Spring 2006 **Steve Tadelis**

Econ 206 - Problem Set 2

Due 03/15/06

Question 1

Consider a monopolist who rents a service to a single customer during T periods. This rental service is available in any non-negative amount (i.e., $q \in [0, \infty)$) and is produced by the seller at a cost of $c(q) = q^2$. The marginal benefit (in monetary terms) from the good is constant at either $\theta_H = 2$ or $\theta_L = 1$. The buyer knows his type, but the seller does not, and it is common knowledge that $\Pr\{\theta = \theta_H\} = \mu$. Assume that both parties are risk neutral, and both have a common discount factor $\delta = \frac{2}{3}$.

a) Assume first that $T = 1$. Derive the monopolist's optimal contract (revelation mechanism) for this problem. For what values of μ will both types be served?

Assume for the remainder of this question that $\mu = \frac{1}{3}$.

b) Assume now that $T = 2$, and only short term (period-by-period) contracts can be written. That is, in every period t the seller can offer a new menu of contracts to the buyer for the service in this period. Compute the Perfect Bayesian equilibrium of this game, and the expected profit of the seller from his "best" PBE contract.

c) Assume again that $T = 2$. Suppose now that the seller can offer a menu of "sales contracts" in period 1. That is, if the buyer accepts this contract, he consumes the service in both periods. If he does not accept it, the seller can offer a new contract in the beginning of period 2 for the consumption of the service in period 2 only. Does the seller have any incentive to offer such a long-term contract in this model, or can she do as well with short-term contracts?

d) Relate your answer in **c)** above to the notion of Long-Term Renegotiation-Proof contracts.

Question 2

Consider a standard moral-hazard problem with the following features:

- The principal (p) and the agent (a) are both risk neutral. Let x be the verifiable output, e be the agent's unobserved effort, and $w(x)$ the payment to the agent. The two parties' final utility levels are given by,

$$\begin{aligned} u_p &= x - w(x), \\ u_a &= w(x) - v(e), \end{aligned}$$

  where $v(\cdot)$ is a strictly increasing function of effort.

- The agent has finite wealth, which constrains the principal to offer incentive schemes $w(x) \geqslant 0$ for all x. Assume that this constraint guarantees that the agent is willing to work for the principal. (That is, no additional participation constraint is needed.)

- Output can take three values: $x_1 = 1$, $x_2 = 2$, and $x_3 = 3$. Effort can take two values: $e_0 = 0$ and $e_1 = 1$. Normalize $v(0) = 0$.

- The probability of x given e, denoted $\pi(x|e)$, satisfies the Monotone Likelihood Ratio Property given by,

$$\frac{\pi(x_j|e=1)}{\pi(x_j|e=0)} > \frac{\pi(x_{j-1}|e=1)}{\pi(x_{j-1}|e=0)} \quad \text{for } j = 2, 3.$$

a) Assume that a second best solution to this problem induces the agent to choose e_1. Show that in such a solution $w(x_1) = w(x_2) = 0$, and $w(x_3) > 0$.

b) Assume now that the slope of the incentive scheme is restricted to lie between 0 and 1, i.e., that,

$$\frac{w(x_j) - w(x_{j-1})}{x_j - x_{j-1}} \in [0, 1] \quad \text{for } j = 2, 3.$$

(This assumption can be rationalized by arguing that the agent could otherwise engage in arbitrage using either free disposal or by using external funds to boost revenues.) Assume again that a second best solution induces the agent to choose e_1. Show that in such a solution $w(x_1) = 0$, and that if $w(x_2) > 0$ then in such a solution $w(x_3) - w(x_2) = 1$.