A Very Brief Introduction to the Time Value of Money

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The time is August of 2011. As you arrive for your first of four years at Berkeley, you begin to think about your tuition payments. Happily, you've just paid the $11,100 tuition due for your first year. Thinking about your years at Cal, you anticipate that you might have an unpaid internship after your third year so you decide to put some savings from this year's summer job away for your last year of tuition.

With $11,100 in hand you go down to your local bank, open up a savings account, and deposit the money. You look forward to earning interest over the next three years until you need the money. You are shocked and disappointed to learn from the bank that interest rates on savings accounts are rather low—0.3 percent.

At the end of the first year, you'll have earned 0.3 percent interest on your deposit of $11,100—interest of $33.30.

$$F_1 = P * (1 + 0.003)$$

P represents the **principal** the amount of money that you deposit. Here, we'll use F_1 to mean the future value after one year, one **time period**. Note that we multiply by 1 + 0.003 because the interest rate is .3 of a percent. (1 percent would be 0.01). The **periodic rate** is 0.3 percent.

So at the end of the first year, you'll have $11,133.30 in your account.

Compound interest and the future value formula

If interest rates remain the same, in the second year, you'll earn 1.003 times the amount in your account at the beginning of the year:

$$F_2 = F_1 * (1 + 0.003)$$

$$F_2 = \$11{,}133.30 * (1 + 0.003) = \$11{,}166.70$$

You could get the same result starting with the initial sum, P1, by multiplying by 1 + 0.003 twice:

$$F_2 = P * (1 + 0.003) * (1 + 0.003) = P * (1 + 0.003)^2$$

Before we multiply three times to find out how much you'd have in the account at the end of three years (to fund your last year of tuition) let's derive the general **future value formula** of a sum of money that earns **compound interest**:

$$F_t = P * (1 + r)^t$$

Where *r* is the periodic rate of interest and t is the number of time periods. In this case, we start with $11,100 and compound interest for 3 years:

$$F_3 = \$11{,}100 * (1 + 0.003)^3 = \$11{,}200.20$$

How inflation affects savers

While you are moving into your rather cramped Unit dorm room, your new roommate kindly inquires whether you are relying on financial aid. You cheerfully reply that you are funding your tuition from summer earnings, you've already put the money for the last year's tuition in the bank and, indeed, you're going to earn $100.20 profit from your compound interest.

Not so fast. Your roomie points out that tuition has been rising rapidly over the last few years. With a few clicks of a web search the roomie shows you that the UC Regents have already announced that unless the University System gets more money from the State of California, then tuition will have to rise by 8 percent each year for at least the next five years. That means 3 years from now, tuition will be like 24 percent more than it is now. You check the website and agree that the plan is an 8 percent per year tuition hike. However, you dispute the effect: "you can't just add 8 percent 3 times—you have to compound it". You realize that you can apply the same formula for the Future Value of a sum of money invested:

$$F_t = P * (1 + r)^t$$

In this case, F_t is the cost of tuition 3 years from now, *P* is the amount paid this year, *r* is the inflation amount per year and t is the number of time periods.

$$F_t = \$11{,}100 * (1 + 0.08)^3 = \$13{,}982.80$$

This is about 26 percent more. Why? Because each year the new tuition is 1.08 times last year's tuition, which was 1.08 times the year before.

Comparing the $11,200.20 you would have in your savings account with the estimated last year tuition of $13,982.80 you can see how *high inflation hurts savers*. Consider the case where you were planning to spend the original $11,100 to purchase an economy entry-level car. If inflation for car prices was running at 8 percent (like tuition), you'd have a choice: You could buy the car now (and have a car in hand), or you can diligently save the money for 3 years in which case you'd be out of luck because the price of the car would be $13,982.80 but you'd only have $11,200.20 in the bank.

This leads to two observations: First, savers want more interest than the prevailing rate of inflation. Second, current interest rates are exceptionally low. They've been deliberately driven lower by government policy in the hope that people will be discouraged from saving and that they'll go out and buy new cars (and keep auto workers employed). Additionally, the government hopes that firms will be willing to borrow more money at low rates and expand their businesses.

