Abstention, ideology and information acquisition[1]

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Abstract

We consider a committee in which each member can collect information of different precision. Voters have asymmetric information and diverse preferences that are two dimensional: one dimension captures the voter's ideology and the other dimension captures the intensity of that voter's preferences. We show that information and abstention are not necessarily negatively correlated at the individual level. In equilibrium, voters collect different qualities of information, and there are informed voters who abstain although they would have voted had they not collected information. Moreover, some voters are more likely to abstain the more informed they are. The larger the electorate, the less information a voter collects, and the higher the turnout is. We also discuss the manner in which incentives to acquire information are non-monotonic in terms of both concern and ideology.

Keywords: Abstention, Information Acquisition, Heterogeneity.
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1 Introduction

Very few papers study equilibrium models of endogenous information in committees.[1] None of them study abstention or *roll-off*. Considering that roll-off is usually explained as an informational phenomenon (Feddersen and Pesendorfer (1996)), a nexus between information acquisition and abstention seems appropriate. In this paper we study that nexus and answer the question, *who abstains in equilibrium?*

We start with a traditional model of costless voting where voters have asymmetric information and private and diverse preferences. Our set-up is based on Austen-Smith and Banks (1996): an election between two candidates, in which one candidate is preferred in one state, while the other candidate is preferred in the remaining state. Voters suffer no utility losses for electing the "correct" candidate, but differ on the utility losses they suffer for mistaken decisions. Our main innovation is that we allow voters to gather information selecting the quality of a binary signal that is correlated with the true state of the world. We assume that more informative signals entail a higher cost.

We also introduce a richer set of preferences. Traditionally, preferences in committees are modeled with a single parameter that captures the ideological bias. This parameter is sufficient to understand the incentives to vote with exogenous information. On the other hand, because incentives to acquire information depend on the absolute level of utility losses, restricting preferences to be single dimensional matters to understand the link between costly information acquisition and abstention. To properly understand this relation, we assume that voters differ not only on ideology, but also on the intensity of utility losses. As a result, and in contrast to other models of endogenous information, voters collect information of different quality in our model.

The existence of an equilibrium with voters endogenously collecting information of different quality does not follow from a straightforward application of fixed-point arguments.

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[1] See Persico (2004), Gerardi and Yariv (2008), Gershkov and Szentes (2009), Feddersen and Sandroni (2006), Martinelli (2006), Martinelli (2007), Cai (2009) and Li (2001).

We resolve this difficulty by transforming the problem of finding a fixed point in the space of best responses to a fixed point problem in the space of "pivotal" probabilities. We then proceed to study the voter's behavior and the connection between information and abstention. First, we show that rational ignorance (making decisions by consciously not acquiring information) is driven by two different forces: 1) extreme ideology and 2) balanced ideology with low intensity.

In equilibrium, abstention takes two different forms. On the one hand there is abstention of the previously mentioned "rationally ignorant" voters with low intensity. On the other hand there are voters who collect information and vote if this information reinforces their bias, but abstain if the information goes against their bias. In a sense, the swing voter's curse happens because such a voter does not have information (Feddersen and Pesendorfer (1996)), or because the signal received creates indifference regarding the candidates at the interim level (Davis et al. (1970)). We show that among these informed voters there are some voters who are more likely to *vote* the more informed they are and some voters who are more likely to *abstain* the more informed they are.

Our model allows us to study in detail the correlation between information and abstention. In particular we can answer the question, do marginally better informed subjects vote with higher probability? While the question of whether *informed* voters show up more often than *uninformed* voters may be answered positively, the effect of being *marginally more informed* depends on the voter's ideology.

The rest of the paper is organized as follows. After reviewing the literature we present our model in Section 3. Section 4 presents an existence result and a partial but detailed characterization. In Section 5 we discuss the importance of our assumption about preferences. All proofs are provided in an Appendix.

2 Literature Review

Our paper is related to voting models with endogenous information[2] and models of abstention where voting is not costly. We discuss each branch of the literature below.

Feddersen and Pesendorfer (1996) is the first paper providing an explanation for roll-off based on the level of information that a voter receives exogenously. They argue that uninformed voters rely on their peers for decisions because, on average, their peers are better informed. In essence, abstention is a type of delegation that occurs when a voter is poorly informed. This is the traditional swing voter's curse.[3] Feddersen and Pesendorfer (1999) by introduce heterogeneity both in preferences and in quality of information.[4] They provide examples where "individuals with better information are more likely to participate than individuals with worse information..."[5] Hence, "because uninformed independents abstain and informed independents vote, the model provides an informational explanation for why better educated individuals are more likely to vote" (Feddersen (2004), page 104). Both Feddersen and Pesendorfer (1996) and Feddersen and Pesendorfer (1999) place the emphasis on differential information. We show that when information is endogenous, the link between information and abstention is more intricate and requires further consideration of the role of the voter's ideological bias.

The existing literature on endogenous information studies cases where voters collect the same level of information in equilibrium.[6] Li (2001), Persico (2004), Feddersen and Sandroni (2006), Martinelli (2007), Gerardi and Yariv (2008), and Gershkov and Szentes (2009) assume that voters are homogenous (at least those willing to collect information) and/or that each

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[2] Gerling et al. (2003) surveys models with information acquisition in committees.

[3] Abstention has been also studied in other decision theoretic models as in Ghirardato and Katz (2006) and Larcinese (2007)). Davis et al. (1970) assume that voters abstain because they do not gain much by switching the winner (indifference) or they do not win much by selecting any winner (alienation); Davis et al. (1970) study elections in which voters behave in that particular way. Shotts (2006) allows voters to signal by abstaining in order to affect the outcome of a second election.

[4] Battaglini et al. (2007) provide evidence that the strategic behavior that leads to the swing voter's curse can be replicated in the lab.

[5] Feddersen and Pesendorfer (1999), page 382.

[6] The only exception is an example in Li (2001) with a very particular type of heterogeneity in a two-member committee.

voter can receive an independent draw from a common distribution. Cai (2009) assumes that voters collect information before knowing their preferences and, consequently, that they are homogenous at the information acquisition stage. Martinelli (2006) allows for heterogeneity and the option of acquiring information of different quality, but restricts the environment so in equilibrium every informed voter has the same incentives to collect information.

There is some evidence that information and turnout are in fact positively correlated. Wattenberg et al. (2000) use survey and aggregate data on Presidential and House races on the same ballot to show that information and abstention are negatively correlated. Coupé and Noury (2004) use data from the National Research Council regarding the quality of different research programs and find that roll-off can be explained by lack of information. Larcinese (2007) and Lassen (2005) argue that information is endogenous and using an instrumental variable approach provide evidence that information and turnout are positively correlated.[7]

In our paper we argue that information may or may not be positively correlated with abstention. The previous studies did not find this correlation because they compare aggregate measures without conditioning for ideology at the individual level (which matters as we show demonstrate) or because they define information in a coarse way. Wattenberg et al. (2000), Larcinese (2007), and Lassen (2005) compare informed voters with uninformed voters; and Coupé and Noury (2004) use three different levels of information quality to classify between informed and uninformed. All of these strategies lead to testing the composition of the electorate as a whole and without regard for the individual voter's behavior.

To our knowledge the closest test regarding the effect of marginal information was conducted by Palfrey and Poole (1987). They found that "[in the distance utility model]...the probability of voting for Reagan increases with information level. The opposite is true for Carter." (Palfrey and Poole (1987), pp. 526). They also found that the effect of information

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[7] On the other hand Gentzkow (2006) finds that more Television exposure reduces turnout. He argues that the correlation between information and turnout is positive given that voters have substituted away other sources of information (newspapers and magazines).

on turnout is positive but they separate the decision "to vote" from the decision on "who to vote for," so they cannot properly analyze the effect of ideology on information acquisition and the overall effect on turnout.

3 The model

There is a set of potential voters $\mathcal{N}$ with $|\mathcal{N}| = n$ who must decide between two options, A and Q; and there are two equally likely states of nature $\omega \in \{a, q\}$. The winner is selected according to the plurality rule, and if there is a tie the winner is selected by tossing a fair coin.[8] The set of possible actions for a voter is $\{Q, \emptyset, A\}$ where Q (A) is a vote for candidate Q (A) and $\emptyset$ stands for abstention.

There are two classes of voters: **non partisan** and **partisan**. With probability $\alpha \in (0, 1)$ a voter i is partisan. Partisans voters are described in terms of their behavior: with probability $\xi_x \in (0, 1)$, a partisan voter is type $x \in \{Q, \emptyset, A\}$ in which case she casts a ballot x, where $\sum_{x \in \{Q, \emptyset, A\}} \xi_x = 1$. Non partisan voters have contingent preferences described by $\theta = \{\theta_q, \theta_a\} \in [0, 1]^2$: if A (Q) is selected in state q (a), then the voter type $\theta = \{\theta_q, \theta_a\}$ suffers a utility loss of θ_q (θ_a) and there is no utility loss for selecting A (Q) in state a (q). Voters' preferences are private information. If the voter is non partisan her preferences are drawn independently from a distribution with a cumulative distribution function F on $[0, 1]^2$ with no mass points. We assume further that no hyperplane of F has positive measure (hyperdiffuse distribution) so if we let $g(\theta_a)$ be any function we have that $\int dF(\theta_a, g(\theta_a)) = 0$.[9] We assume that F, α and $\{\xi_Q, \xi_A\}$ are common knowledge. We refer to non partisan voter i's preferences as her type, and to a "non partisan voter type θ" simply as a "type θ".

After types are privately revealed, she can select the precision of the information she will receive: $p \in \left[\frac{1}{2}, 1\right]$ where p is the parameter of a Bernoulli random variable S which takes

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[8] The existence and characterization results are robust to different rules and to asymmetry across states as long as they verify some regularity conditions. Details can be provided upon request.

[9] We can ignore voters who are indifferent between strategies as in Caplin and Nalebuff (1991).

values on the set $\{s_q, s_a\}$. We assume that signals have the same precision in both states: $\Pr(s_\omega \mid p, \omega) = p$ for $\omega \in \{a, q\}$. Information is costly and the precision cost is given by $C : \left[\frac{1}{2}, 1\right] \to \mathcal{R}_+$, where we assume that:

Assumption 1 *The cost function C is twice continuously differentiable everywhere in $\left[\frac{1}{2}, 1\right]$ and satisfies 1) $C'(p) > 0$ and $C''(p) > 0$ for all $p > \frac{1}{2}$, 2) $C''\left(\frac{1}{2}\right) \geq C\left(\frac{1}{2}\right) = C'\left(\frac{1}{2}\right) = 0$, 3) $\lim_{p \to 1} C'(p) \to \infty$.*

The set of voters ($\mathcal{N}$), the (common) distribution that characterizes these voters' preferences (α, ξ_A, ξ_Q, F), and the cost of information function (C), constitute a committee. Since non partisan voters decide the precision of the signal and how they vote after receiving the signal, voter i's **pure strategy** is both an investment function $P^i : [0,1]^2 \to \left[\frac{1}{2}, 1\right]$ and a voting function $V^i : [0,1]^2 \times \{s_q, s_a\} \to \{Q, \emptyset, A\}$, such that $P^i(\theta)$ is the investment level of non partisan voter i with type θ, and $V^i(\theta, S) = (V^i(\theta, s_q), V^i(\theta, s_a))$ is the vote cast by non partisan voter i with type θ given the signal s.[10] When we refer to a generic voting function, investment function or strategy, we omit the superscripts that indicate types. We will say that, if $V^i(\theta, s) = v$ for all $s \in \{s_q, s_a\}$ player i of type θ uses an **uninformed** voting function, and if $V^i(\theta, s_q) \neq V^i(\theta, s_a)$, player i of type θ uses an **informed** voting function. We will identify strategies by their voting function.

The timing of the game is as follows: 1) Nature draws the profile of types as well as the state, 2) each player i observes her own preferences, 3) non partisan player i privately decides whether or not to acquire information by selecting $p^i \in \left[\frac{1}{2}, 1\right]$, 4) each player draws a private signal from the selected distribution parameterized by p^i, 5) players vote simultaneously after signals are observed and, 6) the winner is elected according to the plurality rule with ties broken by a fair coin toss.

Conditional on the profile of strategies of all voters except i, we define the probability that the winner is x in state ω, when voter i votes v, as $\Pr(x \mid \omega, v)$. We also define the

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[10] $V(\theta, S)$ describes the voter's behavior and $(v_q, v_a) \in \mathbf{X}^2$ is notation to describe arbitrary strategies (vote v_q after receiving s_q and vote v_a after receiving s_a) . When we want to refer to a particular vote, we use just v.

expected utility of player i of type θ when she votes v , and the state is ω, as $u^i(v \mid \theta, \omega) \equiv -\theta_\omega \Pr((-\omega) \mid \omega, v)$ where we let $(-\omega) = Q(A)$ if $\omega = a(q)$. The expected utility of player i of type θ and investment choice p, when she votes v after receiving the signal s is

$$U^i(p, v \mid \theta, s) \equiv \sum_{\omega \in \{q,a\}} u^i(v \mid \theta, \omega) \Pr(\omega \mid s, p) \tag{1}$$

Using (1), the gross expected utility of player i of type θ and investment choice p, for a voting strategy (v_q, v_a) is

$$\mathcal{U}^i(p, (v_q, v_a)) \mid \theta) \equiv \sum_{x \in \{q,a\}} U^i(p, v_x \mid \theta, s_x) \Pr(s_x) \tag{2}$$

We study Bayesian equilibria in symmetric profiles of pure strategies. Although we omit other players' strategies in definitions (1), (2), and $\Pr(x \mid \omega, v)$ the reader should understand that player i's payoffs depend on them.

Definition 1 *A symmetric Bayesian equilibrium for the voting game is a strategy* $(P^*(\theta), V^*(\theta, S))$ *such that:* ***1)*** *for all* $j = 1, ...n$, $V^j(\theta, S) = V^*(\theta, S)$ *and* $P^j(\theta) = P^*(\theta)$ *for every type* θ, ***2)*** *for every type* θ, *and for any other feasible votes* (v_q, v_a) *and* p, *the strategy* $(P^*(\theta), V^*(\theta, S))$ *satisfies*

$$\mathcal{U}^i(P^*(\theta), V^*(\theta, S) \mid \theta) - C(P^*(\theta)) \geq \mathcal{U}^i(p, (v_q, v_a) \mid \theta) - C(p) \tag{3}$$

Note that when voters collect information they are planning on voting in a particular way after every signal, but in our equilibrium definition we did not impose that, after receiving the signal, the voter still wants to vote as planned. The next Lemma shows that the ex ante condition (3) is sufficient for voters to vote as planned. This result is a consequence of statistical decision theory (see DeGroot (2004), page 139) because for a fixed information level the decision problem is separable in the signals which implies

Lemma 1 *If the strategy* $(P^*(\theta), V^*(\theta, S))$ *verifies (3) then* $U^i(P^*(\theta), V^*(\theta, s) \mid \theta, s) \geq U^i(P^*(\theta), v' \mid \theta, s)$ *for each* $s \in \{s_a, s_q\}$.

