Unilateral Effects Calculations[*]

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Introduction

We compare here the prices set for two products, each of which is owned and controlled by a single firm in a differentiated Bertrand duopoly, with the profit-maximizing prices for these two products as set by a single firm controlling both products. The calculations given below apply to the special case in which marginal costs are constant for output levels near pre-merger levels. They also do not account for any efficiencies associated with common ownership of the two products. The first set of calculations applies with linear demand. The second set of calculations applies with constant elasticity demand and symmetry between the two products.

The comparison studied here can be useful in studying horizontal mergers, either for the purposes of market definition or for the purposes of assessing unilateral competitive effects. Obviously, these models are very simple and cannot alone form the basis of any conclusions regarding competitive effects in any specific proposed merger. However, they can provide a very valuable starting point, often more informative than focusing on the shares of the two firms among some collection of products, at least if we can observe with reasonable accuracy the pre-merger gross margins on the products being studied.

This analysis can inform the common situation in which the two firms compete with products offered by other firms, "outside products," as well as each other. With more than two firms, but with the prices of the outside products fixed, no change is needed in any of this analysis. The prices of outside products are just parameters that enter into the demand curves facing the two firms. Of course, if outside products are offered at fixed prices, are not subject to capacity

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[*] This note has been expanded and updated repeatedly over the years. This document is available at http://faculty.haas.berkeley.edu/shapiro/unilateral.pdf.

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constraints, and are close substitutes, we would expect to see a high elasticity of demand for the two inside products. This in turn would imply that the pre-merger margins on the two inside products are small.

If the prices of the outside products adjust as the prices of the two inside products change, the analysis presented below does need to be changed. If outside prices tend to rise with the inside prices, as we might generally expect due to upward-sloping reaction curves in price space, the profit-maximizing price increase for the firm controlling products 1 and 2 will tend to be larger than calculated here. However, other supply responses by rivals, such as product repositioning or new entry, will tend to dampen any incentive to raise price. Furthermore, if one of the firms studied also owns one or more outside products, that will tend to make price increases on the inside products more profitable than shown here due to the internalization of diverted sales from the other firm's inside product to that firm's outside product.

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Basic Duopoly Model

Firm *i* sets price p_i for $i = 1, 2$. Demand for firm *i* is given by $x_i = D_i(p_1, p_2)$. Costs at firm *i* are given by $C_i(x_i)$. Profits of firm *i* are given by $\pi_i = p_i x_i - C_i(x_i)$.

We assume that the two firms set prices independently prior to the proposed merger. Therefore, the pre-merger prices form a Bertrand equilibrium. Therefore, the pre-merger prices $\overline{p}_1$ and $\overline{p}_2$ are the solution to the two equations, $\frac{d\pi_i}{dp_i} = 0$ for $i = 1, 2$.

In general, the post-merger profits of the combined entity are given by $\pi = [p_1 x_1 - C_1(x_1)] + [p_2 x_2 - C_2(x_2)] + S(x_1, x_2)$ where $S(x_1, x_2)$ are the synergies, or cost savings, if the combined firm produces outputs x_1 and x_2. The optimal post-merger prices p_1^* and p_2^*

are the solution to the two equations, $\frac{d\pi}{dp_i} = 0$ for $i = 1, 2$. In this note, we assume that marginal costs are constant, c_1 and c_2, over the relevant range of output. We also take $S(x_1, x_2) = E_1 c_1 + E_2 c_2$, so the synergies take the form of reductions in these marginal costs. For some calculations, we assume no synergies, i.e., $E_1 = E_2 = 0$. The cost savings just necessary to offset the price-increasing effects of a horizontal merger are calculated in Gregory Werden, (1996), "A Robust Test for Consumer Welfare Enhancing Mergers Among Sellers of Differentiated Products," *Journal of Industrial Economics* 44: 409–413.

The profit-maximizing post-merger percentage price increase for product *i* is given by $z_i \equiv \frac{p_i^* - \bar{p}_i}{\bar{p}_i}$. Quantifying this discrete comparison in general is hard, although we do know that there will be some price increase if the products are substitutes and there are no synergies.