Present value of a future sum of money

Now that you know that tuition is likely to be $13,982.80 three years from now as you enter your senior year, you need to know how much money to save right now so that you'll have the correct amount when it's needed. We can calculate this just by rearranging the future value formula to solve for the present value, P_x.

$$F_t = P_x * (1+r)^t$$

Rearranged:

$$P_x = F_t * \frac{1}{(1+r)^t}$$

We know that the future sum we need (F_t) is $13,982.80, the *t* number time periods is 3, and the periodic rate of interest in that savings account is 0.3 percent, so *r* is 0.003. Substituting,

$$P_x = \$13{,}982.80 * \frac{1}{(1+0.003)^3} = \$13{,}857.71$$

The process we've just done is called **discounting a future value to the present** and the term $\frac{1}{(1+r)^t}$ is called the **discount factor**.

The processes of compounding rates of return for several time periods out into the future, or—conversely—discounting a future sum of money to the present are the key principles of all finance.

The effect of different interest rates

The present value $13,857.71 is very close to the sum of money you need ($13,982.80) because interest rates are exceptionally low. Imagine for a moment that you could find an investment that would give you a rate of return of 10 percent. This is not impossible—before the financial crisis of 2008, the US stock markets rewarded investors with rates of return that were typically 11 percent a year when averaged over long time periods (ten years or more). If you could get a 10 percent rate, using the present value formula, you'd only need to put away $10,505.49 this year to have the required $13,982.80 in 3 years time. Note, the sum you need is actually *less* than this year's tuition and that makes sense because the 10 percent return is greater than 8 percent price rise.

Most workers now have to save money for retirement through their own 401(k) plans rather than relying on a company pension. Since pension savers begin saving in their 20s (when they join the workforce) and may invest for 30 years or more, you can see why people pay attention to the rate of return that they can earn on their investments. Compounding exaggerates the effect of small changes in interest rates. Take this example. Suppose that on your 30th birthday you put $10,000 away as a lump sum in your retirement account. Your plan is to retire early, at age 60. That gives you 30 years of compounding. If you can achieve a rate of return of 6 percent, on retirement you'd have $57,435 in the account when you stop working. Not bad. But if you were able to achieve a 10 percent return you'd have $174,494. How does moving the interest rate from 6 to 10 percent (less than double) more than double the size of the account? One word: compounding. In years 2, 3, 4 and so on, there is more money in the account to grow by the rate of return. This is sometimes referred to as "the magic of compounding" or achieving "interest on the interest" (that is, interest earned this year on interest earned in previous years).

Looking for better interest rates

We've already seen that a prudent saver would seek an interest rate that is better than the rate of inflation. Interest rates on savings accounts are at all-time lows, about 0.3 percent a year. You might argue that the 8 percent a year expected increase in tuition is exceptional and that the change in the Consumer Price Index (CPI—you could term this "ordinary inflation") is about 2.5 percent. Subtracting the 0.3 percent earned from 2.5 percent still means that you'd be about 2.1 percent worse off by saving money for the future, rather than buying something now. That is, if you received a small inheritance and were planning to purchase furniture for your first post-college apartment, you'd probably be better off purchasing the furniture now rather than trying to save money for a purchase in the future. The prices of the same furniture would be up a bit (by about 2.5 percent per year) but you'd only be earning 0.3 percent per year.

When you ask your bank for interest rates on savings accounts you learn that the return is insufficient to keep up with inflation. Worse, although 0.3 percent is low, on a savings account the rate of interest can change month-to-month and might be *even lower* in the future. Banks offer a product to appeal to savers who are prepared to **lock up their money** for a set time period in return for a fixed interest rate. A bank will sell a customer a **certificate of deposit (CD)** for a fixed time period at a set interest rate that will be the same for the life of the CD.

If you went to the bank at the moment and asked for a 1-year CD, the offered rate would be about 0.4 percent—still very low by historic standards and only a bit better than the 0.3 offered but it would bring the principal we'd need to save today down to $13,816.34 as compared to $13,857.71 for the savings account earning 0.3 percent in order to achieve our target of $13,982.80. Moreover, each year, at the end of Year 1 and Year 2, we'd have to go to the bank, take our principal and interest and buy a new CD. Perhaps at that time interest rates would be lower. This is called **reinvestment risk**—the possibility that when we need to reinvest money the rates will be even worse than they are today. Fortunately, commercial banks will offer CDs of many different durations up to 10 years. For our 3-year problem of funding your senior year, the offered rate is 1.01 percent. We'd have to plan for principal of $13,567.54.