Proof. See Appendix (A.2). ■

If a strategy maximizes the expected utility of a particular type, that type is willing to carry on with the planned voting behavior after every signal. This result becomes particularly useful when the equilibrium characterization is presented.

It is useful to define the probability that an arbitrary voter $j \neq i$ votes v, in state ω, when all other players but i are using the strategy $(P(\theta), V(\theta, S))$:

$$\Pr(v \mid \omega) = (1-\alpha) \int\limits_{\theta \in [0,1]^2} \sum_{s \in \{s_q, s_a\}} \mathbf{I}(V(\theta, s) = v) \Pr(s \mid P(\theta), \omega)\, dF(\theta) + \alpha \xi_v \tag{4}$$

where $\mathbf{I}(x = y) = 1$ iff $x = y$ and 0 otherwise. The first part of the right side is merely the probability that a voter is non partisan multiplied by the probability that a non partisan votes v. The second part is the probability that a voter is partisan, multiplied by the probability that a partisan votes v.

4 Solving the Model

4.1 Voting Incentives

Recalling that $\Pr(A \mid \omega, X)$ is the probability that in state ω candidate A wins when voter i cast the ballot X we define the change in the probability of A winning when voter i switches her vote from $X \in \{Q, \emptyset\}$ to A in state ω as

$$\Delta \Pr(\omega, X) \equiv \Pr(A \mid \omega, A) - \Pr(A \mid \omega, X) \tag{5}$$

It follows that the change in the probability of A winning when voter i switches her vote from Q to $\emptyset$ in state ω is given by $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \emptyset)$. With some abuse we refer to $\Delta \Pr(\omega, X)$ for $X \in \{\emptyset, Q\}$ and $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \emptyset)$ as the pivotal probabilities in state ω. The existence of partisan voters makes every outcome possible in equilibrium and therefore:[11]

Lemma 2 *In any committee,* $\Delta \Pr(\omega, Q)$*,* $\Delta \Pr(\omega, \emptyset)$ *and* $\Delta \Pr(\omega, Q) - \Delta \Pr(\omega, \emptyset)$ *are positive for each* $\omega \in \{q, a\}$*.*

Let $\frac{\Delta \Pr(a,X)}{\Delta \Pr(q,X)} = L_{X \to A}$ for $X \in \{Q, \emptyset\}$ and $\frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\emptyset)}{\Delta \Pr(q,Q) - \Delta \Pr(q,\emptyset)} = L_{Q \to \emptyset}$. Using the definition of expected utility in (2) and the fact that $U^i(P^*(\theta), A \mid \theta, s) \geq U^i(P^*(\theta), v' \mid \theta, s)$ for any $v' \neq A$ (see Lemma (1)), a necessary condition for a non partisan voter type θ to vote for A after receiving the signal s is

$$\frac{\theta_q}{\theta_a} \frac{\Pr(q \mid s, p)}{\Pr(a \mid s, p)} \leq \min\{L_{Q \to A}, L_{\emptyset \to A}\} \tag{6}$$

and a necessary condition for her to vote for Q is

$$\frac{\theta_q}{\theta_a} \frac{\Pr(q \mid s, p)}{\Pr(a \mid s, p)} \geq \max\{L_{Q \to A}, L_{Q \to \emptyset}\} \tag{7}$$

Strict inequalities give sufficient conditions.

It is immediate to see that if $p = \frac{1}{2}$ the set of types θ that selects $V(\theta, s_a) \neq V(\theta, s_q)$ has no mass: if a voter is willing to switch votes when the signals are uninformative she must be indifferent between the two candidates and by assumption indifference has measure 0. Therefore, only uninformed strategies with $V(\theta, s_a) = V(\theta, s_q)$ and informed strategies with $P(\theta) > \frac{1}{2}$ and $V(\theta, s_a) \neq V(\theta, s_q)$, need to be studied.

Recalling that a voting strategy is a pair $(v_q, v_a) \in \{Q, A, \emptyset\}^2$, there are 9 possible voting strategies. Six of them may be part of an informed strategy: QA, $Q\emptyset$, AQ, $A\emptyset$, $\emptyset Q$, and $\emptyset A$. It is clear that those strategies that involve information being used in the wrong way are not optimal for a positive mass of players.

Lemma 3 *The voting strategies* AQ*,* $A\emptyset$ *or* $\emptyset Q$ *are not optimal for almost all types.*

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[11] For more general rules some care is needed. Details can be provided upon request.

Proof. See Appendix (A.2). ■

Now we need to consider only six voting strategies that may occur in equilibrium with positive probability. In principle voters can be separated into six different groups: **strong supporters for each candidate** ($\mathcal{SS}^A$ for A and $\mathcal{SS}^Q$ for Q), **weak supporters for each candidate** ($\mathcal{WS}^A$ for A and $\mathcal{WS}^Q$ for Q), **abstainers** ($\mathcal{A}$) and **independents** ($\mathcal{I}$). Weak supporters for A (Q) vote for A (Q) if $s = s_a$ ($s = s_q$) and abstain if $s = s_q$ ($s = s_a$) while strong supporters for A (Q) vote for A (Q) without collecting information. Abstainers do not collect information and abstain regardless of the signal received, and independents collect information and follow the signal they receive. The question is which set of voters will emerge in equilibrium and which will not be part of the equilibrium. Surprisingly, we will show that all but independents are present while the presence of independents cannot be guaranteed.

4.2 Information acquisition

It is straightforward to see that abstainers and strong supporters do not invest, while the probability that a type uses a weak supporter's strategy or is independent without performing any investment is 0. Now there are three relevant investment functions: one for each group that collects information (independents and weak supporters for A and Q). We define

Definition 2 *Let* $P^x : [0,1]^2 \to \left[\frac{1}{2}, 1\right]$ *for* $x \in \{QA, \emptyset A, Q\emptyset\}$ *be such that* $P^{\emptyset A}(\theta)$, $P^{Q\emptyset}(\theta)$ *and* $P^{QA}(\theta)$ *are the investment strategy of weak supporters for* A*, weak supporters for* Q*, and independents, respectively.*

Using (2) for each of the possible optimal strategies with investment and the information technology, we derive the optimal investment function implicitly as:

$$\begin{aligned} C'\left(P^{XA}(\theta)\right) &= \sum_{\omega \in \{q,a\}} \theta_\omega \frac{\Delta \Pr(\omega, X)}{2}, X \in \{Q, \emptyset\} \qquad (8) \\ C'\left(P^{Q\emptyset}(\theta)\right) &= C'\left(P^{QA}(\theta)\right) - C'\left(P^{\emptyset A}(\theta)\right) \end{aligned}$$

Since $\lim_{p\to 1} C'(p) \to \infty$, there is some $\eta < 1$ such that $P^x(\theta) \leq \eta$ for all informed voting strategies with $x \in \{QA, \emptyset A, Q\emptyset\}$.[12] The second equation in (8) illustrates that a player type θ using the strategy QA collects more information than she would have collected if she were a weak supporter. Why is this the case? Imagine a voter that is considering switching her vote from A to Q after signal s. That switch will change the outcome in the events when there is a tie, and when A or Q are winning by one vote. Now imagine the same voter considering the effects of a change from A to $\emptyset$. That switch will change the outcome in the events when there is a tie, and when Q is winning by one vote. Note that the situation where A was winning by one vote is not relevant in the case of a switch from A and $\emptyset$. In a sense, abstaining after some signal reduces the marginal value of information.

For independent behavior to be optimal, the level of investment required must be high. This is directly related to non-concavity in the demand for information (Stiglitz and Radner (1984) and Chade and Schlee (2002)). Assume that θ_a and θ_q are low so there is little investment in information acquisition. If θ_a and θ_q are very similar, the risk of introducing noise in the electorate plus the investment entails a high expected cost in utility terms. The non partisan voter prefers delegating to the electorate rather than voting for one or the other candidate with very weak evidence. The next lemma states the result formally.

Lemma 4 *A necessary condition for the independent behavior to be optimal with investment level* p*, is*

$$\left(\frac{p}{1-p}\right)^2 \geq \frac{L_{Q\to\emptyset}}{L_{\emptyset\to A}} \tag{9}$$

Moreover, if there is endogenous abstention with positive probability, independents must invest a strictly positive amount.

Proof. See Appendix (A.2). ■

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[12] It is worth noticing that the restriction of P to the domain $[0,1]^2$ is not needed. This will play an important role when we show that an equilibrium exists.

4.3 Existence and Characterization

The information acquisition rule is a function of the voter's type to the interval $\left[\frac{1}{2}, 1\right]$ so finding an equilibrium among all possible information acquisition rules requires the use of fixed point arguments in functional spaces. Usually a notion of compactness or monotonicity is needed to show the existence of a fixed point in those spaces. Unfortunately compactness in our case is not granted unless we severely restrict the information technology.[13] We will also show that information acquisition is not monotonic on the preference parameters. The key to the existence result is that all cutoff functions and information acquisition rules for almost every type change smoothly as functions of the pivotal probabilities. Since strategies are fully characterized by those pivotal probabilities finding a fixed point in the space of pivotal probabilities turns out to be equivalent to finding a fixed point in the space of strategies. Hence, we first need to describe the equilibrium and then use its properties to actually show that there is an equilibrium.

In order to formally describe the equilibrium we need to define cutoff functions that separate types according to the strategy they use. There are six possibly optimal strategies which imply that a particular type θ must perform 15 comparisons in order to decide which strategy to use. Fortunately, there are some cut off functions that do not intersect in the type space, which means that a voter does not need to make some comparisons in order to decide how to vote. Sometimes if a strategy (v_q, v_a) yields a higher utility than a strategy $\left(v_q', v_a'\right)$ it is the case that (v_q, v_a) is also better than a third strategy $\left(v_q'', v_a''\right)$. Moreover, in some situations if a voter is willing to follow a particular strategy, her preferences must be such that certain other strategies cannot be followed. For example, imagine the cutoff function that describes the collection of types that are indifferent between the strategies AA and $Q\emptyset$. This implies that there is some type θ willing to vote in favor of A with no

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[13] More technically, the quality of information may be a discontinuous mapping of the preference parameters, even among voters who decide to collect information. The best response function is only a C^0 function almost everywhere. This precludes the application of fixed point arguments for infinite dimensional spaces (see Rudin (1973), in particular, the equicontinutity requirement in Schauder's Fixed Point Theorem).

information (strategy AA implies $p = \frac{1}{2}$) but to abstain when there is information in favor of A (strategy $Q\emptyset$ implies $p > \frac{1}{2}$); this contradicts that the information received is in favor of A precluding the existence of such a type.

Each cutoff function will de described by a superscript that identifies which two strategies are being compared. Let $(v_q v_a) \in \{A, Q, \emptyset\}^2$ and $\left(v'_q v'_a\right) \in \{A, Q, \emptyset\}^2$ be a pair of voting functions that describe a strategy; let $\Delta\mathcal{U}^i\left((v_q v_a), \left(v'_q v'_a\right) \mid \theta_q, \theta_a\right)$ be defined as the difference between the utility derived from using $v_q v_a$ and the utility from using the strategy $v'_q v'_a$:

$$\begin{aligned} & \mathcal{U}^i\left(P^{v_q v_a}\left(\theta_q, \theta_a\right), (v_q v_a) \mid \theta_q, \theta_a\right) - C\left(P^{v_q v_a}\left(\theta_q, \theta_a\right)\right) \\ & -\mathcal{U}^i\left(P^{v'_q v'_a}\left(\theta_q, \theta_a\right), \left(v'_q v'_a\right) \mid \theta_q, \theta_a\right) - C\left(P^{v'_q v'_a}\left(\theta_q, \theta_a\right)\right) \end{aligned} \tag{10}$$

We define the function $g^{(v_q v_a)=\left(v'_q v'_a\right)}\left(\theta_a\right)$ such that type θ is indifferent between using the strategy $(v_q v_a)$ and the strategy $\left(v'_q v'_a\right)$ iff $\theta_q = g^{(v_q v_a)=\left(v'_q v'_a\right)}\left(\theta_a\right)$; implicitly $g^{(v_q v_a)=\left(v'_q v'_a\right)}\left(\theta_a\right)$ is defined by $\Delta\mathcal{U}^i\left((v_q v_a), \left(v'_q v'_a\right) \mid g^{(v_q v_a)=\left(v'_q v'_a\right)}\left(\theta_a\right), \theta_a\right) = 0$. For a fixed set of pivotal probabilities the g function is only a function from the space of mistakes in state a to the space of mistakes in state q and its existence follows by the implicit function theorem. In some cases we cannot assure that g exists so we need to work with a function that maps mistakes in state q to mistakes in state a. In such cases we are going to use the letter h such that the type that verifies $\theta_a = h^{(v_q v_a)=\left(v'_q v'_a\right)}\left(\theta_q\right)$ is indifference between the strategies $(v_q v_a)$ and $\left(v'_q v'_a\right)$; the function $h^{(v_q v_a)=\left(v'_q v'_a\right)}\left(\theta_q\right)$ is implicitly defined as $\Delta\mathcal{U}^i\left((v_q v_a), \left(v'_q v'_a\right) \mid \theta_q, h^{(v_q v_a)=\left(v'_q v'_a\right)}\left(\theta_q\right)\right) = 0$.

Four important comments are in order. First, technically g is a function of every voter's strategy but the effect of every voter except i is aggregated via the pivotal probabilities. Second, these functions are defined beyond $[0,1]^2$. Third, we cannot show that, $g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right)$ or $h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)$ always exists. Nevertheless, we can show that, at least one of them exists and, when both are properly defined, they are each other's inverse: $h^{(QA)=(\emptyset\emptyset)}\left(g^{(QA)=(\emptyset\emptyset)}(x)\right) = x$. Finally, contrary to all other cases, it may be that $g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right) > 1$ (or $h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right) > 1$)

for all $\theta_a \in [0,1]$ (or $\theta_q \in [0,1]$). In that case, being an abstainer is always better than following an independent behavior.