Linear Demand

We now consider the case of linear demand. In that case, we can, without loss of generality, define the units of each product so that the slope of each demand curve is -1. These may well not be the units in which the products are naturally measured, so care must be taken in applying the formulas derived below. For closely related calculations that do not redefine the units in this manner, see Jerry Hausman, Serge Moresi, and Mark Rainey, (2010), "Unilateral Effects of Mergers with General Linear Demand," *Economic Letters*, available at http://crai.com/uploadedFiles/Publications/Unilateral%20Effects-of-Mergers-with-General-Linear-Demand-Hausman-Moresi-Rainey.pdf.

With the normalization of units just described, we can write the demand curves as $x_1 = A_1 - p_1 + D_{21} p_2$ and $x_2 = A_2 - p_2 + D_{12} p_1$. Here the parameter D_{12} measures the Diversion Ratio from product one to product two, i.e., the fraction of sales lost by firm 1, when it raises the price of product 1, that are captured by product 2. The parameter D_{21} is the analogous Diversion Ratio from product two to product one.

Pre-Merger Bertrand Equilibrium

The profits of firm 1 are $\pi_1 = (p_1 - c_1)(A_1 - p_1 + D_{21} p_2)$. Differentiating with respect to p_1 and setting this equal to zero, we get firm 1's best-response curve, $2p_1 = A_1 + c_1 + D_{21} p_2$. Likewise, firm 2's best response curve is given by $2p_2 = A_2 + c_2 + D_{12} p_1$. The pre-merger Bertrand equilibrium is the solution to these two equations, i.e., the pair of prices $(\bar{p}_1, \bar{p}_2)$ which satisfy

$$2\bar{p}_1 = A_1 + c_1 + D_{21}\bar{p}_2 \text{ and } 2\bar{p}_2 = A_2 + c_2 + D_{12}\bar{p}_1.$$

These equations can be solved explicitly, giving the Bertrand equilibrium prices

$$\bar{p}_1(4 - D_{12}D_{21}) = 2(A_1 + c_1) + D_{21}(A_2 + c_2)$$

and

$$\bar{p}_2(4 - D_{12}D_{21}) = 2(A_2 + c_2) + D_{12}(A_1 + c_1).$$

Post-Merger Equilibrium: No Synergies

If both products are owned by the same firm, that firm, the merged entity, maximizes $\pi_1 + \pi_2 = (p_1 - c_1)x_1 + (p_2 - c_2)x_2$. Differentiating with respect to p_1 and setting this equal to zero gives $2p_1 = A_1 + c_1 + (D_{21} + D_{12})p_2 - D_{12}c_2$. The analogous equation for p_2 is $2p_2 = A_2 + c_2 + (D_{21} + D_{12})p_1 - D_{21}c_1$. Solving these two equations gives the post-merger price levels, p_1^* and p_2^*.

As noted above, we are interested in the percentage difference between the pre-merger and post-merger prices. Focusing on product 1, we start with $2p_1^* = A_1 + c_1 + (D_{21} + D_{12})p_2^* - D_{12}c_2$, and subtract using $2\bar{p}_1 = A_1 + c_1 + D_{21}\bar{p}_2$ to get $2(p_1^* - \bar{p}_1) = D_{21}(p_2^* - \bar{p}_2) + D_{12}(p_2^* - c_2)$. Likewise, we have $2(p_2^* - \bar{p}_2) = D_{12}(p_1^* - \bar{p}_1) + D_{21}(p_1^* - c_1)$.

Substituting using this last equation into the previous equation gives $2(p_1^* - \bar{p}_1) = \frac{D_{21}}{2}[D_{12}(p_1^* - \bar{p}_1) + D_{21}(p_1^* - c_1)] + D_{12}(p_2^* - c_2)$. Collectively terms and expanding gives $(p_1^* - \bar{p}_1)(4 - D_{21}D_{12}) = D_{21}{}^2(p_1^* - \bar{p}_1 + \bar{p}_1 - c_1) + 2D_{12}(p_2^* - \bar{p}_2 + \bar{p}_2 - c_2)$. Moving the $p_1^* - \bar{p}_1$ term to the left-hand side and substituting again for $p_2^* - \bar{p}_2$ gives

$$(p_1^* - \bar{p}_1)(4 - D_{21}D_{12} - D_{21}{}^2) = D_{21}{}^2(\bar{p}_1 - c_1) + 2D_{12}(\bar{p}_2 - c_2) + D_{12}[D_{12}(p_1^* - \bar{p}_1) + D_{21}(p_1^* - c_1)].$$