Current rates of return are remarkably low. What would the situation be like if it was possible to get a more typical rate of return? Remembering that savers want to at least beat inflation, over very long periods of time, financial historians have observed a consistent result: For financial securities where there is absolutely no risk of losing money, the rate of return is typically 2 – 3 percent above inflation. Since we can measure current inflation at about 2.5 percent, if we make the rather bold assumption that inflation for the next 3 years will run at the same rate, we would normally expect interest rates on 3-year CDs to be about 4.5 to 5.5 percent per year. How would that help us? You could save much less from your summer earnings this year:

Investment	**Interest rate**	**Principal required today Target Future Value $13,982.80**
Savings account	0.3%	$13,857.71
1-year CD	0.4%	$13,816.34
3-year CD	1.1%	$13,531.34
"Typical"	5%	$12,078.87

Guaranteed returns over a long period of time

Our 3-year problem of saving for tuition is a relatively short period of time in the financial world. Many investors are planning for a 30 or 40-year **time horizon**. Consider pension planning. If you take a few years off after graduation to travel the world and don't start work until age 25, even if you take "early retirement" at age 60 (the expected retirement age for full Social Security is 67) that would be a 35 year time frame. Similarly, life insurance companies collect hundreds of millions of dollars in premiums. They know they'll have to pay out when the policyholder dies, but that might be 60 or more years in the future. Life insurance companies and pension plans belong to a category called **institutional investors**. They have billions of dollars to invest and are often looking at very long time horizons. Other types of institutional investors include sovereign wealth funds and mutual funds.

While many institutional investors are prepared to buy common stocks (in the hope that the share price will go up over long periods of time) most investors also need some investments where they can be guaranteed both a specific return and be sure to get their capital back intact at the end of an agreed time period. **Bonds** fulfill this need. An investor gives a bond issuer money in return for a promise to pay back the money at a later date and to pay interest along the way. The interest payments can be monthly, quarterly, semi-annually (which turns out to be how the US Treasury pays on bonds), or annually.

Bonds and shares (common stocks) are types of **financial securities**. A financial security is a thing of value that represents some ownership right including the right to a share of profits or interest. They are bought and sold every **trading day** when the stock and bond markets are open. Because bonds pay a specific periodic interest payment they are known as **fixed income securities**. That implies that stocks could be thought of as "variable income"—a firm does more or less well year to year and pays a greater or smaller amount of money in dividends.

A bond is a promise to repay a specific amount, say $10,000 called the **par value** (or "face amount" that is the amount printed on the face of the bond), a **due date** when the money will be returned (also called **maturity date**) and a **coupon rate**, the periodic interest rate. The "coupon" comes from days when bonds were issued in a physical form on paper and along the side of the bond little coupons were printed that could be clipped and turned in for each interest payment when due. These days, bonds are all managed electronically with an entry in brokerage accounts, so-called **book entry**.

The US Treasury is one of the world's largest issuer of bonds, with about $14.32[1] *trillion* outstanding in June of 2011. Financial firms like US bonds for many reasons. They are considered to be **riskless investments**, that is the US government could never go bankrupt and would always pay money when due. Additionally, they come in many different flavors:

[1] Roughly equal to 1x the US GDP in the same year.

Type of Treasury Instrument	Duration (maturity)	How interest is paid
Bill (“T-bill”)	28-days, 3 months 6 months,1 year	No coupon interest, sold at a discount
Notes (“T-notes”)	2 to 10 years	Coupon payment every 6 months
Bonds (“Long bonds”)	20 years and 30 years	Coupon payment every 6 months

With dozens of different types of fixed income securities, financial professionals need one security to use as a **benchmark** (a baseline to which all others are compared). For this they use the **10-year bond** (technically a Treasury *Note* but referred to informally as a bond). This is the security that you will always see quoted in market reports each day especially as a data point on what sorts of interest rates sophisticated investors are expecting.

Buying bonds at a discount

In the table above you will notice a curious feature of T-bills: They don’t actually pay periodic interest. Instead the government asks professional investors this question: How much will you loan us today if we promise to repay you $10,000 in one year from now? We know the answer as we can calculate it from the present value formula:

$$P_x = \quad F_t * \frac{1}{(1+r)^t}$$

We’d like to know how much to bid now, P_x , in order to receive $10,000 in the future. We know that the time period is 1 (one year from now) but we’ll need to make an assumption about the rate of interest, *r*. Financial professionals spend a lot of time studying **prevailing interest rates** which means simply the rate that they see other firms charging for similar instruments. Rates are currently low (around 1 percent) but for demonstration, let’s assume more normal economic conditions and a prevailing one year rate of 2.9 percent.

$$\text{Amount we should bid} = \quad \$10{,}000 * \frac{1}{(1+0.029)^1} = \$9{,}718.17$$

You can check that this works out: If you give the Treasury $9,718.17 today and get back exactly $10,000 in one year's time that would be interest of ($10,000 - $9,718.17) = $281.83. Now $281.83 divided by $9,718.17 is exactly 2.9 percent. Just as we used the word "discount" to refer to the present value of future sums we can say "We bought this bond at a discount", meaning less than its face value.