Using the cutoff functions described previously, we can define the set of strong supporters as[14]

$$\begin{aligned} \mathcal{SS}^A &\equiv \left\{\theta \in [0,1]^2 : \theta_q \leq \min\left\{g^{(AA)=(\emptyset A)}(\theta_a), g^{(AA)=(QA)}(\theta_a)\right\}\right\} \\ \mathcal{SS}^Q &\equiv \left\{\theta \in [0,1]^2 : \theta_q \geq \max\left\{g^{(QQ)=(Q\emptyset)}(\theta_a), g^{(QQ)=(QA)}(\theta_a)\right\}\right\} \end{aligned} \tag{11}$$

Strong supporters for each candidate are located where $\frac{\theta_a}{\theta_q}$ is extremely low or extremely high, and the set $\mathcal{SS}^X$ is located next to the set of weak supporters in favor of the same candidate and sometimes next to the set of independents. This implies that abstainers or supporters of candidate $-X$ do not share a boundary with the set of strong supporters of X. Consequently, in order to distinguish the set of strong supporters, it is sufficient to compare this strategy with strategies that involve information acquisition. This implies that a type close to that boundary behaves in different ways with respect to information depending on which group she belongs to. Finally, a certain kind, of one shot deviation principle holds for this group of voters. Indeed, we only need to determine that the voters prefer the uninformative strategy, XX, to any informative strategy that always supports candidate X after signals s_x.

The sets of weak supporters are defined as:

$$\begin{aligned} \mathcal{WS}^A &\equiv \left\{\theta \in [0,1]^2 : \min\left\{g^{(\emptyset A)=(QA)}(\theta_a), g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)\right\} \geq \theta_q > g^{(AA)=(\emptyset A)}(\theta_a)\right\} \\ \mathcal{WS}^Q &\equiv \left\{\theta \in [0,1]^2 : g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a) \leq \theta_q < g^{(QQ)=(Q\emptyset)}(\theta_a), \theta_a \leq h^{(Q\emptyset)=(QA)}(\theta_q)\right\} \end{aligned}$$

Note that the ideological dimension ($\frac{\theta_a}{\theta_q}$) of weak supporters is bounded above and below. In addition to being surrounded by strong supporters, weak supporters can be surrounded by independents, abstainers, or both. A one shot deviation principle also holds for this

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[14] Since its measure is zero we can assign types that are indifferent to any of the groups that provide the same expected utility.

group but its intuition is more subtle. Consider the case of weak supporters for candidate A. These voters support A if the signal is s_a and abstain if the signal is s_q. The most immediate deviation from supporting A after s_a is abstaining while there are two possible immediate deviations from abstention after s_q (supporting A or supporting Q). Note that the set of weak supporters for A is determined by the only possible deviation on one side (a comparison with the strong supporter via $g^{(AA)=(\emptyset A)}\left(\theta_a\right)$) and the better of the two other deviations on the other side (via $\min\left\{g^{(\emptyset A)=(QA)}\left(\theta_a\right), g^{(\emptyset A)=(\emptyset\emptyset)}\left(\theta_a\right)\right\}$).

The case of independents and abstainers is more delicate because they are separated by the function $g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right)$ or $h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)$ depending on which one is properly defined. We define the set of abstainers $\mathcal{A}$, when $1 \geq \frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}$ (so $g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right)$ is well defined) as

$$\mathcal{A} \equiv \left\{ \theta \in [0,1]^2 : \left[\begin{array}{c} L_{Q\to\emptyset} > \frac{\theta_q}{\theta_a} > L_{\emptyset\to A}, \theta_q \leq g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right) \\ , g^{(\emptyset A)=(\emptyset\emptyset)}\left(\theta_a\right) < \theta_q < g^{(Q\emptyset)=(\emptyset\emptyset)}\left(\theta_a\right) \end{array} \right] \right\} \qquad (12)$$

while if $1 < \frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)} + \frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}$ (so $h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)$ is well defined) the set of abstainers $\mathcal{A}$ is defined by

$$\mathcal{A} \equiv \left\{ \theta \in [0,1]^2 : \left[\begin{array}{c} L_{Q\to\emptyset} > \frac{\theta_q}{\theta_a} > L_{\emptyset\to A}, \theta_a \leq h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right) \\ , g^{(\emptyset A)=(\emptyset\emptyset)}\left(\theta_a\right) < \theta_q < g^{(Q\emptyset)=(\emptyset\emptyset)}\left(\theta_a\right) \end{array} \right] \right\} \qquad (13)$$

Independents are obviously defined as the complement of all these groups in $[0,1]^2$. Note that abstainers are surrounded by weak partisans and independents, if independents exist. Here the notion of one shot deviation does not apply: in order to reach abstainers from independents two deviations are needed. While potentially abstainers are again surrounded by three groups, independents, if they exist, can be surrounded by all other groups. Although the definition of independents involves all relevant comparisons, the rest of the groups are defined by a reduced set of comparisons. The next Proposition states formally that these comparisons are necessary and sufficient to describe the equilibrium.

Proposition 1 *Let* $P^{\emptyset A}(\theta)$*,* $P^{Q\emptyset}(\theta)$ *and* $P^{QA}(\theta)$ *be defined as in (8) and the sets* $\mathcal{WS}^A$*,* $\mathcal{WS}^Q$*,* $\mathcal{SS}^A$*,* $\mathcal{SS}^Q$*,* $\mathcal{A}$ *and* $\mathcal{I}$ *defined as above. In any committee the strategy* $(P^*(\theta), V^*(\theta, S))$ *with*

1. $P^*(\theta)$ *that prescribes* $P^{\emptyset A}(\theta)$ *for* $\theta \in \mathcal{WS}^A$*,* $P^{Q\emptyset}(\theta)$ *for* $\theta \in \mathcal{WS}^Q$*,* $P^{QA}(\theta)$ *for* $\theta \in \mathcal{I}$*, and* $P^*(\theta) = \frac{1}{2}$ *otherwise,*

2. $V^*(\theta, S)$ *that prescribes the uninformative behavior* $\emptyset\emptyset$ *for* $\theta \in \mathcal{A}$*,* XX *for* $\theta \in \mathcal{SS}^X$ *with* $X \in \{Q, A\}$*, and the informative behavior* $\emptyset A$ *for* $\theta \in \mathcal{WS}^A$*,* $Q\emptyset$ *for* $\theta \in \mathcal{WS}^Q$ *, and* QA *for* $\theta \in \mathcal{I}$*,*

is a symmetric Bayesian equilibrium.

Proof. See Appendix (A.2). ■

Although we cannot prove uniqueness of equilibrium, our partial characterization describes *all* symmetric Bayesian equilibria.

What is the intuition behind the equilibrium? On the one hand independents and abstainers try to treat candidates similarly. The intensity of their preferences determines whether collecting information can help overcome any minor ideological bias they might have. Abstainers are rationally ignorant and do not support any candidate, and independents collect information to follow it. Strong supporters for each candidate are sufficiently biased to overpower the maximum amount of information that they are willing to collect if they were going to follow the signals.

The interesting groups are the weak supporters for A and Q. They are characterized by weakly biased preferences so they do not want to treat both candidates similarly; behaving as abstainers is ruled out. The maximum level of information they are willing to collect is not enough to overpower their ideological bias when the signal contradicts it, so they are not willing to rely solely on information to decide their vote. There are two strategies left to consider. These moderately biased voters could decide not to collect information in which case they would uninformatively support the candidate toward whom they are biased. Note that they could collect information and abstain if the signal favors the candidate they do not favor ideologically. For those who are moderately biased this is the preferred strategy. In essence these voters suffer the swing voter's curse because of the combination of ideology and information when the signal goes against their bias.

Once the description of equilibrium is complete we are ready to prove existence.

Proposition 2 *There exists a symmetric Bayesian equilibrium. Moreover, this equilibrium is characterized by the strategy* $(P^*(\theta), V^*(\theta, S))$ *in Proposition (1).*

Proof. See Appendix (A.2). ■

Note that we have shown that there is an equilibrium and that it is characterized by the set of groups defined in Proposition (1), but nothing guarantees, yet, that there is abstention in equilibrium. To see this let us assume that $\xi_{\emptyset} = 0$[15] and assume that every non partisan voter actually is submitting a positive vote. Since there are an odd number of voters, the only pivotal event is a tie, and relying on information or preferences to decide the vote is preferable to abstention. Fortunately $\xi_{\emptyset} > 0$ is enough for the pivotal events with positive probability to include those in which Q is winning or losing by one vote, which leads to endogenous abstention. We prove this by contradiction in the next Proposition. The intuition is as follows. If we assume that there is no endogenous abstention Proposition (1) asserts that only $\mathcal{SS}^A$, $\mathcal{SS}^Q$, and $\mathcal{I}$ are not empty so information is collected by the average voter. The collection of information insures that in state q (a) the event in which Q is winning by one vote or tying is more (less) likely than the event in which A is winning by one vote or tying which creates too much risk for those voters that collect information of poor quality. Hence being an abstainer is the preferred strategy for those with low intensity and fairly balanced preferences.

Proposition 3 *In any equilibrium there is endogenous abstention since the sets* $\mathcal{WS}^A$*,* $\mathcal{A}$*, and* $\mathcal{WS}^Q$ *are not empty.*

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[15] We assume that $\xi_{\emptyset} > 0$ to guarantee that there is endogenous abstention in equilibrium.

Proof. See Appendix (A.2). ■

Equilibria with abstention take basically three forms that can be inferred from the description of each group of voters. A first type of equilibrium is described in Figure (1). There we have a situation in which strong partisans are surrounded only by weak partisans. On the other hand when weak partisans are surrounded on the outside by strong partisans, weak partisans with high intensity are additionally surrounded also by independents, and weak partisans with low intensity are surrounded by abstainers. Abstainers and independents have more balanced preferences, and they are ordered according to the intensity of preferences. Since it might be possible that behaving like independents is not optimal for some electorates, another form of equilibrium emerges when only abstainers are present in the 45 degree diagonal. Figure (2) describes another class in which strong partisans share a boundary with independents. The main difference here is that weak supporters are not characterized by extreme intensity while some independents are characterized by a fairly strong bias and collect enough information to overpower it. Figures (1) and (2) rely on certain symmetry assumptions. If we rule out those symmetry assumptions we could have one side (say those biased towards A) where strong supporters are only surrounded by weak partisans, while the other side (biased towards Q) has strong supporters who are surrounded by both weak supporters and independents.

It is important to note that, for low values of θ_a and θ_q, we know that the investment condition (9) does not hold, and abstainers exists because $g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a) < \theta_q < g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a)$ for some θ. Recalling that independents collect a strictly positive amount of information(see Lemma (4)), for low values of θ_a and θ_q the separation of types close to the origin is given by the functions $g^{(QQ)=(Q\emptyset)}(\theta_a)$ ($\mathcal{SS}^Q$ from $\mathcal{WS}^Q$) $g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a)$ ($\mathcal{A}$ from $\mathcal{WS}^Q$), $g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)$ ($\mathcal{A}$ from $\mathcal{WS}^A$), and $g^{(AA)=(\emptyset A)}(\theta_a)$ ($\mathcal{SS}^A$ from $\mathcal{WS}^A$). In Proposition (1) we show that,

$$g^{(QQ)=(Q\emptyset)}(\theta_a) \geq g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a) \geq g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a) \geq g^{(AA)=(\emptyset A)}(\theta_a)$$

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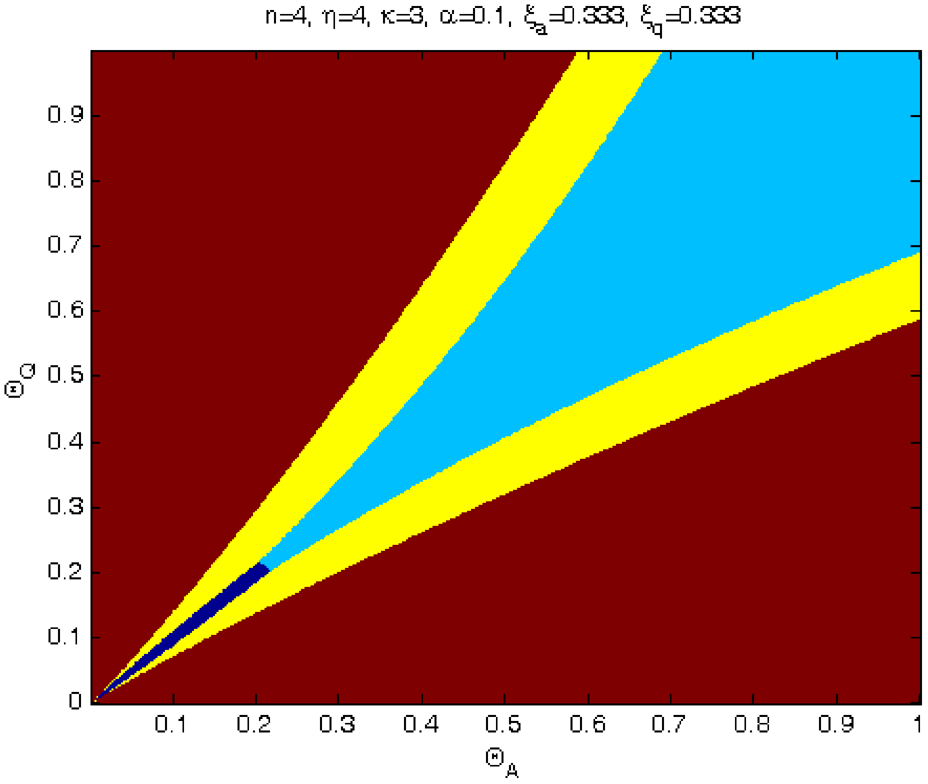

1.*pdf*

Figure 1: Strong supporters are in red, weak supporters in yellow, independents in light blue and abstainers in dark blue. The distribution of θ_ω is beta with parameters $(2,2)$ and the committee consists of 4 $(n=4)$ members that are partisan with 10% probability $(\alpha = 0.1)$ and are evenly split between the voting options $(\xi_a = \xi_q = \xi_\varnothing = \frac{1}{3})$. The cost function is $C(p) = 4\left(p - \frac{1}{2}\right)^3$.

