AN INTERNATIONAL ARBITRAGE PRICING MODEL WITH PPP DEVIATIONS

ROSS LEVINE*

This paper derives an intertemporal, international arbitrage pricing model that relaxes more assumptions than previous asset pricing paradigms. The analysis shows how risk, risk premia, and the translation of these variables between all real and nominal numeraires depend upon a small number of stochastic state variables that define the economy's production and credit opportunities. When the model is applied to the forward exchange market, it highlights the potentially central role of real exchange rates in determining the evolution of forward exchange risk premia.

I. INTRODUCTION

There is a growing literature examining the implications of purchasing power parity (PPP) deviations in international asset pricing models. This literature typically assumes that residents of a country deflate nominal returns by their national price index in order to determine real returns. In this setting, stochastic PPP deviations imply that the real return on an asset differs internationally. This international difference in real returns has forced authors to make important simplifying assumptions in order to derive international asset pricing models. Solnik [1974] develops an international capital asset pricing model (ICAPM) assuming no inflation and no correlation between PPP deviations and local real returns. His model is extended by Sercu [1980] to permit correlation between exchange rates and local real returns, and by Kouri and de Macedo [1978] to allow for nonstochastic inflation.

This paper abandons the ICAPM approach in order to price assets in a setting of stochastic inflation rates, stochastic PPP deviations, *and* correlation between PPP deviations and real asset returns *without* imposing concomitant restrictions on agents' utility functions.[1] The paper extends Ross's [1976] arbitrage pricing theory to an international environment with PPP deviations. It improves upon Solnik's [1983] and Ross and Walsh's [1983] international arbitrage pricing models (IAPM) by (i) allowing both stochastic inflation and stochastic PPP deviations, (ii) demonstrating how returns may be translated into *any* nominal or real numeraire, and (iii) emphasizing the dynamic

* Board of Governors of the Federal Reserve System. The author thanks M. Carkovic, M. Darby, S. Edwards, M. Grinblatt, G. Mossetti, G. Tabellini, an anonymous referee, and especially R. Sweeney for helpful comments. This paper represents the views of the author and should not be interpreted as reflecting those of the Board of Governors of the Federal Reserve System or other members of its staff.

1. Adler and Dumas [1983] comprehensively review the international asset pricing literature.

Economic Inquiry
Vol. XXVII, October 1989, 587–599

nature of international asset pricing. In addition, the model specifies risk, the price of risk, and the translation of these variables between all nominal and real numeraires as functions of the model's fundamental state variables. This yields testable restrictions not found in domestic arbitrage pricing models (APM).[2]

After deriving an IAPM with PPP deviations, this paper uses the model to examine the forward exchange market's risk premium. The model's state variables determine the stochastic structure of PPP deviations and national price levels, which in turn define the dynamics of risk and the price of risk. The analysis demonstrates the potentially central role of PPP deviations in determining the evolution of forward exchange risk premia. Since the source of the bias between forward and future spot exchange rates has not been convincingly identified and since PPP deviations have not been highlighted in studies of this bias, future empirical investigations concerning the systematic discrepancies between forward and future spot exchange rates should incorporate PPP deviations.[3]

The model's equations are discussed in section II. Section III derives an intertemporal IAPM with PPP deviations, demonstrates how to translate risk and the price of risk into any nominal or real numeraire using the model's state variables, and identifies testable restrictions not found in the domestic APM. Section IV derives and examines the forward exchange market's risk premium, and section V presents a brief summary.

II. THE MODEL

The world economy consists of $n+1$ countries each with its own currency. Currency 0 is arbitrarily chosen as the nominal numeraire. Every investor is able to issue and purchase assets in any country through freely floating international exchange markets. It is assumed that country i has N_i assets, and the total number of assets ($N = \sum_{i=0}^{n} N_i$) is much larger than the number of countries ($n+1$). For convenience, the first $n+1$ assets are nominally riskless in local currency terms so that asset 0 is country 0's nominally riskless asset.

