B. PROOFS OF OTHER RESULTS

1. FINITE PUNISHMENT PERIODS IN THE *RG*

PROPOSITION 1 WITH FINITE PUNISHMENT PERIOD. *Given δ and γ, if β is sufficiently high, cooperation can be sustained under the following strategies for both players: if the game is not in a punishment phase, play NO if the other player plays NO in the previous period; if the other player has played O in the previous period, initiate a punishment phase; if the game is in a punishment phase, and it has been initiated by you, play O for T periods when given the opportunity; else play NO.*

PROOF. In this case, we must only analyze a single period deviation to determine when cooperation can be sustained. Here, *A*'s expected utility from playing *O* in a non-punishment phase is:

$$EU_A(A_A = O) = 1 + \gamma\sum_{t=1}^{T}\delta^t$$

The expected payoff, then, can be written:

$$EU_A(A_A = O) = 1 + \frac{\delta\gamma(1-\delta^T)}{1-\delta} = \frac{1-\delta+\delta\gamma(1-\delta^T)}{1-\delta} \tag{18}$$

If *A* plays *NO*, his expected payoff in the punishment period is:

$$EU_A(A_A = N) = \beta\sum_{t=0}^{T}\delta^t$$

which can be rewritten:

$$EU_A(A_A = N) = \frac{\beta\delta(1-\delta^T)}{1-\delta} \tag{19}$$

Thus, *A* will cooperate if and only if (19) is greater than (18). Thus, I obtain the condition that *A* cooperates if and only if

$$\beta > \frac{1 - \delta + \delta\gamma(1 - \delta^T)}{\delta(1 - \delta^T)} = \beta_A^* \tag{20a}$$

From the symmetry of the game I can obtain a similar result for *B*, substituting 1 - γ for γ. Namely, *B* will cooperate if an only if

$$\beta > \frac{1 - \delta + \delta(1 - \gamma)(1 - \delta^T)}{\delta(1 - \delta^T)} = \beta_B^* \tag{20b}$$

Since cooperation is sustained only if both players do not have an incentive to play O, and both (20a) and (20b) are lower bounds, I therefore can define a condition such that cooperation can be sustained:

$$\beta > \max(\beta_A^*, \beta_B^*) = \beta^* \tag{21}$$

The proposition follows from the fact that β* is a function of δ and γ.■

Notice that grim trigger, which is equivalent to *T* approaching infinity, is simply a special case of this more general result.

PROPOSITION 2 WITH FINITE PUNISHMENT PERIOD. *With a T period punishment strategy, as γ approaches 0.5, cooperation can be sustained over a wider range of the parameters.*

PROOF. Solving for the maximum condition given in (5), we obtain the following:

$$\beta^* = \begin{cases} \beta_A^* & if \quad \gamma > \dfrac{1}{2} \\ \\ \beta_B^* & if \quad \gamma < \dfrac{1}{2} \end{cases} \tag{22}$$

The proposition follows from the fact that the β* is a decreasing function of γ if γ < 0.5 and increasing if γ > 0.5. ■

2. THE *RG* WITH NEGATIVE QUADRATIC UTILITY FUNCTIONS

PROOF THAT PROPOSITIONS 1 AND 2 HOLD WITH NEGATIVE QUADRATIC UTILITY FUNCTIONS.

Let the structure of the game be identical to the *RG* except that $u_i = -(x - x_i)^2 \quad i \in \{A, B\}$ where, without loss of generality, the players' ideal points are $x_A = 0$ and $x_B > 0$. Substituting into (3) above and following the analysis, we obtain that *A* will cooperate iff

$$x \in \{-x_B\sqrt{\delta(1-\gamma)}, x_B\sqrt{\delta(1-\gamma)}\} \tag{23}$$

and *B* will cooperate iff

$$x \in \{x_B(1-\sqrt{\delta\gamma}), x_B(1+\sqrt{\delta\gamma})\}. \tag{24}$$

Since the set of cooperative regions must overlap, the upper bound in (23) must be higher than the lower bound in (24), a condition we can write

$$x_B\sqrt{\delta(1-\gamma)} - x_B(1-\sqrt{\delta\gamma}) \geq 0. \tag{25}$$

Taking the first derivative of the left hand side of (25), it is clear that holing the other parameters constant, the expression is increasing in δ, implying that as the players' become more patient, it is easier to sustain cooperation (or alternatively, the set of cooperative equilibria expands), which is Proposition 1. Further, we can also consider the conditions on γ in which the cooperative region is maximized. Here, if we take

$$\max_{\gamma} x_B\sqrt{\delta(1-\gamma)} - x_B(1-\sqrt{\delta\gamma}).$$

we obtain the condition

$$-\frac{1}{2}\frac{x_B\delta}{\sqrt{\delta(1-\gamma)}}-\frac{1}{2}\frac{x_B\delta}{\sqrt{\delta\gamma}}=0 \qquad \Rightarrow \gamma = 0.5 \tag{26}$$

which is analogous to Proposition 2. ■

3. THE *RG* WITH *N* VETO PLAYERS

PROOF THAT PROPOSITION 2 HOLDS WITH *N* VETO PLAYERS.