which gives

Proposition 4 *In any equilibrium there is some* $\alpha > 0$ *such that for every type that verifies* $\theta_a + \theta_q \leq \alpha$ *the group of voters are ordered clockwise as* $\mathcal{SS}^A$, $\mathcal{WS}^A$, $\mathcal{A}$, $\mathcal{WS}^Q$ *and* $\mathcal{SS}^Q$*.*

The fact that voters are ordered in this way leads to information that is non monotonic on ideology for low intensity. First of all, let's concentrate on types with the same level of intensity; say $\theta_a + \theta_q = \beta$ and assume that β is sufficiently low. Consider the case in Figure (1) starting from $\theta_a = 0$ and $\theta_q = \beta$ and increasing θ_a down the line $\theta_a + \theta_q = \beta$. Information is nil first (when $\theta \in \mathcal{SS}^Q$), grows when $\theta \in \mathcal{WS}^Q$ and is nil again when $\theta \in \mathcal{A}$; then information is positive when $\theta \in \mathcal{WS}^A$ to be nil again when $\theta \in \mathcal{SS}^A$. Clearly information is non-monotonic on the ideological level. On the other side if β is sufficiently

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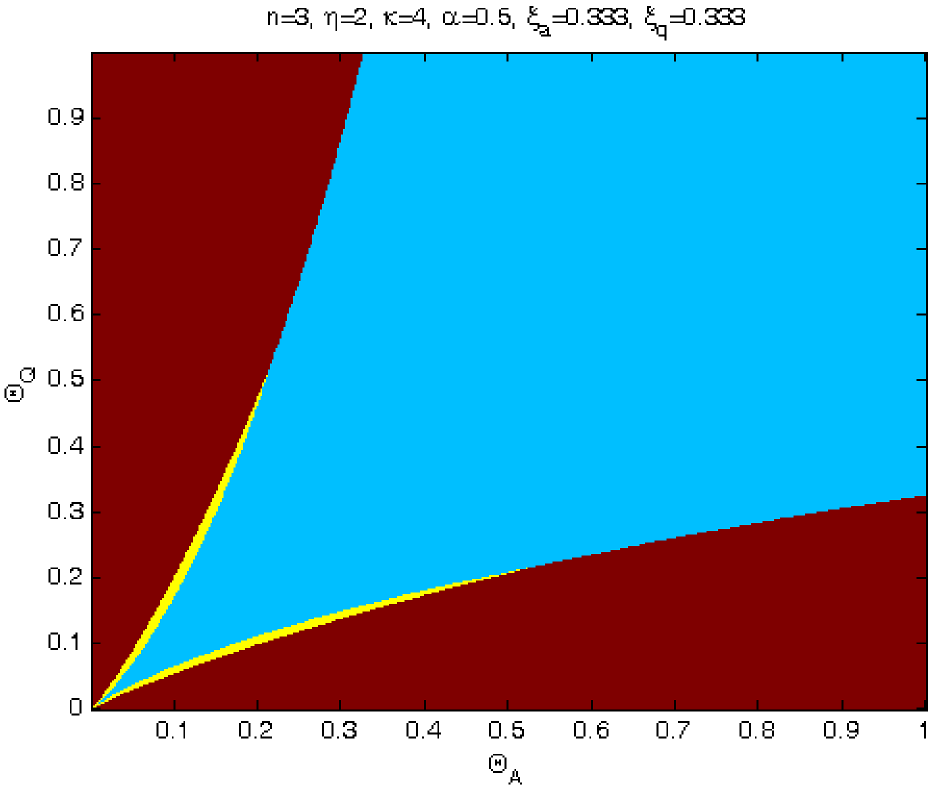

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Figure 2: Strong supporters are in red, weak supporters in yellow, independents in light blue and abstainers in dark blue. The distribution of θ_ω is beta with parameters $(1, 2)$ and the committee consists of 3 $(n = 3)$ members that are partisan with 10% probability $(\alpha = 0.1)$ and are evenly split between the voting options $(\xi_a = \xi_q = \xi_\varnothing = \frac{1}{3})$. The cost function is $C(p) = 2\left(p - \frac{1}{2}\right)^4$. The size of abstainers is significantly small.

large we move from $\mathcal{WS}^Q$ to $\mathcal{I}$ and then to $\mathcal{WS}^A$. In this case information is not monotonic either and it is possible to argue that more centrist voters will collect more information. The relation between ideology, information, and abstention is more complex. In particular, we cannot rule out that $\mathcal{I}$ and $\mathcal{SS}^A$ (or $\mathcal{SS}^Q$) are adjacent to each other. That is, we cannot rule out that the functions $g^{(QQ)=(QA)}(\theta_a)$ and $g^{(AA)=(QA)}(\theta_a)$ are necessary to describe the equilibrium as presented in Figure (2).

4.4 Some comparative statics

For a fixed N we have only provided a partial characterization because the "parameters" of the cutoff functions g are endogenous variables. Performing comparative statics for a fixed N turns out to be significantly difficult because we require a complete and full characterization of equilibrium. In this section we discuss properties of the equilibrium when the number of voters is large. In particular we focus on the following questions, what is the ideology of the voters that collect information, and what is the ideology of the voters that abstain? Unlike Feddersen and Pesendorfer (1996) and Feddersen and Pesendorfer (1999), in our model nobody abstains when the electorate becomes large. The intuition hinges on the fact that investment is 0 in the limit which directly implies that weak supporters disappear when N is large. The smaller the quantity of information collected by the average player, the more a player relies on her own private ideological bias. In such case it becomes probable that a player would rather follow her bias than abstain and delegate the decision to the rest of the committee.

Proposition 5 *When $n \to \infty$ investment goes to 0 and the probability of a non partisan voter abstaining goes to 0.*

Proof. See Appendix (A.2). ■

We now focus on identifying the regions that are not red in Figures (1) and (2) when the number of voters is large. The following result links the presence of partisan voters with the response of non- partisan voters in electorates that are sufficiently large.

Lemma 5 *When the electorate is sufficiently large the set of strong supporters for X verifies*

$$\int_{\theta \in \mathcal{SS}^X} f(\theta)\, d\theta \approx \frac{1}{2} + \alpha \frac{\left(\xi_{-X} - \xi_X\right)}{2} \tag{14}$$

Proof. See Appendix (A.2). ■

The intuition for this result can be traced back to Feddersen and Pesendorfer (1997): for a sufficiently large number of voters the proportion of votes in favor of one or the other candidate is close to being evenly split. Moreover, (14) shows that non partisans voters compensate for the bias introduced by partisan voters. Indeed, when ξ_A increases (while ξ_Q is constant, which means less partisan abstention), the set of strong supporters for A

decreases when N is large. In terms of Figures (1) and (2), there is a clockwise shift. This implies that for a sufficiently large number of voters, the higher the partisan vote in favor of A, the more ideologically biased toward that candidate are the voters who abstain and collect information.

Note that an increment on the proportion of non partisan voters (the higher α) has a differential effect depending on the proportion of non partisan supporters for each one of the candidates.

Assume the case $\xi_A > \xi_Q$ which implies that increasing α increases the probability of a vote for A by a partisan voter more that increases the probability of a vote for Q by a partisan voter. Following the compensation intuition, we have a situation in which the set of strong supporters for Q increases, while the set of strong supporters for A decreases; and, again we have a clockwise shift in Figures (1) and (2).

To sum up, a change in the electorate that raises the proportion of partisan voters who support candidate A with respect to the proportion of partisan voters that support candidate Q, implies that the non partisan voters who are closer ideologically to candidate A will be the voters that actually collect information in equilibrium. And this, in turn, implies that these same voters that are closer to candidate A are also those who might decide to abstain. In other words, the higher the proportion of partisan voters supporting candidate A, the higher is the proportion of voters who rationally decide to become strong supporters for candidate Q. Consequently, the notion of independent voters and abstainer is directly related to the relative proportions of partisan voters.

5 Applications

5.1 The role of flexible preferences

In the model presented here, preferences are described by two parameters. It is traditional in voting models to assume that utility losses are perfectly and inversely correlated ($\theta_q^i + \theta_a^i = \delta_1$

for all i).[16] This assumption is sufficient to describe the voting strategy (see expressions (6) and (7)), but the levels of these losses are relevant in terms of information acquisition (see expression (8)). Behaviorally, when θ_q and θ_a are imperfectly correlated, we have voters who care about both types of political mistakes (false positives and true negatives) and care differently about them. We now illustrate why allowing for flexible preferences matters theoretically: restricting preferences diminishes the model's capacity of properly capturing optimal abstention as a social phenomenon, and this restriction is not innocuous when information is endogenous. In the next subsection we show why restricting preferences may lead to undesirable conclusions and predictions about information acquisition and abstention in committees.[17]

Condition (9) in Lemma (4) implies that there is some $\underline{p} > \frac{1}{2}$ such that every independent voter selects information such that $P^{QA}(\theta) \geq \underline{p}$. Let $\Theta_\epsilon = \left\{\theta \in [0,1]^2 : |\theta_a + \theta_q - 1| < \epsilon\right\}$ and let's assume that $\widetilde{F}$ is such that $\widetilde{F}(\theta \in \Theta_\epsilon) = 1$ for every $\epsilon > 0$ so all the mass is concentrated around the counter diagonal.[18] Imagine also that in any equilibrium for every $\theta \in \Theta_\epsilon$, the information collected is not that good so assume $P^{QA}(\theta) < \underline{p}$. Independents will not be part of any equilibrium, and every centrist voter will be an abstainer. In this case we will conclude that only "intermediate levels" of ideology collect information (see Larcinese (2009)).

Alternatively if $P^{QA}(\theta) > \underline{p}$ for every $\theta \in \Theta_\epsilon$, abstainers will not be part of the equilibrium, and every centrist will be an independent. Moreover, if some extra conditions hold,[19] there may be no equilibrium with abstention by non partisan voters. If $\widetilde{F}$, α or $\left(\xi_A, \xi_Q\right)$ are such that the equilibrium is described in Figure (2) weak supporters will be driven away, and

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[16] Assumptions presenting heterogeneity as $\theta_q^i - \theta_a^i = \phi^i \in [-1,1]$ or $\frac{\theta_q^i}{\theta_a^i} = \phi^i \in [0,\infty]$ suffer the same drawback presented here.

[17] We do not provide formal statements about these claims, but illustrate the potential problems that might arise when we restrict attention to a particular level of intensity.

[18] Although our assumptions prevent this situation when $\epsilon \to 0$ (the hyperdiffuse requirement on F), it is easy to show that the existence and characterization results hold when we reduce the dimension of the preference parameters.

[19] In particular, the set of weak supporters must be small and close to the origin.

only partisan voters will abstain when we use restricted preferences. This restriction leads us to conclude that abstention is not an equilibrium phenomenon: non partisan voters never abstain.

5.2 The correlation between information and abstention

Let $\Pr(v \neq \emptyset \mid P, \omega)$ be the probability of voting conditional on the quality of the information P and on the state ω. It is obvious that $\frac{d\Pr(v \neq \emptyset \mid P, \omega)}{dP} = 0$ for all those who strictly prefer not to collect information ($\mathcal{SS}^A$, $\mathcal{SS}^Q$ and $\mathcal{A}$) as well as for those who strictly prefer to be independent voters ($\mathcal{I}$). On the other hand in state a (q), $\mathcal{WS}^A$ present a negative (positive) correlation between information and abstention while $\mathcal{WS}^Q$ present a positive (negative) correlation between information and abstention. At the aggregate level the correlation between information and abstention depends on the relative size of the weak supporters for one candidate or the other.

Another question concerns the difference between the probability of voting with and without information: $\Delta\Pr(v \neq \emptyset \mid \omega) = \Pr(v \neq \emptyset \mid P > 0, \omega) - \Pr(v \neq \emptyset \mid P = 0, \omega)$. In this case, only independents and strong supporters always vote, weak supporters abstain with some probability, and abstainers do not vote. Clearly, $\Delta\Pr(v \neq \emptyset \mid \omega)$ also measures the proportion of voters in each group and captures the structure of the electorate more than the actual correlation between information and abstention. Moreover, depending on which is the actual state, the measure can yield stronger or weaker results.

6 Conclusions

Few papers study abstention as optimal behavior, and none of them allow for information acquisition which contrasts with roll off being an informational phenomenon. Following this idea, we presented a model of committees with *abstention* and *endogenous information acquisition* using two interdependent innovations: we allowed voters to *select the precision* *of the signal* they receive and committee members' preferences to incorporate differences on the levels of *both ideology* and *intensity.*

In equilibrium, there are two classes of uninformed voters: first, **abstainers,** with balance preferences and low intensity, and **strong supporters** for each one of the candidates, who are extremely biased ideologically. Rational ignorance takes on two different forms. In the first case, abstainers decide not to collect information and delegate on the other members by abstaining and, in the second case, strong supporters always vote, although their votes are not based on any information. There are also two classes of informed voters: **weak supporters** for each candidate with a relatively low ideological bias, and **independents** with balanced preferences and high intensity.

We show that information acquisition is not a monotonic function of ideology and intensity: voters who have more at stake in an election may decide to collect less information. Moreover, the optimal information acquisition function is discontinuous even among voters who collect some information. This happens when voters, endogenously, use different voting strategies because the value of information changes discontinuously.

Restricting preferences to a single dimension is not insignificant when information is endogenous and abstention is possible. Because one- dimensional preferences do not allow for intensity, strategies that depend on differing intensity may not arise in equilibrium when preferences are restricted. Moreover, if those strategies that are dominated in the equilibrium of the model without intensity utilize abstention as part of an optimal voting strategy, restricted models fail to capture abstention as an equilibrium behavior. Therefore, we conclude that restricting preferences may give misleading characterizations of abstention.

We show that correlation patterns between information and abstention are present as long as we condition this correlation on particular groups of voters: some voters are *more likely to vote* the more informed they are, while other voters are *more likely to abstain* the more informed they are. Some voters abstain even if they have much at stake in the election and have strong evidence in favor of one candidate. Abstention is not simply the result of poor information, but a more complex interaction between preferences and information. In our model some well informed voters may abstain preventing this good information to reach the electorate. Unfortunately a result regarding information aggregation is beyond the scope of the paper. The principle problem when the electorate becomes large is to understand who the "geometry" of the equilibrium changes.