Although all goods are available in each country, residents of country i can consume only goods purchased in country i. Transactions costs, storage costs, tariffs, nontraded goods, and taste differences imply that representative consumption baskets differ internationally. Therefore, the real return on an asset depends upon the country in which returns are evaluated.

2. On the difficulties inherent in testing capital asset pricing models see Roll [1977]. On the empirical testability of domestic APMs see Roll and Ross [1980].

3. See Hansen and Hodrick [1980], Frankel and Froot [1987], Krasker [1980] and Sweeney [1986] for different explanations of the systematic differences between forward and future spot exchange rates. See Roll and Solnik [1977], Korajczyk [1985], and Levine [1989] for investigations into the role of real exchange rates in explaining this bias.

Assume that there exists a $K \times 1$ vector of state variables (Θ) which describes the state of the world and provides the fundamental dynamic relations of the model. Let Θ follow a continuous time vector Markov process of the Ito type:

$$d\Theta / \Theta = \mu_\Theta[\Theta(t), t]dt + \Sigma_\Theta[\Theta(t), t]dZ \tag{1}$$

where throughout this paper $\mu_l[\Theta(t), t]$ represents the expected rate of change in variable l at time t, given that the state vector is $\Theta(t)$. $\Sigma_\Theta[\Theta(t), t]$ is a $K \times K$ diagonal matrix of state-dependent instantaneous standard deviations. The main diagonal consists of $\sigma_1[\Theta(t), t], \ldots, \sigma_K[\Theta(t), t]$, where $\sigma_s[\Theta(t), t]$ is state variable s's instantaneous standard deviation at time t when the state of the world is described by $\Theta(t)$. The $K \times 1$ vector dZ is composed of elements $dz_1, \ldots, dz_K$ that are correlated increments of standard Wiener processes.[4] Thus, the unanticipated change in state variable s at time t is equal to $\sigma_s[\Theta(t), t]dz_s(t)$, where $dz_s(t)$ is the realization of dz_s at time t.

The state vector includes such variables as technological growth, the money supply, legal arrangements, and other important processes which may be considered exogenous or at least predetermined. The intertemporal development of the state variables defines the production and credit opportunities available to the economy. More specifically, the probability distributions of variables such as aggregate price levels, exchange rates, and PPP deviations depend on the current level of the state variables (Θ) that are themselves changing randomly over time.[5]

Each country's inflation rate is assumed to follow an Ito-type continuous time Markov process.

$$dP_i / P_i = \mu_{P_i}[\Theta(t), t]dt + \sum_{s=1}^{K} b^s_{P_i} [\Theta(t), t]\, \sigma_s[\Theta(t), t]dz_s, \tag{2}$$

$$i = 0, \ldots, n$$

where P_i is country i's domestic price level, and where throughout $b^s_l[\Theta(t), t]$ quantifies the sensitivity of variable l's rate of change to unanticipated movements in state variable s at time t when the state of the

4. A real valued function on (t, t') is a standard Wiener process if (i) z is a continuous process with independent increments, and (ii) $z(t') - z(t)$ has a normal distribution with zero mean and variance $t' - t$. For a more rigorous definition of Wiener processes and their applications in economics and finance see Malliaris and Brock [1982].

5. Some of the "fundamental" variables that this state vector represents are unlikely to follow continuous sample paths. As Ross [1976] shows, however, this is not important in applications of the APT. This point is discussed further below.

world is $\Theta(t)$. Each $b_l^s[\Theta(t), t]$ term is assumed to follow an Ito-type continuous time Markov process that is only a function of the state vector $\Theta(t)$. Note that the expected inflation rate in country i at time t ($\mu_{P_i}[\Theta(t), t]$) is time-varying and conditional on all information available at time t.