Moving the $p_1^* - \bar{p}_1$ term to the left-hand side and expanding the $(p_1^* - c_1)$ terms gives

$$(p_1^* - \bar{p}_1)(4 - D_{21}D_{12} - D_{21}{}^2 - D_{12}{}^2) = D_{21}{}^2(\bar{p}_1 - c_1) + 2D_{12}(\bar{p}_2 - c_2) + D_{12}D_{21}(p_1^* - \bar{p}_1 + \bar{p}_1 - c_1).$$

Again moving the $p_1^* - \bar{p}_1$ term to the left-hand side, we get

$$(p_1^* - \bar{p}_1)[4 - (D_{21} + D_{12})^2] = 2D_{12}(\bar{p}_2 - c_2) + D_{21}(D_{21} + D_{12})(\bar{p}_1 - c_1).$$

Dividing by the pre-merger price of product 1, we have

$$\frac{p_1^* - \bar{p}_1}{\bar{p}_1}[4 - (D_{21} + D_{12})^2] = 2D_{12}\frac{\bar{p}_2 - c_2}{\bar{p}_2}\frac{\bar{p}_2}{\bar{p}_1} + D_{21}(D_{21} + D_{12})\frac{\bar{p}_1 - c_1}{\bar{p}_1}. \tag{1}$$

Defining the pre-merger margins as $M_i \equiv \frac{\bar{p}_i - c_i}{\bar{p}_i}$, and likewise for product 2, we have

$$\frac{p_1^* - \bar{p}_1}{\bar{p}_1}[4 - (D_{21} + D_{12})^2] = 2D_{12}M_2\frac{\bar{p}_2}{\bar{p}_1} + D_{21}(D_{21} + D_{12})M_1. \tag{1'}$$

This can also be written as

$$\frac{p_1^* - \bar{p}_1}{\bar{p}_1} = \frac{2D_{12}M_2\frac{\bar{p}_2}{\bar{p}_1} + D_{21}(D_{21} + D_{12})M_1}{4 - (D_{21} + D_{12})^2}. \tag{1''}$$

This formula is related to the Gross Upward Pricing Pressure Index (GUPPI) for product 1, which is defined as $GUPPI_1 \equiv \frac{D_{12}(\bar{p}_2 - c_2)}{\bar{p}_1}$. See Carl Shapiro, (2010), "The 2010 Horizontal Merger Guidelines: From Hedgehog to Fox in Forty Years," *Antitrust Law Journal*, available at http://faculty.haas.berkeley.edu/shapiro/hedgehog.pdf, and the references therein. *GUPPI* measures the opportunity cost of selling one unit of product 1, due to ownership of product 2, measured as a fraction of the price of product 1, taking as given the price of product 2 (and all other prices and products). In particular, the first term in the numerator is twice $GUPPI$, so if the other "feedback" terms are ignored (i.e., if we substitute zeroes the second term in the numerator and for the second term in the denominator, as in the single-product problem discussed below), the indicated price increase is $GUPPI/2$. This corresponds to the pass-through rate of 50% for a single firm facing linear demand (holding all other prices as fixed), applied to the opportunity cost term associated with the ownership of product 2. As equation (1'') shows, the equilibrium price increase for product 1 with linear demand is larger than this amount because the price of product 2 will also rise (without any efficiencies) and because of feedback effects between the two prices.

Hausman, Moresi, and Rainey (2010) provide a similar formula that does not require normalizing the units of the two products so that the slope of each demand curve is -1. See their Proposition 1. They argue that in many cases "it is reasonable to assume that the cross-price derivatives of the demand functions are equal or approximately equal (i.e., $\partial Q_2/\partial p_1 \cong \partial Q_1/\partial p_2$)." With that assumption, their Proposition 2 reports that

$$\frac{p_1^* - \overline{p}_1}{\overline{p}_1} = \frac{GUPPI_1}{2} \times \frac{1 + \dfrac{\overline{p}_1 - c_1}{\overline{p}_2 - c_2} D_{21}}{1 - D_{12}D_{21}}.$$

This expression tells us by how much $GUPPI_1/2$ under-estimates the equilibrium price increase for product 1 with linear demand and no efficiencies.