Existing empirical studies of abstention and information only capture the relative size of the different groups that emerge in equilibrium. In essence, the strength of such a test depends on which equilibrium is actually represented in the data. Our model suggests that this approach does not provide the entire story. Empirical tests need to consider the ideological dimension to capture the differential effect of information acquisition on voting.

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A Appendix

A.1 Properties of cutoff functions

In characterizing the equilibrium we use heavily the difference in utilities between two strategies $\Delta\mathcal{U}^i\left((v_q v_a),(v'_q v'_a) \mid \theta_q,\theta_a\right)$ defined in (10), and the functions $g^{(v_q v_a)=\left(v'_q v'_a\right)}(\theta_a)$ and $h^{(v_q v_a)=\left(v'_q v'_a\right)}(\theta_q)$ implicitly defined by $\Delta\mathcal{U}^i\left((v_q v_a),(v'_q v'_a) \mid g^{(v_q v_a)=\left(v'_q v'_a\right)}(\theta_a),\theta_a\right) = 0$ and $\Delta\mathcal{U}^i\left((v_q v_a),(v'_q v'_a) \mid \theta_q, h^{(v_q v_a)=\left(v'_q v'_a\right)}(\theta_q)\right) = 0$ respectively. The next Remarks are useful to define each one of the sets of voters. The proofs are provided immediately below the statements.

Remark 1 *$g^{(AA)=(\emptyset A)}(\theta_a) < g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)$ for every $\theta_a > 0$*

Let $\mathcal{L}(x) = C'(x)\,x - C(x)$ and using (2), (8) and (10) we have that $g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)$ and $g^{(AA)=(\emptyset A)}(\theta_a)$ are respectively defined as

$$\frac{\mathcal{L}\left(P^{(\emptyset,A)}\left(g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a),\theta_a\right)\right)}{g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)} \equiv \frac{\Delta\Pr(q,\emptyset)}{2} \tag{15}$$

$$\frac{\mathcal{L}\left(P^{(\emptyset,A)}\left(g^{(AA)=(\emptyset A)}(\theta_a),\theta_a\right)\right)}{\theta_a} \equiv \frac{\Delta\Pr(a,\emptyset)}{2} \tag{16}$$

Applying the implicit function theorem to (15) and (16) it follows that $g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)$ is increasing and strictly convex and $g^{(\emptyset A)=(AA)}(\theta_a)$ is increasing and strictly concave. Using properties of convex and concave functions we have that $\frac{g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)}{\theta_a} > L_{\emptyset\to A} > \frac{g^{(AA)=(\emptyset A)}(\theta_a)}{\theta_a}$.

Remark 2 *$g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a) < g^{(Q\emptyset)=(QQ)}(\theta_a)$ for every $\theta_a > 0$*

It follows the same line used in Remark (1).

Remark 3 *$g^{(\emptyset A)=(QA)}(\theta_a) < g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a)$ for every $\theta_a > 0$*

Let the pair $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ be such that $\widetilde{\theta}_q = g^{(Q\emptyset)=(\emptyset\emptyset)}\left(\widetilde{\theta}_a\right)$ so (2), (10) and the definition of $\mathcal{L}(x)$ give that

$$\frac{\mathcal{L}\left(P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)}{\widetilde{\theta}_a} = \frac{\Delta\Pr(a,Q) - \Delta\Pr(a,\emptyset)}{2} \tag{17}$$

Using again (2), (10) we have that the type $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ prefers (QA) to $(\emptyset A)$ if

$$\frac{\mathcal{L}\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - \mathcal{L}\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)}{\widetilde{\theta}_a} \geq \frac{\Delta\Pr(a,Q) - \Delta\Pr(a,\emptyset)}{2} \tag{18}$$

Using the second line of (8) we have that the condition (18) is equivalent to

$$\begin{aligned}
&\widetilde{\theta}_a \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\emptyset)}{2} + C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
\leq\ & C'\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) + C'\left(P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)
\end{aligned}$$

Using now that $\widetilde{\theta}_q = g^{(Q\emptyset)=(\emptyset\emptyset)}\left(\widetilde{\theta}_a\right)$ verifies (17) we have that the type $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ prefers (QA) to $(\emptyset A)$ if

$$\begin{aligned}
& C'\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
\geq\ & C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
& - C'\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned} \tag{19}$$

or

$$\begin{aligned}
& C'\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
\geq\ & C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
& + C'\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned} \tag{20}$$

Note that if $P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) > P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ condition (19) gives that it is sufficient if

$$\begin{aligned}
-C\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \leq\ & C\left(P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
& + C'\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned}$$

and if $P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) \leq P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ condition (20) implies that it is sufficient if

$$\begin{aligned}
-C\left(P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \leq\ & C\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\
& + C'\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)
\end{aligned}$$

Convexity of C gives that the right hand side of the previous two conditions is always positive. Since $P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) \geq P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ and $P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) \geq P^{Q\emptyset}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$, if the type $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ verifies $\widetilde{\theta}_q = g^{(Q\emptyset)=(\emptyset\emptyset)}\left(\widetilde{\theta}_a\right)$ we must have that $\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ prefers QA to $\emptyset A$ which implies $\widetilde{\theta}_q \geq g^{(\emptyset A)=(QA)}\left(\widetilde{\theta}_a\right)$ by definition of $g^{(\emptyset A)=(QA)}\left(\theta_a\right)$.

Remark 4 *$g^{(\emptyset A)=(QA)}\left(\theta_a\right) > (<)\, g^{(AA)=(Q,A)}\left(\theta_a\right) > (<)\, g^{(AA)=(\emptyset A)}\left(\theta_a\right)$. Moreover, if $L_{Q\to\emptyset} \leq L_{Q\to A} \leq L_{\emptyset\to A}$ then $g^{(\emptyset A)=(QA)}\left(\theta_a\right) < g^{(AA)=(Q,A)}\left(\theta_a\right) < g^{(AA)=(\emptyset A)}\left(\theta_a\right)$*

Assume first that $g^{(\emptyset A)=(QA)}(\theta_a) < g^{(AA)=(\emptyset A)}(\theta_a)$ so every type with $\theta_q \leq g^{(\emptyset A)=(QA)}(\theta_a)$ weakly prefers $\emptyset A$ to QA and strictly prefers AA to $\emptyset A$; so AA is strictly preferred to QA so for indifference between AA and QQ we must have that $g^{(AA)=(Q,A)}(\theta_a) > g^{(\emptyset A)=(QA)}(\theta_a)$. On the other hand we have that every type that verifies $g^{(AA)=(\emptyset A)}(\theta_a) \leq \theta_q$ weakly prefers $\emptyset A$ to AA and strictly prefers QA to $\emptyset A$ and therefore strictly prefers QA to AA and we must have $g^{(AA)=(\emptyset A)}(\theta_a) > g^{(AA)=(Q,A)}(\theta_a)$.

Assume now that $g^{(\emptyset A)=(QA)}(\theta_a) > g^{(AA)=(\emptyset A)}(\theta_a)$ so every type that verifies $\theta_q \geq g^{(\emptyset A)=(QA)}(\theta_a)$ weakly prefers QA to $\emptyset A$ and strictly prefers $\emptyset A$ to AA so QA is strictly preferred to AA and we must have that $g^{(AA)=(Q,A)}(\theta_a) < g^{(\emptyset A)=(QA)}(\theta_a)$. On the other hand we have that every type that verifies $g^{(AA)=(\emptyset A)}(\theta_a) \geq \theta_q$ weakly prefers AA to $\emptyset A$ and strictly prefers $\emptyset A$ to QA and therefore strictly prefers AA to QA and we must have $g^{(AA)=(\emptyset A)}(\theta_a) < g^{(AA)=(Q,A)}(\theta_a)$.

For the second part we have that it is sufficient if $g^{(\emptyset A)=(QA)}(\theta_a) < g^{(AA)=(Q,A)}(\theta_a)$ when $L_{Q\to\emptyset} \leq L_{Q\to A} \leq L_{\emptyset\to A}$. Using (18) we have that every type with $\widetilde{\theta}_q = g^{(\emptyset A)=(QA)}\left(\widetilde{\theta}_a\right)$ verifies

$$\begin{aligned} & \widetilde{\theta}_q \frac{\Delta \Pr(q,Q) - \Delta \Pr(q,\emptyset)}{2} P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) \\ & -\widetilde{\theta}_a \frac{\Delta \Pr(a,Q) - \Delta \Pr(a,\emptyset)}{2}\left(1 - P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\ = \; & C\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) - C\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \\ & -C'\left(P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)\left(P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) - P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right) \end{aligned}$$

By concavity of C and the fact that $P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right) > P^{\emptyset A}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)$ we have that

$$\frac{\widetilde{\theta}_q}{\widetilde{\theta}_a} < L_{Q\to\emptyset}\frac{\left(1 - P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)}{P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)} \leq L_{Q\to A}\frac{\left(1 - P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)\right)}{P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)}$$

where the inequality on the right follows by assumption of $L_{Q\to\emptyset} \leq L_{Q\to A}$. Using now (2), (8) and (10) we have that $g^{(AA)=(QA)}(\theta_a)$ is defined as

$$\theta_a \frac{\Delta \Pr(a,Q)}{2} = \mathcal{L}\left(P^{QA}\left(g^{(AA)=(QA)}(\theta_a),\theta_a\right)\right) \tag{21}$$

which implies that $g^{(AA)=(QA)}(\theta_a)$ is strictly concave with $\frac{\partial g^{(AA)=(QA)}(\theta_a)}{\partial\theta_a} = L_{Q\to A}\frac{1-P^{QA}\left(g^{(AA)=(QA)}(\theta_a),\theta_a\right)}{P^{QA}\left(g^{(AA)=(QA)}(\theta_a),\theta_a\right)}$; using properties of concave functions we have that $\frac{g^{(AA)=(QA)}\left(\widetilde{\theta}_a\right)}{\widetilde{\theta}_a} > L_{Q\to A}\frac{1-P^{QA}\left(g^{(AA)=(QA)}\left(\widetilde{\theta}_a\right),\widetilde{\theta}_a\right)}{P^{QA}\left(g^{(AA)=(QA)}\left(\widetilde{\theta}_a\right),\widetilde{\theta}_a\right)}$ which implies that

$$\frac{g^{(AA)=(QA)}\left(\widetilde{\theta}_a\right)}{\widetilde{\theta}_a}\frac{P^{QA}\left(g^{(AA)=(QA)}\left(\widetilde{\theta}_a\right),\widetilde{\theta}_a\right)}{1 - P^{QA}\left(g^{(AA)=(QA)}\left(\widetilde{\theta}_a\right),\widetilde{\theta}_a\right)} > \frac{\widetilde{\theta}_q}{\widetilde{\theta}_a}\frac{P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)}{1 - P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)}$$

Note now that $\frac{\widetilde{\theta}_q}{\widetilde{\theta}_a}\frac{P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)}{1-P^{QA}\left(\widetilde{\theta}_q,\widetilde{\theta}_a\right)}$ is increasing in $\widetilde{\theta}_q$ which implies that $g^{(AA)=(QA)}\left(\widetilde{\theta}_a\right) > \widetilde{\theta}_q = g^{(\emptyset A)=(QA)}\left(\widetilde{\theta}_a\right)$.

A.2 Proofs

Proof of Lemma (1). The proof follows by contradiction. Let $(P^*(\theta), V^*(\theta, S))$ verify (3) but assume that there is some v' such that for s_a we have that $U^i(P^*(\theta), V^*(\theta, s_a) \mid \theta, s_a) < U^i(P^*(\theta), v' \mid \theta, s_a)$. This implies that the strategy $(P^*(\theta), (V^*(\theta, s_q), v'))$ yields expected utility $\mathcal{U}^i(P^*(\theta), (V^*(\theta, s_q), v') \mid \theta) =$

$$\begin{aligned} & U^i(P^*(\theta), v' \mid \theta, s_a)\Pr(s_a) + U^i(P^*(\theta), V^*(\theta, s_q) \mid \theta, s_q)\Pr(s_q) \\ > \; & \mathcal{U}^i(P^*(\theta), V^*(\theta, S) \mid \theta) \end{aligned}$$

which contradicts that $(P^*(\theta), V^*(\theta, S))$ verifies (3) ■

Proof of Lemma (3). We prove the case $A\emptyset$; the cases $\emptyset Q$ and AQ are analogous. If a non partisan voter uses $A\emptyset$, (6) gives

$$\frac{\Pr(q \mid s_q, p)}{\Pr(a \mid s_q, p)} \leq \frac{\theta_a}{\theta_q}\min\{L_{Q\to A}, L_{\emptyset\to A}\} \leq \frac{\Pr(q \mid s_a, p)}{\Pr(a \mid s_a, p)}$$

which is a contradiction since $\Pr(\omega \mid s_\omega, p) > \Pr(\omega \mid s_{-\omega}, p)$ for $p > \frac{1}{2}$. If $p = \frac{1}{2}$, it is optimal only for types that satisfy $\frac{\theta_q}{\theta_a} = \min\{L_{Q\to A}, L_{\emptyset\to A}\}$. ■

Proof of Lemma (4). The proof uses this Lemma that is of future interest:

Lemma 6 *A necessary condition for abstention is*

$$L_{Q\to\emptyset} > L_{Q\to A} > L_{\emptyset\to A} \tag{22}$$

Proof. *Note that some algebra gives that $L_{Q\to A} > (<)\, L_{\emptyset\to A}$ iff $L_{Q\to\emptyset} > (<)\, L_{Q\to A}$. Assume then that $L_{Q\to A} \leq L_{\emptyset\to A}$ so the inequality (22) does not hold. Then (6) and (7) become*

$$\frac{\Pr(q \mid s, p)}{\Pr(a \mid s, p)} \leq \frac{\theta_a}{\theta_q}L_{Q\to A} \leq \frac{\Pr(q \mid s, p)}{\Pr(a \mid s, p)}$$

which implies for almost all types that, a positive vote, either for A or Q, is preferred to abstaining. ■

Using the optimal conditions for voting after receiving the signals, (6) and (7), we have that it is necessary for independents that $\frac{\Pr(a|s_q,p)}{\Pr(q|s_q,p)}L_{Q\to\emptyset} \leq \frac{\theta_q}{\theta_a} \leq \frac{\Pr(a|s_a,p)}{\Pr(q|s_a,p)}L_{\emptyset\to A}$. Using that $\frac{\Pr(q|s_q,p)}{\Pr(a|s_q,p)} = \frac{\Pr(a|s_a,p)}{\Pr(q|s_a,p)} = \frac{p}{1-p}$, it is necessary that $\frac{1-p}{p}L_{Q\to\emptyset} \leq \frac{\theta_q}{\theta_a} \leq \frac{p}{1-p}L_{\emptyset\to A}$ which gives (9). Using the condition (22) in (9) we have that $\left(\frac{p}{1-p}\right)^2 > 1$ and, $p > \frac{1}{2}$ is necessary. ■

Proof of Proposition 1. We focus on the sets of strong supporters for A ($\mathcal{SS}^A$) and weak supporters for A ($\mathcal{WS}^A$), and on the set of abstainers ($\mathcal{A}$). For these sets be check that voters are maximizing expected utility by selecting those strategies and using the optimal information (condition (3)). The sets of strong supporters for Q ($\mathcal{SS}^Q$) and weak supporters for Q ($\mathcal{WS}^Q$) can be analyzed by reversing the order of A and Q and the signals s_a and s_q in the analysis. Finally the set of independents ($\mathcal{I}$) can be defined as the complement of the remaining sets of voters.