The instantaneous nominal rate of change in the value of asset i is

$$dR^i / R^i = \mu_{R^i}[\Theta(t), t]dt + \sum_{s=1}^{K} b^s_{R^i}[\Theta(t), t]\, \sigma_s[\Theta(t), t]dz_s + d\varepsilon_i, \qquad i = 0,\ldots,N \tag{3}$$

where R^i is the nominal value of asset i in currency 0 terms. The instantaneous expected return equals the expected rate of change in the price plus the expected dividend divided by the price. The dividend decision of the firm will not be explicitly modelled; $\mu_{R^i}[\Theta(t), t]$ will be referred to as the instantaneous conditional expected nominal rate of return on asset i. The terms $\sum_{s=1}^{K} b^s_{R^i}[\Theta(t), t]\, \sigma_s[\Theta(t), t]dz_s$ represent the unexpected percentage change in the nominal value of asset i arising from unanticipated realizations of the state vector. The term $d\varepsilon_i$ is a zero mean, unit variance increment of a Wiener process that is uncorrelated across assets and uncorrelated with the state variables (i.e., $E(d\varepsilon_i | dz_s) = 0$, and $E(d\varepsilon_i | d\varepsilon_j) = 0$, for $i \neq j$).

The nominal exchange rate is defined as:

$$S_{ij} = P_i D_{ij} / P_j, \tag{4}$$

where S_{ij} is the amount of currency i exchangeable for a unit of currency j in the spot exchange market, and deviations of D_{ij} from unity represent PPP deviations between countries i and j.

Isard [1977] and Kravis and Lipsey [1978] show that exact PPP does not hold. Evidence regarding the predictability of real exchange rate movements is less conclusive.[6] This paper recognizes the empirical evidence and models PPP deviations as a subsystem of stochastic differential equations:

$$dD_{ij} / D_{ij} = \mu_{D_{ij}}[\Theta(t), t] + \sum_{s=1}^{K} b^s_{D_{ij}}[\Theta(t), t]\, \sigma_s[\Theta(t), t]dz_s. \tag{5}$$

6. See, for example, Roll [1979], Adler and Lehman [1983], Darby [1983], Huizinga [1987], and Levine [1989].

Note that $dD_{ii} / D_{ii} = 0$ by definition, and each country has a different (though not independent) stochastic PPP relationship with other countries.[7] The expected rate of change in PPP deviations is not necessarily zero. Thus, while a random walk is consistent with the above formulation, PPP deviations are not restricted to a random walk.

This model is partial equilibrium in the sense that unspecified state variables provide the fundamental dynamic relations of the model. It is not general equilibrium in the sense of Arrow-Debreu because technological sources of uncertainty are not explicitly related to the equilibrium prices. Cox, Ingersoll, and Ross [1985], and Brock [1982] construct general equilibrium asset pricing models in a domestic setting. Production possibilities are explicitly modelled as a set of linear stochastic activities where it is these direct technological shocks that ultimately induce stochastic contingent claim prices. They, however, construct their models in a completely real setting with no aggregate price level. Since the model used in this paper incorporates stochastic inflation rates in each of $n+1$ countries, a money market and aggregate price level would have to be added: a nontrivial task left for future work.

It is not necessary, however, to model explicitly the microeconomic components of risk for the framework to be consistent with a general equilibrium model. In order to be consistent with general equilibrium, prices must be endogenously determined through the equilibrium of supply and demand. Since all random shocks are captured as elements of the state vector (Θ) and assuming asset supply and demand schedules are functions of the same state variables, the resulting equilibrium prices will also follow Ito processes.[8] Thus, although the model presented above does not embody the full range of relationships which would be captured by a general equilibrium model, the model is consistent with endogenously determined prices.

III. AN INTERNATIONAL ARBITRAGE PRICING MODEL WITH PPP DEVIATIONS

This section demonstrates four important features of the model outlined above. First, the nominal return structure for assets in country 0 follows a linear return generating process. This characteristic and the assumptions stated in section II permit application of Ross's [1976] APT, and the derivation of an IAPM in nominal country 0 terms. Second, any nominally riskless arbitrage portfolio in country 0 is riskless in nominal and real terms for any international investor. Thus, arbitrage cannot occur between arbitrage

7. PPP deviations co-vary, but there is a fixed relationship between some PPP deviations, i.e., $D_{ij} = D_{iN} / D_{jN}$.

This restriction is not exploited because it does not importantly influence the pricing of assets in an arbitrage pricing setting.