Analogous formulas apply to product 2:

$$\frac{p_2^* - \overline{p}_2}{\overline{p}_2}[4 - (D_{21} + D_{12})^2] = 2D_{21}\frac{\overline{p}_1 - c_1}{\overline{p}_1}\frac{\overline{p}_1}{\overline{p}_2} + D_{12}(D_{21} + D_{12})\frac{\overline{p}_2 - c_2}{\overline{p}_2}, \qquad (2)$$

and

$$\frac{p_2^* - \overline{p}_2}{\overline{p}_2}[4 - (D_{21} + D_{12})^2] = 2D_{21}M_1\frac{\overline{p}_1}{\overline{p}_2} + D_{12}(D_{21} + D_{12})M_2, \qquad (2')$$

and

$$\frac{p_2^* - \bar{p}_2}{\bar{p}_2} = \frac{2D_{21}M_1 \dfrac{\bar{p}_1}{\bar{p}_2} + D_{12}(D_{21} + D_{12})M_2}{4 - (D_{21} + D_{12})^2} . \qquad (2'')$$

If the pre-merger prices and incremental costs are observed with reasonable accuracy, then the variables M_1, M_2, and $\dfrac{\bar{p}_2}{\bar{p}_1}$ can be calculated.

The remaining parameters, which are key, are the Diversion Ratios, D_{21} and D_{12}. In practice, these Diversion Ratios are more difficult to observe than prices and costs, so it is usually desirable to conduct sensitivity analysis to see how the post-merger price increases depend upon these Diversion Ratios. A simple spreadsheet that can be used to display the price increase for product 1, using equation (1), as a function of these two Diversion Ratios, is available at http://faculty.haas.berkeley.edu/shapiro/unilateral.xls.

Equations (1) and (2) simplify in the symmetric case where $\dfrac{\bar{p}_1 - c_1}{\bar{p}_1} = \dfrac{\bar{p}_2 - c_2}{\bar{p}_2} = M$, $\bar{p}_1 = \bar{p}_2$ and $D_{12} = D_{21} = D$. In that case we get

$$\frac{p^* - \bar{p}}{\bar{p}} = \frac{DM}{2(1-D)} . \qquad (3)$$

This expression is reported in Carl Shapiro, (1996), "Mergers with Differentiated Products," *Antitrust*, 10:23-30. To illustrate, with a pre-merger margin of $M = 0.4$ and a Diversion Ratio of $D = .20$, the post-merger price increase is 6.25%.

Post-Merger Equilibrium with Synergies

We are now ready to introduce the possibility of synergies. We noted above that the post-merger prices are the solution to the two equations $2p_1 = A_1 + c_1 + (D_{21} + D_{12})p_2 - D_{12}c_2$ and $2p_2 = A_2 + c_2 + (D_{21} + D_{12})p_1 - D_{21}c_1$. There we were taking c_1 and c_2 as given. But we can use these equations to see how the post-merger prices vary with these two costs levels. Totally differentiating with respect to c_1 and c_2 and solving, gives

$$\frac{dp_1^*}{dc_1}[4-(D_{21}+D_{12})^2] = 2 - D_{21}(D_{12}+D_{21})$$

and

$$\frac{dp_1^*}{dc_2}[4-(D_{21}+D_{12})^2] = D_{21} - D_{12}\,.$$

With synergies of E_1c_1 for product 1 and synergies of E_2c_2 for product 2, the equilibrium price for product 1 is the level computed above, with no synergies, less the amount $\frac{dp_1^*}{dc_1}E_1c_1 + \frac{dp_1^*}{dc_2}E_2c_2$. The additional decrement to the price of product 1 resulting from the synergies thus equals

$$\frac{1}{4-(D_{21}+D_{12})^2}\{[2-D_{21}(D_{12}+D_{21})]E_1c_1 + (D_{21}-D_{12})E_2c_2\}\,.$$