Changes in the prevailing interest rates change the value of a bond

One of the most difficult concepts for young business professionals to grasp is how the price of fixed instruments changes day to day in the financial markets—didn't you just tell me that a bond is a promise to repay a specific certain amount on the due date? Well, that's true—you will get the par value on the due date. But prevailing interest rates change from day to day and that affects how much a bond is worth.

First consider this analogy: You are driving a 1996 Toyota Corolla while you are in college. You love the car but have to admit it's nothing special; it's just the same as any other Corolla on the road. If you run short of cash you could sell the car. That is, the car is a "thing of value". The money that you'd get for it would not be what you paid for it, would not be what *you* think it's worth, but what other people who are in the market for a used Corolla are prepared to pay. For example, in general prices of used cars deteriorate over time but during early 2011 when gas prices rose sharply, the prices that used-car buyers were prepared to pay for fuel-efficient cars like the Corolla went *up*.

Although bonds have a due date the original purchaser is not required to hold the bond until maturity, the time when the money will be paid back. Just like your "thing of value" Corolla, a bond is also a thing of value and can be sold at any time on what is called the **secondary market** (that is, a sale to someone who is not purchasing from the original issuer).

Suppose for a moment you owned a bond that had no maturity date and lasted forever. Such an instrument would be called a *perpetuity* and some British government bonds are of this type. Suppose that the bond pays 4 percent, regular as clockwork, every year. If the bond was denominated in $1,000s, if prevailing interest rates are 4 percent on the day you buy the bond then you'd pay exactly $1,000.

Now imagine that a year or so later, you check the prevailing interest rates and see that they have fallen and that the markets are now settling on 3 percent as an acceptable rate. Is your bond still worth the same? No. Someone going into the markets with $1,000 will only be able to earn 3 percent a year, or $30 on $1,000 invested. But your bond pays $40 a year. If you re-sell your bond you'll likely get $1,333.33 for it. The interest payment of $40 divided by the $1,333.33 investment amounts to 3 percent. That is, the markets for fixed income securities come into balance. The purchaser of at $1,333 would be said to have purchased a $1,000 face value bond **at a premium**.

Conversely, if the prevailing interest rate goes up, investors would pay less for your bond. This concept is somewhat unintuitive because at the back of your mind you're probably thinking: Well, most bonds do have a *due date* and even if the price in the secondary market went down, I wouldn't have to sell. That's true—a bond held to maturity ignores the price risk due to changing interest rates. If the logic behind changing prices for fixed income securities is impossible to grasp, just learn the rules:

1. When prevailing interest rates go UP, the price that an investor would pay for a bond in the secondary market goes DOWN.

2. When prevailing interest rates go DOWN, the price at which you could sell a fixed income security goes UP.

In practice, very few bonds are perpetuities—they have a due date. If you think about a 30-year bond (that is a long bond), you can imagine that its price in the secondary market would fluctuate inversely with prevailing interest rates as there are many more years to go before the par value is paid back. Imagine being stuck with 29 years left on a bond that was paying 5 percent if market rates are up to 8 percent! Investors would not be willing to buy your bond at the full price.

The exact price that the bond would trade at is a difficult calculation. In the secondary markets a buyer would buy the 29-year bond "at a discount" (less than $1,000); for sake of argument, let's guess that the price would be $900. The investor would then receive $50 a year until the due date and then, yes, would receive the face value of $1,000. The investor would have an additional profit of $100 ($1,000 paid out minus $900 spent to buy the bond). Figuring out the rate of return over the 29 years would involve a complex formula (a financial calculator or spreadsheet will do this for you). However, fortunately newspapers and online financial sites do the work for you and publish two figures: the **current yield** (in this case, say, 5.5 percent) and the **yield to maturity** (including the extra money when the bond is paid off), say 6.1 percent. Note that if we pay a *premium* for a bond (say, $1,050 for $1,000 par value) then the *yield to maturity* is actually less than the coupon rate.