8. See Breeden [1979] and Richard [1979].

portfolios of different countries. Third, the linear arbitrage pricing relationship derived in nominal country 0 terms holds regardless of the nominal or real numeraire in which returns are defined. Thus, the IAPM is indeed international. Finally, the translation of risk and the price of risk between all nominal and real numeraires is defined in terms of the model's fundamental state variables. This yields testable restrictions not found in the domestic APT.

To facilitate exposition, let

$$\delta_s = \sigma_s[\Theta(t), t]dz_s \quad s = 1, \ldots, K$$

$$b_j^s = b_j^s[\Theta(t), t] \text{ for all } s \text{ and } j$$

and

$$\mu_j = \mu_j\,[\Theta(t), t].$$

Note that (1) δ_s is the unanticipated movement in state variable s; (2) b_j^s is variable j's time varying, state-dependent sensitivity to changes in state variable s;[9] and (3) μ_j is the time varying, state-dependent anticipated rate of change in variable j.

Equation (3) then becomes

$$dR^i/R^i = \mu_{R^i}\,dt + \sum_{s=1}^{K} b_{R^i}^s\,\delta_s + d\varepsilon_i.^{10} \tag{3'}$$

Given the assumptions of section II, the APT may be applied to (3′) for an investor who is concerned with nominal returns in country 0. Consider an arbitrage portfolio consisting of investment proportions x_i, which is the currency zero amount purchased or sold of asset i as a fraction of total wealth. This portfolio uses no wealth, has no systematic risk, and is well diversified. This is expressed formally by

9. Recall that this sensitivity is assumed to follow a continuous time Ito process, and depends only on the state vector.

10. Ross [1976, 347] demonstrates that the δ's "need not be jointly independent or even independent of the (dε$_i$)'s, they need not possess variances, and none of the random variables need be normally distributed." Moreover, Ross [1976, 355] argues that agents "can hold a variety of views on the distribution of (δ) without violating the basic arbitrage condition, ..." The δ's must, however, have an expected value of zero. Thus, deriving an asset pricing model with, for example, diffusion and jump processes is feasible. Relating risk and risk premia to the state variables and translating these variables internationally, however, is beyond the scope of this paper.

$$\sum_{i=0}^{N} x_i = 0,$$

$$\sum_{i=0}^{N} x_i b^s_{R^i} = 0, \text{ for } s = 1,\dots,K, \text{ and} \tag{6}$$

$$\sum_{i=0}^{N} x_i \, d\varepsilon_i \approx 0.$$

Given (6), the expected return on this portfolio is

$$\sum_{i=0}^{N} x_i \, (dR^i/R^i) \approx \sum_{i=0}^{N} x_i \, (\mu_{R^i}).$$

Since this arbitrage portfolio is riskless and uses no wealth, the expected return must be zero. As shown by Ross, this implies that expected returns are a linear combination of a constant and each asset's sensitivity to unexpected movements in the state vector, i.e., the $b^s_{R^i}$'s. The algebraic expression of this result is that there exist $K+1$ constants such that

$$\mu_{R^i} - \overline{\lambda}^0_0 \approx \sum_{s=1}^{K} \overline{\lambda}^0_s \, b^s_{R^i}, \tag{7}$$

where $\overline{\lambda}^0_0$ is the nominal return on asset 0 (currency 0's nominally riskless asset) evaluated in country 0. Intuitively, (7) expresses the expected nominal return on asset i in country 0 above the nominally riskless rate as a weighted average of asset i's systematic risk. Systematic risk is defined as the sensitivity to common shocks.[11] The weights are factor risk premia. That is, $\overline{\lambda}^0_s$ represents the market price of a unit of systematic risk of type s. These factor risk premia must be equal across all assets evaluated in country 0 nominal terms to rule out riskless arbitrage opportunities. Since the above specification does not specify preferences, the factor risk premia emerge as simple constants. The analysis conducted by Cox, Ingersoll, and Ross [1985]

11. Since equation (7) is the expected nominal return on i, the sensitivity terms, the $b^s_{R^i}$'s, are *ex ante* expressions. This is different from equation (3′) where the sensitivities are realized values. Since the sensitivities themselves are assumed to follow Ito-type processes, the difference between *ex ante* and *ex post* sensitivities could be expressed in an equation similar to (2). This distinction is not made to simplify notation.

suggests that these prices will be related to marginal indirect utilities with respect to the state variables.