To show that Proposition 2 holds for n veto players, it is sufficient to show that the minimum β required for the stronger player to cooperate is increasing in her electoral probability. Without loss of generality assume $\gamma > 0.5$. Let p_{At} be the probability that A's program will be in place in period t. Thus, we can write A's payoff for defection as

$$u_{A\infty} = 1 + \sum_{t=1}^{\infty} \delta^t p_{At} \tag{27}$$

This implies that if p_{At} is increasing in γ then β∗ is increasing in γ. The general form for p_{At} can be written

$$p_{At} = p_{A(t-1)}(1-(1-\gamma)^n) + (1-p_{A(t-1)})\gamma^n \tag{28}$$

Taking the first derivative of (28), we have

$$\frac{\partial p_{At}}{\partial\gamma} = \frac{\partial p_{A(t-1)}}{\partial\gamma}(1-(1-\gamma)^n) + np_{A(t-1)}(1-\gamma)^{n-1} - \frac{\partial p_{A(t-1)}}{\partial\gamma}\gamma^n + n(1-p_{A(t-1)})\gamma^{n-1}$$

which can be rewritten

$$\frac{\partial p_{At}}{\partial\gamma} = \frac{\partial p_{A(t-1)}}{\partial\gamma}(1-(1-\gamma)^n - \gamma^n) + np_{A(t-1)}(1-p_{A(t-1)})(1-\gamma)^{n-1}\gamma^{n-1} \tag{29}$$

Thus, (29) implies that

$$\frac{\partial p_{At}}{\partial \gamma} > 0 \quad if \quad \frac{\partial p_{A(t-1)}}{\partial \gamma} > 0. \tag{30}$$

By induction, (30) holds if $\frac{\partial p_{A1}}{\partial \gamma} > 0$. As argued previously,

$$p_{A1} = 1-(1-\gamma)^n \tag{31}$$

which is increasing in γ. The result when γ > 0.5 follows from (27) and for γ < 0.5 by symmetry. ■

4. A ZERO-SUM *IG*

Here I introduce a modification to the *IG* which changes the payoff-structure to a single dimension. This allows the modeling of one form of a zero-sum game, which might be appropriate for certain empirical situations. To wit, this zero-sum insulation game (*ZIG*) is identical to the *IG* in all but one respect. Now, the payoffs if neither is insulating are as follows: if *A*'s program is in place by itself, the payoffs are (1,0); if both *A* and *B*'s programs are in place, the payoffs are (β, 1−β); and if *B*'s program is in place by itself, the payoffs are (0,1). As before, if a player plays *I*, he ensures that her program is in place in every stage thereafter, but her payoffs are modified by a factor α, where α < 1. In this case, the proofs are similar to those for Propositions 3a, 3b, 3c, and 4.

PROPOSITION A3a. *In the ZIG, if α is sufficiently large, both players will play {(I,O)}.*

PROOF. Here, the expected payoffs for *A* are identical to those given in (7). Thus, it is still the case that if the condition given in (9) holds, *A*'s best response to *B* playing (*I,O*) is (*I,O*). We call the cutpoint for *A* $\alpha_A^{'}$. The difference in the *ZIG* is that *B*'s payoffs are no longer the same as *A*'s around γ=0.5. While *A*'s cutpoint for insulation remains the same, it is necessary to reconsider the conditions under which *B* will insulate. We can solve for the condition under which *B*'s best response to (*I,O*) is (*I,O*) by noticing that since the game form is the same as in the *IG* except for the payoffs and reelection probabilities, we must only substitute (1-γ) for γ and (1- β) for β into (9) to obtain a similar condition for *B*. Making these substitutions, we obtain, *B*'s best response to *A* playing (*I,O*) is (*I,O*), if and only if

$$\alpha > \frac{1-\gamma+2\beta\delta\gamma-\delta-\delta\gamma}{1-\gamma-\delta+\beta\delta\gamma} = \alpha_B^{'} \qquad (32)$$

Since α must be greater than both α_A^* and α_B^*, we have the result:

$$\{(I,\mathrm{O});(I,O)\} \text{ is an equilibrium} \Leftarrow \alpha > \max(\alpha_A^{'},\alpha_B^{'}) = \alpha^{'} \qquad (33)$$

which is the proposition.■

PROPOSITION A3b. *In the ZIG, if α is not sufficiently large for both players to play {(I,O)}, then an equilibrium for the IG is either {(I,O);(NI,O)} or {(NI,O);(I,O)} if α is sufficiently large.*

PROOF. The case for *A* is the same as in Proposition 3b. From (13a) we have

$$\alpha > \frac{1-\delta+\delta\gamma}{1-\delta+\delta\gamma+\delta\beta(1-\gamma)} = \alpha_A^{''} \qquad (34a)$$

Thus, if $\alpha < \alpha_B^{'}$, and (34a) holds, then *A* will insulate while *B* will not. By substituting (1 - γ) for γ and (1-β) for β in (34a), we can derive a similar result in which *B* will insulate and *A* will not. In particular, if α < $\alpha_A^{'}$, *A* will play (*NI,O*) whether or not *B* plays (*I,O*) or (*NI,O*). Given that *A* plays (*NI,O*), B will play (*I,O*) if and only if

$$\alpha > \frac{1-\delta\gamma}{1-\delta\beta\gamma} = \alpha_B^{''} \tag{34b}$$

(34a) and (34b) constitute the proposition.■

PROPOSITION A3c. *In the ZIG, if α is sufficiently small, then neither player will insulate.*

PROOF. This follows from (32), (33), (34a) and (34b). The proof is perfectly analogous to that of Proposition 3c.■

Notice that in general, Proposition 4 does not have an analog in the *ZIG*, For example, $\alpha^{'}$ might not have a vertex at 0.5. Instead, it is possible that because of the asymmetric nature of the payoffs, the vertex will be either lesser or greater than 0.5 depending on the values of β and δ. The same is true for $\alpha^{''}$. This means that in a zero-sum setting, while for most values of γ, as political uncertainty increases, both the both insulate and neither insulate equilibria can be supported by a larger set of the parameters, there can be a region (γ*,0.5) for which the cooperative space *decreases*.