$\mathcal{SS}^A$ is well defined by (11)

Since $\frac{\partial \Delta \mathcal{U}^i((AA),(\emptyset,A)|\theta_q,\theta_a)}{\partial \theta_q} < 0$, if $\theta_q < g^{(AA)=(\emptyset,A)}(\theta_a)$ the strategy AA is preferred to the strategy $(\emptyset, A)$. On the other hand, since $\frac{\partial \Delta \mathcal{U}^i((\emptyset,A),(\emptyset\emptyset)|\theta_q,\theta_a)}{\partial \theta_q} < 0$, if $\theta_q < g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)$ the strategy $(\emptyset, A)$ is preferred to the strategy $(\emptyset, \emptyset)$. Using Remark (1) if a type prefers AA to the strategy $\emptyset A$ ($\theta_q < g^{(AA)=(\emptyset A)}(\theta_a)$) that type also prefers the strategy AA to the strategy $\emptyset\emptyset$. It is easy to see that $\frac{\partial \Delta \mathcal{U}^i((AA),(Q,A)|\theta_q,\theta_a)}{\partial \theta_q} < 0$ so if $\theta_q < g^{(AA)=(Q,A)}(\theta_a)$ the strategy AA is preferred to the strategy QA. Putting all together a type that verifies $\theta_q < \min\left\{g^{(AA)=(Q,A)}(\theta_a), g^{(AA)=(\emptyset A)}(\theta_a)\right\}$ prefers the strategy AA to QA, $\emptyset A$ and $\emptyset\emptyset$.

Since $\frac{\partial \Delta \mathcal{U}^i((\emptyset\emptyset),(Q\emptyset)|\theta_q,\theta_a)}{\partial \theta_q} < 0$, if $\theta_q < g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a)$, the strategy $\emptyset\emptyset$ is preferred to the strategy $Q\emptyset$. On the other hand, since $\frac{\partial \Delta \mathcal{U}^i((Q\emptyset),(QQ)|\theta_q,\theta_a)}{\partial \theta_q} < 0$, if $\theta_q < g^{(Q\emptyset)=(QQ)}(\theta_a)$ the strategy $Q\emptyset$ is preferred to the strategy QQ. Using Remark (2) we have that a if $\theta_q < g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a)$ then a type that prefers $\emptyset\emptyset$ to the strategy $Q\emptyset$ also prefers the strategy $\emptyset\emptyset$ to the strategy QQ.

Using Remark (3) we have that if $\theta_q < g^{(\emptyset A)=(QA)}(\theta_a)$ then $\emptyset\emptyset$ is preferred to both $Q\emptyset$ and using Remark (2) we have that $\emptyset\emptyset$ is also preferred to QQ. To finish the proof that the set of strong supporters for A is well defined by (11) we have that the first part of Remark (4) implies that $\min\left\{g^{(AA)=(Q,A)}(\theta_a), g^{(AA)=(\emptyset A)}(\theta_a)\right\} \leq g^{(\emptyset A)=(QA)}(\theta_a)$ so we must have that $\theta_q < \min\left\{g^{(AA)=(Q,A)}(\theta_a), g^{(AA)=(\emptyset A)}(\theta_a)\right\}$ implies that AA is also preferred to $Q\emptyset$ and QQ since the strategy AA is preferred to $\emptyset\emptyset$.

Using Remark (4) we have that when $L_{Q\to\emptyset} \leq L_{Q\to A} \leq L_{\emptyset\to A}$ the only relevant comparison is between AA and QA since $\emptyset A$ is not going to be used in equilibrium. In terms of the cutoffs functions we have that if $L_{Q\to\emptyset} \leq L_{Q\to A} \leq L_{\emptyset\to A}$ then $g^{(AA)=(\emptyset A)}(\theta_a) \geq g^{(AA)=(QA)}(\theta_a)$ which implies that $\mathcal{SS}^A \equiv \left\{\theta \in [0,1]^2 : \theta_q \leq g^{(AA)=(QA)}(\theta_a)\right\}$.

$\mathcal{WS}^A$ is well defined

Using Remarks (2) and (1) we have $g^{(\emptyset A)=(QA)}(\theta_a) < g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a) < g^{(Q\emptyset)=(QQ)}(\theta_a)$ so if $\theta_q \leq g^{(\emptyset A)=(QA)}(\theta_a)$, the strategy $\emptyset A$ is preferred to the strategy QA and the strategy $\emptyset\emptyset$ is preferred to the strategies $Q\emptyset$ and QQ. Now using that if $g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a) \geq \theta_q$, the strategy $\emptyset A$ is preferred to the strategy $\emptyset\emptyset$, and by the previous line it is preferred to the strategies $Q\emptyset$ and QQ. Finally, if $\theta_q > g^{(AA)=(\emptyset A)}(\theta_a)$, the strategy $\emptyset A$ is preferred to the strategy AA.

Using now Remark (4) we have that $L_{Q\to\emptyset} \leq L_{Q\to A} \leq L_{\emptyset\to A}$ implies $g^{(\emptyset A)=(QA)}(\theta_a) \leq g^{(AA)=(\emptyset A)}(\theta_a)$ so the set $\left\{\theta \in [0,1]^2 : \min\left\{g^{(\emptyset A)=(QA)}(\theta_a), g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a)\right\} \geq \theta_q > g^{(AA)=(\emptyset A)}(\theta_a)\right\} = \emptyset$ which implies that $\mathcal{WS}^A = \emptyset$. Similarly $L_{Q\to\emptyset} > L_{Q\to A} > L_{\emptyset\to A}$ assures that $\mathcal{WS}^A \neq \emptyset$.

$\mathcal{A}$ is well defined

We want to show that the set of abstainers defined either as in (12) or in (13) is well defined whenever $g^{(QA)=(\emptyset\emptyset)}(\theta_a)$ and/or $h^{(QA)=(\emptyset\emptyset)}(\theta_a)$ are well defined.

It is immediate to see that $g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a) < \theta_q < g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a)$ implies that $\emptyset\emptyset$ is preferred to any strategy followed by a weak supporter. Remark (1) which implies $g^{(\emptyset A)=(\emptyset\emptyset)}(\theta_a) > g^{(AA)=(\emptyset A)}(\theta_a)$ and Remark (2) which implies $g^{(Q\emptyset)=(\emptyset\emptyset)}(\theta_a) < g^{(Q\emptyset)=(QQ)}(\theta_a)$ we have that $g^{(AA)=(\emptyset A)}(\theta_a) < \theta_q < g^{(Q\emptyset)=(QQ)}(\theta_a)$ implies that $\emptyset A$ is preferred to AA and $Q\emptyset$ is preferred to QQ, so using the previous argument we have that $\emptyset\emptyset$ is preferred to any strategy followed by a strong supporter.

It remains to show that abstaining without information is preferred to voting following the signal. Using (2), (8), (10), and the function $\mathcal{L}$ we define $g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right)$ and $h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)$, whenever possible, as

$$\begin{aligned}\mathcal{L}\left(P^{QA}\left(g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right),\theta_a\right)\right) &= g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right)\frac{\Delta\Pr\left(q,\emptyset\right)}{2}+\theta_a\frac{\Delta\Pr\left(a,Q\right)-\Delta\Pr\left(a,\emptyset\right)}{2} \\ \mathcal{L}\left(P^{QA}\left(\theta_q,h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)\right)\right) &= \theta_q\frac{\Delta\Pr\left(q,\emptyset\right)}{2}+h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)\frac{\Delta\Pr\left(a,Q\right)-\Delta\Pr\left(a,\emptyset\right)}{2}\end{aligned} \tag{23}$$

and using the implicit function theorem we have that $g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right)$ is well defined when $\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}\neq P^{QA}\left(g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right),\theta_a\right)$ and $h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)$ is well defined when $\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}\neq 1-P^{QA}\left(\theta_q,h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)\right)$.

Using (6) and (7) we have that a necessary condition for the strategy $\emptyset\emptyset$ to be used is $L_{Q\to\emptyset}>\frac{\theta_q}{\theta_a}>L_{\emptyset\to A}$ where we used that abstention is only possible if $L_{Q\to\emptyset}>L_{Q\to A}>L_{\emptyset\to A}$. First note that if the left hand side of any of the equations in (23) is bigger than the right hand side, we have that the strategy QA is preferred to $\emptyset\emptyset$. Assume that $\emptyset\emptyset$ is indifferent to QA so we must have the first line of (23) gives

$$\frac{\theta_q}{\theta_a}\frac{\Delta\Pr\left(q,Q\right)}{\Delta\Pr\left(a,Q\right)}\left(P^{QA}\left(\theta_q,\theta_a\right)-\frac{\Delta\Pr\left(q,\emptyset\right)}{\Delta\Pr\left(q,Q\right)}\right)=\left(\left(1-P^{QA}\left(\theta_q,\theta_a\right)\right)-\frac{\Delta\Pr\left(a,\emptyset\right)}{\Delta\Pr\left(a,Q\right)}\right)+C\left(P^{QA}\left(\theta_q,\theta_a\right)\right)$$

Assume that $\left(\theta_q,\theta_a\right)$ verifies that $P^{QA}\left(\theta_q,\theta_a\right)\leq\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}$ and since $L_{Q\to\emptyset}>L_{Q\to A}>L_{\emptyset\to A}$ by (22) implies that $P^{QA}\left(\theta_q,\theta_a\right)>\frac{1}{2}$ we must have that

$$\begin{aligned}\frac{L_{\emptyset\to A}}{L_{Q\to A}}\left(P^{QA}\left(\theta_q,\theta_a\right)-\frac{\Delta\Pr\left(q,\emptyset\right)}{\Delta\Pr\left(q,Q\right)}\right) &> \left(\left(1-P^{QA}\left(\theta_q,\theta_a\right)\right)-\frac{\Delta\Pr\left(a,\emptyset\right)}{\Delta\Pr\left(a,Q\right)}\right) \\ P^{QA}\left(\theta_q,\theta_a\right) &> \frac{\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}}{\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(q,\emptyset)}+\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}}\end{aligned} \tag{24}$$

Using again the assumption that $P^{QA}\left(\theta_q,\theta_a\right)\leq\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}$ we have that

$$\begin{aligned}\frac{\Delta\Pr\left(q,\emptyset\right)}{\Delta\Pr\left(q,Q\right)} &> \frac{\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}}{\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(q,\emptyset)}+\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)}} \\ \frac{\Delta\Pr\left(a,\emptyset\right)}{\Delta\Pr\left(a,Q\right)}+\frac{\Delta\Pr\left(q,\emptyset\right)}{\Delta\Pr\left(q,Q\right)} &> 1\end{aligned}$$

Therefore, if $\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}+\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}\leq 1$ every type that it is indifferent between $\emptyset\emptyset$ and QA verifies that $P^{QA}\left(\theta_q,\theta_a\right)>\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}$ whenever $L_{Q\to\emptyset}>L_{\emptyset\to A}$. Now assume that $\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}+\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}>$

1 and we have that the first line of (24) implies that

$$-\frac{L_{\emptyset\to A}}{L_{Q\to A}}\left(1-P^{QA}\left(\theta_q,\theta_a\right)-\frac{\Delta\Pr\left(a,\emptyset\right)}{\Delta\Pr\left(a,Q\right)}\right)>\left(1-P^{QA}\left(\theta_q,\theta_a\right)-\frac{\Delta\Pr\left(a,\emptyset\right)}{\Delta\Pr\left(a,Q\right)}\right)$$

so we must have that $\left(1-P^{QA}\left(\theta_q,\theta_a\right)\right)<\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}$.

Recalling that $g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right)$ is well defined when $\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}\neq P^{QA}\left(g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right),\theta_a\right)$ and $h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)$ is well defined when $\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}\neq 1-P^{QA}\left(\theta_q,h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)\right)$ we have that $\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}+\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}\leq 1$ is sufficient to apply the implicit function theorem for $g^{(QA)=(\emptyset\emptyset)}\left(\theta_a\right)$ and $\frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(a,Q)}+\frac{\Delta\Pr(q,\emptyset)}{\Delta\Pr(q,Q)}>1$ is sufficient to apply the implicit function theorem for $h^{(QA)=(\emptyset\emptyset)}\left(\theta_q\right)$.

Finally Lemma (6) implies that if $L_{Q\to\emptyset}\leq L_{Q\to A}\leq L_{\emptyset\to A}$ no abstention is possible and the set of abstainers is empty.

It is easy to verify that the sets $\mathcal{SS}^A$, $\mathcal{SS}^Q$, $\mathcal{WS}^A$, $\mathcal{WS}^Q$, $\mathcal{A}$ and $\mathcal{I}$ cover all types in $[0,1]^2$ without intersecting each other. ■

Proof of Proposition 2. Let $\phi=\frac{\left(1-\left(\xi_A+\xi_Q\right)\alpha\right)^{n-1}}{2}$ and define the spaces

$$\begin{aligned}X_1 &\equiv \left\{(x,y)\in\left[\xi_A\alpha,1-\left(\xi_\emptyset+\xi_Q\right)\alpha\right]\times\left[\xi_Q\alpha,1-\left(\xi_\emptyset+\xi_A\right)\alpha\right]\right\}\\ X_2\left(\phi\right) &\equiv \left\{(x,y,v,z)\in[\phi,1]^2\times[\phi,1]^2:x+\phi\leq v,y+\phi\leq z\right\}\end{aligned}$$

Let $\left(x_1^\omega,x_2^\omega\right)\in X_1$ be a generic element of the set X_1 if we let $\omega=a$ $(\omega=q)$ so $\left(x_1^\omega,x_2^\omega\right)$ plays the role of the probabilities a random non partisan voter supports A (Q) in different states. Let $\left(y_\emptyset^a,y_\emptyset^q,y_Q^a,y_Q^q\right)$ by a generic element of the space $X_2\left(\phi\right)$ so $y_\emptyset^\omega$ plays the role of $\Delta\Pr\left(\omega,\emptyset\right)$ and y_Q^ω plays the role of $\Delta\Pr\left(\omega,Q\right)$ for $\omega\in\{a,q\}$.