Since application of the APT requires the definition of a riskless portfolio, it is imperative to show that the arbitrage portfolio used to derive (7) $(x_0, \ldots, x_N)$ is riskless for any investor evaluating returns in nominal or real terms. Consider, for example, an investor in country j evaluating the real return of asset i. By Ito's lemma the real return on this asset is

$$\begin{aligned} \tilde{r}^i_j = \mathrm{d}(R^i / P_0 D_{0j}) / (R^i / P_0 D_{0j}) &= \mathrm{d}R^i / R^i - \mathrm{d}P_0 / P_0 - \mathrm{d}D_{0j} / D_{0j} \\ &- (\mathrm{d}R^i / R^i)(\mathrm{d}P_0 / P_0) - (\mathrm{d}R^i / R^i)(\mathrm{d}D_{0j} / D_{0j}) \\ &+ (\mathrm{d}P_0 / P_0)(\mathrm{d}D_{0j} / D_{0j}) + (\mathrm{d}D_{0j} / D_{0j})^2 \\ &+ (\mathrm{d}P_0 / P_0)^2 \end{aligned} \tag{8}$$

Substitution of (3′) yields

$$\begin{aligned} = \mu_{R^i}\,\mathrm{d}t - \mathrm{d}P_0 / P_0 - \mathrm{d}D_{0j} / D_{0j} + \sum_{s=1}^{K} b^s_{R^i}\,[\delta_s - \delta_s(\mathrm{d}P_0 / P_0) \\ - \delta_s(\mathrm{d}D_{0j} / D_{0j})] + \mathrm{d}\varepsilon_i - \mathrm{d}\varepsilon_i(\mathrm{d}P_0 / P_0) - \mathrm{d}\varepsilon_i(\mathrm{d}D_{0j} / D_{0j}) \\ + (\mathrm{d}P_0 / P_0)(\mathrm{d}D_{0j} / D_{0j}) + (\mathrm{d}P_0 / P_0)^2 + (\mathrm{d}D_{0j} / D_{0j})^2. \end{aligned}$$

Recalling condition (6), the return on arbitrage portfolio, $x_0, \ldots, x_N$, evaluated in real country j terms is

$$\sum_{i=0}^{N} x_i\,\tilde{r}^i_j = \sum_{i=0}^{N} x_i\,\mu_{R^i} = 0.$$

Thus, an arbitrage portfolio in 0 is an arbitrage portfolio in j, and the derivation of (7) is valid.

The arbitrage pricing relationship defined in (7) prices expected returns for a country 0 resident evaluating nominal returns. In order to be a viable and testable pricing model, the structure of this pricing relationship must hold for all arbitrarily chosen numeraires, real or nominal. This may be demonstrated with a few tedious substitutions.[12]

12. This demonstration is very similar to that of Ross and Walsh [1983].

Taking the expectation of (8) and letting

$$\zeta_0^i = -(dR^i / R^i)(dP_0 / P_0) - (dR^i / R^i)(dD_{0j} / D_{0j}) + (dP_0 / P_0)(dD_{0j} / D_{0j})$$
$$+ (dD_{0j} / D_{0j})^2 + (dP_0 / P_0)^2,$$

yields the expected return on asset i evaluated in country j real terms,

$$r_j^i = \mu_{R^i} - \mu_{P_0} - \mu_{D_{0j}} + \zeta_0^i, \tag{9}$$

where ζ_0^i is the covariance of the return on asset i (evaluated in numeraire terms) with the deflator used to express returns in real country j terms. In order to translate the arbitrage pricing relationship, (7), it is useful to note that the covariance of the return on asset i evaluated in real country j terms with the relevant deflator is

$$\text{cov}\{\tilde{r}_j^i,\ [(dP_0 / P_0)(dD_{0j} / D_{0j})]^{-1}\} = \zeta_j^i = \zeta_0^i + [(dP_0 / P_0)(D_{0j} / D_{0j})]^{-2}. \tag{10}$$

