There is a continuum of consumers interested in purchasing one unit of this good with a maximum willingness to pay of $v > m$. There exists in this market a clearinghouse. Firms must choose what price to specify and whether or not to list their price, p_i , at the clearinghouse. The clearinghouse charges firms, $\phi \geq 0$ to list their good at the clearinghouse. We further assume that there exists a mass $S > 0$ of consumers, that we will denote "shoppers." These consumers access the clearinghouse and buy at the lowest posted price provided this price is less than v . If there are no prices posted in the clearinghouse or all prices are greater than v , the shopper then chooses a firm at random and purchases the good provided that $p < v$. A mass of consumers $L \geq 0$ is "loyal" consumers. These consumers always purchase from the same firm given that the firm's price is less than v .

Given this setup, it is shown that if $L > 0$ or $\phi > 0$ that there exists a price dispersion equilibrium. This price dispersion equilibrium is characterized by the probability that a firm lists its price at the clearinghouse, α . Where,

$$\alpha = 1 - \left(\frac{\frac{n}{n-1} \phi}{(v-m)S} \right)^{\frac{1}{n-1}}$$

In this price dispersion equilibrium, a firm that lists its price at the clearinghouse charges a price drawn from the distribution

$$F(p) = \frac{1}{\alpha} \left(1 - \left(\frac{\frac{n}{n-1} \phi + (v-p)L}{(p-m)S} \right)^{\frac{1}{n-1}} \right) \text{ on } [p_0, v]$$

If a firm does not list its price at the clearing it sets a price equal to v . Finally, each firm regardless of if it lists its price at the clearinghouse earns expected profits

$$E[\pi] = (v-m)L + \frac{1}{n-1} \phi$$

We shall now examine three various models of clearinghouse price dispersion equilibrium. The first such model was presented by Rosenthal in 1980. Rosenthal demonstrates that equilibrium price dispersion exists in a clearinghouse model when some mass of consumers has exogenous preferences for a particular firm. Rosenthal further demonstrates that in his model, as the number of competing firms increase, the expected transaction price paid by all consumers goes up. However, this result hinges on the assumption that entry of a new firm into the market increases the fraction of loyal consumers to shoppers in the market, as each new firm that enters brings its own loyal consumers. The usefulness of this model is limited due to the very strong assumptions adopted by Rosenthal. We are able to easily see that equilibrium price dispersion arises due to heterogeneous preferences

of consumers. The heterogeneous preferences of consumers are taken as exogenous, and do not arise due to rational maximizing behavior. Interestingly, when $n=2$, the Rosenthal model describes an identical price dispersion to the Burdett and Judd model. This actually provides some interesting justification for assuming a positive mass of loyal consumers, since Burdett and Judd demonstrated that expected surplus is equivalent for the consumer making two searches (the shopper) and the consumer making only one search (the loyal consumer). So, an optimizing consumer should not care whether he is assigned to be a shopper or loyal consumer.

Varian (1980) rebrands loyal consumers as uninformed consumers, and shoppers and informed consumers, and shows that a price dispersion equilibrium exists when consumers have different ex ante information sets. In light of the results we just discussed, this is not particularly interesting. However, Varian also demonstrates that equilibrium price dispersion can persist when consumers are making optimal decisions provided that consumers have different costs of accessing the clearinghouse. To quickly see this result, let VOI be the value of information to the consumer, so

$$VOI = E[p] - E[p_{\min}^{(n)}]$$

Now, suppose the cost of accessing the is C_i where $i=\{H,L\}$, with $C_H > C_L$. Then so long as $C_L \leq VOI < C_H$ the low cost, consumers will optimally become informed, while the high cost consumer will optimally remain uninformed. A price dispersion equilibrium will persist. This leads to a valuable conclusion that in general, price

dispersion is not a monotonic function of consumers' information costs or the fraction of the "shoppers" in the market.

Baye and Morgan (2001) take a different approach to analyzing the clearinghouse model than Rosenthal or Varian. Baye and Morgan adopt a most likely valid assumption that in general it is costly for a firm to advertise their price on the clearinghouse. Specifically, Baye and Morgan specify a model with n homogeneous profit maximizing firms, a continuum of homogeneous optimizing consumers, and a profit-maximizing clearinghouse. In this setup, the clearinghouse maximizes expected profits when it sets the subscription fee to consumers sufficiently low, so all consumers subscribe to the clearinghouse, but charges firms a positive fee to post prices. Price dispersion in this model arises due to the incentives for the clearinghouse to set positive fees for the firms. In contrast to the other two clearinghouse models, which adopt strong assumptions on consumer's preferences or types, the Baye and Morgan model analyzes a very realistic environment, under the assumption of consumer and firm homogeneity. While consumer and firm homogeneity are most likely not valid real world assumptions, the use of them as an economic tool pays dividends. We are able to clearly see that at least some level of price dispersion in clearinghouse markets may be due to the positive cost for firms to post price information on the clearinghouse.

The wide variety of models presented above make it very clear that the level of price dispersion depends on the structure of the market including, the number of sellers, the distribution of costs, consumer's demand and so on. Many empirical papers have investigated price dispersion in various "real world" scenarios and have

concluded that price dispersion is ubiquitous and persistent. These empirical papers have provided us with clear evidence that widespread adoption of technologies such as the Internet, that greatly reduce or even eliminate search costs, have not lead to a reduction in levels of price dispersion.

While the literature makes it very clear that equilibrium price dispersion exists across markets for homogenous goods, it does not explore the related issue of price dispersion for a good within a firm, more usually known as price discrimination. This phenomenon is often considered in the framework of bundling, dynamic pricing, where prices change over time, or with respect to favored pricing for repeat costumers. However, I think that viewing price discrimination in the framework of search theoretic models could provide interesting economic intuition. It only seems natural, to think of price discrimination as an equilibrium characteristic that arises due to heterogeneous preferences or information asymmetries of consumers.

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