Using the present value formula to price stocks

Common stocks don't have a maturity (due date). They represent a fractional ownership right in a firm (hence "share") and that right exists far off into the future. Many large US companies (IBM, General Electric) are more than 100 years old. As firms grow, with more customers and more sales, they become more valuable. You can see why pension managers put some of their money under management into stocks. For example, as of June 2011, Zynga (the developer of games such as *Farmville*) is not yet currently traded (it's a private company). But when the company sells shares to the public (some time in the near future) you might decide to buy with this rationale: Zynga has grown from nothing in 2007 to $850 million in sales by 2010; it's likely to grow even more in the next few years.

The growth in the value of a share over time is an important source of wealth creation. For example, if you invested money in the US cigarette maker Altria 5 years ago in 2006, you would've paid $13.84 per share. An investment of $10,000 would have bought you 722 shares. You could sell those shares in June 2011 for $26.73 each and receive $19,314, a profit of $9,314 over 5 years. This 93 percent return is about 14 percent compounded annually over the 5 years.

Let's go back to Zynga. If the company conducts an IPO you'd know how many shares were issued and thus also know what portion of the company each share represents. As a financial analyst, you'd observe that Zynga is still fast-growing and you could look ahead 5 years and construct a financial model of Zynga's operations: How many customers, how much they would each spend, how much it costs the firm to operate and the resulting profit. You could then figure out how much a share of Zynga stock would be worth in the future; let's say your result is $200.

What price per share would you offer in 2011? You could use the present value formula.

$$P_x = \quad F_t * \frac{1}{(1+r)^t}$$

We want to find P_x, given that F_t is $200 and we are looking 5 years into the future so t is 5. But what to do about r? What interest rate should we use? If you thought your only alternative was to put money in Treasury 5-year Notes you'd use the rate of 1.6 percent (the current yield on 5-year Treasuries) and you'd bid $184.74. However, a more sensible alternative would be to consider that you could just put your money into Altria shares; if the next 5 years are like the last 5 years (not always true, of course) you'd use 14 percent (it's the rate of return from an alternative investment).

$$\text{Price you'd be prepared to pay} = \quad \$200 * \frac{1}{(1+0.14)^5}$$
$$= \$104$$

As a practical matter, due to market frenzy and a phenomenon called "herding" (a lot of investors want to buy a stock only because they see other people buying the stock and the price is going up), on the first day of trading an IPO share price might get bid up a lot higher. However, a professional investor would be able to say that it's "worth" about $104 (and might refuse to buy if the price went higher).

Back to Altria: If you looked up the firm using a source such as YAHOO FINANCE you'd see that in addition to the substantial rise in price, the firm also paid out a total of $4.18 in dividends over the same 5 years. That makes the **total return** (price appreciation + dividends received) on Altria even better than the 14 percent we calculated. Of course, the $4.18 didn't come at once at the end of the time period but rather every three months during the time you'd owned the stock. That makes including the dividends in the rate of return calculation rather complicated and something you can put off until you take a Finance class[2].

However, the general idea of *discounting future sums back to the present* is very useful. Remember that in the Zynga example, we considered a valuation 5 years in the future of $200. Where did that figure come from? You'll learn in class that sophisticated investors don't care about dividends as such—they're happy just so long as the firm is earning money. You can imagine that if Zynga was highly profitable but chose not to pay a dividend to shareholders, the firm would have a lot of money to invest in its own projects or to buy other firms; so either way, its valuation would go up. Sophisticated investors do look at the stream of estimated earnings off into the future. With reasonable estimates of the future earnings of a firm, analysts discount those numbers back to the present to calculate what a stock is worth. That explains how some stocks trade at $10 and others at $200. Active participants in stock markets use the present value formula to make an assessment of the future earnings stream of the firm.

Conclusion and summary

The fundamental concepts of finance are on the one hand investors who seek a good return on invested capital and on the other firms that are looking for capital and are prepared to pay for it. The highly-developed capital markets match up these two groups with a wide variety of financial instruments that meet the needs of market participants.

Common sense tells us that "money today is worth more than money tomorrow" and worth substantially more than the same amount of money to be received in 1 or 5 years from now. We can make a rational calculation of the value today by using the present value formula. The present value formula is just a rearrangement of the future value formula that enables us to compute the effect of compounding interest over several time periods.

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[2] Price appreciation (capital gain) and dividends have different tax rates for US investors and that can become important in selecting investments. But that's beyond an introductory treatment of the topic.