Now, substitute (9) and (10) into (7), and note that (i) asset 0 is country 0's nominally riskless asset, and (ii) $b^s_{r_j^i} = b^s_{R^i} - b^s_{D_{0j}} - b^s_{P_0}$ in order to obtain[13]

$$r_j^i - r_j^0 - (\zeta_j^i - \zeta_j^0) = \sum_{s=1}^{K} \overline{\lambda}_s^0 (b^s_{r_j^i} + b^s_{P_0} + b^s_{D_{0j}}). \tag{11}$$

From (11), the return on a riskless asset in real j terms, λ_j^j, is

$$\lambda_j^j - r_j^0 + \zeta_j^0 = \sum_{s=1}^{K} \overline{\lambda}_s^0 (b^s_{P_0} + b^s_{D_{0j}}).$$

Subtracting this equation from (11) yields

$$r_j^i - \lambda_j^j = \sum_{s=1}^{K} \overline{\lambda}_s^0 (b^s_{R^i} - b^s_{P_0} - b^s_{D_{0j}}) + \zeta_j^i. \tag{12}$$

All that remains is to translate the factor risk premia into real country j terms. Since

13. This second point is easily verified using Ito's Lemma.

$$\zeta_j^i = \operatorname{cov}\{\tilde{r}_j^i,\ [(dP_0/P_0)(dD_{0j}D_{0j})]^{-1}\}$$

$$= \sum_{s=1}^{K}\sum_{l=1}^{K} (-b_{P_0}^s - b_{D_{0j}}^s)\operatorname{cov}(\Theta_s, \Theta_l)\, b_{r_j^i}^l$$

$$= \sum_{l=1}^{K} \Phi^l(P_0, D_{0j})(b_{r_j^i}^l)$$

where

$$\Phi^l(P_0, D_{0j}) = \sum_{s=1}^{K} (-b_{P_0}^s - b_{D_{0j}}^s)\ \operatorname{cov}(\Theta_s, \Theta_l),$$

equation (12) may be rewritten as

$$r_j^i - r_j^i = \sum_{s=1}^{K} \lambda_s^j(b_{R^i}^s - b_{P_0}^s - b_{D_{0j}}^s), \tag{13}$$

where

$$\lambda_s^j = \bar{\lambda}_s^0 + \Phi^s(P_0, D_{0j}). \tag{14}$$

Thus, the simple linear arbitrage pricing relationship holds regardless of the nominal or real numeraire in which real returns are defined. Equation (13) expresses the expected real return on asset i evaluated in country j above the real riskfree rate as a weighted average of asset i's systematic risk in real country j terms. The weights are real country j risk premia. Moreover, (13) and (14) demonstrate how the international arbitrage pricing model translates risk and the price of risk internationally. Asset i's systematic risk in real j terms ($b^s_{r_j^i}$) consists of three components. The first is the sensitivity of the nominal return of asset i to changes in state variable s ($b^s_{R^i}$). The second is the sensitivity of country 0's price level to changes in state variables s ($b^s_{P_0}$). The third term indicates the sensitivity of purchasing power parity deviations between country 0 and j ($b^s_{D_{0j}}$), the country in which real returns are being evaluated. Equation (13) shows that the price of each type of systematic risk evaluated in one numeraire is related to the price of risk in an alternative numeraire by a term depending on the covariance of state variables and the sensitivity of prices and PPP deviations to unanticipated movements in the state variables. This cross-country relationship among the risk prices produces additional testable implications.[14]

IV. THE FORWARD EXCHANGE MARKET'S RISK PREMIUM

One of the many issues that the IAPM can be used to analyze is the forward exchange market's risk premium. Although it is well established that forward exchanges rates systematically differ from corresponding future spot prices, the source of this bias has not been convincingly identified.[15] This section uses the insights gained above to investigate the sources of risk premia in international exchange markets.