Using $c_1 = p_1(1-M_1)$ and $c_2 = p_2(1-M_2)$, we can express this decrement as a fraction of the pre-merger price of product 1 as

$$\frac{1}{4-(D_{21}+D_{12})^2}\{E_1(1-M_1)[2-D_{21}(D_{12}+D_{21})] + E_2(1-M_2)(D_{21}-D_{12})\frac{p_2}{p_1}\}\,.$$

Without synergies, we had

$$\frac{p_1^* - \overline{p}_1}{\overline{p}_1} = \frac{2D_{12}M_2\frac{\overline{p}_2}{\overline{p}_1} + D_{21}(D_{21}+D_{12})M_1}{4-(D_{21}+D_{12})^2}\,.$$

With synergies, we deduct the decrement from this expression and simplify to get:

$$\frac{p_1^* - \overline{p}_1}{\overline{p}_1} = \frac{[2D_{12}M_2 - E_2(1-M_2)(D_{21}-D_{12})]\frac{\overline{p}_2}{\overline{p}_1} + [D_{21}(D_{21}+D_{12})M_1 - E_1(1-M_1)(2-D_{21}(D_{12}+D_{21}))]}{4-(D_{21}+D_{12})^2}$$

With no synergies this simplifies to formula (1') above. With symmetry, it simplifies to

$$\frac{p^* - \overline{p}}{\overline{p}} = \frac{DM}{2(1-D)} - \frac{E(1-M)}{2}.$$

The first term is the expression derived above for the symmetric price increase *without* synergies. The second term is the single-firm pass-through rate with linear demand, namely one-half, times the reduction in the marginal cost of each product, which equals EC, measured as a fraction of its price, which becomes $E(C/P) = E(1-M)$.

Alternative Single-Product Calculation

A simpler calculation can be performed by focusing on a single product, say product 2, taking as given the price of product 1, and asking how much the price of product 2 will rise due to the merger. This will be a lower bound for the calculations given above, since with upward sloping reaction curves the profit-maximizing price of product 1 will also rise, making a further increase in the price of product 2 optimal.

A single firm's pricing problem can be written as choosing output x to maximize profits of $\pi(x) = R(x) - cx$, under our assumption of constant marginal cost in the relevant range. The first-order condition is $R'(x) = c$. Now ask how output (and thus price) change if the marginal cost changes. We get $R''(x)\frac{dx}{dc} = 1$. Since $\frac{dp}{dc} = \frac{dp}{dx}\frac{dx}{dc}$, we have $\frac{dp}{dc} = p'(x)\frac{dx}{dc} = \frac{p'(x)}{R''(x)}$. Now $R(x) = xp(x)$, so $R'(x) = p(x) + xp'(x)$, and $R''(x) = 2p'(x) + xp''(x)$. Thus we get in general $\frac{dp}{dc} = \frac{p'(x)}{2p'(x) + xp''(x)} = \frac{1}{2+E}$, where $E \equiv \frac{xp''(x)}{p'(x)}$. The variable E, known as Seade's *E,* is discussed further in Carl Shapiro, "Theories of Oligopoly Behavior," in the *Handbook of Industrial Organization*, Volume I, R. Schmalensee and R. Willig, eds., 1989. In the case of linear demand, $E = 0$ and $\frac{dp}{dc} = \frac{1}{2}$. The pass-through rate is greater than one-half if $p''(x) > 0$ so $E < 0$, as with constant elasticity demand, and smaller than one-half if $p''(x) < 0$ so $E > 0$.

Taking as given the price of product 2, the ownership of product 2 means that each incremental unit of product 1 comes with an extra (opportunity) cost, namely the loss of D_{12} units of product 2. Each incremental unit of product 1 thus reduces the merged firm's profits on product 2 by $D_{12}(\overline{p}_2 - c_2)$. Because ownership of product 2 raises the (opportunity) cost of selling more units of product 1, the merger has the same effect on the price of product 1 (here, where we are fixing the price of product 2) as would raising the cost of selling a unit of product 1 by $T_1 \equiv D_{12}(\overline{p}_2 - c_2)$, or imposing a per-unit tax on product 1 of this magnitude. With the liner demand approximation, the pass-through rate is one-half, so the increase in the price of product 1 is $p_1^* - \overline{p}_1 \approx D_{12}(\overline{p}_2 - c_2)/2$. Expressing this in percentage terms gives

$$\frac{p_1^* - \overline{p}_1}{\overline{p}_1} \approx \frac{1}{2} D_{12} M_2 \frac{\overline{p}_2}{\overline{p}_1}.$$

This is an under-estimate of the equilibrium effect so long as each firm's reaction function is upward sloping, since the price of product 2 will also go up in equilibrium.