Let $p^i:[0,1]^2\times X_2\left(\phi\right)\to\left[\frac{1}{2},1-\eta\right]$, $i=1,2,3$ be implicitly defined by $C'\left(p^1\right)=\frac{\theta_a y_\emptyset^a+\theta_q y_\emptyset^q}{2}$, $C'\left(p^2\right)=\frac{\theta_a y_Q^a+\theta_q y_Q^q}{2}$, and $C'\left(p^3\right)=\frac{\theta_a\left(y_Q^a-y_\emptyset^a\right)+\theta_q\left(y_Q^q-y_\emptyset^q\right)}{2}$, and let η be such that $C'\left(1-\eta\right)>1$. So p^1 plays the role of $P^{\emptyset A}$, p^2 plays the role of P^{QA} and p^3 plays the role of $P^{Q\emptyset}$ as defined in (8). Now consider an element $\left(y_\emptyset^a,y_\emptyset^q,y_Q^a,y_Q^q\right)\in X_2\left(\phi\right)$ and using $\left(p^1,p^2,p^3\right)$, we can define the cutoff functions used in the characterization of equilibrium . Therefore, the sets of strong and weak supporters, independents and abstainers are well defined. Using Proposition (1) we have that $P\left(X^\omega\right)$, the probability of a vote for $X\in\{Q,A\}$ in state $\omega\in\{q,a\}$, is

$$\begin{aligned}\Pr\left(A^a\right) &\equiv \int_{\theta\in\mathcal{WS}^A}p^1\left(\theta\right)dF\left(\theta\right)+\int_{\theta\in\mathcal{SS}^A}dF\left(\theta\right)+\int_{\theta\in\mathcal{I}}p^2\left(\theta\right)dF\left(\theta\right) \qquad (25)\\ \Pr\left(A^q\right) &\equiv \int_{\theta\in\mathcal{WS}^A}\left(1-p^1\left(\theta\right)\right)dF\left(\theta\right)+\int_{\theta\in\mathcal{SS}^A}dF\left(\theta\right)+\int_{\theta\in\mathcal{I}}\left(1-p^2\left(\theta\right)\right)dF\left(\theta\right)\end{aligned}$$

$$\begin{aligned}\Pr\left(Q^{q}\right) &\equiv \int\limits_{\theta\in\mathcal{WS}^{Q}} p^{3}\left(\theta\right)dF\left(\theta\right)+\int\limits_{\theta\in\mathcal{SS}^{Q}} dF\left(\theta\right)+\int\limits_{\theta\in\mathcal{I}} p^{2}\left(\theta\right)dF\left(\theta\right) \qquad (26)\\ \Pr\left(Q^{a}\right) &\equiv \int\limits_{\theta\in\mathcal{WS}^{Q}} \left(1-p^{3}\left(\theta\right)\right)dF\left(\theta\right)+\int\limits_{\theta\in\mathcal{SS}^{Q}} dF\left(\theta\right)+\int\limits_{\theta\in\mathcal{I}} \left(1-p^{2}\left(\theta\right)\right)dF\left(\theta\right)\end{aligned}$$

For functions $\left(p^{1},p^{2},p^{3}\right)$ and $\left(y_{\emptyset}^{a},y_{\emptyset}^{q},y_{Q}^{a},y_{Q}^{q}\right)\in X_{2}\left(\phi\right)$ we define the functions $G_{X}^{\omega}:X_{2}\left(\phi\right)\rightarrow X_{1}$ for $X=A,Q$ such that

$$\begin{aligned}G_{A}^{\omega}\left(y_{\emptyset}^{a},y_{\emptyset}^{q},y_{Q}^{a},y_{Q}^{q}\right) &\equiv \xi_{A}\alpha+\left(1-\alpha\right)\Pr\left(A^{\omega}\right)\\ G_{Q}^{\omega}\left(y_{\emptyset}^{a},y_{\emptyset}^{q},y_{Q}^{a},y_{Q}^{q}\right) &\equiv \xi_{Q}\alpha+\left(1-\alpha\right)\Pr\left(Q^{\omega}\right)\end{aligned}$$

Let $\left(x_{1}^{\omega},x_{2}^{\omega}\right)\in X_{1}$ and we can define for $l\in\left\{x_{1}^{\omega},x_{2}^{\omega}\right\}$

$$\frac{\tau_{1}\left(\omega\right)}{\left(1-\left(x_{1}^{\omega}+x_{2}^{\omega}\right)\right)^{n-1}} \equiv \sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{\left(n-1\right)!}{k!k!\left(n-1-2k\right)!}\left(\frac{x_{1}^{\omega}}{\left(1-\left(x_{1}^{\omega}+x_{2}^{\omega}\right)\right)}\frac{x_{2}^{\omega}}{\left(1-\left(x_{1}^{\omega}+x_{2}^{\omega}\right)\right)}\right)^{k} \qquad (27)$$

$$\frac{\tau_{2}\left(l,\omega\right)}{\left(1-\left(x_{1}^{\omega}+x_{2}^{\omega}\right)\right)^{n-1}} \equiv l\sum_{k=0}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{\left(n-1\right)!}{k!\left(k+1\right)!\left(n-2k-2\right)!}\left(\frac{x_{1}^{\omega}}{\left(1-\left(x_{1}^{\omega}+x_{2}^{\omega}\right)\right)}\frac{x_{2}^{\omega}}{\left(1-\left(x_{1}^{\omega}+x_{2}^{\omega}\right)\right)}\right)^{k}$$

where $\lfloor T\rfloor$ is the biggest integer smaller than T. Clearly $\tau_{1}\left(\omega\right)$ is the situation where A and Q are tied in state ω and $\tau_{2}\left(x_{1}^{\omega},\omega\right)$ $\left(\tau_{2}\left(x_{2}^{\omega},\omega\right)\right)$ is the probability that candidate A (Q) has an advantage of 1 vote in state ω.

Recalling $\Delta\Pr\left(\omega,\emptyset\right)$ and $\Delta\Pr\left(\omega,Q\right)$ we define the function $K^{\omega}:X_{1}\times X_{1}\rightarrow X_{2}\left(\phi\right)$ as $K_{1}^{\omega}\left(x_{1}^{a},x_{2}^{a},x_{1}^{q},x_{2}^{q}\right)=\frac{\tau_{2}\left(x_{2}^{\omega},\omega\right)+\tau_{1}\left(\omega\right)}{2}$ and $K_{2}^{\omega}\left(x_{1}^{a},x_{2}^{a},x_{1}^{q},x_{2}^{q}\right)=\tau_{1}\left(\omega\right)+\frac{\tau_{2}\left(x_{1}^{\omega},\omega\right)+\tau_{2}\left(x_{2}^{\omega},\omega\right)}{2}$ so K_{1}^{ω} gives $\Delta\Pr\left(\omega,\emptyset\right)$ and K_{2}^{ω} gives $\Delta\Pr\left(\omega,Q\right)$.[20]

Now we have all the elements to show that an equilibrium actually exists. Take an arbitrary element of $\mathcal{S}\equiv\left(X_{1}\right)^{2}\times X_{2}\left(\phi\right)$, define the function $\Gamma:\mathcal{S}\rightarrow\mathcal{S}$ such that $\Gamma\equiv\left\{G_{A}^{a},G_{Q}^{a},G_{A}^{q},G_{Q}^{q},K_{1}^{\omega},K_{2}^{\omega}\right\}$, where the components are defined as above. We are going to show first that actually Γ is a continuous function.

For continuity of $\left(G_{A}^{a},G_{Q}^{a},G_{A}^{q},G_{Q}^{q}\right)$ we first observe that all the cutoff functions that determine the types (weak and strong supporters, abstainers and independents), are well defined and continuous for $\left(y_{\emptyset}^{a},y_{\emptyset}^{q},y_{Q}^{a},y_{Q}^{q}\right)$ and $\left(p^{1},p^{2},p^{3}\right)$ as defined above. Therefore $\Pr\left(A^{\omega}\right)$ and $\Pr\left(Q^{\omega}\right)$ are continuous on $\left(y_{\emptyset}^{a},y_{\emptyset}^{q},y_{Q}^{a},y_{Q}^{q}\right)$ when we consider that $\left(p^{1},p^{2},p^{3}\right)$ are also continuous and well defined for $y_{\emptyset}^{\omega}\in\left[\phi,1\right]$, $y_{Q}^{\omega}\in\left[\phi,1\right]$. The fact that K is continuous in $\left(x_{1}^{a},x_{2}^{a},x_{1}^{q},x_{2}^{q}\right)$ follows trivially by continuity of $\tau_{2}\left(l,\omega\right)$ and $\tau_{1}\left(\omega\right)$ in $\left(x_{1}^{a},x_{2}^{a},x_{1}^{q},x_{2}^{q}\right)$. X_{1} and $X_{2}\left(\phi\right)$ are convex and compact, so Brouwer's fixed point theorem holds (Border (1985)) and there is some $x\in\mathcal{S}$ such that $\Gamma\left(x\right)=x$. ■

Proof of Proposition (3). We need to show now that there is abstention in equilibrium.

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[20] Imagine that x_{2}^{ω} $\left(x_{1}^{\omega}\right)$ is the probability of a vote in favor of Q (A) in state ω and imagine a voter considering switching her vote from $\emptyset$ to A. Then $\frac{\tau_{1}(\omega)}{2}$ is the increment in the probability of A winning because a tie is *broken* and $\frac{\tau_{2}\left(x_{2}^{\omega},\omega\right)}{2}$ is increment in the probability of A winning because a tie is *created.*

First we prove the following Lemma

Lemma 7 *Let* $T\left(y;h\right)=\sum_{k=0}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!h!(n-1-k-h)!}\left(y\right)^{k}$*;* $\frac{T(k+1)}{T(k)}$ *is increasing in* y*.*

Proof. *Using the definition of* $T\left(y;h\right)$ *we have that*

$$\frac{T\left(y;k+1\right)}{T\left(y;k\right)}=\left(n+1\right)\frac{1+\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}\left(y\right)^{k}}{1+\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}}-2$$

Taking derivatives with respect to y *we have*

$$\begin{aligned}\frac{d\left(\frac{T(y;k+1)}{T(y;k)}\right)}{dy} &= \frac{\left(n+1\right)}{y}\frac{\left(\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{(k-1)!(k+1)!(n-1-2k)!}\left(y\right)^{k}\right)\left(1+\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)}{\left(1+\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)^{2}}\\ &\quad-\frac{\left(n+1\right)}{y}\frac{\left(1+\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}\left(y\right)^{k}\right)\left(\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{(k-1)!k!(n-1-2k)!}\left(y\right)^{k}\right)}{\left(1+\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)^{2}}\end{aligned}$$

$$
\begin{aligned}
\frac{d\left(\frac{T(y;k+1)}{T(y;k)}\right)}{dy} \quad = \quad & \frac{(n+1)}{y}\frac{\left(\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}k\left(y\right)^{k}\right)\left(1+\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}\left(k+1\right)\left(y\right)^{k}\right)}{\left(1+\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)^{2}} \\
& -\frac{(n+1)}{y}\frac{\left(1+\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}\left(y\right)^{k}\right)\left(\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}k\left(k+1\right)\left(y\right)^{k}\right)}{\left(1+\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\left(\frac{T(y;k+1)}{T(y;k)}\right)}{dy} \quad = \quad & -\frac{(n+1)}{y}\frac{\left(\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}k^{2}\left(y\right)^{k}\right)}{\left(1+\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)^{2}} \\
& +\frac{(n+1)}{y}\frac{\left(\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}k\left(y\right)^{k}\right)\left(\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}\left(k+1\right)\left(y\right)^{k}\right)}{\left(1+\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)^{2}} \\
& -\frac{(n+1)}{y}\frac{\left(\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}\left(y\right)^{k}\right)\left(\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!(k+1)!(n-1-2k)!}k\left(k+1\right)\left(y\right)^{k}\right)}{\left(1+\sum_{k=1}^{\left\lfloor\frac{n}{2}-1\right\rfloor}\frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)^{2}}
\end{aligned}
$$

so

$$\frac{d\left(\frac{T(y;k+1)}{T(y;k)}\right)}{dy} \leq \frac{(n+1)}{y}\frac{\begin{array}{c}\left(\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{(k-1)!(k+1)!(n-1-2k)!}\left(y\right)^{k}\right)\left(\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!(k+1)!(n-1-2k)!}\left(k+1\right)\left(y\right)^{k}\right) \\ -\left(\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!(k+1)!(n-1-2k)!}\left(y\right)^{k}\right)\left(\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!(k+1)!(n-1-2k)!}k\left(k+1\right)\left(y\right)^{k}\right)\end{array}}{\left(1+\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!k!(n-1-2k)!}\left(y\right)^{k}\right)^{2}}$$

so it is sufficient to prove the result if

$$\begin{aligned} &\left(\sum_{j=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}j\left(y\right)^{j}\right)\left(\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\left(k+1\right)\left(y\right)^{k}\right) \\ \leq\ &\left(\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\left(y\right)^{k}\right)\left(\sum_{j=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}j\left(j+1\right)\left(y\right)^{j}\right)\end{aligned}$$

$$\begin{aligned} &\sum_{j=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}j\left(k+1\right)\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\left(y\right)^{k+j} \\ \leq\ &\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\sum_{j=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}j\left(j+1\right)\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\left(y\right)^{k+j}\end{aligned}$$

$$0 \geq \sum_{j=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1\right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\left(k-j\right)j\left(y\right)^{j+k} \tag{28}$$

where we have replaced the index k for j appropriately in the first expression. Note that the *right hand side of (28) is equivalent to*

$$\begin{aligned}
&\sum_{j=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\,(k-j)\,j\,(y)^{j+k} \\
= & -\sum_{j=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\,(k-j)^2\,(y)^{j+k} \\
& +\sum_{j=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\,k\,(k-j)\,(y)^{j+k}
\end{aligned}$$

$$\begin{aligned}
&\sum_{j=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\,(k-j)\,j\,(y)^{j+k} \\
= & -\sum_{j=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\,(k-j)^2\,(y)^{j+k} \\
& +\sum_{j=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\,j\,(j-k)\,(y)^{k+j}
\end{aligned}$$

$$\begin{aligned}
&2\sum_{j=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\,(k-j)\,j\,(y)^{j+k} \\
= & -\sum_{j=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor}\sum_{k=1}^{\left\lfloor \frac{n}{2}-1 \right\rfloor} \frac{(n-1)!}{k!\,(k+1)!\,(n-1-2k)!}\frac{(n-1)!}{j!\,(j+1)!\,(n-1-2j)!}\,(k-j)^2\,(y)^{j+k}
\end{aligned}$$