If one defines the risk premium, *RP*, as the difference between the forward exchange rate and the corresponding expected future spot exchange rate, then (assuming a continuous time framework and covered interest rate parity) it is easy to show that

$$RP = \bar{\lambda}_0^0 - \bar{\lambda}_j^j - \mathrm{E}(\mathrm{d}S_{0j} / S_{0j}), \tag{15}$$

where $\bar{\lambda}_0^0$ is the nominal return in country 0 on asset 0 (country 0's nominally risk-free asset), $\bar{\lambda}_j^j$ is the nominal return on country j's riskfree asset in j, and $\mathrm{E}(\mathrm{d}S_{0j} / S_{0j})$ is the expected depreciation of currency 0 against currency j. In addition, the expected nominal return on country j's risk-free asset in country 0 is

$$\bar{\lambda}_j^j + \mathrm{E}(\mathrm{d}S_{0j} / S_{0j}) = \bar{\lambda}_0^0 + \sum_{s=1}^{K} \bar{\lambda}_s^0 (b_{P_0}^s + b_{D_{0j}}^s - b_{P_j}^s). \tag{16}$$

Combining (15) and (16) yields an expression for the foreign exchange risk premium

$$RP = \sum_{s=1}^{K} \bar{\lambda}_s^0 (b_{P_j}^s - b_{D_{0j}}^s - b_{P_0}^s). \tag{17}$$

14. Since the b^s terms are permitted to evolve according to an Ito process, these sensitivities will co-vary with the state variables and thus should be incorporated into the $\mathrm{cov}(\Theta_s, \Theta_l)$ term. This would imply changing the term $\Phi^l(P_0, D_{0j}) = \sum_{s=1}^{K} (-b_{P_0}^s + b_{D_{0j}}^s)\,\mathrm{cov}(\Theta_s, \Theta_l)$ such that the new b^s's are the fixed sensitivities of $b_{P_0}^s$ and $b_{D_{0j}}^s$ to changes in the state variables. The term, $\Phi^l(P_0, D_{0j})$, is left as is for simplicity.

15. See, for example, the papers cited in footnote 3.

The risk premium is a function of risk and the price of risk. The risks associated with entering the forward exchange market equal the systematic risks associated with the price levels in the respective countries and the real exchange rate. Risk prices in nominal currency 0 terms are simply composed of the price of each type of systematic risk. This foreign exchange risk premium is thus directly related to the covariance of national price levels and real exchange rates with movements in the fundamental state variables.

If purchasing power parity holds perfectly, then $b^s_{D_{0j}} = 0$. The foreign exchange risk premium may still exist and evolve intertemporally because the systematic risks associated with national price levels may be time-varying. Nonetheless, the systematic risk associated with the sensitivity of real exchange rates to unanticipated movements in the state variables may be an integral component of the risk premium. Consequently, empirical investigations into the discrepancies between forward and future spot exchange rates should incorporate the potentially central role of real exchange rates.

V. CONCLUSIONS

This paper derives an IAPM with PPP deviations that relaxes more assumptions than previous international pricing models. That is, stochastic PPP deviations, stochastic, imperfectly correlated inflation rates, and time-varying covariances are incorporated without concomitant restrictions on agents' utility functions. Moreover, the model is driven by a small number of stochastic state variables which define the economy's production and credit opportunities. The evolution of these variables determine risk and the price of risk in each country.

The model is used to examine the forward exchange market's risk premium. The risk premium is shown to be directly related to the covariance of national price levels and real exchange rates with movements in the fundamental state variables. The lack of an empirically tractable model of time-varying risk premia has hindered empirical inquiry into the nature of systematic discrepancies between forward and future spot prices. Future work based on the IAPM may help uncover the sources of this bias.

REFERENCES

Adler, M. and B. Dumas. "International Portfolio Choice and Corporation Finance: A Synthesis." *Journal of Finance*, June 1983, 925–84.