In the symmetric case, the single-product under-estimate becomes $\frac{p^* - \overline{p}}{\overline{p}} = \frac{1}{2} DM$. We can easily compare this with the equilibrium price increase in the linear demand duopoly model given above, which is $\frac{p^* - \overline{p}}{\overline{p}} = \frac{1}{2} \frac{DM}{1-D}$. The duopoly (equilibrium) value is a multiple $\frac{1}{1-D}$ of the single-firm figure. To illustrate, if the Diversion Ratio is one-third, the duopoly value is 1.5 times the single-firm figure. If the Diversion Ratio is 20%, the duopoly value is 1.25 times the single-firm figure. For moderately large Diversion Ratios, the duopoly value is distinctly larger than the single-firm figure. So, while the single-firm calculation is instructive and intuitive, it gives a significant underestimate unless the Diversion Ratio is quite low.

Constant Elasticity Demand

For constant elasticity of demand, and assuming symmetry, the elasticity falls from its pre-merger level of ε to a post-merger level of $\varepsilon(1-D)$. Pre-merger profit maximization implies that $M = 1/\varepsilon$, or $\frac{\overline{p}-c}{\overline{p}} = \frac{1}{\varepsilon}$. Solving for $\overline{p}$ gives $\overline{p} = \frac{c}{1-\frac{1}{\varepsilon}}$. Post-merger, we have

$\frac{p^*-c}{p^*} = \frac{1}{\varepsilon(1-D)}$, so $p^* = \frac{c}{1-\frac{1}{\varepsilon(1-D)}}$. For this to make sense, we need the elasticity of demand post-merger to exceed one, i.e., $\varepsilon(1-D) > 1$. Using $M = 1/\varepsilon$, this necessary condition can be written as $1-D > M$, or $D < 1-M$. In words, the Diversion Ratio, D, must not be too large, especially if the pre-merger margins are large. If this condition is not satisfied, the assumption of constant elasticity of demand would imply that the post-merger firm could make arbitrarily large profits by setting higher and higher prices. This is a reminder that the assumption of constant elasticity of demand cannot hold up for large price increases and should be considered a convenient and standard special case useful for small price increases.

Substituting, the percentage post-merger price increase $\frac{p^*-\overline{p}}{\overline{p}}$ is given by $\frac{\frac{c}{1-\frac{1}{\varepsilon(1-D)}} - \frac{c}{1-\frac{1}{\varepsilon}}}{\frac{c}{1-\frac{1}{\varepsilon}}}$.

Canceling the *c* terms and multiplying by $\varepsilon(1-D)$ gives $\frac{\frac{1}{\varepsilon(1-D)-1} - \frac{1}{\varepsilon(1-D)-(1-D)}}{\frac{1}{\varepsilon(1-D)-(1-D)}}$, which can be written as $\frac{\frac{1}{\varepsilon(1-D)-1} - \frac{1}{(\varepsilon-1)(1-D)}}{\frac{1}{(\varepsilon-1)(1-D)}}$. Multiplying numerator and denominator by

$(\varepsilon-1)(1-D)$ and collecting terms gives $\frac{(\varepsilon-1)(1-D)}{\varepsilon(1-D)-1}-1$. Combining these two terms gives $\frac{D}{\varepsilon(1-D)-1}$. Again using $M=1/\varepsilon$, this becomes $\frac{D}{\frac{(1-D)}{M}-1}$, so we have

$$\frac{p^*-\overline{p}}{\overline{p}}=\frac{DM}{1-D-M}. \qquad (4)$$

This expression is reported in Carl Shapiro, (1996), "Mergers with Differentiated Products," *Antitrust*, 10:23-30. As noted earlier, all of this requires that $D+M<1$, so the Diversion Ratio, D, is bounded above by $1-M$. This is a tight bound if pre-merger margins are large.

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