■

The next Corollary follows from Lemma (6) and presents sufficient conditions from abstention that are easier to check:

Corollary 1 *There is abstention for some positive mass of types if*

$$\frac{\Pr(A\mid a)}{\Pr(Q\mid a)} > 1 > \frac{\Pr(A\mid q)}{\Pr(Q\mid q)} \tag{29}$$

Proof. *Using the definition of* $L_{X\to A}$ *for* $X\in\{\emptyset,Q\}$ *we have that* $L_{Q\to A} > L_{\emptyset\to A}$ *is equivalent to* $\frac{\Delta\Pr(a,Q)}{\Delta\Pr(q,Q)} > \frac{\Delta\Pr(a,\emptyset)}{\Delta\Pr(q,\emptyset)}$. *Using (27) and replacing* x_1^{ω} *with* $\Pr(A\mid\omega)$ *and* x_2^{ω} *with*

$\Pr(Q \mid \omega)$ *we have that*

$$\begin{aligned} \Delta \Pr(\omega, \emptyset) &= \frac{\tau_2(\Pr(Q \mid \omega)) + \tau_1(\omega)}{2} \\ \Delta \Pr(\omega, Q) &= \Delta \Pr(\omega, \emptyset) + \frac{\tau_2(\Pr(A \mid \omega), \omega) + \tau_1(\omega)}{2} \end{aligned} \tag{30}$$

so $L_{Q \to A} > L_{\emptyset \to A}$ *is equivalent to*

$$\frac{\tau_2(\Pr(A \mid a), a) - \tau_2(\Pr(Q \mid a), a)}{\tau_1(a) + \tau_2(\Pr(Q \mid a), a)} > \frac{\tau_2(\Pr(A \mid q), q) - \tau_2(\Pr(Q \mid q), q)}{\tau_1(q) + \tau_2(\Pr(Q \mid q), q)}$$

so it is sufficient if $\frac{\tau_2(\Pr(A|a),a)}{\tau_2(\Pr(Q|a),a)} > 1 > \frac{\tau_2(\Pr(A|q),q)}{\tau_2(\Pr(Q|q),q)}$. *Using (27) we have that condition (29) is necessary and sufficient for a positive mas of non partisan voters abstaining in equilibrium.* ■

Now we are ready to show that there is abstention. We are going to prove by contradiction so assume that there is no endogenous abstention in equilibrium. For any fixed N Proposition (1) we have that $\mathcal{I} \neq \emptyset$ so there is information acquisition in equilibrium and since abstention is exogenous and the same in both states we must have that

$$\begin{aligned} \Pr(A \mid a) &> \Pr(A \mid q) \\ \Pr(Q \mid a) &< \Pr(Q \mid q) \end{aligned} \tag{31}$$

Note that, since there is no endogenous abstention, (22) cannot hold which implies one of the next three possibilities: $\frac{\Pr(A|q)}{\Pr(Q|q)} \geq \frac{\Pr(A|a)}{\Pr(Q|a)}$, $\frac{\Pr(A|a)}{\Pr(Q|a)} \leq 1$, or $\frac{\Pr(A|q)}{\Pr(Q|q)} \geq 1$.

Case 1:

Note that we must have $\Pr(Q \mid a)\Pr(A \mid q) \geq \Pr(Q \mid q)\Pr(A \mid a)$ which contradicts that there is information acquisition in equilibrium so (31) must hold.

Case 2:

Assume the second case ($\frac{\Pr(A|a)}{\Pr(Q|a)} \leq 1$) which implies the following order of probabilities of voting for candidates given the state

$$\Pr(A \mid q) < \Pr(A \mid a) \leq \Pr(Q \mid a) < \Pr(Q \mid q) \tag{32}$$

and also that

$$\begin{aligned} \frac{\Pr(A \mid q)}{\Pr(Q \mid q) + \Pr(A \mid q)} &< \frac{\Pr(A \mid a)}{\Pr(Q \mid a) + \Pr(A \mid a)} \leq \frac{1}{2} \\ \frac{1}{2} &\leq \frac{\Pr(Q \mid a)}{\Pr(Q \mid a) + \Pr(A \mid a)} < \frac{\Pr(Q \mid q)}{\Pr(Q \mid q) + \Pr(A \mid q)} \end{aligned}$$

since there is no endogenous abstention and abstention does not depend on the state. Note that the function $f_1(x) = x(1-x)$ is single peaked which implies that

$$f_1\left(\frac{\Pr(A \mid a)}{\Pr(Q \mid a) + \Pr(A \mid a)}\right) > f_1\left(\frac{\Pr(A \mid q)}{\Pr(Q \mid q) + \Pr(A \mid q)}\right) \tag{33}$$

so we must have that

$$\begin{aligned}
&\left(\frac{\Pr(A\mid a)}{(1-(\Pr(A\mid a)+\Pr(Q\mid a)))}\frac{\Pr(Q\mid a)}{(1-(\Pr(A\mid a)+\Pr(Q\mid a)))}\right)^{k}\\
=&\left(f_1\left(\frac{\Pr(A\mid a)}{\Pr(Q\mid a)+\Pr(A\mid a)}\right)\right)^{k}\left(\frac{(\Pr(A\mid a)+\Pr(Q\mid a))}{(1-(\Pr(A\mid a)+\Pr(Q\mid a)))}\right)^{2k}\\
>&\left(f_1\left(\frac{\Pr(A\mid q)}{\Pr(Q\mid q)+\Pr(A\mid q)}\right)\right)^{k}\left(\frac{(\Pr(A\mid q)+\Pr(Q\mid q))}{(1-(\Pr(A\mid q)+\Pr(Q\mid q)))}\right)^{2k}
\end{aligned}$$

where the second line follows because $\Pr(Q\mid q)+\Pr(A\mid q)=\Pr(Q\mid a)+\Pr(A\mid a)=1-\left(1-\xi_A-\xi_q\right)\alpha$, the third line follows by (33), and the last line follows since abstention is constant across states. Using (27) from Proposition (2) with $\Pr(A\mid\omega)$ instead of x_1^{ω} and $\Pr(Q\mid\omega)$ instead of x_2^{ω} we must have that $\tau_1(a)>\tau_1(q)$ and $\tau_2(l,a)>\tau_2(l,q)$.

Recalling (1) we have that is necessary and sufficient for abstention that the following inequality holds:

$$\frac{\tau_1(a)+\Pr(A\mid a)\widetilde{\tau}_2(a)}{\tau_1(a)+\Pr(Q\mid a)\widetilde{\tau}_2(a)}>\frac{\tau_1(q)+\Pr(A\mid q)\widetilde{\tau}_2(q)}{\tau_1(q)+\Pr(Q\mid q)\widetilde{\tau}_2(q)} \tag{34}$$

where

$$\frac{\widetilde{\tau}_2(\omega)}{(1-(x_1^{\omega}+x_2^{\omega}))^{n-1}}=\frac{\tau_2(l,\omega)}{(1-(x_1^{\omega}+x_2^{\omega}))^{n-1}}\frac{1}{l}$$

Note that we can manipulate the sufficient condition (34) to get:

$$\begin{aligned}
\frac{\Pr(Q\mid a)-\Pr(A\mid a)}{\frac{\tau_1(a)}{\widetilde{\tau}_2(a)}+\Pr(Q\mid a)}&<\frac{\Pr(Q\mid q)-\Pr(A\mid q)}{\frac{\tau_1(q)}{\widetilde{\tau}_2(q)}+\Pr(Q\mid q)}\\
\frac{\tau_1(q)}{\widetilde{\tau}_2(q)}(\Pr(Q\mid a)-\Pr(A\mid a))+\Pr(Q\mid a)\Pr(A\mid q)&<\frac{\tau_1(a)}{\widetilde{\tau}_2(a)}(\Pr(Q\mid q)-\Pr(A\mid q))+\Pr(Q\mid q)\Pr(A
\end{aligned}$$

Recalling that $0<\Pr(Q\mid a)-\Pr(A\mid a)<\Pr(Q\mid q)-\Pr(A\mid q)$ by (32), and $\Pr(Q\mid q)\Pr(A\mid a)>\Pr(Q\mid a)\Pr(A\mid q)$ by the fact that there is information acquisition, if $\frac{\tau_1(a)}{\widetilde{\tau}_2(a)}\geq\frac{\tau_1(q)}{\widetilde{\tau}_2(q)}$ we have that (34) hold.

Recalling the definition of $T(y;h)$ in Lemma (7) we have that $\frac{T\left(\frac{\Pr(A|\omega)\Pr(Q|\omega)}{(1-(\Pr(A|\omega)+\Pr(Q|\omega)))^2};k+1\right)}{T\left(\frac{\Pr(A|\omega)\Pr(Q|\omega)}{(1-(\Pr(A|\omega)+\Pr(Q|\omega)))^2};k\right)}=\frac{\widetilde{\tau}_2(\omega)}{\tau_1(\omega)}$. Using the result in (33) we have that Lemma (7) gives that $\frac{\widetilde{\tau}_2(a)}{\tau_1(a)}<\frac{\widetilde{\tau}_2(q)}{\tau_1(q)}$ since $\frac{\Pr(A|a)\Pr(Q|a)}{(1-(\Pr(A|a)+\Pr(Q|a)))^2}>\frac{\Pr(A|q)\Pr(Q|q)}{(1-(\Pr(A|q)+\Pr(Q|q)))^2}$.

Case 3:

The case $\frac{\Pr(A|q)}{\Pr(Q|q)}\geq 1$ follows uses that it must be that

$$\Pr(Q\mid a)<\Pr(Q\mid q)\leq\Pr(A\mid q)<\Pr(A\mid a)$$

and (33) implies that $\tau_1(a) < \tau_1(q)$ and $\tau_2(l,a) < \tau_2(l,q)$. Since (34) cane be written as

$$\begin{aligned} \frac{\Pr(A \mid a) - \Pr(Q \mid a)}{\frac{\tau_1(a)}{\widetilde{\tau}_2(a)} + \Pr(Q \mid a)} &> \frac{\Pr(A \mid q) - \Pr(Q \mid q)}{\frac{\tau_1(q)}{\widetilde{\tau}_2(q)} + \Pr(Q \mid q)} \\ \left(\frac{\tau_1(q)}{\widetilde{\tau}_2(q)} + \Pr(Q \mid q)\right)(\Pr(A \mid a) - \Pr(Q \mid a)) &> \left(\frac{\tau_1(a)}{\widetilde{\tau}_2(a)} + \Pr(Q \mid a)\right)(\Pr(A \mid q) - \Pr(Q \mid q)) \end{aligned}$$

and since $\Pr(A \mid a) - \Pr(Q \mid a) > \Pr(A \mid q) - \Pr(Q \mid q)$ it is sufficient if $\frac{\tau_1(q)}{\widetilde{\tau}_2(q)} > \frac{\tau_1(a)}{\widetilde{\tau}_2(a)}$. Using again that $\frac{T\left(\frac{\Pr(A|\omega)\Pr(Q|\omega)}{(1-(\Pr(A|\omega)+\Pr(Q|\omega)))^2};k+1\right)}{T\left(\frac{\Pr(A|\omega)\Pr(Q|\omega)}{(1-(\Pr(A|\omega)+\Pr(Q|\omega)))^2};k\right)} = \frac{\widetilde{\tau}_2(\omega)}{\tau_1(\omega)}$, Lemma (7) gives that $\frac{\widetilde{\tau}_2(a)}{\tau_1(a)} > \frac{\widetilde{\tau}_2(q)}{\tau_1(q)}$ since now (33) implies that $\frac{\Pr(A|a)\Pr(Q|a)}{(1-(\Pr(A|a)+\Pr(Q|a)))^2} < \frac{\Pr(A|q)\Pr(Q|q)}{(1-(\Pr(A|q)+\Pr(Q|q)))^2}$. ■

Proof of Proposition (5). The first part follows by noting that $\Delta \Pr(\omega, Q) \to 0$ when $n \to \infty$.

Using that no information is collected in the limit we have that $\Pr(A \mid \omega) = \Pr(A)$ and $\Pr(Q \mid \omega) = \Pr(Q)$ which implies that inequality (29) in Corollary (1) does not hold and abstention by non partisans voters has measure 0. ■

Proof of Lemma (5). Note that if there is information acquisition (31) must hold. Note also that the argument in Proposition (3) to rule out the orders $\frac{\Pr(A|q)}{\Pr(Q|q)} \geq \frac{\Pr(A|a)}{\Pr(Q|a)}$, $\frac{\Pr(A|a)}{\Pr(Q|a)} \leq 1$, or $\frac{\Pr(A|q)}{\Pr(Q|q)} \geq 1$ does not depend on the fact that there is no endogenous abstention but just on (31). This implies directly that if there is abstention it must be the case that condition (29) also holds.

Using now that in the limit we must have that almost no information is used we must have that $\frac{\Pr(A|a)}{\Pr(Q|a)} = 1 = \frac{\Pr(A|q)}{\Pr(Q|q)}$ and for sufficiently large N it must be that there is some $\psi > 0$ such that

$$\psi > \Pr(A \mid a) - \Pr(Q \mid a) > 0 > \Pr(A \mid q) - \Pr(Q \mid q) > -\psi$$

Since in the limit there is almost no information we must have that

$$\alpha\xi_Q + \int\limits_{\theta \in \mathcal{SS}^Q} f(\theta)\, d\theta \approx \alpha\xi_A + \int\limits_{\theta \in \mathcal{SS}^A} f(\theta)\, d\theta$$

which gives the result ■