Adler, M. and B. Lehman. "Deviations from Purchasing Power Parity in the Long Run." *Journal of Finance*, December 1983, 1471–87.

Breeden, D. T. "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities." *Journal of Financial Economics*, September 1979, 265–96.

Brock, W. A. "Asset Prices in a Production Economy," in *The Economics of Information and Uncertainty*, edited by J. J. McCall. Chicago: University of Chicago Press, 1982, 1–46.

Cox, J. C., J. Ingersoll and S. Ross. "An Intertemporal General Equilibrium Model of Asset Prices." *Econometrica*, March 1985, 363–84.

Darby, M. R. "Movements in Purchasing Power Parity: The Short and Long Runs," in *The International Transmission of Inflation*, edited by M. R. Darby and J. R. Lothian, Chicago: University of Chicago Press, 1983, 462–77.

Frankel, J. and K. A. Froot. "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations." *American Economic Review*, March 1987, 133–53.

Hansen, L. P. and R. J. Hodrick. "Forward Exchange Rates as Optimal Predictors of Future Spot Rates: An Econometric Analysis." *Journal of Political Economy*, October 1980, 829–53.

Huizinga, J. "An Empirical Investigation of the Long Run Behavior of Real Exchange Rates." Photocopy, University of Chicago, 1987.

Isard, P. "How Far Can We Push the Law of One Price?" *American Economic Review*, December 1977, 942–48.

Korajczyk, R. A. "The Pricing of Forward Contracts for Foreign Exchange." *Journal of Political Economy*, April 1985, 346–68.

Kouri, P. and J. B. Macedo. "Exchange Rates and the International Adjustment Process." *Brookings Papers on Economic Activity* 1, 1978, 111–50.

Krasker, W. S. "The 'Peso' Problem in Testing the Efficiency of Forward Exchange Markets." *Journal of Monetary Economics*, April 1980, 269–76.

Kravis, I. and R. Lipsey. "Price Behavior in the Light of Balance of Payments Theories." *Journal of International Economics*, May 1978, 193–246.

Levine, R. "The Pricing of Forward Exchange Rates." *Journal of International Money and Finance*, June 1989, 163–79.

Malliaris, A. G. and W. A. Brock. *Stochastic Methods in Economics and Finance*. New York: North-Holland Press, 1982.

Richard, S. F. "A Generalized Capital Asset Pricing Model," in *Portfolio Theory 25 Years After: Essays in Honor of Harry Markovitz*, edited by E. J. Elton and M. J. Gruber. Amsterdam: North-Holland, 1979, 215–32.

Roll, R. "A Critique of the Asset Pricing Theory's Tests; Part I: On Past and Potential Testability of the Theory." *Journal of Financial Economics*, March 1977, 129–76.

______. "Violations of Purchasing Power Parity and Their Implications for Efficient International Commodity Markets," in *International Finance and Trade*, edited by M. Sarnat and G. Szego. Cambridge, MA: Ballinger, 1979, 133–76.

Roll, R. and S. A. Ross. "An Empirical Investigation of the Arbitrage Pricing Theory." *Journal of Finance*, December 1980, 1073–103.

Roll, R. and B. Solnik. "A Pure Foreign Exchange Asset Pricing Model." *Journal of International Economics*, May 1977, 161–79.

Ross, S. A. "The Arbitrage Theory of Capital Asset Pricing." *Journal of Economic Theory*, December 1976, 341–60.

Ross, S. A. and M. W. Walsh. "A Simple Approach to the Pricing of Risky Assets with Uncertain Exchange Rates." *Research in International Business and Finance* 3, 1983, 39–54.

Sercu, P. "A Generalization of the International Asset Pricing Model." *Reveu de l'Association Francaise de Finance*, June 1980, 91–135.

Solnik, B. "An Equilibrium Model of the International Capital Market." *Journal of Economic Theory*, August 1974, 500–24.

______. "International Arbitrage Pricing Theory." *Journal of Finance*, May 1983, 449–57.

Sweeney, R. J. "Beating the Foreign Exchange Market." *Journal of Finance*, March 1986, 163–82.